Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

Consensus Based Multi-Agent Control Structures

Miloš S. Stanković, Srdjan S. Stanković and Dušan M. Stipanović

Abstract— **In this paper a new approach to decentralized overlapping control of complex systems has been proposed in the form of multi-agent network were the agents are using dynamic consensus strategy in order to reach the agreement upon the control actions. Several structures and algorithms have been proposed which differ, on one hand, on the choice of the variables upon which the agreement among the agents is made (control or estimation variables), and, on the other hand, on the local controller structures derived from the decentralized control laws. Properties and performance of the proposed algorithms have been discussed and illustrated by several examples.**

I. INTRODUCTION

Control of complex systems can be achieved via hierarchical multilayered agent-based structures benefiting from their inherent properties such as *modularity, scalability, adaptability, flexibility and robustness*. The agent-based structures consist of a number of simpler *subsystems* (or "*agents*"), each of which addresses in a coordinated manner a specific *sub-objective* or sub-task so as to attain the overall design objectives. The complexity of the behavior of such systems arises as a result of *interactions* between the multiple agents and the environment in which they operate. More specifically, multi-agent control systems are fundamental components in a wide range of safety-critical engineering systems, and are commonly found in aerospace, traffic control, chemical processes, power generation and distribution, flexible manufacturing, robotic system design and self-assembly structures. A multi-agent system can be considered as a loosely coupled network of problem-solver entities that work together to find answers to problems that are beyond the individual capabilities or knowledge of each entity, where local control law has to satisfy decentralized information structure constraints (see *e.g.* [1]), and where no global system control is desired. Different aspects of multi-agent control systems are covered by a vast literature within the frameworks of computer science, artificial intelligence, network and system theory; for some aspects of multi-agent control systems and sensor networks see *e.g.* [2], [3], [4], [5].

Considering methodologies for achieving *agreement* between the agents upon some decisions, important results were obtained in relation to *distributed iterations* in parallel computation and distributed optimization as early as in the 1980s, *e.g.* [6], [7], [8], [9], [10]. A very intensive research has been carried out recently in this direction, including numerous applications (see, *e.g.* [11], [12], [13], [14], [15], [16], [2], [3]). The majority of the cited references share a common general methodology: they all use some kind of dynamic *consensus* strategy.

In this paper an attempt is made to approach the problem of *overlapping decentralized control of complex systems* by using a *multi-agent strategy*, where the agents (subsystems) communicate in order to achieve agreement upon a control action by using a dynamic consensus methodology. The aim of the paper is to propose several different novel control structures derived from: a) the choice of the variables upon which the agreement is made; b) basic local controller structures derived from the decentralized control laws implemented by the agents.

The paper is organized as follows. Section 2 deals with the problem definition, including the subsystem models and the distribution of control tasks among the agents. In Section 3 several new control structures are proposed based on the agreement between the agents upon the *control variables*. In the most general setting, it is assumed that each agent is able to formulate its local feedback control law starting from the local information structure constraints in the form a general four-term dynamic output controller. The subsystem inputs generated by the agents by means of the local controllers enter the consensus process which generates the control signals to be applied to the system by some a priori specified agents. In the general case, the consensus scheme, determining, in fact, the control law for the whole system, is constructed on the basis of an *aggregation* of the local dynamic controllers. It is shown how the proposed scheme can be adapted to either static local output feedback controllers, or static local state feedback controllers. In Section 4 an alternative to the approach presented in Section 3 is proposed, based on the introduction of a dynamic consensus at the level of *state estimation* [17], [18], [19]. Namely, it is assumed that the agents are able to generate local estimates of parts of the overall state vector using their own subsystem models. The dynamic consensus scheme is introduced to provide each agent with a reliable estimate of the whole system state. The control signal is obtained by applying the known global LQ optimal state feedback gain to the locally available estimates. A number of selected examples illustrate the applicability of all the proposed consensus based control schemes.

This work has been supported by the Boeing Company via the Information Trust Institute.

Miloš S. Stanković and Dušan M. Stipanović are with the Department of Industrial and Enterprise Systems Engineering and the Coordinated Science Laboratory, University of Illinois at Urbana-Champaign, Illinois, USA. email: mstanko3@uiuc.edu, dusan@uiuc.edu

Srdjan S. Stanković is with Faculty of Electrical Engineering, University of Belgrade, Serbia. email: stankovic@etf.bg.ac.yu

II. Problem formulation

Let a complex system be represented by a linear model

$$\mathbf{S}: \quad \begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx, \end{aligned} \tag{1}$$

where $x \in \mathcal{R}^n$, $u \in \mathcal{R}^m$ and $y \in \mathcal{R}^\nu$ are the state, input and output vectors, respectively, while A, B and C are constant matrices of appropriate dimensions.

Assume that N *agents* have to control the system $\mathbf{S}$ according to their own resources. The agents have their local models of parts of $\mathbf{S}$

$$\mathbf{S}_i: \quad \begin{aligned} \dot{\xi}^{(i)} &= A^{(i)}\xi^{(i)} + B^{(i)}v^{(i)} \\ y^{(i)} &= C^{(i)}\xi^{(i)} \end{aligned} \tag{2}$$

where $\xi^{(i)} \in \mathcal{R}^{n_i}$, $v^{(i)} \in \mathcal{R}^{m_i}$ and $y^{(i)} \in \mathcal{R}^{\nu_i}$ are the corresponding state, input and output vectors, and $A^{(i)}$, $B^{(i)}$ and $C^{(i)}$ constant matrices, $i = 1, \dots, N$. Components of the input vectors $v^{(i)} = (v_1^{(i)}, \dots, v_{m_i}^{(i)})^T$ represent subsets of the global input vector u of $\mathbf{S}$, so that $v_j^{(i)} = u_{p_j^i}$, $j = 1, \dots, m_i$, and $p_j^i \in \mathcal{V}^i$, where $\mathcal{V}^i = \{p_1^i, \dots, p_{m_i}^i\}$ is the *input index set* defining $v^{(i)}$. Similarly, for the outputs $y^{(i)}$ we have $y_j^{(i)} = y_{q_j^i}$, $j = 1, \dots, \nu_i$, and $q_j^i \in \mathcal{Y}^i$, where $\mathcal{Y}^i = \{q_1^i, \dots, q_{p_i}^i\}$ is the *output index set*; according to these sets, it is possible to find such constant $p_i \times n$ matrices C_i that $y^{(i)} = C_i x$, $i = 1, \dots, N$. The state vectors $\xi^{(i)}$ do not necessarily represent parts of the global state vector x. They can be chosen, together with the matrices $A^{(i)}$, $B^{(i)}$ and $C^{(i)}$, according to the local criteria for modelling the input-output relation $v^{(i)} \to y^{(i)}$. In the particular case when $\xi^{(i)} = x^{(i)}$, $x_j^{(i)} = x_{r_j^i}$, $j = 1, \dots, n_i$, $n_i \leq n$ and $r_j^i \in \mathcal{X}^i$, where $\mathcal{X}^i = \{r_1^i, \dots, r_{n_i}^i\}$ is the *state index set* defining $x^{(i)}$. In the last case, models $\mathbf{S}_i$, in general, represent *overlapping subsystems* of $\mathbf{S}$ in a more strict sense; matrices $A^{(i)}$, $B^{(i)}$ and $C^{(i)}$ can represent in this case submatrices of A, B and C.

The task of the *i*-th agent is to generate the control vector $v^{(i)}$ and to implement the control action $u^{(i)} \in \mathcal{R}^{\mu_i}$, satisfying $u_j^{(i)} = u_{s_j^i}$, $j = 1, \dots, \mu_i$, and $s_j^i \in \mathcal{U}^i$, where $\mathcal{U}^i = \{s_1^i, \dots, s_{\mu_i}^i\}$ is the *control index set* defining $u^{(i)}$. It is assumed that $\mathcal{U}^i \subseteq \mathcal{V}^i$ and $\mathcal{U}^i \cap \mathcal{U}^j = \emptyset$, so that $\sum_{i=1}^N \mu_i = m$, that is, the control vector $u^{(i)}$ of the *i*-th agent is a part of its input vector $v^{(i)}$, while one and only one agent is responsible for generation of each component of u within the considered control task. Consequently, all agents include the entire vectors $v^{(i)}$ of $\mathbf{S}_i$ in their control design considerations, but they have to implement only those components of $v^{(i)}$ for which they are responsible.

In the case when the inputs $v^{(i)}$ do not overlap, the agents perform their task autonomously, without interactions with each other; that is we have the case of decentralized control of $\mathbf{S}$, when the control design is based entirely on the local models $\mathbf{S}_i$. However, in the case when the model inputs $v^{(i)}$ overlap, more than one model $\mathbf{S}_i$ can be used for calculation of a particular component of the input vector u. Obviously, it would be beneficial for the agent responsible for implementation of that particular input component to use different suggestions about the control action and to calculate the numerical values of the control signal to be implemented on the basis of an agreement between the agents. The agents that do not implement any control action ($\mathcal{U}^i = \emptyset$) could, in this context, represent "advisors" to the agents responsible for control implementation. The aim of the paper is to propose several overlapping decentralized feedback control structures for $\mathbf{S}$ based on a *consensus* between multiple agents.

We will classify different control structures which can be used for solving the above problem in two main groups: (1) the structures based on the consensus at the control input level; (2) the structures based on the consensus at the state estimation level.

III. Structures based on consensus at the control input level

A. Algorithms derived from the local dynamic output feedback control laws

We assume that all the agents are able to design their own local dynamic controllers which generate the input vectors $v^{(i)}$ in $\mathbf{S}_i$ according to

$$\mathbf{C}_i: \quad \begin{aligned} \dot{w}^{(i)} &= F^{(i)}w^{(i)} + G^{(i)}y^{(i)} \\ v^{(i)} &= K^{(i)}w^{(i)} + H^{(i)}y^{(i)} \end{aligned} \tag{3}$$

where $w^{(i)} \in \mathcal{R}^{\rho_i}$ represents the controller state, and matrices $F^{(i)}$, $G^{(i)}$, $K^{(i)}$ and $H^{(i)}$ are constant, with appropriate dimensions. Local controllers are designed according to the local models and local design criteria, $i = 1, \dots, N$. Assuming that the agents can communicate between each other, the goal is to generate the control signal u for $\mathbf{S}$ based on mutual agreement, starting from the inputs $v^{(i)}$ generated by $\mathbf{C}_i$. The idea about reaching an agreement upon the components of u stems from the fact that the index sets $\mathcal{V}^{(i)}$ are, in general, overlapping, so that the agents responsible for control implementation according to the index sets $\mathcal{U}^{(i)}$ can improve their local control laws by getting "suggestions" from the other agents.

Algorithm A.1. The second relation in (3) gives rise to $\dot{v}^{(i)} = K^{(i)}\dot{w}^{(i)} + H^{(i)}\dot{y}^{(i)}$, wherefrom we get

$$\begin{aligned} \dot{v}^{(i)} = K^{(i)}[F^{(i)}w^{(i)} + G^{(i)}y^{(i)}] + \\ H^{(i)}C^{(i)}[A^{(i)}\xi^{(i)} + B^{(i)}v^{(i)}] = K^{(i)}F^{(i)}w^{(i)} + \\ +K^{(i)}G^{(i)}y^{(i)} + H^{(i)}C^{(i)}A^{(i)}\xi^{(i)} + H^{(i)}C^{(i)}B^{(i)}v^{(i)}. \end{aligned} \tag{4}$$

Since $y^{(i)}$ are the available signals, and $v^{(i)}$ vectors to be locally generated for participation in the agreement process, we will use the following approximation

$$\begin{aligned} \dot{v}^{(i)} \approx [K^{(i)}F^{(i)}K^{(i)+} + H^{(i)}C^{(i)}B^{(i)}]v^{(i)} + \\ +[K^{(i)}G^{(i)} + H^{(i)}C^{(i)}A^{(i)}C^{(i)+} - \\ -K^{(i)}F^{(i)}K^{(i)+}H^{(i)}]y^{(i)}, \end{aligned} \tag{5}$$

where $F_*^{(i)} = K^{(i)}F^{(i)}K^{(i)+}$ and $A_*^{(i)} = C^{(i)}A^{(i)}C^{(i)+}$ are approximate solutions of the *aggregation* relations $K^{(i)}F^{(i)} = F_*^{(i)}K^{(i)}$ and $C^{(i)}A^{(i)} = A_*^{(i)}C^{(i)}$, respectively, where A^+ denotes the pseudoinverse of a given matrix A [1], [20].

We will assume for the sake of presentation clarity that all the agents can have their "suggestions" for all the components of u; that is, we assume that the vector $U_i \in \mathcal{R}^m$ is a "local version" of u proposed by the i-th agent to the other agents. Furthermore, we introduce $m \times \rho_i$ and $m \times \nu_i$ constant matrices K_i and H_i, obtained by taking the rows of $K^{(i)}$ and $H^{(i)}$ at the row indices defined by the index set $\mathcal{V}^{(i)}$ and leaving zeros elsewhere, and $n_i \times m$ matrix B_i obtained from $B^{(i)}$ by taking its columns at the indices defined by $\mathcal{V}^i$. Let $U = \mathrm{col}\{U_1, \ldots, U_N\}$, $Y = \mathrm{col}\{y^{(1)}, \ldots, y^{(N)}\}$, $\tilde{K} = \mathrm{diag}\{K_1, \ldots, K_N\}$, $\tilde{H} = \mathrm{diag}\{H_1, \ldots, H_N\}$ $\tilde{A} = \mathrm{diag}\{A^{(1)}, \ldots, A^{(N)}\}$, $\tilde{B} = \mathrm{diag}\{B_1, \ldots, B_N\}$, $\tilde{C} = \mathrm{diag}\{C^{(1)}, \ldots, C^{(N)}\}$, $\tilde{F} = \mathrm{diag}\{F^{(1)}, \ldots, F^{(N)}\}$, and $\tilde{G} = \mathrm{diag}\{\tilde{G}^{(1)}, \ldots, \tilde{G}^{(N)}\}$. Assume that the agents communicate between each other in such a way that they send current values of U_i to each other. Accordingly, we define the *consensus matrix* as $\tilde{\Xi} = [\Xi_{ij}]$, where Ξ_{ij}, $i, j = 1, \ldots, N$, $i \neq j$, are $m \times m$ diagonal matrices with positive entries and $\Xi_{ii} = -\sum_{i=1, i\neq j}^{N} \Xi_{ij}$, $i = 1, \ldots, N$. Then, the algorithm for generating U, *i.e.* the vector containing all the agent input vectors U_i, $i = 1, \ldots, N$, representing the result of the overall consensus process, is given by

$$\begin{aligned} \dot{U}_i = \textstyle\sum_{j=1, j\neq i}^{N} \Xi_{ij}(U_j - U_i) + [K_i F^{(i)} K_i^+ + \\ + H_i C^{(i)} B^{(i)}] U_i + [K_i G^{(i)} + H_i C^{(i)} A^{(i)} C^{(i)+} - \\ - K_i F^{(i)} K_i^+] y^{(i)}, \end{aligned} \quad (6)$$

$i = 1, \ldots, N$, or

$$\dot{U} = [\tilde{\Xi} + \tilde{K}\tilde{F}\tilde{K}^+ + \tilde{H}\tilde{C}\tilde{B}]U + [\tilde{K}\tilde{G} + \tilde{H}\tilde{C}\tilde{A}\tilde{C}^+ - \tilde{K}\tilde{F}\tilde{K}^+\tilde{H}]Y. \quad (7)$$

The vector U generated by (7) is used for control implementation in such a way that the i-th agent picks up the components of U_i selected by the index set $\mathcal{U}^{(i)}$ and applies them to the system $\mathbf{S}$. If Q is an $m \times mN$ matrix with zeros everywhere except one place in each row, where it contains 1; for the j-th row with $j \in \mathcal{U}^{(i)}$, 1 is placed at the column index $(i-1)m + j$. Then, we have $u = QU$, and system (1) can be written as

$$\dot{x} = Ax + BQU. \quad (8)$$

Also, according to the adopted notation, $y^{(i)} = C_i x$, so that $Y = \bar{C}x$, where $\bar{C}^T = \left[C_1^T \vdots \cdots \vdots C_N^T \right]$. Therefore, the whole closed-loop system is represented by

$$\begin{bmatrix} \dot{U} \\ \dot{x} \end{bmatrix} = \left[\begin{array}{c:c} \tilde{\Xi} + \tilde{K}\tilde{F}\tilde{K}^+ + \tilde{H}\tilde{C}\tilde{B} & (\tilde{K}\tilde{G} + \tilde{H}\tilde{C}\tilde{A}\tilde{C}^+ - \\ & -\tilde{K}\tilde{F}\tilde{K}^+\tilde{H})\bar{C} \\ \hdashline BQ & A \end{array} \right] \begin{bmatrix} U \\ x \end{bmatrix}. \quad (9)$$

Obviously, the system is stabilized by the controller (7) if the state matrix in (9) is asymptotically stable. ■

Algorithm A.2. One alternative for the above algorithm is the algorithm depending explicitly on the regulator state $w^{(i)}$. It has the disadvantage of being of higher order than A.1; however, it does not utilize any approximation of $w^{(i)}$ with $v^{(i)}$. Recalling (4), we obtain equation

$$\begin{aligned} \dot{v}^{(i)} \approx K^{(i)} F^{(i)} w^{(i)} + H^{(i)} C^{(i)} B^{(i)} v^{(i)} \\ + [K^{(i)} G^{(i)} + H^{(i)} C^{(i)} A^{(i)} C^{(i)+}] y^{(i)}, \end{aligned}$$

since $w^{(i)}$ is generated by the first relation in (3). If $W = \mathrm{col}\{w^{(1)}, \ldots, w^{(N)}\}$, then the whole closed-loop system can be represented as

$$\begin{bmatrix} \dot{U} \\ \dot{W} \\ \dot{x} \end{bmatrix} = \left[\begin{array}{c:c:c} \tilde{\Xi} + \tilde{H}\tilde{C}\tilde{B} & \tilde{K}\tilde{F} & (\tilde{K}\tilde{G} + \tilde{H}\tilde{C}\tilde{A}\tilde{C}^+)\bar{C} \\ \hdashline 0 & \tilde{F} & \tilde{G}\bar{C} \\ \hdashline BQ & 0 & A \end{array} \right] \begin{bmatrix} U \\ W \\ x \end{bmatrix}. \quad (10)$$

Both control algorithms A.1 and A.2 have the structure which reduces to the local controllers when $\tilde{\Xi} = 0$. In the case of A.1, the local controllers are derived from $\mathbf{C}_i$ after aggregating (3) to one vector-matrix differential equation for $v^{(i)}$, while in the case of A.2 the differential equation for $v^{(i)}$ contains explicitly the term $w^{(i)}$, generated by the local observer in $\mathbf{C}_i$. The form of these controllers is motivated by the idea to introduce a first order dynamic consensus scheme. Namely, without the local controllers, relation $\dot{U} = \tilde{\Xi}U$ provides asymptotically a weighted sum of the initial conditions $U_i(t_0)$, if the graphs corresponding to the particular components of U_i have a center node (see *e.g.* [5], [19]). Combination of the two terms provides a possibility to improve the overall performance by exploiting potential advantages of each local controller. However, the introduction of additional dynamics required by the consensus scheme may deteriorate the performance, and makes the choice of the local controller parameters dependable upon the overall control scheme.

Example 1. An insight into the possibilities of the proposed algorithms can be obtained from a simple example in which the system $\mathbf{S}$ is represented by (1), with $A = \begin{bmatrix} 0.8 & 2 & 0 \\ -2.5 & -5 & -0.3 \\ 0 & 10 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Assume that we have two agents characterized by $\mathbf{S}_1$ with $A^{(1)} = \begin{bmatrix} 0.8 & 2 \\ -2.5 & -5 \end{bmatrix}$, $B^{(1)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $C^{(1)} = [\,1 \;\; 0\,]$, and $\mathbf{S}_2$ with $A^{(2)} = \begin{bmatrix} -5 & -0.3 \\ 10 & -2 \end{bmatrix}$, $B^{(2)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $C^{(2)} = [\,0 \;\; 1\,]$. Obviously, there is only one control signal u. Assume that the second agent is responsible for control implementation, so that $u = u^{(2)} = v^{(2)}$, according to the adopted notation. Assume that both agents have their own controllers $\mathbf{C}_1$ and $\mathbf{C}_2$, obtained by the LQG methodology, assuming a low measurement noise level, so that $F^{(1)} = \begin{bmatrix} 1.6502 & 2.0000 \\ -2.4717 & -2.8223 \end{bmatrix}$, $G^{(1)} = \begin{bmatrix} -0.8502 \\ -0.26970 \end{bmatrix}$, $K^{(1)} = [\,0.7414 \;\; 0.82231\,]$ and $H^{(1)} = 0$, and $F^{(2)} = \begin{bmatrix} -2.2361 & -24.3071 \\ 0.1000 & 1.1200 \end{bmatrix}$, $G^{(2)} = \begin{bmatrix} 24.2068 \\ -3.1200 \end{bmatrix}$, $K^{(2)} = [\,0.2361 \;\; 0.0003\,]$ and $H^{(2)} = 0$. The system $\mathbf{S}$ with the local controller $\mathbf{C}_2$ is unstable. Algorithm

A.1 has been applied according to (7), after introducing $Q = [\,0\ 1\,]$ and $\Xi_{12} = \Xi_{21} = 100I_2$. Fig. 1 presents the impulse response for all three components of the state vector x for $\mathbf{S}$. Algorithm A.2 has then been applied according to (10); the corresponding responses are presented in Fig. 2.

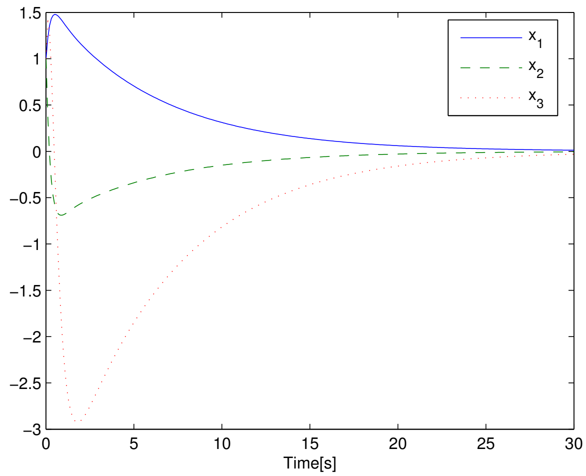

Fig. 1. Algorithm A.1

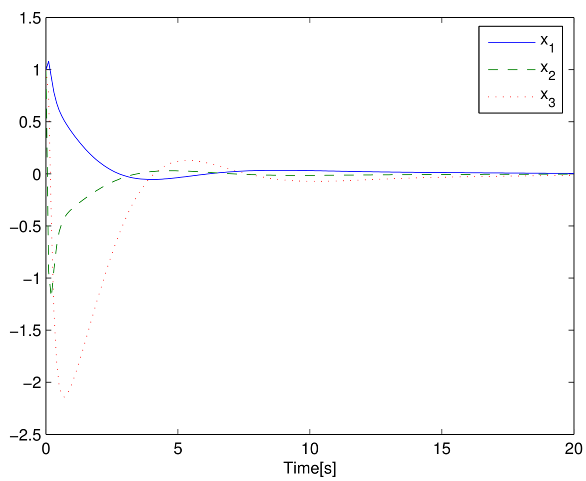

Fig. 2. Algorithm A.2

It is to be emphasized that the consensus scheme puts together two local controllers, influencing in such a way both performance and robustness. Here, the role of the first controller is only to help the second controller in defining the control signal. The importance of the consensus effects can be seen from Fig. 3 in which the responses in the case when $\tilde{\Xi} = 0$ is presented for algorithm A.1. It is obvious that the response is worse than in Fig. 1. In the case of A.2 the system without consensus is even unstable. ■

The problem of stabilizability of $\mathbf{S}$ by A.1 and A.2 is, in general, very difficult having in mind the supposed diversity of local models and dynamic controllers. Any analytic insight

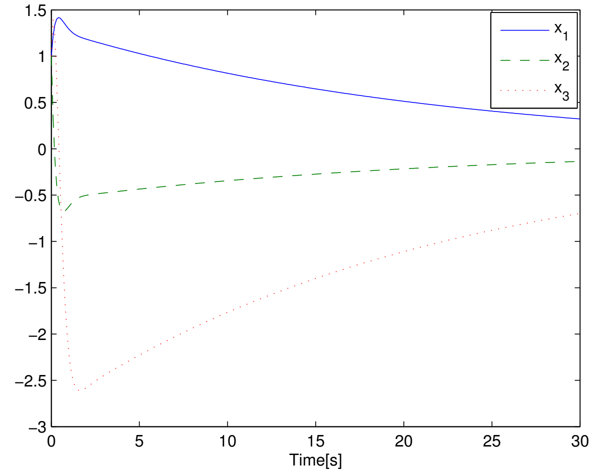

Fig. 3. Algorithm A.1 local controllers without consensus

from this point of view into the system matrices in (9) and (10) seems to be very complicated. It is, however, logical to expect that the introduction of the consensus scheme can, in general, contribute to the stabilization of $\mathbf{S}$. Selection of the elements of $\tilde{\Xi}$ can, obviously, be done in accordance with the expected performance of the local controllers and the confidence in their suggestions (see, for example, an analogous reasoning related to the estimation problem [19]). In this sense, connectedness of the agents network contributes, in general, to the overall control performance.

B. Algorithms derived from local static feedback control laws

Algorithm A.3. Assume now that we have static local output controllers, obtained from $\mathbf{C}_i$ in (3) by introducing $F^{(i)} = 0$, $G^{(i)} = 0$ and $K^{(i)} = 0$, so that we have $v^{(i)} = H^{(i)}y^{(i)}$. Both algorithms A.1 and A.2 give in this case

$$\dot{U} = \tilde{\Xi}U + \tilde{H}\tilde{C}[\tilde{B}U + \tilde{A}\tilde{C}^{+}Y]. \tag{11}$$

The closed-loop system is now given by

$$\begin{bmatrix} \dot{U} \\ \dot{x} \end{bmatrix} = \left[\begin{array}{c:c} \tilde{\Xi} + \tilde{H}\tilde{C}\tilde{B} & \tilde{H}\tilde{C}\tilde{A}\tilde{C}^{+}\bar{C} \\ \hdashline BQ & A \end{array}\right] \begin{bmatrix} U \\ x \end{bmatrix}. \tag{12}$$

Remark 1. The proposed multi-agent control schemes can be compared to those overlapping decentralized control schemes for complex systems that are derived by using the expansion / contraction paradigm and the inclusion principle (especially in the case of Algorithm A.3) *e.g.* [1], [20], [21], [22], [23], [24], having in mind that both approaches follow analogous lines of thought, starting from similar information structure constraints (the above presented approach is, however, much more general). From this point of view, formulation of the local controllers connected to the agents corresponds to the controller design in the expanded space in the case of inclusion based systems, and the application of a dynamic consensus strategy to the contraction to the original space for control action implementation, see *e.g.* [1], [21], [22]. The proposed methodology offers, evidently, much more flexibility (local model structure, agreement strategy), at the expense of additional closed loop dynamics introduced by the consensus scheme itself. Moreover, it is interesting to notice that numerous numerical simulations show a pronounced advantage of the proposed scheme (smoother and even faster responses). The reason could be found in the advantage of the consensus strategy over the contraction transformation, which seems to be overly simplified and unsatisfactory for putting together locally designed overlapping decentralized controllers. ∎

IV. STRUCTURES BASED ON CONSENSUS AT THE STATE ESTIMATION LEVEL

The previous section was devoted to general structures with consensus at the input level in systems where multiple agents with overlapping resources and different competences participate in defining the global control law. The algorithms start from the local models and the local controllers, and the consensus scheme tends to make equal the overlapping components of the local input vectors. The next section will approach the problem in a different way, where the consensus strategy is introduced at the level of state estimation. This estimation scheme itself has been proposed and generally discussed in [17], [19].

Algorithm A.4. Assume that the local models are such that $\xi^{(i)} = x^{(i)}$, so that the dynamic systems $\mathbf{S_i}$ are *overlapping subsystems* of $\mathbf{S}$. Therefore, starting from the model $\mathbf{S_i}$ and the accessible measurements $y^{(i)}$, each agent is able to generate autonomously the local estimate $\hat{x}^{(i)}$ of the vector $x^{(i)}$ using an estimator which can be defined in the following Luenberger form:

$$\bar{\mathbf{E}}_\mathbf{i}: \quad \dot{\hat{x}}^{(i)} = A^{(i)}\hat{x}^{(i)} + B^{(i)}v^{(i)} + L^{(i)}(y^{(i)} - C^{(i)}\hat{x}^{(i)}) \tag{13}$$

where $L^{(i)}$ is a constant matrix, which can be taken to be the steady state Kalman gain [25], and $v^{(i)}$ is supposed to be known.

The overlapping decentralized estimators defined by (13) provide a set of *overlapping estimates* $\hat{x}^{(i)}$. If the final goal is to get an estimate $\hat{x}$ of the whole state vector x of $\mathbf{S}$, a consensus scheme can be introduced which would enable all the agents to get reliable estimates of the whole state vector x on the basis of: (1) the local estimates $\hat{x}^{(i)}$, and (2) communications between the nodes based on a decentralized strategy uniform for all the nodes. If X_i is an estimate of x generated by the i-th agent, we propose the following set of estimators to be attached to all the agents in the network:

$$\begin{aligned}\mathbf{E_i}: \quad \dot{X}_i = A_iX_i + B_i^*u + \Sigma_{\substack{j=1\\ j\neq i}}^N \Xi_{ij}(X_j - X_i)+\\ +L_i(y^{(i)} - C_iX_i),\end{aligned} \tag{14}$$

$i = 1, \ldots, N$, A_i is an $n \times n$ matrix with $n_i \times n_i$ nonzero elements being equal to those of $A^{(i)}$, but being placed at the indices defined by $\mathcal{X}^i \times \mathcal{X}^i$, L_i is an $n \times \rho_i$ matrix obtained similarly as A_i in such a way that its nonzero elements are those of $L^{(i)}$ placed row by row at row-indices defined by $\mathcal{X}^i$, B_i^* is an $n \times m$ matrix obtained from B_i by putting its rows at the indices defined by $\mathcal{X}^i$, and Ξ_{ij}, $i \neq j$, are constant $n \times n$ diagonal matrices with positive entries. The algorithm is based on a combination of *decentralized overlapping estimators* and a *consensus scheme* with matrix gains Ξ_{ij}, tending to make the local estimates X_i as close as possible. If $X = \mathrm{col}\{X_1, \ldots, X_N\}$ is the vector composed of all the state estimates in the agents network, the following model describes its global behavior:

$$\mathbf{E}: \quad \dot{X} = (\tilde{\Xi} + \tilde{A}^* - \tilde{L}^*\tilde{C}^*)X + \tilde{B}^*U^* + \tilde{L}^*Y \tag{15}$$

where $\tilde{\Xi}$ represents now an $nN \times nN$ matrix composed of the blocks Ξ_{ij}, $i \neq j$, with $\Xi_{ii} = -\sum_{i=1, i\neq j}^N \Xi_{ij}$, $i = 1, \ldots, N$, $\tilde{A}^* = \mathrm{diag}\{A_1, \ldots, A_N\}$, $\tilde{L}^* = \mathrm{diag}\{L_1, \ldots, L_N\}$, $\tilde{C}^* = \mathrm{diag}\{C_1, \ldots, C_N\}$, $\tilde{B}^* = \mathrm{diag}\{B_1^*, \ldots, B_N^*\}$ and $U^* = \mathrm{col}\{u, \ldots, u\}$.

Moreover, we shall assume that all the agents have the a priori knowledge about the optimal state feedback for $\mathbf{S}$, expressed as $u = K^o x$. Using this knowledge and the estimation scheme (14), the agents can calculate the corresponding inputs $U_i = K^o X_i$; implementation of the control signals is done according to the index sets $\mathcal{U}^i$.

The decentralized overlapping estimation scheme with consensus, providing state estimates of the whole state vector x to all the agents, together with the globally optimal control law, represents a control algorithm, denoted as A.4, which provides a solution to the posed multi-agent control problem of $\mathbf{S}$.

Defining $\tilde{K}^o = \mathrm{diag}\{K^o, \ldots, K^o\}$, we have, according to the above given notation, that $u = Q\tilde{K}^oX$, so that the whole closed-loop system becomes

$$\begin{bmatrix} \dot{X} \\ \dot{x} \end{bmatrix} = \left[\begin{array}{c:c} \tilde{\Xi} + \tilde{A}^* - \tilde{L}^*\tilde{C}^* + \tilde{B}^*\bar{K} & \bar{L} \\ \hdashline BQ\tilde{K}^o & A \end{array}\right] \begin{bmatrix} X \\ x \end{bmatrix}, \tag{16}$$

where $\bar{K} = \mathrm{col}\{QK^o, \ldots, QK^o\}$ and $\bar{L} = \mathrm{col}\{L_1C_1, \ldots, L_NC_N\}$. A simplified version of the above algorithm, from the point of view of communications, is obtained by replacing the actual input u by the local estimates of the input vector $U_i = K^oX_i$ in (14), having in mind the local availability of X_i. ∎

Example 2. In this example the performance of the above algorithm is demonstrated on the same system as in Example 1. The local estimators are performing the local state estimation using the gains $L_1 = [-4 \;\; 9]^T$ and $L_2 = [2 \;\; -7]^T$. The consensus gains in the matrix $\tilde{\Xi}$ are selected to be $\Xi_{12} = \Xi_{21} = 1000I_2$. The global LQ optimal control matrix K^o is implemented by both agents. Since only the second agent implements the input u, we assume that the first one uses the estimate $U_i = K^oX_i$ in the local state estimation algorithm. The impulse response of the proposed control algorithm, which is shown on Figure 4, is comparable to the the impulse response of the globally LQ optimal controller shown on the same figure. ∎

Stability analysis of Algorithm A.4 represents in general a very complex task. It is possible to apply the methodology

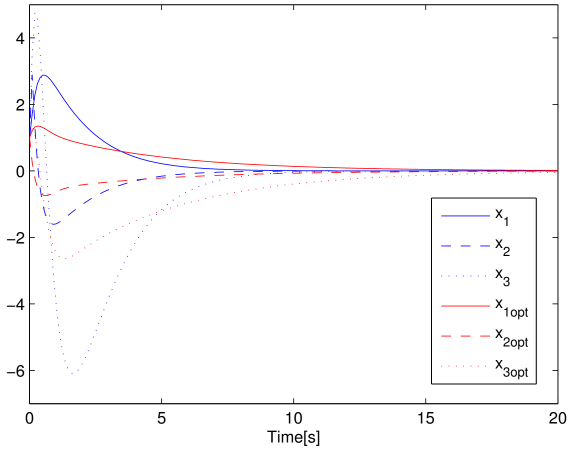

Fig. 4. Algorithm A.4 and globally LQ optimal controller

of [26] under very simplifying assumptions, and to show that the eigenvalues of (16) are composed of the eigenvalues of $\tilde{A}^* - \tilde{L}^*\tilde{C}^*$, $\tilde{A}^* + \tilde{B}^*\bar{K}$ and $\tilde{A}^* - \tilde{L}^*\tilde{C}^* + \tilde{B}^*\bar{K}$ modified by a term depending on the eigenvalues of the Laplacian of the network and the consensus gain matrices. However, the underlying assumptions include the one that all the agents have the exact system model, as well as that the control inputs are transmitted throughout the network; in the overlapping decentralized case, which is in the focus of this work, these assumptions are violated, making the stability problem much more complex, dependent on the accuracy of the local models and the related estimators.

V. Conclusion

In this paper several structures for multi-agent control based on a dynamic consensus strategy have been proposed. After formally defining the problem of multi-agent control with information structure constraints on the basis of overlapping decomposition of a given complex system, two novel classes of overlapping decentralized control algorithms based on consensus are presented. In the first class, an agreement between the agents is required at the level of control inputs. Three distinct control schemes are described and discussed: dynamic output feedback (Algorithms A.1 and A.2), static state feedback and static output feedback (Algorithm A.3). In the second class, when the agreement is required at the estimation level, a control scheme based on state estimation with consensus, coupled with a globally optimal state feedback, is presented and analyzed (Algorithm A.4). The proposed control algorithms have been illustrated by several examples which demonstrate their effectiveness.

Finally, according to some of our initial results, the proposed methodology for multi-agent consensus based control can be efficiently applied to formations of autonomous vehicles.

References

[1] D. D. Šiljak, *Decentralized Control of Complex Systems*. New York: Academic Press, 1991.
[2] H. Gharavi and S. Kumar, Eds., *Proceedings of the IEEE: Special Isssue on Sensor Networks and Applications*, vol. 91, August 2003.
[3] C. G. Cassandras and W. Li, "Sensor networks and cooperative control," *European Journal of Control*, vol. 11, pp. 436–463, 2005.
[4] P. Yang, R. Freeman, and K. Lynch, "Multi-agent coordination by decentralized estimation and control," - submitted to *IEEE Trans. Autom. Control*, 2006.
[5] W. Ren, R. W. Beard, and E. M. Atkins, "A survey of consensus problems in multi-agent coordination," in *Proc. American Control Conference*, 2005, pp. 1859–1864.
[6] D. P. Bertsekas and J. N. Tsitsiklis, *Parallel and Distributed Computation: Numerical Methods*. Prentice Hall, 1989.
[7] J. N. Tsitsiklis, "Problems in decentralized decision making and computation," Ph.D. dissertation, Dep. Electrical Eng. Comput. Sci., M.I.T., Cambridge, MA, 1984.
[8] J. N. Tsitsiklis, D. P. Bertsekas, and M. Athans, "Distributed asynchronous deterministic and stochastic gradient optimization algorithms," *IEEE Trans. Autom. Control*, vol. 31, pp. 803–812, 1986.
[9] B. Baran, E. Kaszkurewicz, and A. Bhaya, "Parallel asynchronous team algorithms: convergence and performance analysis," *IEEE Trans. Parallel and Distributed Syst.*, vol. 7, pp. 677–688, 1996.
[10] V. D. Blondel, J. M. Hendrickx, A. Olshevsky, and J. N. Tsitsiklis, "Convergence in multiagent coordination, consensus and flocking," in *Proc. IEEE Conf. Decision and Control*, 2005.
[11] A. Fax and R. Murray, "Information flow and cooperative control of vehicle formations," *IEEE Trans. Automat. Contr.*, vol. 49, pp. 1465–1476, 2004.
[12] A. Jadbabaie, J. Lin, and A. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Trans. Automat. Contr.*, vol. 48, pp. 988–1001, 2003.
[13] Z. Lin, B. Francis, and M. Maggiore, "Necessary and sufficient conditions for formation control of unicycles," *IEEE Trans. Automat. Contr.*, vol. 50, pp. 121–127, 2005.
[14] R. Olfati-Saber and R. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. Automat. Contr.*, vol. 49, pp. 1520–1533, 2004.
[15] W. Ren and R. Beard, "Consensus seeking in multi-agent systems using dynamically changing interaction topologies," *IEEE Trans. Autom. Control*, vol. 50, pp. 655–661, 2005.
[16] W. Ren, R. W. Beard, and D. B. Kingston, "Multi-agent Kalman consensus with relative uncertainty," in *Proc. American Control Conference*, 2005.
[17] S. S. Stanković, M. S. Stanković, and D. M. Stipanović, "Consensus based overlapping decentralized estimator," in *Proc. American Control Conference*, 2007.
[18] ——, "Consensus based overlapping decentralized estimation with missing observations and communication faults," - to be presented at the IFAC Congress, 2008.
[19] ——, "Consensus based overlapping decentralized estimator," Accepted for publication in *IEEE Trans. Autom. Control*, 2007.
[20] M. Ikeda, D. D. Šiljak, and D. White, "An inclusion principle for dynamic systems," *IEEE Trans. Autom. Control*, vol. 29, pp. 244–249, 1984.
[21] M. Ikeda and D. D. Šiljak, "Decentralized control with overlapping information sets," *J. Optimization Theory and Applications*, vol. 34, pp. 279–310, 1981.
[22] ——, "Overlapping decentralized control with input, state and output inclusion," *Control Theory and Advanced Technology*, vol. 2, pp. 155–172, 1986.
[23] S. S. Stanković, M. J. Stanojević, and D. D. Šiljak, "Decentralized overlapping control of a platoon of vehicles," *IEEE Trans. Control Syst. Techn.*, vol. 8, pp. 816–832, 2000.
[24] D. M. Stipanović, G. İnhalan, R. Teo, and C. Tomlin, "Decentralized overlapping control of a formation of unmanned aerial vehicles," *Automatica*, vol. 40, pp. 1285–1296, 2004.
[25] B. D. O. Anderson and J. B. Moore, *Optimal Filtering*. New York: Prentice Hall, 1979.
[26] R. S. Smith and F. Y. Hadaegh, "Closed-loop dynamics of cooperative vehicle formationswith parallel estimators and communication," *IEEE Trans. Autom. Control*, vol. 52, pp. 1404–1414, 2007.