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Design of Reconfigurable and Fault-Tolerant Suspension Systems Based on LPV Methods

Péter Gáspár, Zoltán Szabó and József Bokor

Abstract— **This paper proposes the design of reconfigurable suspension systems in road vehicles. The purpose of the suspension system is to improve passenger comfort and road holding during travel and improve safety during various maneuvers. In the modeling the nonlinearities of the suspension system, the changes in the forward velocity and the change of the adhesion coefficient between the tire and the road are taken into consideration. The effects of the longitudinal or lateral load transfers during maneuvers or abrupt hard brakings are monitored in order to reduce their harmful effects on handling and comfort. When a fault (loss in effectiveness) occurs at one of the suspension actuators a reconfiguration is required in order to guarantee fault-tolerant operation. The design of the proposed reconfiguration and fault-tolerant control is based on an $\mathcal{H}_\infty$ Linear Parameter Varying (LPV) method that uses monitored scheduling variables of the travel.**

I. INTRODUCTION

The main purpose of an active suspension system is to provide good handling characteristics and improve passenger comfort while harmful vibrations caused by road irregularities and on-board excitation sources act upon the vehicle, see [2], [7], [11]. By using an active suspension system not only the effects of road irregularities can be eliminated but road holding can also be improved, see e.g. [1], [12], [13].

This function is based on the fact that the system is able to generate a stabilizing moment to balance the overturning moment in such a way that the control torque leans the vehicle into the bend. Moreover, it is able to generate a moment to balance the pitching moment during abrupt and hard brakings.

The design of an active suspension in these problems significantly differs from that of the conventional active suspension design, where the performance specifications for passenger comfort, suspension deflections and tire deflections are met simultaneously. However, in these cases the controller is able to focus only on one of the performance specifications and thus neglect other performances. When the vehicle is cruising, the performances are the same as in the conventional system. When the vehicle is in an emergency, i.e. it is coming close to rolling over or significant pitching dynamics is generated, the performance demands significantly differ from those in the conventional case.

When a rollover is imminent the active suspension system generates a stabilizing moment to balance the overturning moment. When this dangerous situation persists, the active brake system must generate unilateral brake forces in order to reduce the risk of the rollover. This is an integrated control system, since several actuators co-operate and meet different performance requirements. Moreover, during abrupt brakings pitch dynamics increases significantly. The active suspension is also able to generate a moment and improve the pitch stability of the vehicle.

The reconfigurable control is extended with a fault-tolerant property in order to guarantee performances even if a hydraulic actuator fault occurs in the active suspension system. In case of a detected failure the operation of the control mechanism must be modified in order to guarantee roll stability. The solution of the fault-tolerant operation requires the reconfigurability of the active brake. The basis of this solution is that the active brake is able to change the yaw dynamics and reduce the rollover risk.

In this paper the model for control design is constructed in an LPV structure, in which the nonlinearities of the suspension system, the changes in forward velocity and other varying variables are selected as scheduling variables. The LPV modeling techniques allow us to take into consideration the nonlinear effects in the state space description, thus the model structure is nonlinear in the parameters, but linear in the states. In the control design the performance specifications for vertical, rolling and pitching dynamics, and the model uncertainties are also taken into consideration.

The structure of the paper is as follows. In Section II the control-oriented modeling of the vertical, yaw and roll dynamics are considered. In Section III the performance specifications are formalized in an LPV design framework. In Section IV the integrated control mechanism is demonstrated through simulation examples. Finally, Section V contains some concluding remarks.

II. CONTROL-ORIENTED MODELING OF VEHICLE DYNAMICS

A. Modeling of vehicle dynamics based on an LPV framework

The full-car vehicle model comprises five parts: the sprung mass and four unsprung masses (Figure 1). All suspensions consist of a spring, a damper and an actuator, which generate pushing forces between the body and the axle. Let the front and rear displacement of the sprung mass and of the

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P. Gáspár, Z. Szabó and J. Bokor are with the Systems and Control Laboratory, Computer and Automation Research Institute, Hungarian Academy of Sciences, Hungary, Phone: 361-2796171, Fax: 361-4667503, E-mail: {gaspar;szabo;bokor}@sztaki.hu

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unsprung masses on the left and right side be denoted by z_{1ij}, z_{2ij} and $(ij) \in (fl, fr, rl, rr)$, respectively. In the full-car model the disturbances, w_{ij} are caused by road irregularities. The control forces f_{ij} are generated by the actuators.

In vehicle modelling the motion differential equations of the combined vertical, yaw, pitch and roll dynamics of the single unit vehicle are formalized. The vehicle can translate longitudinally and laterally. The sprung mass can rotate around a horizontal axis. The unsprung masses can also roll, permitting the vertical compliance of the tires. The suspension springs, dampers and active suspensions generate forces and moments between the sprung and unsprung masses in response to vertical, roll and pitch motions. The tires produce lateral forces that vary linearly with the side slip angles.

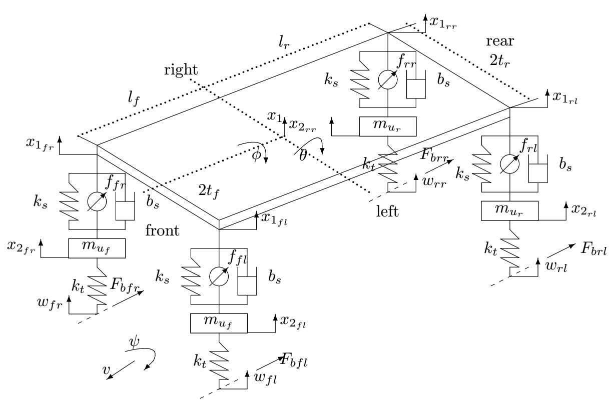

Fig. 1. The dynamics of a full-car model

The sprung mass is assumed to be a rigid body and has freedoms of motion in the vertical, pitch and roll directions. z_s is the vertical displacement at the center of gravity. The signals are the side slip angle of the vehicle body β, the heading angle of the sprung mass ψ, the roll angle ϕ and the pitch angle ψ. δ_f is the front wheel steering angle. v is the forward velocity. The braking forces are denoted by F_{bij}. Linear approximations are applied to the front and rear displacements of the sprung mass on the left and right-hand side.

The vertical suspension forces of the vehicle depend on the suspension damping forces F_{bij} and the suspension spring forces F_{kij}:

$$F_{zij} = F_{bij} + F_{kij} - f_{ij}. \tag{1}$$

where the suspension damper and spring forces are expressed using nonlinear terms:

$$F_{bij} = (b_f^l - b_f^{sym}\rho_{bij})(\dot{z}_{2ij} - \dot{z}_{1ij}) + b_f^{nl}\rho_{bij}\sqrt{\rho_{bij}(\dot{z}_{2ij} - \dot{z}_{1ij})}, \tag{2a}$$

$$F_{kij} = k_f^l(z_{2ij} - z_{1ij}) + k_f^{nl}(z_{2ij} - z_{1ij})^3. \tag{2b}$$

with the notation $\rho_{bij} = sgn(\dot{z}_{2ij} - \dot{z}1ij)$. The vertical tire forces are: $F_{tij} = k_t(z_{2ij} - w_{ij})$. The b_f^l coefficient affects the damping force linearly while b_f^{nl} has a nonlinear impact on the damping characteristics. b_f^{sym} describes the asymmetric behavior of the characteristics.

The lateral tire forces F_{yf} and F_{yr} in the direction of the wheel ground contact are approximated linearly to the tire slide slip angles α_f and α_r, respectively:

$$F_{yfl} = \mu C_f \alpha_f, \quad F_{yrl} = \mu C_r \alpha_r, \tag{3}$$

where μ is the side force coefficient and C_f and C_r are tire side slip constants and $F_{yfr} = F_{yfl}$, $F_{yrr} = F_{yrl}$. The chassis and the wheels have identical velocities at the wheel ground contact points. At stable driving conditions, the tire side slip angle α_i is normally small and it can be approximated as $\alpha_f = -\beta + \delta_f - l_f \cdot \dot{\psi}/v$ and $\alpha_r = -\beta + l_r \cdot \dot{\psi}/v$.

The motion differential equations, i.e. the lateral dynamics and the yaw dynamics of the vehicle, the pitch dynamics and the roll dynamics of the sprung mass are the following. The additional equations for the four unsprung masses can be formalized.

$$mv(\dot{\beta} + \dot{\psi}) - m_s h\ddot{\phi} = F_{yfl} + F_{yfr} + F_{yrl} + F_{yrr}, \tag{4a}$$

$$I_\psi \ddot{\psi} = l_f(F_{yfl} + F_{yfr}) - l_r(F_{yrl} + F_{yrr}) - t_r \Delta F_b, \tag{4b}$$

$$(I_\theta + m_s h^2)\ddot{\theta} - m_s gh\theta = l_f(F_{zfl} + F_{zfr}) + -l_r(F_{zrl} + F_{zrr}) - hF_b, \tag{4c}$$

$$(I_\phi + m_s h^2)\ddot{\phi} - m_s gh\phi - m_s vh(\dot{\beta} + \dot{\psi}) = t_f(F_{zfr} - F_{zfl}) + t_r(F_{zrr} - F_{zrl}). \tag{4d}$$

These equations are expressed in a state space representation. Let the state vector be the following:

$$x = \begin{bmatrix} \beta & \dot{\psi} & z_s & \theta & \phi & z_2 & \dot{z}_s & \dot{\theta} & \dot{\phi} & \dot{z}_2 \end{bmatrix}^T \tag{5}$$

with the notation: $z_2 = \begin{bmatrix} z_{2fl} & z_{2fr} & z_{2rl} & z_{2rr} \end{bmatrix}^T$. The disturbance vector contains the front wheel steering angle δ_f, the sum of the brake forces F_b and the road disturbances at the front and rear-hand side of the vehicle:

$$d = \begin{bmatrix} \delta_f & F_b & w_{fl} & w_{fr} & w_{rl} & w_{rr} \end{bmatrix}^T \tag{6}$$

with the notations $w = \begin{bmatrix} w_{bfl} & w_{bfr} & w_{brl} & w_{brr} \end{bmatrix}^T$. The control inputs are the suspension forces and the difference of the brake forces:

$$u = \begin{bmatrix} f_{fl} & f_{fr} & f_{rl} & f_{rr} & \Delta F_b \end{bmatrix}^T. \tag{7}$$

The difference between the brake forces ΔF_b provided by the compensator is assumed to be apply to the rear axle. This means that only one wheel is decelerated at the rear axle.

Then the state equation arises in the following form:

$$\dot{x} = A(\rho)x + B_1(\rho)d + B_2(\rho)u. \tag{8}$$

where $A(\rho) = A_0 + \sum_{i=1}^n \rho_i A_i$, $B_1(\rho) = B_{10} + \sum_{i=1}^n \rho_i B_{1i}$, $B_2(\rho) = B_{20} + \sum_{i=1}^n \rho_i B_{2i}$, in which n is the number of the scheduling variables ρ_i.

In this paper, the nonlinear effects of the forward velocity and that of the adhesion coefficient are taken into consideration in the vehicle dynamics. The adhesion coefficients depend on the type of road surface. There are several factors that can affect the value of the adhesion coefficient, which is a nonlinear and time varying function. It is very difficult to accurately quantify and measure the effect of all of the external factors on μ. In this paper the changes of the adhesion coefficient occurring at only one of the tires are ignored, their influence is taken into consideration in terms of the whole vehicle. Since the model contains a time-varying adhesion coefficient, an adaptive observer-based grey-box identification method has been proposed for its estimation, [10].

The nonlinear characteristics of the suspension system are also taken into consideration. These nonlinear effects are considered by selecting the square of the relative displacement and the signum of the relative velocity as the components of the scheduling vector in the corresponding LPV model: $\rho_{bij} = sgn(\dot{z}_{2ij} - \dot{z}_{1ij})$ and $\rho_{kij} = (z_{2ij} - z_{1ij})^2$. Parameter ρ_{bij} depends on the relative velocity, parameter ρ_{kij} is equal to the relative displacement. In practice, the relative displacement is a measured signal. The relative velocity is then determined by numerical differentiation from the measured relative displacement.

Thus, the scheduling vector is the following:

$$\rho = \begin{bmatrix} \rho_r & \rho_b & \rho_k \end{bmatrix}^T \tag{9}$$

with the notations $\rho_r = \begin{bmatrix} \frac{1}{v} & \mu & \frac{\mu}{v} & \frac{\mu}{v^2} \end{bmatrix}$, $\rho_b = \begin{bmatrix} \rho_{bfl}, \rho_{bfr}, \rho_{brl}, \rho_{brr} \end{bmatrix}$ and $\rho_k = \begin{bmatrix} \rho_{kfl}, \rho_{kfr}, \rho_{krl}, \rho_{krr} \end{bmatrix}$.

B. Monitored parameters in the control design

In the reconfigurable and fault-tolerant control of the suspension system several scheduling variables must be monitored and added to the scheduling vector in order to improve the safety of the vehicle: a variable is needed to reduce the rollover risk; a variable is needed to reduce the harmful effects of the abrupt braking; and a variable is also required to take a detected failure of an active component into consideration.

When the vehicle is in a normal cruising mode the suspension system guarantees passenger comfort and road holding. When the vehicle is close to a rollover, the suspension system generates a stabilizing moment to balance the overturning moment in such a way that the control torque leans the vehicle into the corner.

Roll stability is achieved by limiting the lateral load transfers on both axles, i.e. at the front and rear axles ΔF_{zyi}, $i \in (f, r)$, to below the levels for wheel lift-off. The lateral load transfer is calculated: $\Delta F_{zyi} = k_t \phi_{ti}$, where ϕ_{ti} is the roll angle of the unsprung mass. The tire contact force is guaranteed if $\frac{m_i g}{2} \pm \Delta F_{zyi} > 0$ for both sides of the vehicle, where m_i is the mass of the vehicle at the front and rear. This requirement leads to the definition of the normalized lateral load transfer, which is the ratio of the lateral load transfer and the mass: $R_i = \frac{\Delta F_{zyi}}{m_i g} = \frac{k_t \phi_{ti}}{m_i g}$. If the R_i takes on the value ± 1 then the inner wheels in the bend lift off. The limit cornering condition occurs when the load on the inside wheels has dropped to zero and all the load has been transferred onto the outside wheels. This event does not necessary result in the rolling over of the vehicle. However, the aim of the control design is to prevent rollover in all cases and thus the lift-off of the wheels must also be prevented. Thus, the normalised load transfer is also critical when the vehicle is stable but the tendency of the dynamics is unfavourable in terms of a rollover. The maximal value of the normalized lateral load transfers $\rho_R = max\{R_f, R_r\}$ is selected as a scheduling variable. The aim of the control design is to reduce the normalized lateral load transfer ρ_R if it exceeds a predefined critical value.

The pitch angle of the sprung mass can increase significantly during a sudden and hard braking. Pitch stability is achieved by limiting the longitudinal load transfers to below a predefined level. The normalized longitudinal load transfer is the normalized value of the pitch angle: $\rho_P = \theta / \theta_{max}$ where θ_{max} is the maximal value of the pitch angle. The aim of the control design during braking is to reduce the pitching dynamics if the normalized longitudinal load transfer exceeds a critical value. Thus, ρ_P is selected as a scheduling variable.

In practice the roll rates (pitch rates) of the unsprung masses are measured and the roll angles (pitch angles) are calculated by using a numerical integration. A method was proposed for the estimation of the roll angles of the unsprung masses based on an observer design, see [9].

Since the fault-tolerant control requires fault information in order to guarantee performances and modify its operation. Thus, a fault detection and isolation (FDI) filter is also designed. The scheduling vector is augmented with three variables:

$$\rho_a = \begin{bmatrix} \rho_R & \rho_P & \rho_D \end{bmatrix}^T \tag{10}$$

Here the fault information provided by a fault detection filter is given by $\rho_D = \frac{f_{act}}{f_{max}}$, where f_{act} is an estimation of the failure (output of the FDI filter) and f_{max} is an estimation of the maximum value of the potential failure (fatal error). The value of a possible fault is normalized into the interval $\rho_D = [0, 1]$. The estimated value f_{act} means the measure of the performance degradation of an active suspension component.

C. Design of an FDI filter

The fault detection of the actuator dynamics is based on quarter car models of the suspension system at the front and rear on the left- and right-hand side. Using (1) and (2) the force equations of the quarter-car model are:

$$m_{sij} \ddot{z}_{1ij} = F_{zij}, \tag{11}$$

$$m_{uij} \ddot{z}_{2ij} = -F_{zij} - F_{tij}. \tag{12}$$

Possible faults of the actuators (loss of effectiveness) can be detected by reconstructing the actual suspension forces f_{ij}. Since the real actuators might have a saturation effect it is necessary to check, in addition, if the actual forces are lower than those corresponding to the saturation level of the actuators.

Having measured the signals $y_{1ij} = \ddot{z}_{1ij}$, $y_{2ij} = \ddot{z}_{2ij}$ and $y_{3ij} = z_{2ij} - z_{1ij}$ an inversion-based detection filter is proposed, [3], [14]. In the constructions of the filter the first step is to express f_{ij} from the above equations and in these expression we plug in the known values y_i:

$$\begin{aligned} f_{ij} &= (b_f^l - b_f^{sym}\rho_{bij})(\dot{z}_{2ij} - \dot{z}_{1ij}) \\ &+ b_f^{nl}\rho_{bij}\sqrt{\rho_{bij}(\dot{z}_{2ij} - \dot{z}_{1ij})} \\ &+ \left(k_f^l + k_f^{nl}\rho_{kij}\right) y_{3ij} - m_{sij} y_{1ij}. \end{aligned} \tag{13}$$

By plugging back the obtained expressions in the original equations the resulting LPV system will have the same states as the original ones and it will be observable with the output y_{3ij}.

For a LPV system that depends affinely on the scheduling variables an LPV observer can be designed using LMI techniques: let us recall that an LPV system is said to be quadratically stable if there exist a matrix $P = P^T > 0$ such that $A(\rho)^T P + PA(\rho) < 0$ for all the parameters ρ, see [8].

In order to obtain a quadratically stable observer the LMI $A_o^T(\rho)P + PA_o(\rho) < 0$ must hold for suitable $K(\rho)$ and $P = P^T > 0$, with $A_o = A + KC$. By introducing the auxiliary variable $L(\rho) = PK(\rho)$, one has to solve the following set of $LMIs$ on the corner points of the parameter space:

$$A(\rho)^T P + PA(\rho) - C^T L(\rho)^T - L(\rho)C < 0. \tag{14}$$

By using the estimated state signals the values of the actual suspension forces f_{ij} are computed using (13).

III. Design of reconfigurable and fault-tolerant control

A. Construction of the LPV model for control design

In the control design besides the vertical dynamics, the roll and the pitch dynamics are also taken into consideration. The performance equation and the output equation in the state space representation are the following:

$$z = C_1(\varrho)x + D_{11}(\varrho)d + D_{12}(\varrho)u \tag{15}$$
$$y = C_2(\varrho)x + D_{21}(\varrho)d \tag{16}$$

where $\varrho = \begin{bmatrix}\rho & \rho_a\end{bmatrix}^T$. The controller is designed to meet several performance specifications, such as enhancing passenger comfort, increasing roll stability and pitch stability, guaranteeing suspension working space and reducing energy consumption. Thus, during travel the aim is to minimize the heave acceleration a_z, the suspension deflections $z_{sij} = z_{2ij} - z_{1ij}$ the tire deflections $z_{tij} = z_{1ij} - w_{ij}$ with $ij \in (fl, fr, rl, rr)$. During vehicle maneuvers the aim is to minimize the lateral acceleration a_y, the lateral load transfers at the front and the rear $\Delta F_{zyf}, \Delta F_{zyr}$ and during braking the aim is to minimize the pitch angle θ. The performance vector contains the following components:

$$z = \begin{bmatrix} a_y & \theta & a_z & z_{sij} & z_{tij} & \Delta F_{zyf} & \Delta F_{zyr}\end{bmatrix}^T. \tag{17}$$

The measured outputs are the longitudinal and lateral accelerations of the sprung mass, the yaw rate, the roll rate and the suspension deflections at the suspension components: $y = \begin{bmatrix} a_y & \dot{\psi} & \dot{\phi} & z_{sij}\end{bmatrix}^T$.

The reconfiguration and fault-tolerant control structure is solved by a weighting strategy, which is presented in this section. As an illustration the closed-loop interconnection structure is shown in Figure 2.

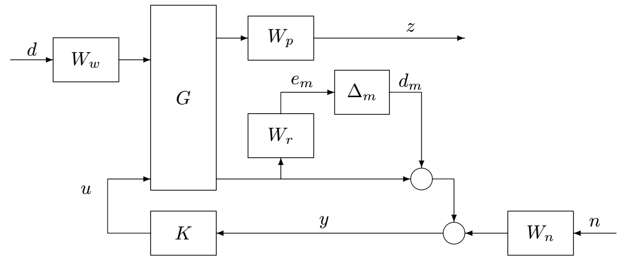

Fig. 2. The closed-loop interconnection structure

The weighting functions chosen for performance outputs can be considered as penalty functions: they are selected large in a frequency range where small signals are desired, and small where larger performance outputs can be tolerated. The weighting function for lateral acceleration is selected in such a way that in the low frequency domain the steering angle at the lateral acceleration should be rejected by factors of ϕ_{ay}.

The weighting functions for longitudinal acceleration is selected in such a way that in the low frequency domain the braking forces at the longitudinal acceleration should be rejected by factors of ϕ_{ax}. At the same time the weighting functions for the vertical acceleration and the suspension are selected by a factor of ϕ_{az}.

$$W_{p,ay} = \phi_{ay}\frac{A_1(\frac{s}{T_a}+1)}{(\frac{s}{T_b}+1)} \tag{18}$$
$$W_{p,\theta} = \phi_{\theta}\frac{A_2(\frac{s}{T_c}+1)}{(\frac{s}{T_d}+1)} \tag{19}$$
$$W_{p,az} = \frac{A_3(\frac{s}{T_e}+1)}{(\frac{s}{T_f}+1)} \tag{20}$$
$$W_{p,zs} = \frac{A_4(\frac{s}{T_g}+1)}{(\frac{s}{T_h}+1)} \tag{21}$$

with time constants T_i and proportional coefficients A_i. The gains ϕ_{ay} and ϕ_θ in the weighting functions are selected as function of parameters ρ_R and ρ_P in the following way:

$$\phi_{ay} = \begin{cases} 0 & \text{if } |\rho_R| < R_1 \\ \frac{(|\rho_R| - R_1)}{(R_2 - R_1)} & \text{if } R_1 \le |\rho_R| \le R_2 \\ 1 & \text{if } |\rho_R| > R_2 \end{cases} \tag{22}$$
$$\phi_{\theta} = \begin{cases} 0 & \text{if } |\rho_P| < P_1 \\ \frac{(|\rho_P| - P_1)}{(P_2 - P_1)} & \text{if } P_1 \le |\rho_P| \le P_2 \\ 1 & \text{if } |\rho_P| > P_2 \end{cases} \tag{23}$$

where R_1, R_2, P_1, P_2 are constants.

The gain ϕ_{ay} (ϕ_θ) is increased in order to minimize the lateral acceleration (the pitch angle). In the lower range of ρ_R (ρ_P) the gain must be small, and in the upper range of ρ_R (ρ_P) the gains must be large. Consequently, the weighting functions must be selected in such a way that they minimize the lateral load transfers in an emergency (the pitch angle during an abrupt brake). However in normal cruising the control does not focus on the these signals since the weight is small.

When the vehicles is in an emergency, i.e. it is coming close to rolling over an integrated control system with an active suspension and an active brake are applied in order to reduce the risk. In the event of a fault the range of the operation of the brake system must be extended and the wheels are decelerated gradually rather than rapidly if the normalized lateral load transfer has reached its critical value. A small value of R_1 corresponds to activating the brake system early and gradually, whereas a large value of R_1 corresponds to activating the brake system abruptly, see equation (22). Thus, the design parameter R_1 is chosen to be scheduled on the fault information ρ_D:

$$R_1 = R_1 - \frac{\rho_D}{\alpha} \tag{24}$$

where α is a constant factor.

B. Control design

In order to describe the control objective, the parameter dependent augmented plant $P(\varrho)$ must be built up using the closed-loop interconnection structure. The augmented plant $P(\varrho)$ includes the parameter dependent yaw-roll dynamics and the weighting functions.

$$\left[\begin{array}{c}\tilde{z}\\ \hline y\end{array}\right] = \left[\begin{array}{c|c}P_{11}(\varrho) & P_{12}(\varrho)\\ \hline P_{21}(\varrho) & P_{22}(\varrho)\end{array}\right]\left[\begin{array}{c}w\\ \hline u\end{array}\right], \tag{25}$$

where $w = \begin{bmatrix} d & n & d_m\end{bmatrix}$ and $\tilde{z} = \begin{bmatrix} z & e_m\end{bmatrix}$. The signals d_m, e_m are the output of the uncertainty block Δ_m and its input, respectively. The closed-loop system $M(\varrho)$ is given by a lower linear fractional transformation (LFT) structure:

$$M(\varrho) = \mathcal{F}_\ell(P(\varrho), K(\varrho)), \tag{26}$$

where $K(\varrho)$ depends on the scheduling parameter ϱ. The purpose of the control design is to minimize the induced $\mathcal{L}_2$ norm of a LPV system $M(\varrho)$ with zero initial conditions, which is given by

$$\inf_K \sup_{\varrho\in\mathcal{F_P}} \sup_{\|w\|_2\neq 0, w\in\mathcal{L}_2} \frac{\|z\|_2}{\|w\|_2}. \tag{27}$$

The $\mathcal{H}_\infty$ controller synthesis is extended to LPV systems using parameter dependent Lyapunov functions, see [4], [5], [6], [15]. The control design leads to infinite dimensional convex feasibility conditions. In practice these conditions can, in general, only be obtained approximately, by selecting grid points from the whole set, thus it is converted into finite dimensional LMIs. The number of grid points depends on the nonlinearity and the operation range of the system.

If parameter-dependent Lyapunov functions are used, the controller designed depends explicitly on $\dot{\rho}$. Thus, in order to construct a parameter-dependent controller, both ρ and $\dot{\rho}$ must be measured or available. When $\dot{\rho}$ is not measured in practice, a suitable extrapolation algorithm must be used to achieve an estimation of the parameter $\dot{\rho}$. An algorithm based on ρ-dependent change of variables to remove $\dot{\rho}$ dependence is also proposed, see [4].

For the interconnection structure, $\mathcal{H}_\infty$ compensators are synthesized for several values of velocity in a range $v = [20\text{kph}, 120\text{kph}]$. The scheduling variable μ is gridded with several values $\mu = [0.1, ..., 1.1]$. The scheduling variable F, which is the fault information provided by the FDI filter, can be taken from interval $\rho_D = [0, 1]$. The zero value of ρ_D corresponds to the non-faulty operation and the value 1 to the full hydraulic actuator failure. The parameter space of the normalized longitudinal load transfer is grided as $\rho_P = [0, P_1, P_2, 1]$. The parameter space of the normalized lateral load transfer is grided as $\rho_R = [0, R_1, R_2, 1]$.

IV. Simulation examples

In the simulation example the operation of a conventional suspension system is compared with a reconfigurable suspension system. In the example a cornering maneuver with 70 kph velocity is presented. The cornering maneuver starts at the 1^{st} second and at the 4^{th} second a huge bump with 10 cm maximal value disturbs the motion of the vehicle. Figure 3 shows the suspension forces u_{fl}, u_{rl}, u_{fr} and u_{rr}. The suspension system operating in the conventional manner generates suspension forces in order to reduce the effects of harmful vertical vibrations. Thus, it focuses on the large bump which disturbs the motion at the 4^{th} second and so, the minimization of the heave acceleration as it is illustrated by the dashed line.

In the reconfigurable case when the vehicle maneuver causes a critical value regarding rolling over, the suspension system generates moments to balance the overturning moments, thus the control force focuses only on reducing the normalized lateral load transfer and guaranteeing passenger comfort is no longer a priority (solid line). The purpose of reconfigurable active suspensions is to meet conventional performances in normal cruising and to guarantee rollover prevention and improve safety in emergencies.

In the second example the operation of the fault-tolerant integrated control is illustrated. The vehicle performs the same maneuver as in the first example. However, it is assumed that an actuator failure has already been detected at the front and rear. The time responses of the normalized lateral load transfer, the braking force at the rear and the suspension forces at the left and the right are presented in Figure 4. The solid line illustrates the fault operation and the dashed line illustrates the fault-free case.

It is observed that the normalized load transfer increases due to the reduced power of the actuators. According to the detected actuator fault the brake is activated at a smaller value of the critical normalized load transfer. Moreover, the duration of the required brake force is longer in the case of a suspension fault. Because of the braking action the suspension system generates the same forces (except in the faulty component) as the ones in the fault-free case.

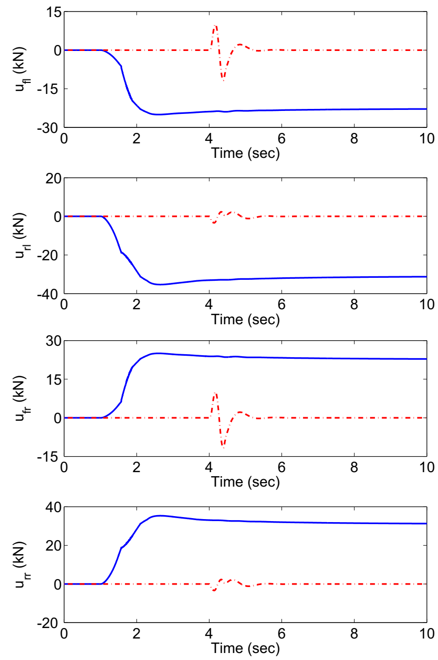

Fig. 3. The operation of the reconfigurable suspension system

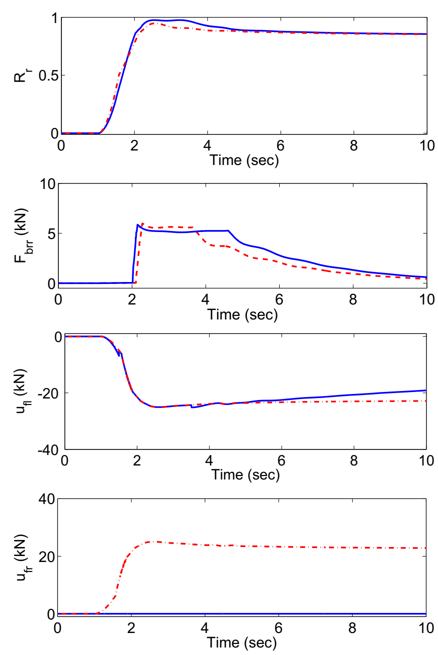

Fig. 4. The operation of the fault-tolerant control system

V. Conclusion

In this paper a fault-tolerant reconfigurable controller which includes an active suspension system and an active brake has been proposed. With this structure roll stability, pitch stability and passenger comfort can be guaranteed. Moreover, if a fault occurs in the active suspension system and it is detected by the FDI filter, the active brake assumes the role of the active suspension to enhance rollover prevention. A weighting strategy is applied in the closed-loop interconnection structure, in which the normalized lateral and longitudinal load transfers and the residual output of the FDI filter play an important role. This control mechanism guarantees the balance between roll stability, pitch stability and passenger comfort. Due to the nonlinear components present in the model but whose values are available or can be measured an LPV modeling is applied. The control design is based on $\mathcal{H}_\infty$ LPV method using parameter dependent Lyapunov functions

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