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Generalized Piecewise Linear Feedback Stabilizability of Controlled Linear Switched Systems

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Abstract— **The paper consider the (closed–loop) stabilizability problem of controlled linear switched systems. It is shown that if the switching system is completely controllable then it is stabilizable. Moreover, it is shown that for these systems it can be found a closed–loop (event driven) switching strategy with suitable linear feedbacks that (weakly) stabilizes the system, i.e. the switching system is stabilizable by a generalized piecewise linear feedback. These results holds for systems where the control inputs are sign constrained, too.**

I. INTRODUCTION

Motivated by the need of dealing with physical systems that exhibit a more complicated behavior, hybrid systems have become very popular nowadays. In particular, there has been a relevant interest in the analysis and synthesis of so-called *switching systems* intended as the simplest class of hybrid systems.

A switching system is composed of a family of different (smooth) dynamic modes such that the switching pattern gives continuous, piecewise smooth trajectories. Moreover, we assume that one and only one mode is active at each time instant.

The study of the switching systems is closely related to some investigations on differential inclusions. By using techniques from convex nonsmooth analysis a series of very powerful results can be deduced, for an overview of the most important ideas related to differential inclusions and stability problems related to switching systems see [5], [25], [27].

Stability issues of switched systems, especially switched linear systems, have been of increasing interest in the recent decade, see for example [10], [15], [16], [17], [19], [31].

In the study of the stability of switched systems one may consider switched systems under given switching signals or tries to synthesise stabilizing switching signals for a given collection of dynamical systems. Concerning the first class a lot of papers focus on the asymptotic stability analysis for switched homogeneous linear systems under arbitrary switching (strong stability, robust stabilization), and provide necessary and sufficient conditions, see [4], [12], [22].

The requirement of (robust) stability imposes very strict conditions on the dynamics, e.g. all the subsystems must be stable or stabilizable. Even under this condition, one has, in general, further restrictions on the allowable switching frequency (dwell time), determined by the spectrum of the matrices, [34], [33].

For strongly stabilizable linear controlled switching systems the feedback control always can be chosen as a "patchy", linear variable structure controller, see [4]. The control is defined by a conic partition $\mathbb{R}^n = \bigcup_{k=1}^{N} \mathcal{C}_k$ of the state space while on each cone $\mathcal{C}_k$ the feedback is given by $u = F_k x$.

In the more general situation, when one has unstable modes, more severe conditions on the switching sequence have to be imposed. In this respect one of the most elusive problems is the switching stabilizability problem, i.e., under what condition is it possible to stabilize a switched system by properly designing autonomous (event driven) switching control laws. For autonomous switchings the vector field changes discontinuously when the state hits certain "boundaries". This problem corresponds to the weak asymptotic stability notion of the associated differential inclusions.

Based on the ideas presented in [23] it was proved that the (weak) asymptotic stabilizability of switched autonomous linear systems by means of an event driven switching strategy can be formulated in terms of a conic partition of the state space, see [18],[20]. This result can be seen as a generalization of the corresponding theorem for strong stability. However, in contrast to the strong stability results, the corresponding Lyapunov function is not always convex, see [6].

Completely controllable linear time invariant (LTI) systems $\dot{x} = Ax + Bu$ are stabilizable and the stabilization can be always done by a static state feedback $u = Kx$. Similar result, with a suitable set of linear state feedbacks, is valid for the case when the inputs are sign constrained, see [26], [32].

The paper gives a generalization of these fundamental results for the weak stabilizability of the class of completely controllable linear switching systems, where the control inputs might be sign constrained, i.e. it is shown that a completely controllable linear switching system is closed–loop stabilizable, moreover, the stabilization can be performed by using a generalized piecewise linear feedback. The structure of the paper is the following: after fixing the notation and giving a short overview of the results concerning

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controllability in Section II, in Section III it is proved that complete controllability implies stabilizability. In Section IV it is shown that the stabilization can be achieved by applying a finite set of suitable linear state feedbacks with a closed–loop switching strategy. The next section contains the Conclusions together with some topics for further research.

II. General considerations

Consider the class of (open-loop) linear switched systems:

$$\dot{x}(t) = A(\sigma(t))x(t) + B(\sigma(t))u(t) \tag{1}$$

where $x \in \mathbb{R}^n$ is the state variable and $u \in \Omega$ is the input variable. $\sigma : \mathbb{R}^+ \to S$ is a measurable switching function mapping the positive real line into $S = \{1, \cdots, s\}$, i.e. the matrices $A(\sigma)$ and $B(\sigma)$ are measurable. The input set might be unconstrained $\Omega = \mathbb{R}^m$ or constrained $\Omega = \mathbb{R}^m_+$.

A solution (Carathéodory) of (1) on an interval I is an almost everywhere (a.e.) differentiable function $\varphi(t) : I \to \mathbb{R}^n$ that satisfies (1) a.e. on I.

Following classical lines, (1) is said to be *completely controllable* if every point in the state space is reachable from any other point in the state space by using bounded measurable controls and a suitable switching function. As for LTI systems, algebraic conditions that guarantees complete controllability can be given in terms of the state matrices of the individual subsystems, see [31], [7].

It is of fundamental importance for our further investigations that for completely controllable linear switching systems every point pair of the state space can be joined by a trajectory with a finite number of switchings. Actually there is a universal (but not necessarily unique) finite switching sequence, for details, see e.g. [31] for the unconstrained case and [7] for the sign constrained case.

A. Vector fields, differential inclusions

A control system on a smooth n-dimensional manifold M is a collection $\mathcal{F}$ of smooth vector fields depending on independent parameters $w = [w_1, \cdots, w_m] \in \Xi \subset \mathbb{R}^m$ called control inputs such that $w(t)$ belongs to a suitable class of real valued functions $\mathcal{W}$, called admissible controls. The set of admissible control functions usually are taken to be the set of measurable functions.

A switching system can be considered as a nonlinear polysystem of the form

$$\dot{x} = f(x(t), w(t)), \quad x(0) = 0 \tag{2}$$

where in general, it is assumed that $x \in M$ and $f(., w)$, $w \in \Xi$ is an analytic (smooth) vector field on M. It is supposed that M is an n-dimensional real analytic manifold (paracompact and connected).

In our case $\Xi = S \times \Omega$ with

$$f_w(x) = f(x(t), w(t)) = A_i x(t) + B_i u$$

where $w = (i, u)$. System (2) can be associated in a natural way with the collection of vector fields

$$F(x) = \{f_w(x) \,|\, w \in \Xi\},$$

i.e. instead of the original switching system (1) one can consider the the (convexified) differential inclusion

$$\dot{x} \in A_c(x)$$

where

$$A_c(x) = \{\sum_{i=1}^{s} \alpha_i (A_i x + B_i u),\, \alpha_i \geq 0,\, \sum_{i=1}^{s} \alpha_i = 1 \,|\, u \in \Omega\}.$$

By weak stabilizability of the linear switching system is meant the weak stabilizability of the associated differential inclusion, for details see [27].

System (2) is *globally asymptotically controllable* (GAC) provided that for each x_0 there is a bounded measurable control such that for the corresponding trajectory $\lim_{t\to\infty} x(t) = 0$ and $|x(t)| \leq \theta(|x_0|)$ for all $t \geq 0$ for a nondecreasing function $\theta : [0, \infty) \to [0, \infty)$ with $\lim_{t\to\infty} \theta(t) = 0$, i.e. for each initial state, there exists a control such that the corresponding solution is defined and converges to zero with "small overshoot" and also that the input remains bounded for x near zero, for details see [24].

III. Stabilizability of completely controllable linear switching systems

In order to prove stabilizability of completely controllable linear switching systems it is sufficient to show that they are globally asymptotically controllable.

Lemma 1: A completely controllable linear switching system is globally asymptotically controllable.

Proof: Let us consider the unit sphere $\mathcal{S}$ and a point $x \in \mathcal{B}$. By complete controllability it follows that there is a finite switching sequence $\tau_x = (\tau_{L_x}, \ldots, \tau_2, \tau_1)$ and a bounded measurable control sequence (actually a piecewise constant control) $u_x = (u_{L_x}, \ldots, u_2, u_1) \in \Omega^{L_x}$ such that the corresponding trajectory steers the point x to the origin, i.e.

$$\Phi(\tau_x, u_x)x = \prod_{j=1}^{L_x} e^{(A_{l_j}\xi + B_{l_j}u_j)\tau_j} x = 0,$$

where, for notational convenience $e^{(A_{l_j}\xi + B_{l_j}u_j)\tau}\zeta$ denotes the flow associated to the vector field $A_{l_j}\xi + B_{l_j}u_j$ that passes through the initial state ζ at $t = 0$.

By the continuity of the map $\Phi(\tau_x, u_x)$ for the fixed pair (τ_x, u_x) for every $\epsilon > 0$ there is a neighborhood $\mathcal{V}_x$ of x such that

$$||\Phi(\tau_x, u_x)\xi|| < \epsilon, \quad \forall \xi \in \mathcal{V}_x,$$

hence for all $\xi \in \mathcal{W}_x = \mathcal{V}_x \cap \mathcal{S}$, see Fig. 1.

Since the unit sphere is compact, there is a finite covering $\mathcal{S} = \cup_{j\in J}\mathcal{W}_{x_j}$. It follows that there is a control strategy that maps the unit sphere into the sphere with radius $\epsilon < 1$ defined by this finite partition.

Since the linear maps $\Phi(\tau_{x_j}, u_{x_j})$ are bounded one has a uniform bound for the "overshoot",

$$\Theta = \max_{j\in J} ||\Phi(\tau_{x_j}, u_{x_j})||.$$

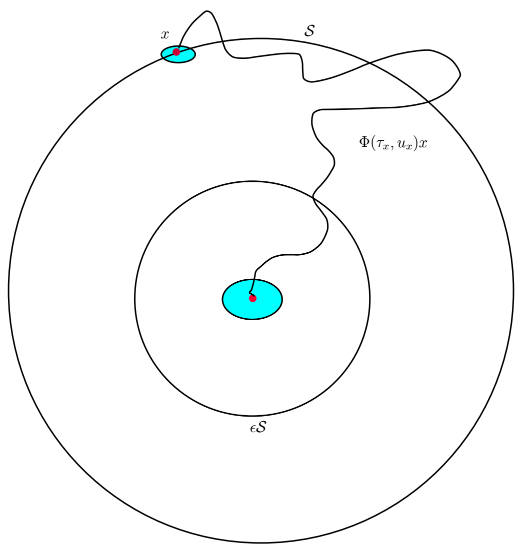

Fig. 1.

Since the vector fields are linear the reachable spaces are cones, therefore the control strategy can be extended from the unit sphere to the whole state space, i.e. one can construct a trajectory with the bound $||x(t)|| < \Theta||x_0||$ that converges to the origin. It follows that a completely controllable linear switching system is globally asymptotically controllable.

The importance of global asymptotical controllability is that it implies closed–loop stabilizability, i.e. it implies the existence of a feedback control such that the resulting closed loop–system is stable, see [9] (in terms of the so–called π trajectories) and [1], [24] (Carathéodory trajectories).

Corollary 1: The completely controllable linear switching system (1) is closed–loop stabilizable.

Remark 1: For discrete–time linear switched systems with unconstrained inputs the assertion of Lemma 1 was proved recently, see [35]. The switching strategy in the proposed solution is a periodic one, based on the universal switching sequence. In contrast to the continuous time case the proof is constructive, moreover the necessary linear feedbacks can be obtained by a linear matrix inequality.

The continuous–time result for the unconstrained input case can be obtained directly from the discrete–time one by using the fact that generically the discretized linear switched system preserves the complete controllability property, see [31]. The resulting control will be a stabilizing control with a periodic (open–loop) switching strategy and a "feedback–like" control for u – a feedback implemented in a sample and hold way.

The assertion of Lemma 1 is also valid for the sign constrained control input case, when the proof based on the discrete–time result is not applicable.

IV. Stabilizability by Generalized Piecewise Linear Feedback

While the general nonlinear theory guarantees the existence of a not too pathological feedback and control Lyapunov function, see [1], [11], [24], the results are hard to be applied to construct directly the required feedback for the switching system, i.e. to obtain the closed–loop switching strategy and necessary control inputs or even to infer that the control inputs are given by linear feedbacks.

Given an autonomous linear switching system

$$\dot{x} = A_i x, \quad i \in S$$

it is a nontrivial task to decide if the system is (weakly) stabilizable or not, in general. There are only a few sufficient conditions that guarantee stabilizability and provide a relatively simple closed-loop switching strategy. One such situation is when the convex hull of the system matrices contains a stable (Hurwitz) matrix, i.e. when there are $\alpha_i > 0$, $\sum_{i=1}^{s} \alpha_i = 1$ such that $\sum_{i=1}^{s} \alpha_i A_i$ is stable.

For the nonautonomous case with unconstrained inputs it is known that if the sum of the individual controllability subspaces gives the whole state space, then there are linear state feedbacks $u = K_i x$ such that the resulting linear switching system

$$\dot{x} = (A_i + B_i K_i)x, \quad i \in S$$

is stable with a suitable closed–loop switching strategy, see [31]. It is not hard to figure out that the required condition is sufficient to guarantee that for any convex combination $\alpha_i > 0$, $\sum_{i=1}^{s} \alpha_i = 1$ there exist feedbacks K_i such that $\sum_{i=1}^{s} \alpha_i (A_i + B_i K_i)$ is stable.

As it can be concluded through simple examples, see [31], there are completely controllable switching systems that are not stabilizable by merely applying a single linear state feedback for the individual subsystems. However, as it will be shown in this Section, if the number of linear feedbacks is increased, one can obtain a set of autonomous linear systems that are (weakly) stabilizable.

For a given set of non–autonomous (controlled) linear switched systems (1) we call *Generalized Piecewise Linear Feedback Stabilizability* (GPLFS) the problem of finding a closed-loop switching strategy with

- suitable linear feedbacks $u_i = K_{l_i} x, \quad i \in S$
- a switching law $\kappa(x) \in S$, $x \in \mathbb{R}^n$

that (weakly)stabilizes the system.

The reasoning behind introducing the concept of generalized piecewise linear feedback stabilizability is to separate the task of finding a suitable switching strategy and that of finding suitable control inputs with low complexity that stabilizes the system in closed–loop.

The main idea is to substitute the original stabilizable nonautonomous system by a stabilizable autonomous linear switched system that might contain more modes then the original one, by applying as control inputs a number of suitable static linear control feedbacks.

Theorem 1: The completely controllable linear switching system (1) is generalized piecewise linear feedback stabilizable.

Proof: In proving the assertion we will apply ideas of the Nagano–Sussmann_Jurdjevic theory of attainability.

The first observation is that the vector field

$$f(x) = \{f_u(x) = Ax + Bu\}$$

can be replaced by the vector field

$$\mathcal{F}(x) = \{f_K(x) = Ax + BKx\},$$

if $x \neq 0$. Indeed, for any $u \in \Omega$ one can chose a nonzero component x_i of x and a $K = [k_{l,j}]$ such that $k_{l,j} = 0$ if $j \neq i$ and $k_{l,i} = \frac{u_l}{x_i}$, then $u = Kx$. Actually one has

$$F(x) = \mathcal{F}(x), \quad \text{if} \quad x \neq 0.$$

Moreover, for any $y, z \in \mathbb{R}^n \setminus 0$ there is a trajectory of the original system that does not pass through the origin. This follows from the fact that the origin is normally reachable from any point, see [14], [30]. Then by the surjective mapping theorem, [3], follows that a neighborhood of the origin is reachable by the same switching sequence. Hence, by the linearity of the vector fields, the whole space is reachable with the given switching sequence.

Since the trajectory $x(t)$ does not pass through the origin, the original vector fields ($F(x)$) can be replaced by the new one ($\mathcal{F}(x)$). Moreover, since a given component of $x(t)$ might vanish only a finite times on a finite interval, it follows that the controls K_i of the vector field $F_K(x)$ are piecewise continuous. It follows that every point pair of the manifold $\mathbb{R}^n \setminus 0$ can be joined by a trajectory corresponding to the vector field $\mathcal{F}$ by admissible controls.

It follows that the vector field $\mathcal{F}$ is completely controllable on the manifold $\mathbb{R}^n \setminus 0$. Since complete controllability implies controllability by piecewise constant controls, see [13], [14], it follows that every point pair of the space $\mathbb{R}^n \setminus 0$ can be joined by a trajectory of suitable autonomous switched systems $A_l + B_l K_l$.

Remark 2: Complete controllability of the vector field $\mathcal{F}$ has a very intuitive geometrical background. Since the solutions of a linear autonomous differential equations realizes some rotations and dilations/compressions in $\mathbb{R}^n$, it means that for a given point pair (y, z) it is possible to select a finite set of feedbacks such that the resulting set of autonomous systems transform the point y into z for a suitable (finite) switching sequence.

In order to show that it is possible to select a finite set of autonomous systems that has the (weak) stabilizability property, the compactness argument applied in the proof of Lemma 1 can be repeated.

Indeed, selecting a point y on the unit sphere $\mathcal{S}$ and fixing a point z on the sphere ϵ_1, there is a trajectory formed by suitable autonomous systems $A_l + B_l K_l$ that steers y to z, i.e.

$$\Psi(\tau_y, K_y)y = \prod_{j=1}^{L_y} e^{(A_{l_j} + B_{l_j} K_j)\tau_j} y = z.$$

By continuity of $\Psi(\tau_y, K_y)$ for fixed τ_y and K_y there is a neighborhood of y that is mapped in a sufficiently small neighborhood of z, such that $||\Psi(\tau_y, K_y)\xi|| < \epsilon_2$, with $0 < \epsilon_1 < \epsilon_2 < 1$, see Fig. 2.

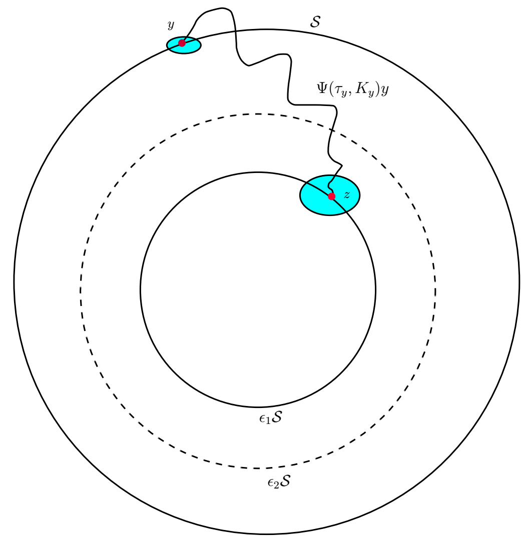

Fig. 2.

These neighborhoods form a covering of the unit sphere, from which it is possible to select a finite one. It follows that it is possible to select a finite set of linear static state feedbacks such, that the resulting set of autonomous system is stabilizable.

Concerning the switching strategy the existence of the suitable closed–loop switching rule is guaranteed by the general results for nonlinear globally asymptotically controllable systems, [24]. However, for nonautonomous switching systems with unconstrained controls slightly more can be asserted.

In [21] it was shown that the existence of an asymptotically stabilizing switching strategy (without sliding motion) of an autonomous linear switched system implies the existence of a conic partition based switching law which globally asymptotically stabilizes the closed–loop switching system. The control is defined by a conic partition $\mathbb{R}^n = \bigcup_{l=1}^{L} \mathcal{C}_l$ of the state space while on each cone $\mathcal{C}_l$ the system defined by $A_{i_l} + B_{i_l} K_l$ with $i_l \in S$ is active.

Remark 3: Since for linear autonomous switching systems asymptotical stability and exponential stability are equivalent, see [31], Theorem 1 shows that completely controllable linear switching systems with (unconstrained input) are exponentially stabilizable.

The sign constrained case is more delicate. The resulting autonomous systems correspond to certain regions of the state space, i.e. the resulting switching system is an autonomous state constrained linear switching system. Therefore the result from [31] is not applicable directly and the case needs further investigation.

Remark 4: Theorem 1 guarantees the generalized piecewise linear feedback stabilizability but does not give a method to compute such feedbacks. However – for the unconstrained input case – the property of complete controllability is feedback invariant. It is known that any controllable unconstrained multi–input linear switching system can be changed into a controllable single–input system via suitable non–regular state feedbacks, see [31]. Moreover, the controllable single–input system can be put into the form $(A_1, b_1), A_2, \cdots, A_s$. Theorem 1 guarantees that by these transformations not only controllability but also stabilizability is preserved. Hence one can obtain a switching system with a reduced complexity for which one might find suitable stabilizing feedbacks more easily, e.g. the resulting BMI or LMI equations in finding suitable piecewise quadratic Lyapunov functions will be simpler.

V. Conclusion

This paper considers the problem of event driven stabilization of linear controlled switched systems and claims that if the system is completely controllable then it is stabilizable by generalized piecewise linear feedback, i.e. it is stabilizable by applying event driven switchings for a finite set of linear autonomous systems obtained by applying a suitable set of linear feedback.

The obtained results are general enough to include the class of linear controlled switched systems with sign constrained control inputs.

For the class of completely controllable unconstrained linear switching systems one has not only closed–loop stabilizability but also closed–loop exponential stabilizability. It is a subject of further research to explore what type of performance bounds exists concerning the convergence rate.

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