Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

High–Gain Adaptive Control: a Derivative-Based Approach

Franco Blanchini, Thomas Parisini, Felice Andrea Pellegrino, Gilberto Pin

***Abstract*— We propose an adaptive scheme which is a counterpart of existing high gain control techniques based on control Lyapunov functions. Given a control Lyapunov function, the main idea is that of tuning the feedback gain according to a suitably–chosen Lyapunov time–derivative. The control gain is not monotonically non–decreasing as in existing techniques, but it is increased or decreased depending on the imposed derivative, thus avoiding the well-known issue of actuator over–exploitation. We are able to show robust convergence of the proposed adaptive control scheme as well as other interesting properties. For instance, it is possible to guarantee an a–priori given upper bound for the transient mode of behavior during adaptation. Furthermore, if the control Lyapunov function is designed based on an optimal control problem, then the control action is nominally optimal, precisely it yields the optimal trajectory for any initial condition, if the actual plant matches the nominal system.**

I. Introduction

Consider a nonlinear dynamic system of the form

$$\dot{x}(t) = F(x(t)) + Bu(t)$$

and assume that a smooth control Lyapunov function Ψ is available. It is well established [1], [15], [6] that, for such systems, it is always possible to design a gradient–based control law of the form

$$u = -\kappa B^T \nabla\Psi(x)^T$$

which stabilizes the system for a suitably high value of κ. The appropriate magnitude of the gain κ depends on the plant characteristics which often are uncertain. Therefore, the choice of κ can be either ineffective (too small) or too actuator–exploiting (too high).

In previous works (see [7], [10]) the so called λ–tracking control has been presented which is a mechanism to increase the gain in an adaptive way. More specifically, let us consider the following adaptive scheme:

$$u(t) = -\kappa(t) B^T \nabla\Psi(x(t))^T , \quad (1)$$

$$\dot{\kappa}(t) = \mu_\lambda \sigma_\epsilon(\|x\|) , \quad \text{with} \quad \kappa(0) = \kappa_0 \geq 0, \quad (2)$$

where $\mu_\lambda > 0$ and $\sigma_\epsilon(\xi) = \max\{0, \xi - \epsilon\}$, for a given $\epsilon > 0$. Function $\sigma_\epsilon(\|x\|)$ represents the distance from the ball of radius ϵ. This means that the gain κ is increased as long as $x(t)$ is outside such a ball, proportionally to the distance. This "increasing" trend of the gain is expected to have a stabilizing effect, so that the state is eventually confined in the ϵ–ball. The reader is referred to specialized literature for the theoretical analysis (see again [7], [10], [11]) and for some interesting applications [4], [8], [9]).

However, the λ–tracking control architecture has some disadvantages, too. First, the gain κ may become too large (note that the gain growth depends on the coefficient μ_λ which is arbitrary) thus requiring an excessive control exploitation. Conversely, μ_λ too small may compromise performances. Moreover, despite the guaranteed convergence to the ϵ–ball, namely practical stability, no bounds have been given to the transient modes of behavior, so that the trajectory may be driven arbitrarily far from the origin. If the adaptive control algorithm is used for repeated tracking operations the gain becomes larger and larger, so that there must be a distinction between the "training session" and the "working session" when the adaptation is stopped.

An alternative approach is the so–called funnel control [11], [12]. That idea can be applied in our context by adapting the gain as follows

$$\kappa(t) = \mu_f \frac{1}{1 - \Psi(x(t))\varphi(t)} ,$$

where $\mu_f > 0$ and $\varphi(t)$ is a *properly selected* strictly increasing positive function converging to $1/\epsilon$ from below. This kind of adaptive scheme ensures the decreasing condition $\Psi(x(t)) \leq 1/\varphi(t)$ and hence convergence of $x(t)$ to the ϵ–ball of Ψ. The reader is referred to [11] for a nice survey. This scheme looks quite interesting, however the resulting controller is time–varying. Furthermore, "jumps" and measurement errors (for instance due to disturbances) may lead to singularity.

Corresponding author: Thomas Parisini. This work has been partially supported by the Italian Ministry for University and Research and by the Italian National Council of Research (C.N.R.).

Franco Blanchini is with the Dept. of Mathematics and Computer Science, University of Udine, Italy. (blanchini@uniud.it)

T. Parisini, F. A. Pellegrino, and G. Pin are with the Dept. of Electrical, Electronic and Computer Engineering, DEEI, University of Trieste, Italy. ({fapellegrino,parisini,gilberto.pin}@units.it).

978-1-4244-3124-3/08/$25.00 ©2008 IEEE

The control law we propose in the present paper is based on the estimation of the Lyapunov derivative, an idea already pursued in [16], [17]. However, our approach is basically different since we are considering a high–gain type of adaptive algorithm, basically combining ideas from [7], [10] and [16]. The main features of the proposed control law are:

- the gain increases or decreases depending on the current mode of behavior and there is no need for interruption of the adaptation;
- there are no requirements of control reset;
- upper bounds on the state transient during adaptation are provided;
- for nominally–linear systems, the control scheme is optimal as long as the model matches the plant (while robust convergence is ensured anyway).

II. Model description and assumptions

In the following, given a function $\Psi : \mathbb{R}^n \to \mathbb{R}$, we denote the α–sublevel set as $\mathcal{N}[\Psi, \alpha] \triangleq \{x : \ \Psi(x) \leq \alpha\}$.

Consider a dynamic system of the form

$$\dot{x}(t) = F(x(t), \Delta_F(t)) + G(\Delta_B(t))u(t)\,, \tag{3}$$

where the following assumptions characterize the various terms involved in (3).

Assumption 1: Function F is continuous and Δ_F is an uncertain term bounded as

$$\|\Delta_F(t)\| \leq \bar{\Delta}_F\,, \quad \forall t \geq 0\,,$$

and $G(\Delta_B(t)) = B(I + \Delta_B(t))$, where Δ_B is an uncertain term bounded as

$$\|\Delta_B(t)\| \leq \bar{\Delta}_B < 1\,, \quad \forall t \geq 0\,.$$

Assumption 2: System (3) admits a control Lyapunov function $\Psi(x)$, i.e. a radially–unbounded smooth positive–definite function such that

$$\dot{\Psi}(x) = \nabla\Psi(x)[F(x, \Delta_F) + G(\Delta_B)\Phi_0(x)] \leq -\phi(x)\,, \tag{4}$$

where $\Phi_0(x)$ is a locally–Lipschitz robustly stabilizing state–feedback controller (not necessarily known) and ϕ is a positive–definite radially–unbounded $\mathscr{C}^1$ function.

The following well–known property holds [15].

Proposition 2.1: There always exists a stabilizing gradient–based control of the form

$$u = -\bar{\kappa}(x)B^T\nabla\Psi(x)^T \tag{5}$$

where $\bar{\kappa}(x)$ is a a properly-selected continuous function.

In rather qualitative terms, $\bar{\kappa}(x)$ must be "large enough" so as to ensure that

$$\nabla\Psi(x)F - \bar{\kappa}(x)\nabla\Psi(x)BB^T\nabla\Psi(x)^T \leq \nabla\Psi(x)\left[F + B\Phi_0(x)\right] \leq -\phi(x)\,.$$

In this paper, we address a different type of control law that can be implemented as long as both functions Ψ and ϕ are exactly known (see Fig. 1):

$$u(t) = -\kappa(t)B^T\nabla\Psi(x(t))^T\,, \tag{6}$$

$$\dot{\kappa}(t) = \mu[\dot{\Psi}(x(t)) + \phi(x(t))]\,,\ \kappa(0) = \kappa_0 \geq 0. \tag{7}$$

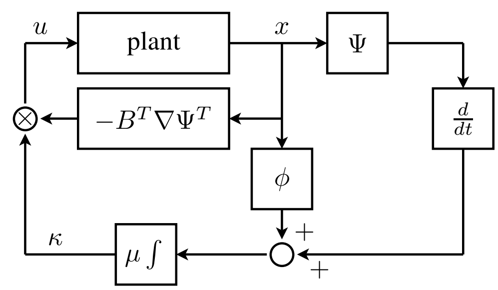

Fig. 1. The proposed adaptive control architecture.

The basic idea is to increase/decrease the gain if the Lyapunov function decreases slower/faster than the prescribed behavior $-\phi(x)$.

Remark 2.1: It is worth noting that the time-derivative $\dot{\Psi}(x)$ is not available, unless the actual process model is known. In principle, we could use a derivator, which, of course, is quite sensible to disturbances. However, from a practical perspective, the gain adaptation can be implemented as

$$\kappa(t) = \mu[\Psi(x(t)) + \xi(t)]$$

$$\dot{\xi}(t) = \phi(x(t)), \quad \xi(0) = \kappa_0/\mu - \Psi(x(0))\,.$$

Remark 2.2: Although for easiness of notation we consider the case of constant B, the following results hold with no changes for more general control–affine systems of the type

$$\dot{x}(t) = F(x(t), \Delta_F(t)) + B(x)\left(I + \Delta_B(t)\right)u(t)\,,$$

where the nominal input matrix is a function of the state.

A. Motivations

Before presenting the analysis and some simulation results, a few basic motivations are provided to gain more insight into the proposed control mechanism.

To fix ideas, let us consider the following simple nonlinear system (e.g., an inverted pendulum):

$$\ddot{\theta}(t) = \alpha\sin(\theta) + (1 + \beta)u(t)$$

where α and β are unknown (possibly time–varying) parameters subject to the bounds $|\alpha| \leq \bar{\alpha}, \quad |\beta| \leq \bar{\beta} < 1$, where $\bar{\alpha}$ and $\bar{\beta}$ are known.

In the ideal case where α and β are exactly known, the control problem can be easily addressed by means of feedback linearization, that is, we can pick the following control action:

$$u(t) = \frac{-\alpha \sin(\theta(t)) + v(t)}{1 + \beta}, \tag{8}$$

where v is an external input variable. Hence, the resulting closed-loop system takes on the form

$$\dot{x}(t) = Ax(t) + Bv(t) \tag{9}$$

with

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{10}$$

Clearly, if α is not known or time–varying, the above scheme cannot be applied.

Now, according to the robust control theory for systems with matched uncertainties [13], [1], [6], a Lyapunov function for the linear plant can be determined as $\Phi(x) = x^T Px$, along with the linear control law $v = -\rho B^T Px$, where ρ is a properly chosen scaling factor (e.g., the inverse of the control weighting matrix in LQ control). This can be done by solving an LQ optimal problem for the nominal linear system. Then we use this function as a control Lyapunov function for the original system.

For the sake of brevity and without loss of generality, let us assume that $\beta = 0$ and let us rewrite the system as

$$\dot{x}(t) = [A + B\Delta E]x(t) + Bu(t)$$

where $E = [1 \ \ 0]^T$ and Δ is defined as follows:

$$\alpha \sin(\theta) = \alpha \frac{\sin(x_1)}{x_1} x_1 \triangleq \Delta x_1$$

with $|\Delta| \leq \bar{\alpha}$.

If we take $u = -\kappa B^T Px$, the corresponding Lyapunov time-derivative is

$$\begin{aligned} \dot{\Psi} &= 2x^T P[(A + B\Delta E)x - \kappa BB^T Px] \\ &= 2x^T P[A - \rho BB^T P]x \\ &+ 2x^T [PB\Delta Ex - (\kappa - \rho)PBB^T P]x. \end{aligned}$$

Thus, we have

$$\begin{aligned} &\dot{\Psi} \leq -x^T Q_{LQ} x + \epsilon \|\Delta Ex\|^2 \\ &\quad -[\epsilon \|\Delta Ex\|^2 - \|x^T PB\| \|\Delta Ex\| + (\kappa - \rho)\|B^T Px\|^2], \end{aligned}$$

where $x^T Q_{LQ} x$ is the Lyapunov time-derivative of the linear optimal control law applied to the ideal plant. If Q_{LQ} is positive definite, and $\epsilon > 0$ small, there exists $\bar{\kappa}$ such that for $\kappa \geq \bar{\kappa}$ the above derivative is negative and then convergence is ensured. Therefore, we could just take any of such gain κ. However if we consider the worst case, the control might be over–exploited if the actual value of α is smaller than the worst case one. Hence, we consider the following adaptation law

$$\dot{\kappa}(t) = \mu[\dot{\Psi}(x(t)) + x(t)^T Q_{LQ} x(t)],$$

for $\mu > 0$, thus “expecting” the same Lyapunov derivative we could ideally achieve by means of the control law (8) with $v = -\rho B^T P$ that we cannot on–line compute without information about the current value taken on by α.

Notice that, in the general case where $\beta \neq 0$, this further source of uncertainty may have strong effects on the control effort. Specifically, for example, a negative value of β might represent a partial failure. Then the value of $\bar{k}$ previously derived must be augmented as $\bar{\kappa} \triangleq \bar{\kappa}/(1 - \bar{\beta})$ to face this “worst case” scenario. Clearly, increasing the gain could be absolutely useless under normal modes of behavior.

This kind of reasoning can be applied to other classes of processes such as those satisfying the so called strict feedback form [6] or the convex processes [3].

III. Main results

A. Convergence and transient upper bound

First, we show that, by means of the proposed control algorithm, we are able to impose bounds on both the transient behaviors of the state $x(t)$ and of the gain $\kappa(t)$, according to the next result.

Proposition 3.1: Under Assumptions 1 and 2, for any arbitrary $\zeta > 0$ and $\eta > 0$, there exists $\hat{\mu} > 0$ such that, for any $\mu \geq \hat{\mu}$, the control law with the adaptive mechanism (6)–(7) ensures the condition

$$\Psi(x(0)) \leq \zeta \Rightarrow \Psi(x(t)) \leq \zeta + \eta.$$

Furthermore, there exist $\underline{\kappa} > 0$ and $\bar{\kappa} > 0$ such that $\underline{\kappa} \leq \kappa(t) \leq \bar{\kappa}$ for all $t \geq 0$.

Proof: Define $\bar{\kappa}_{\eta,\zeta}$ as the maximum of $\bar{\kappa}(x)$ (see Proposition 2.1) on the sublevel set $\mathcal{N}[\Psi, \eta + \zeta]$:

$$\bar{\kappa}_{\eta,\zeta} \triangleq \max_{x \in \mathcal{N}[\Psi, \eta + \zeta]} \bar{\kappa}(x)$$

As a first step, we note that if $\kappa(t) \geq \bar{\kappa}_{\eta,\zeta}$ in (6), then the time-derivative is negative inside $\mathcal{N}[\Psi, \eta + \zeta]$. Indeed, by using the simple notation $F \triangleq F(x, \Delta_F)$ and $G \triangleq B(I +$

Δ_B), we obtain

$$\begin{aligned}\dot{\Psi}(x) &= \nabla\Psi(x)F - \kappa(t)\nabla\Psi(x)GB^T\nabla\Psi(x)^T \\ &\leq \nabla\Psi(x)F - \bar{\kappa}_{\eta,\zeta}\nabla\Psi(x)GB^T\nabla\Psi(x)^T \\ &\leq \nabla\Psi(x)F - \bar{\kappa}(x)\nabla\Psi(x)GB^T\nabla\Psi(x)^T \\ &\leq -\phi(x).\end{aligned}$$

On the other hand, if $\Psi(x(0)) \leq \zeta$, from (7) it follows that

$$\begin{aligned}\kappa(t) = \kappa(0) + \mu[\Psi(x(t)) - \Psi(x(0))] + \mu\int_0^t \phi(x(\sigma))d\sigma \\ \geq \mu[\Psi(x(t)) - \Psi(x(0))]\,. \quad (11)\end{aligned}$$

Now, we show that $\Psi(x(t))$ cannot reach values greater than $\zeta + \eta$ if we take μ as

$$\mu > \bar{\mu} \triangleq \frac{\bar{\kappa}_{\eta,\zeta}}{\eta}. \qquad (12)$$

By contradiction, let us assume that $\Psi(x(t)) = \zeta + \eta$ for some $t > 0$; then, from (11) it follows that

$$\kappa(t) \geq \mu[\Psi(x(t)) - \Psi(x(0))] \geq \mu\eta > \bar{\kappa}_{\eta,\zeta}$$

which, in turn, implies that $\dot{\Psi}(x(t)) < 0$ and hence $\Psi(x(t))$ cannot cross the value $\eta + \zeta$.

To show the boundedness of the gain $\kappa(t)$, we have to notice that, as long as $\kappa(t) \geq \bar{\kappa}_{\eta,\zeta}$, we have

$$\dot{\Psi}(t) \leq -\phi(x(t))\,,$$

which implies that

$$\dot{\kappa}(t) = \mu[\dot{\Psi}(x(t)) + \phi(x(t))] \leq 0\,.$$

This means that $\kappa(t)$ cannot exceed $\bar{\kappa} \triangleq \bar{\kappa}_{\eta,\zeta}$.

Finally, to prove that κ is lower bounded consider again (11) and eliminating all nonnegative terms we have

$$\kappa(t) \geq -\mu\Phi(x(0)) \geq -\zeta\mu.$$

Then, $\underline{\kappa} \triangleq -\zeta\mu$, hence concluding the proof. ∎

The next corollary shows that the previous result takes on a simpler form if we know a global bound on $\bar{\kappa}(x)$. In this case, we achieve an uniform upper bound for the difference $\Psi(x(t)) - \Psi(x(0))$.

Corollary 3.1: Assume that under Assumptions 1 and 2 we can find $\bar{\kappa}_\infty$ such that $\bar{\kappa}(x) \leq \bar{\kappa}_\infty$. Then, for any $\eta > 0$, the value $\bar{\mu} = \bar{\kappa}_\infty/\eta$ is such that, for all $\mu \geq \bar{\mu}$, it turns out that

$$\Psi(x(t)) \leq \Psi(x(0)) + \eta.$$

The next result concerns the convergence of the state modes of behavior.

Proposition 3.2: Under Assumptions 1 and 2, let $\eta > 0$, $\zeta > 0$ and $\mu \geq \bar{\mu}$ be as in Proposition 3.1. Then

$$\lim_{t\to\infty} x(t) = 0\,, \quad \forall\, x(0) \in \mathcal{N}[\Psi, \zeta]\,.$$

Proof: In view of Proposition 3.1, we can fix $\eta > 0$ and confine $x(t)$ inside $\mathcal{N}[\Psi, \zeta + \eta]$, by taking $\mu \geq \bar{\mu}$. Then $\Psi(x(t)) \leq \zeta + \eta$ and hence $x(t)$ is bounded. Function $\kappa(x)$ is bounded by $\bar{\kappa}_{\zeta,\eta}$ on such a set. This implies that $\dot{x}(t)$ is bounded because

$$\dot{x}(t) = F(x, \Delta_F) - \kappa G(\Delta_B)B^T\nabla\Psi(x)^T\,.$$

Furthermore, boundedness of $x(t)$ implies boundedness of $\phi(x(t))$ and of its gradient. Now, we have

$$\begin{aligned}&\mu\int_0^t \phi(x(\sigma))d\sigma \\ &\quad= \kappa(t) - \kappa(0) - \mu[\Psi(x(t)) - \Psi(x(0))] \\ &\quad\leq \kappa(t) + \mu\Psi(x(0)) \leq \bar{\kappa}_{\zeta,\eta} + \mu\zeta\end{aligned}$$

for all t. The composed function $\varphi(\cdot) \triangleq \phi(x(\cdot))$ is nonnegative and its integral is clearly bounded:

$$\int_0^t \varphi(\sigma)d\sigma < +\infty.$$

If we consider the time-derivative of such a function, we obtain $\dot{\varphi}(t) = \nabla\phi(x(t))\dot{x}(t)$ which is bounded for all t. In view of Barbălat's Lemma [2], we have that $\varphi(t) \to 0$ as $t \to \infty$. ∎

B. Nominal optimality

In this subsection, it will be shown that also the control mode of behavior, if generated by an optimal non–adaptive controller designed for the nominal process, remains nominally optimal, a property which is not ensured by the high–gain adaptive schemes in the current literature.

Let us consider a linear system of the form

$$\dot{x}(t) = [A + \Delta_F]x(t) + B[I + \Delta_B]u(t)$$

and consider the LQ controller for the nominal plant given by $u = -R^{-1}B^TPx$. Moreover, consider the function x^TPx where P is the solution of the Riccati equation

$$A^TP + PA - PBR^{-1}B^TP + Q = 0$$

with Q positive definite. Denoting by $A_{CL} = A - BR^{-1}B^TP$ the closed loop matrix, we get

$$A_{CL}^TP + PA_{CL} = -[PBR^{-1}B^TP + Q] \triangleq -Q_{LQ}.$$

Then, the following result can be easily proved.

Proposition 3.3: Assume that $x^T P x$ is a robust control Lyapunov function so that $-x^T Q_{LQ} x$ is the Lyapunov time-derivative for the nominal system under the action of the optimal controller. Then:

- the adaptive control scheme applied with $\phi(x) = x^T Q_{LQ} x$ and with $u(t) = -\kappa(t) R^{-1} B^T P x(t)$ ensures convergence of the state modes of behavior to $\mathcal{N}[\Psi, \epsilon]$ for μ large enough;
- for $\Delta_F = 0$ and $\Delta_B = 0$, this control is optimal provided that $\kappa(0) = 1$.

C. Effects of disturbances and practical stability results

We introduce next a modified version which ensures the gain limitation even in the presence of bounded disturbances.

Consider the system

$$\dot{x}(t) = F(x(t), \Delta_F(t)) + G(\Delta_B(t))u(t) + Ed(t)$$

where $\|d(t)\| \leq \bar{d}$ is a bounded disturbance and E is a given matrix.

For the sake of simplicity, we strengthen a little bit our assumptions, introducing the following

Assumption 3: For any $\bar{d}$ there exists $\delta > 0$ and a state–feedback controller $\Phi_0(x)$ such that

$$\begin{aligned} \dot{\Psi}(x) = \nabla\Psi(x)[F(x, \Delta) + G(\Delta_B)\Phi_0(x) + Ed] \\ \leq -\beta\Psi(x) \end{aligned} \quad (13)$$

for all $x \notin \mathcal{N}[\Psi, \delta]$ ($\Psi(x) > \delta$) and some $\beta > 0$.

Remark 3.1: Note that Assumption 3 does not implies exponential stabilizability of the nominal system, since inequality (13) is not required to hold in a neighborhood of the origin.

To face the new problem, we consider a clipped version of Ψ, namely

$$\Psi_\delta(x) = \max\{0, \Psi(x) - \delta\}\,. \quad (14)$$

Since Ψ_δ and Ψ differ by a constant outside $\mathcal{N}[\Psi, \delta]$, we have

$$\dot{\Psi}_\delta(x(t)) \leq -\beta\left(\Psi_\delta(x(t))\right)$$

hence $\Psi_\delta(x(t))$ is strictly decreasing outside $\mathcal{N}[\Psi, \delta]$. The new adaptation scheme is thus given by

$$\dot{\kappa}(t) = \mu\left[\dot{\Psi}_\delta(x(t)) + \beta\left(\Psi_\delta(x(t))\right)\right].$$

Note that no adaptation occurs as long as $x \in \mathcal{N}[\Psi, \delta]$. The following proposition (whose proof is omitted for brevity) holds.

Proposition 3.4: Under Assumptions 1 and 13, let $\zeta > \delta$ and $\eta > 0$ be arbitrary. Then, there exists $\hat{\mu}$ such that, for all $\mu > \hat{\mu}$, the control law with the adaptive mechanism (6)–(7) ensures that the gain $\kappa(t)$ is upper and lower bounded, $\Psi(x(t)) \leq \zeta + \eta$ and $\Psi_\delta(x(t)) \to 0$, as $t \to \infty$ (namely $x(t)$ converges to $\mathcal{N}[\Psi, \delta]$).

IV. Application to the wing rock problem

Wing rock is an instability phenomenon which can occur when high performance aircrafts fly in high angle of attack. It consists of a limit cycling oscillation of the pair $\phi = x_1$ = roll angle and $p = x_2$ = roll rate. According to [6] and neglecting the actuator dynamics, we considered the following model:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= \theta_1 x_1(t) + \theta_2 x_2(t) + \theta_3 |x_1(t)| x_2(t) + \\ &\quad + d(t) + u(t) \end{aligned}$$

where $\theta_1 = -26.67$, $\theta_2 = 0.76485$, $\theta_3 = -2.9225$.

The term $u(t)$ is the control action while the term $d(t)$ is a persistent disturbance (representing vertical wind gusts) having a proper spectral density [14]. An LQ optimal problem has been solved for the undisturbed, feedback–linearized system of type (9)-(10) (using the weights $Q = I_2$, $R = 1$) obtaining the Lyapunov function

$$\Psi(x) = x^T \begin{bmatrix} \sqrt{3} & 1 \\ 1 & \sqrt{3} \end{bmatrix} x = x^T P x.$$

Then we considered the problem of tracking a square–wave reference for ϕ, as shown in Fig.2 (the references are $\phi^+ = 0.1745$ and $\phi^- = -0.1745$) by means of the proposed controller and the λ–tracker (1)–(2). Actually, due to the disturbance, we used the clipped function $\Psi_\delta(x)$, with $\delta = 0.1$ (see (14)) and deadzone $\epsilon = 0.1$ for the λ–tracker, the adaption gains being $\mu = 4000$, $\mu_\lambda = 10$. The obtained roll angle trajectory, adapted gain and control action are reported, respectively, in Figs. 2,3 and 4 (solid line: proposed controller, dash-dot line: λ–tracker). As far as the tracking error only is considered, it is clear from Fig.2 that, as time increases, the proposed controller produces the same performance while the λ-tracker reduces both the steady-state error and the settling time that become smaller than those of the first transient (where the two controllers are comparable). This, however, is achieved at the expense of an increasing control effort. Indeed, in the case of the λ–tracker, the gain is monotonically increasing in time (Fig.3). As a consequence, the control actions become stronger and stronger (see Fig.4). To emphasize the much stronger control efforts shown by the λ-tracker controller, a part of the control mode of behavior around $t = 10s$ is shown in enlarged form. This happens

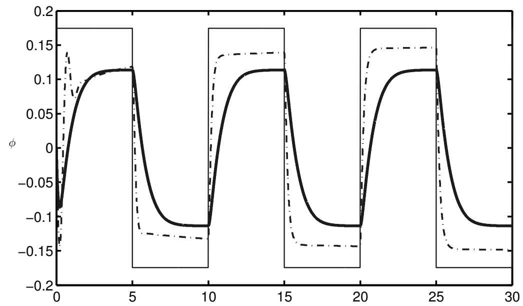

Fig. 2. Trajectory of the roll angle during tracking of a square–wave reference (solid line: proposed controller, dash-dotted line: $\lambda-$tracker).

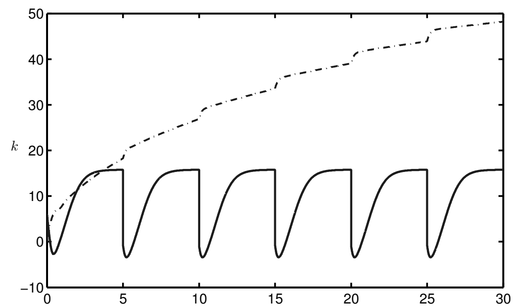

Fig. 3. Adapted gain during tracking for the proposed controller (solid line) and the λ-tracker (dash-dotted line).

because the adaption law is based on the distance between the current state and the reference. The proposed controller does not suffer this problem resulting in transient profiles that do not change over time and in far less actuator exploitation.

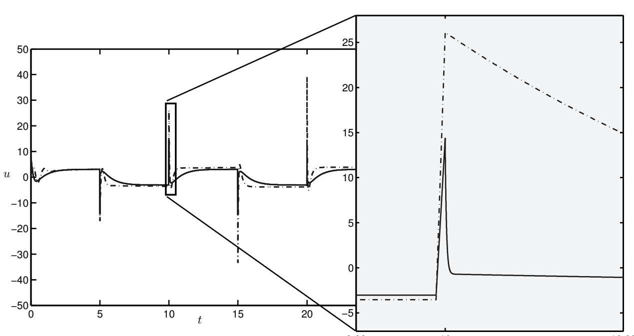

Fig. 4. Control action during tracking for the proposed controller (solid line) and the λ-tracker (dash-dotted line).

V. Conclusions

The proposed adaptive scheme overcomes some limitations of existing high–gain controllers such as λ–tracking and funnel control. The idea is that of adapting the gain based on the Lyapunov derivative. An a–priori upper bound for the transient can be determined and the scheme can be used jointly with standard optimal control techniques guaranteeing optimality if the model matches the system exactly. This in practice means that the control is nearly optimal under accurate modeling and when the system is close to 0. An open problem is the output feedback one. We think that under standard minimum–phase and relative degree assumptions the extension is possible.

References

[1] B. R. Barmish, M. Corless, and G. Leitmann. “A new class of stabilizing controllers for uncertain dynamical systems”, SIAM J. Control Optim., 21(2), March 1983.

[2] I. Barbălat, “Systèmes d’équations différentielles d’oscillations non linéaires”, Revue de Mathématiques Pures et Appliquées, Bucharest, IV, 267-270, 1959.

[3] F. Blanchini. “The gain scheduling and the robust state feedback stabilization problems”, IEEE Trans. Automat Control, 45(11):2061–2070, 2000.

[4] F. Blanchini and P. Giannattasio. Adaptive control of compressor surge instability. *Automatica J. IFAC*, 38(8):1373–1380, 2002.

[5] F. Blanchini, E.P. Ryan, “A Razumikhin–type proposition for functional differential equations with applications to adaptive control”, *Automatica*, Vol. 35, no. 5, pp. 809–818, May 1999.

[6] R. A. Freeman and P. V. Kokotović. “Robust nonlinear control design. State-space and Lyapunov techniques”, Systems and Control: Foundations and Applications. Birkhäuser Inc., Boston, MA, xii+256 pp. edition, 1996. ISBN: 0-8176-3930-6.

[7] A. Ilchmann, “Non-Identifier-Based High-Gain Adaptive Control”, Springer-Verlag, New York, 1993.

[8] A. Ilchmann and M. Pahl, ““Adaptive multivariable regulation of a biogas tower generator”, European Journal Control, Vol. 4, pp.116–131, 1998.

[9] A. Ilchmann, M Thuto and S. Townley, “Input constrained adaptive tracking with application to exotherim chemical reaction systems”, SIAM Journal on Contr. and Optim., Vol. 43, pp. 154–173, 2004.

[10] A. Ilchmann and E. P. Ryan, “Universal λ-tracking for nonlinearly-perturbed systems in the presence of noise”, Automatica, 30 , pp. 337-346, 1994.

[11] A. Ilchmann and E. P. Ryan, “High-gain control without identification: a survey”, GAMM-Mitt. 31, No. 1, 115–125, (2008)(Ges. Angew. Math. Mech. Mitteilungen der Gesellschaft fÃ$\frac{1}{4}$r Angewandte Mathematik und Mechanik. WILEY-VCH, Berlin)

[12] Ilchmann, A. Ryan, E. P. and Trenn, S. “Tracking control: performance funnels and prescribed transient behaviour”, Systems Control Lett. 54 (2005), no. 7, 655–670.

[13] G. Leitmann. “Guaranteed asymptotic stability for some linear systems with bounded uncertainties”, Transactions of the ASME, 101, 212–216, 1979.

[14] F.L. Lewis. “Optimal Control”, Wiley-Interscience; 2 edition, 1995.

[15] Qu, Z, “Robust control of nonlinear uncertain systems”, Wiley, New York, 1988.

[16] Rovithakis, G. A. “Stable adaptive neuro-control design via Lyapunov function derivative estimation”, Automatica J. IFAC 37 (2001), no. 8, 1213–1221.

[17] Karayiannidis, Y., Rovithakis, G. and Doulgeri Z. “Force/position tracking for a robotic manipulator in compliant contact with a surface using neuro-adaptive control”, Automatica J. IFAC 43 (2007), no. 7, 1281–1288.