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Security Constrained Emergency Voltage Stabilization: A Model Predictive Control Based Approach

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Abstract— **Voltage instability impacts power system transfer limits and its reliability. This paper presents an approach to determine a voltage control scheme based on the Model Predictive Control (MPC) theory. MPC is used to develop an optimal control strategy, consisting of a sequence of amounts of the shunt capacitors to switch. The resulting control strategy prevents voltage instability and maintains a desired amount of post-transient voltage stability margin (an index of system security). The objective of optimization is to minimize a weighted sum of the cumulative voltage deviations and the cumulative cost of capacitive controls. The effect of the capacitive control on voltage behavior is computed by means of trajectory sensitivity. The sensitivity of voltage stability margin with respect to the capacitive control is used to construct a security constraint for post-fault operation in the MPC formulation. The efficacy of the proposed approach is illustrated through applications to the 39-bus New England system for preventing voltage collapse.**

Index Terms— **Model predictive control, trajectory sensitivity, voltage stabilization, voltage stability margin, switching control, power system**

I. Introduction

Voltage instability takes the form of a dramatic drop in bus voltages in a transmission system, which may result in system collapse. Voltage control is accomplished by adjusting the production, absorption, and flow of reactive power at various locations in a power system. Reactive power compensation devices including shunt capacitors, shunt reactors, synchronous condensers, and static var compensators (SVCs) can be used to control voltage. Several prior works, such as [1], [2], [3], [4], [5], have studied the problem of determining locations and amounts of reactive power compensation devices to maintain voltage stability while minimizing the cost. The above works, however, are based on *static analysis*. They assume that a post-contingency stable equilibrium point can be reached. However, if disturbances are severe, the power system may not reach a post-contingency stable equilibrium point since the post-contingency trajectory may deviate out of the stability region. In this case, *dynamic analysis* is needed to ensure stable post-contingency trajectories. For example, in the 39-bus New England system considered in Section IV.B, the post-fault power system, according to a static analysis, has a voltage stability margin of 32.4%. However as can be seen from simulation (which considers the dynamic evolution), the system is unable to reach the associated post-fault equilibrium point. This illustrates the limitation of the control design based on a purely static analysis.

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In this paper, we design a control scheme to restore voltage following a contingency and to maintain a pre-specified amount of post-transient voltage stability margin. Voltage stability margin is an indication of how far the post-transient operating point is from the voltage collapse point. It is an index of system security. The derived control strategy not only considers the dynamic performance of voltages after a contingency, but also takes into account the degree of post-transient power system security. The computation of the control strategy is based on *model predictive control* (MPC). Shunt capacitors are adopted as reactive power compensation devices because they have been widely used to enhance voltage stability. The control design problem is to determine a capacitor switching sequence and amounts given their locations and capacities to satisfy the requirements of voltage performance and voltage stability margin. The objective of optimization is to minimize a weighted sum of the cumulative voltage deviations and the cumulative cost of capacitive controls. Trajectory sensitivities are used to estimate the effect of controls on voltage trajectories, whereas voltage stability margin sensitivities are applied to estimate the effect of controls on the voltage stability margin.

The paper is organized as follows. Some fundamental concepts about model predictive control (MPC), trajectory sensitivity as well as voltage stability margin sensitivities are introduced in Section II. The procedure to determine and implement the control strategy is proposed in section III. In Section IV a test case is provided to illustrate the efficacy of the proposed algorithm. Section V provides some discussions and the conclusions.

II. Background

A. Model predictive control

Model Predictive Control (MPC) is a class of algorithms that compute a sequence of control variable adjustments in order to optimize the future behavior of a plant (system). In [6], [7], an emergency voltage control scheme using tree search and model predictive control is presented. The problem is formulated as a nonlinear optimization problem with discrete control variables. Exhaustive tree search is used to solve this problem. MPC is also employed in [8], where

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an optimal coordinated voltage control for voltage stability is proposed and solved by a pseudo gradient evolutionary programming (PGEP) technique. In [9], [10], the authors present a method to compute an emergency voltage control strategy based on MPC. In these works the authors employ a simplified version of model predictive control, which only has one control step in the control horizon. A voltage stabilization control strategy is also proposed in [11] based on load shedding, where the objective function is to minimize the amount of load shedding required to restore the voltages. [12] presents a MPC based voltage control design. The controls are reference voltage of automatic voltage regulators and load shedding.

An introduction to the basic concepts and formulations of MPC can be found in [13]. The principle of MPC is graphically depicted in Fig. 1. Here x represents the state variable that needs to be controlled to a specific range. The available control is represented by variable u.

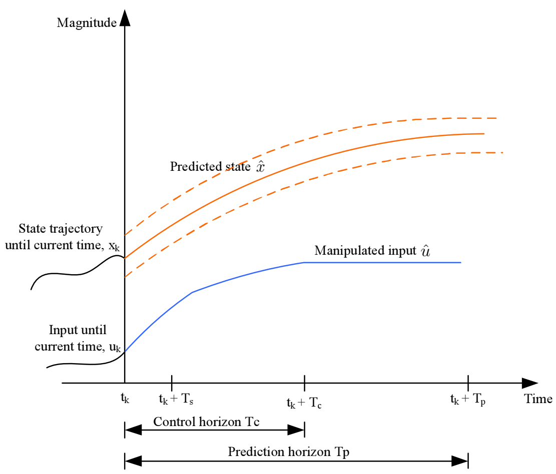

Fig. 1. Principle of MPC

At a current time t_k, the MPC solves an optimization problem over a finite prediction horizon $[t_k, t_k + T_p]$ with respect to a predetermined objective function such that the predicted state variable $\hat{x}(t_k+T_p)$ can optimally stay close to a reference trajectory. The control is computed over a control horizon $[t_k, t_k + T_c]$, which is smaller than the prediction horizon ($T_c \leq T_p$). If there were no disturbances, no model-plant mismatch and the prediction horizon is infinite, one could apply the control strategy found at current time t_k for all times $t \geq t_k$. However, due to the disturbances, model-plant mismatch and finite prediction horizon, the true system behavior is different from the predicted behavior. In order to incorporate the feedback information about the true system state, the computed optimal control is implemented only until the next measurement instant ($t_k + T_s$), at which point the entire computation is repeated.

In a MPC, the optimization problem to be solved at time t_k can be formulated as follows:

$$\min_{\hat{u}} \quad \int_{t_k}^{t_k+T_p} F(\hat{x}(\tau), \hat{u}(\tau))d\tau \qquad (1)$$

subject to

$$\dot{\hat{x}}(\tau) = f(\hat{x}(\tau), \hat{u}(\tau)), \quad \hat{x}(t_k) = x(t_k) \qquad (2)$$

$$u_{min} \leq \hat{u}(\tau) \leq u_{max}, \quad \forall \tau \in [t_k, t_k + T_c] \qquad (3)$$

$$\hat{u}(\tau) = \hat{u}(t_k + T_c), \quad \forall \tau \in [t_k + T_c, t_k + T_p] \qquad (4)$$

$$x_{min}(\tau) \leq \hat{x}(\tau) \leq x_{max}(\tau), \quad \forall \tau \in [t_k, t_k + T_p] \qquad (5)$$

Here, T_c and T_p are the control and prediction horizon with $T_c \leq T_p$. $\hat{x}$ denotes the estimated state and $\hat{u}$ represents "estimated" control (The true state may be different and the true control matches the estimated control only during the first sampling period).

Equation (1) represents the cost function of the MPC optimization. Equation (2) represents the dynamic system model with initial state $x(t_k)$. Equations (3) and (4) represent the constraints on the control input during the prediction horizon. Equation (5) indicates the state operation requirement during the prediction horizon.

B. Trajectory sensitivity

Consider a differential algebraic equation (DAE) of a system,

$$\dot{x} = f(x, y, u), \quad x(0) = x_0 \qquad (6)$$

$$0 = g(x, y, u) \qquad (7)$$

where x is a vector of state variables, y is a vector of algebraic variables, and u is a vector of control variables. Trajectory sensitivity considers the influence of small variations in the control u (and any other variable of interest) on the solution of the state equations (6) and (7). Let u_0 be a nominal value of u, and assume that the nominal system in (8) and (9) has a unique solution $x(t, x_0, u_0)$ over $[t_0, t_1]$.

$$\dot{x} = f(x, y, u_0), \quad x(0) = x_0 \qquad (8)$$

$$0 = g(x, y, u_0) \qquad (9)$$

Then the system in Equations (6) and (7) has a unique solution $x(t, x_0, u)$ over $[t_0, t_1]$ that is related to $x(t, x_0, u_0)$ as:

$$x(t, x_0, u) = x(t, x_0, u_0) + x_u(t)(u - u_0) + \text{high-order terms}$$

$$y(t, x_0, u) = y(t, x_0, u_0) + y_u(t)(u - u_0) + \text{high-order terms}$$

Here $x_u(t) = \frac{\partial x(t,x_0,u)}{\partial u}$ is called the trajectory sensitivities of state variables with respect to variable u and $y_u(t) = \frac{\partial y(t,x_0,u)}{\partial u}$ is the trajectory sensitivities of algebraic variables with respect to variable u.

The evolution of trajectory sensitivities can be obtained by differentiating Equations (6) and (7) with respect to the control variables u and is expressed as:

$$\dot{x}_u(t) = f_x(t)x_u(t) + f_y(t)y_u(t) + f_u(t)$$

$$0 = g_x(t)x_u(t) + g_y(t)y_u(t) + g_u(t)$$

The trajectory sensitivity can be solved numerically. An efficient methodology is presented in [14] for the computation of trajectory sensitivities for a system represented by DAE equations. When time domain simulation of a power system is based on trapezoidal numerical integration, the calculation of trajectory sensitivity requires solving a set of linear equations, thus costing a little time. In our work, we extended the Power System Analysis Tool [15] (a MATLAB based tool) to do trajectory sensitivity calculation and the MPC optimization.

C. Voltage stability margin sensitivity

Consider a system with DAE model

$$\dot{x} = f(x, y, u, \lambda)$$
$$0 = g(x, y, u, \lambda)$$

where x represents a vector of state variables, y represents a set of algebraic variables, u is a vector of control variables and λ is a parameter.

Let $r(\lambda) \in \Re^{N\times 1}$ be a vector of variables which are parameterized by λ and a change in which (due to a change in λ) affects the system stability. (For the power system application, this will consist of load and generation power.) The l^{th} component of $r(\lambda)$ is denoted as $r_l(\lambda)$ which increases linearly with λ as:

$$r_l(\lambda) = (1 + K_l\lambda)r_l(0)$$

Here, K_l is a constant and $r_l(0)$ represents the base case value of the l^{th} component of $r(\lambda)$.

If λ increases slowly and continuously, a bifurcation point is reached beyond which the system loses stability. Let λ^* be the value of λ at this point, then this implies that

$$0 = f(x, y, u, \lambda), 0 = g(x, y, u, \lambda)$$

has no solution when $\lambda > \lambda^*$. The stability margin is defined as

$$SM = \sum_{l=1}^{l=L}(r_l(\lambda^*) - r_l(0)) = \lambda^* \sum_{l=1}^{l=L} K_l r_l(0)$$

The rate change of stability margin with respect to the control variable u is known as the margin sensitivity with respect to u

$$SM_u = \frac{\partial SM}{\partial u} = \frac{\partial \lambda^*}{\partial u}\sum_{l=1}^{l=L} K_l r_l(0)$$

[16] presented a detailed derivation of the sensitivity calculation.

III. Problem formulation and implementation

A. Problem formulation

The purpose of this work is to determine an optimal capacitor switching sequence and amounts given their locations and capacities to satisfy the requirements of voltage performance and voltage stability margin. Detect whether a certain pre-identified contingency has occurred (Note the approach can also work for contingencies that are not necessarily pre-identified, as long as they can be detected in real-time.). If the system performance is not satisfactory, for instance, voltages are out of the their limits or voltage collapse happens, an optimal control strategy is identified based on a decreasing horizon MPC algorithm consisting of the amount and sequence of shunt capacitor switching. This control strategy not only stabilizes system voltages within acceptable ranges following the contingency, but also ensures a desired voltage stability margin. The control changes only at the sampling instants. Let T_p be the prediction horizon, T_c be the control horizon, T_s be the control sampling interval, and $N = \frac{T_c}{T_s}$ be the total number of control steps. The procedure to determine the control strategy at time t_k based on MPC is as follows:

(1) At time t_k (i.e. the $(k+1)^{th}$ sampling instant), an estimate of the current state $x(t_k)$ is obtained. The nominal power system evolves according to Equations (6) and (7). Here, $u = \{B_m^0 + \sum_{i=0}^{k-1}\Delta B_{m1}^i\}_{m=1}^{m=M}$ is the control variable (i.e. amounts of shunt capacitors currently in use). B_m^0 is the amounts of shunt capacitors that exist at time 0. $\sum_{i=0}^{k-1}\Delta B_{m1}^i$ is the amounts of shunt capacitors that were added over time $[0, t_k - T_s]$. Time domain simulation is used to obtain the trajectory of the nominal system (6) and (7), starting from the state $x(t_k)$ at time t_k to the end of prediction horizon $t_k + T_p$. At the same time, the trajectory sensitivity of bus voltages with respect to the shunt capacitors to be added at instants $t_k + (n-1)T_s, n = 1 \ldots N-k$ is obtained and denoted as $V_{B_{mn}}^{kj}(t)$ (see below for the explanation of notation).
Also the sensitivity of voltage stability margin with respect to shunt capacitor at location m is calculated based on a continuation power flow program. It is expressed as $SM_{B_m}^k$ in the optimization.

(2) At time t_k, solve the optimization problem over the prediction horizon $[t_k, t_k + T_p]$ and a control horizon $[t_k, t_k + (N-k)T_s]$ as stated in (10)-(15). The objective of optimization is to minimize a weighted sum of the cumulative voltage deviations and the cumulative cost of capacitive controls as shown in Equation (10). Equation (11) constraints the amount of control m to be added at time $t_k + (n-1)T_s$. Equation (12) constraints the total amount of control m to be added over $[t_k, t_k + (N-k)T_s]$. Equation (13) constraints the voltage fluctuation at time $t \in [t_k, t_k + T_p]$. Equation (15) constraints the voltage stability margin.

Minimize (with respect to ΔB_{mn}^k)

$$\int_{t_k}^{t_k+T_p} (\widehat{V}^k(t) - V_{ref})' R(\widehat{V}^k(t) - V_{ref})dt + \sum_{mn} W_{mn}\Delta B_{mn}^k \quad (10)$$

Subject to

$$\Delta B_m^{min} \le \Delta B_{mn}^k \le \Delta B_m^{max} \quad (11)$$

$$B_m^{min} \le B_m^0 + \sum_{i=0}^{k-1}\Delta B_{m1}^i + \sum_{n=1}^{N-k}\Delta B_{mn}^k \le B_m^{max} \quad (12)$$

$$V_{min}^{kj}(t) \le V^{kj}(t) + \sum_{m=1}^{M}\sum_{n=1}^{N-k} V_{B_{mn}}^{kj}(t)\Delta B_{mn}^{k} \le V_{max}^{kj}(t) \quad (13)$$

$$SM^{k-1} + \sum_{m=1}^{M} SM_{B_m}^{k}(\sum_{n=1}^{N-k}\Delta B_{mn}^{k}) \ge SM_D \quad (14)$$

$$\Delta B_{mn}^{k} \ge 0 \quad (15)$$

Here,

- R is the weighting matrix. $\widehat{V}^k(t)$ is the voltage vector at time $t \in [t_k, t_k + T_P]$ as predicted at the sampling instant t_k.
- W_{mn} is the weight for the cost of control m to be added at time $t_k + (n-1)T_s$.
- M is the total number of control variables, i.e. the number of shunt capacitor locations.
- N is the total number of control steps.
- ΔB_{mn}^k is the amount of control m to be added at time $t_k + (n-1)T_s$ in iteration k.
- $\Delta B_m^{min} \in \Re$ is the minimum amount of control m to be added at any step, typically 0.
- $\Delta B_m^{max} \in \Re$ is the maximum amount of control m to be added at any step.
- ΔB_{m1}^i is the amount of control m implemented at the control sampling point $t_i, i = 0, ...k-1$.
- $B_m^{min} \in \Re$ is the minimum amount of control m that must be used, typically 0.
- $B_m^{max} \in \Re$ is the maximum available amount of control m.
- $V^{kj}(t) \in \Re$ is the voltage of bus j at time $t(t_k \le t \le t_k + T_p)$ of the nominal system at time t_k.
- $V_{min}^{kj}(t)$ is the minimum voltage at bus j desired at time $t_k \le t \le t_k + T_p$.
- $V_{max}^{kj}(t)$ is the maximum voltage at bus j desired at time $t_k \le t \le t_k + T_p$.
- $V_{B_{mn}}^{kj}(t)$ is the trajectory sensitivity of voltage at bus j at time $t_k \le t \le t_k + T_p$ with respect to control m added at time $t_k + (n-1)T_s$.
- SM^{k-1} is the voltage stability margin at time $t_k - T_s$.
- $SM_{B_m}^k$ is the stability margin sensitivity with respect to capacitor m at time t_k.
- SM_D is the desirable stability margin for the system.

(3) At time t_k, a solution of the optimization problem (10)-(15) computes a sequence of controls ΔB_{mn}^k. Add only the first control ΔB_{m1}^k at time t_k and observe or estimate the system state $x(t_{k+1})$ at time $t_{k+1} = t_k + T_s$.

(4) Increase k to $k+1$ and repeat steps (1)-(3) until the $k = N-1$.

B. Implementation

The functional structure of implementing the MPC based coordinated voltage control is shown in Figure 2. Line flow, bus voltage information, switch status as well as phase measurement unit (PMU) measurements are sent to a control center through communication channels of a SCADA system. These measurements plus a network model are used by the state estimator (SE) for filtering out the noise and making best use of the measured data. The results from the state estimator are used for power flow analysis. A power flow solution is then used by an on-line dynamic security assessment program to initialize the state variables of the dynamic models. Further, it uses system models and disturbance information to perform the contingency analysis to evaluate the security margins of the power system. If a contingency is identified where the system will becomes unstable, the MPC based voltage control computation will get triggered at the time an identified critical contingency occurs. The steps of the MPC computation in the k^{th} iteration include:

- Estimate static variables such as voltage magnitudes and angles at time t_k as well as the dynamic variables $x(t_k)$ such as generator angles, velocities and real and reactive load recovery.
- Run time-domain simulation to compute the system trajectory given the current state. This step also requires the knowledge of a complete system model (including both dynamic and static components).
- Obtain trajectory sensitivities of voltage with respect to the control variables and the sensitivities of voltage stability margin with respect to the control variables.
- Solve the quadratic programming optimization problem and implement the first step of the control.
- Repeat the above steps at each sampling point until the end of control horizon.

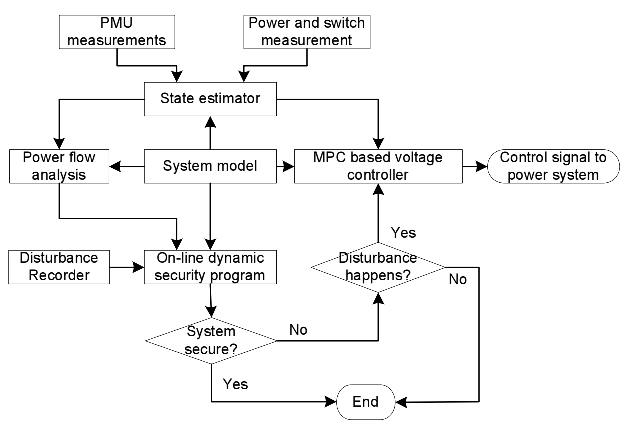

Fig. 2. Structure of implementing a MPC based Voltage stabilization

Remark: While we have suggested an on-line computation of MPC based voltage stabilization control, it is also possible to do the computation off-line based on the predicted (rather estimated) values of the states and trajectory sensitivities.

IV. Application to New England Systems

The proposed voltage stabilization control method is illustrated by applying to the New England 39-bus system. The desired voltage stability margin is chosen to be 35%. The exponential recovery load model is used in both cases. The parameters of the load model are chosen as following:

$$T_P = T_Q = 30, \alpha_s = 0, \alpha_t = 1, \beta_s = 0, \beta_t = 4.5.$$

The parameters in MPC optimization are determined based on the following considerations. Any voltage instability following a contingency must be stabilized in a certain time duration (typically the time in which voltage will decrease by 15%). This is the prediction horizon T_p. The control should be exercised on a time horizon T_c, which is shorter than the prediction horizon, typically the time in which voltage will decrease by 10% (if no control is applied). A discrete-time control must be applied within this duration T_c at a sample-rate high enough to adequately react to the changing voltage trajectory, as well as to allow accurate enough predictions of the voltage trajectory based on the linearization of the trajectory-sensitivity. This dictates the sampling duration T_s. The number of sampling point N is then determined as the ratio of T_c and the sampling duration T_s.

A. New England 10-generator 39-bus test system

1) System description: Fig. 3 shows the New England 10-generator 39-bus system. A fourth-order generator model is used with the exception that a third-order model is used for the generator at bus 39. In addition, all generators excluding those at buses 34, 37 have automatic voltage regulators (AVRs), which are represented by fourth-order models. The loads are represented by the exponential recovery dynamic models. The control variables are the shunt capacitors at buses 16, 17, 19, 21 and 24. Under normal conditions, none of the shunt capacitors are in use.

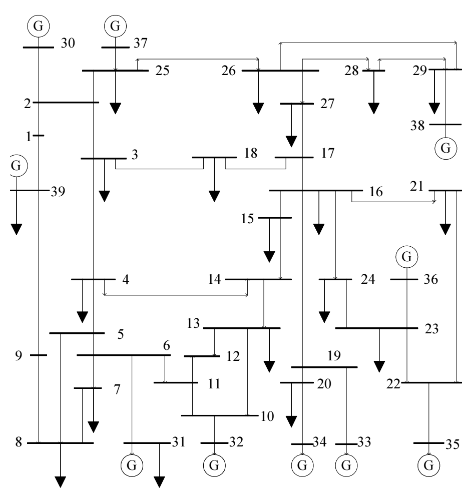

Fig. 3. New England 10-generator 35-bus test system

2) Fault scenario: The contingency considered here is a three-phase-to-ground fault at bus 21 at $t = 1.0$ second, which is cleared at $t = 1.02$ second by tripping of the transmission line between buses 21 and 22. Bus voltages drop dramatically when the fault occurs as shown in Fig. 4. After the fault is cleared at 1.02 second, the voltages recover around 0.95 p.u, although some oscillations follow. About 20 seconds later, the oscillations are damped out, but the voltages start to decline slowly because of the exponential recovery of the loads. Around 2 minutes later, the voltages collapse. According to a continuation power-flow based analysis, the post-fault power system has a voltage stability margin of 32.4%. However as can be seen from simulation (which considers the dynamic evolution), the system is unable to reach the associated post-fault equilibrium point. This illustrates the limitation of the control design based on a purely static analysis. Through our MPC based approach (which incorporates the dynamic analysis) we are able to ensure that the post-fault system has a desired voltage stability margin of 35%, and the system is able to reach the associated post-fault equilibrium point.

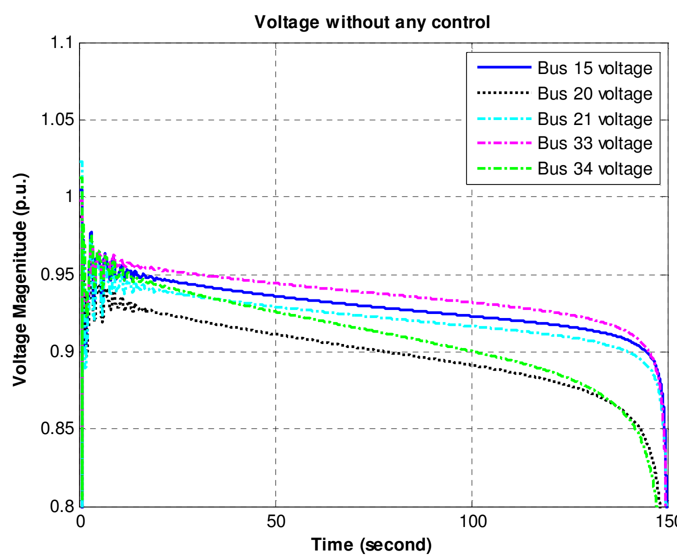

Fig. 4. New England system voltage behavior without MPC control

3) Simulation result: In this example, we have chosen prediction horizon T_p to be 130 seconds (the time in which voltage drops by nearly 15% at bus 20). T_c has been chosen to be 120 seconds. We found that a sample duration of T_s = 20 seconds works well for this example, and so we have the number of control steps: $N = \frac{T_c}{T_s} = \frac{120}{20} = 6$. An optimal control strategy that stabilizes voltage and ensures the security of post-transient power system is found based on the algorithm introduced in Section III. The final control strategy is indicated in Table I. Fig. 5 shows the voltage response with the security constrained control strategy. The post-fault power system has a voltage stability margin of 35.0%, which is the required value.

Time(second)	1.2	21.2	41.2	61.2	81.2	101.2
Capacitor at bus 16 (p.u.)	0	0.2	0.2	0	0.025	0.1
Capacitor at bus 17 (p.u.)	0.2	0	0	0	0.025	0
Capacitor at bus 19 (p.u.)	0	0.1919	0.0556	0	0.2	0
Capacitor at bus 21 (p.u.)	0.2	0.0200	0	0.2	0.025	0
Capacitor at bus 24(p.u.)	0.0333	0.2	0.15	0.1	0.025	0.2

TABLE I

CONTROL STRATEGY FOR NEW ENGLAND SYSTEM

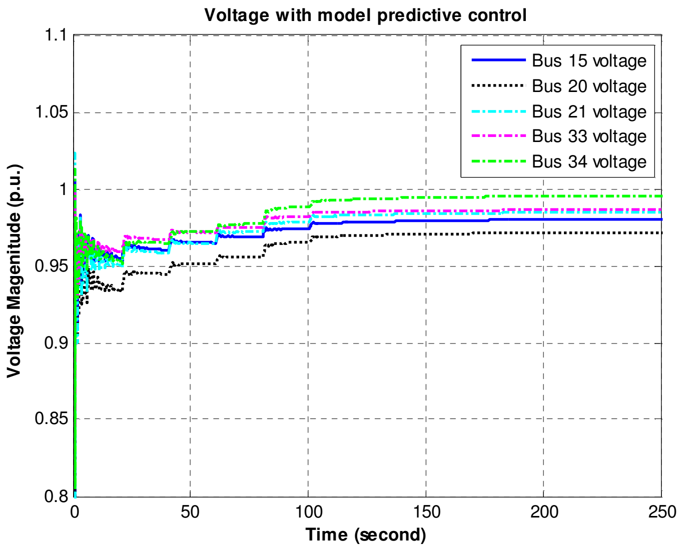

Fig. 5. New England voltage behavior with security constrained MPC control

V. CONCLUSION

This paper proposes a model predictive control design scheme to restore voltage following a contingency and to maintain a pre-specified amount of post-transient voltage stability margin. The distinguishing contribution of our MPC formulation is as follows:

- The proposed MPC approach involves a *dynamic* analysis for the computation of a desired control. This is important since as the 39-bus New England system example considered in section IV.B illustrates, a design based on a purely *static* (power flow) analysis is inadequate.
- The control strategy not only prevents voltage instability, but also maintains a desired amount of post-transient voltage stability margin. Voltage stability margin sensitivities are used to characterize the effect of control variables on stability margin enhancement. Prior works involving dynamic analysis for voltage stabilization did not include voltage stability margin as part of the control objective.
- Use of trajectory sensitivities for determining the effect of control on voltage, which is a more accurate way of determining the effectiveness of control (as opposed to the less accurate linerarization around an operating point or more time-consuming computations based on time-domain simulations).
- A decreasing horizon MPC is used. The control horizon decreases from one iteration to the next. This modification not only reduces the computation time, but also helps the convergence of the optimization process. This feature of MPC has not been explored in prior works on stabilization of power systems.
- Optimization performed at each step involves a quadratic cost function together with linear constraints, which makes the formulation scalable to large-sized practical systems. This is evident by the application to the 39-bus New England system.
- The iterative optimization process helps ensure that errors introduced by sensitivities based computation and model inaccuracies are minimized.

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