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Adaptive Fuzzy Fault-Tolerant Control for Unknown Nonlinear Systems with Disturbances

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Abstract— **A class of unknown nonlinear systems subject to uncertain actuator faults and external disturbances will be studied in this paper with the help of fuzzy approximation theory. Using backstepping technique, a novel adaptive fuzzy control approach is proposed to accommodate the uncertain actuator faults during operation and deal with the external disturbances. The considered faults are modeled as both loss of effectiveness and lock-in-place (stuck at some unknown place). It is proved that the proposed control scheme can guarantee all signals of the closed-loop system semi-globally uniformly ultimately bounded and the tracking error between system output and reference signal converge to a small neighborhood of zero, though the nonlinear functions of the controlled system as well as the actuator faults and the external disturbances are all unknown. Simulation results demonstrate the effectiveness of the control approach.**

I. Introduction

Since adaptive fuzzy systems are proved to be universal approximators [1], and stable adaptive fuzzy control was proposed in [2], many researchers have been interested in studying nonlinear systems with unknown system functions. Adaptive fuzzy control in the existing literature is designed by using fuzzy logic systems to approximate the unknown nonlinear functions adaptively, the corresponding adaptive laws for updating the parameters of the fuzzy systems online can be developed on the basis of Lyapunov stability theory [3]-[13]. In the earlier years, several stable adaptive fuzzy control schemes were introduced for single-input-single-output (SISO) systems [2]-[6], then, [7] extends the corresponding results to multiple-input-multiple-output (MIMO) systems. However, the early works have the restriction that the controlled system is feedback linearizable. Later, [8]-[11] combined adaptive fuzzy control with backstepping technique to control SISO nonlinear systems without the restriction of feedback linearizable. And [12]-[13] developed control method for unknown MIMO nonlinear systems which cannot be feedback linearized with the help of backstepping adaptive fuzzy approach. Although much progress has been achieved in this field, there are still few results on adaptive fuzzy control of nonlinear systems with actuator faults except [14] studied feedback linearizable nonlinear system with additive faults.

Recently, adaptive control has been widely used to deal with actuator faults in various systems. In [15], actuator lock-in-place (stuck at some unknown place) problem is accommodated by adaptive redundant control structure for linear and nonlinear SISO systems. Then [16] extended the results to MIMO parametric-strict-feedback nonlinear systems. Loss of effectiveness of actuator is considered in [17]-[18] for linear systems in the framework of linear matrix inequality (LMI) to guarantee not only the stability, but also the robust performance of the failed system. The common advantage of these adaptive control approach against actuator fault is that they are independent of fault detection and diagnosis (FDD). With the development of intelligent control, some researcher employ neural networks to design control scheme for accommodating some kinds of system faults. [19] presented a general framework for constructing fault diagnosis and accommodation architectures using online approximators and adaptive schemes, then [20] designed an adaptive neural network fault tolerant flight control in the framework proposed in [19]. Also, [21]-[23] gave several adaptive neural network control approaches to construct fault tolerant control. However, the existing results based on neural network need FDD subsystems for fault information. Compared with neural networks, fuzzy logic systems can achieve faster convergence because they are capable of accommodating both numerical data and expert knowledge. So adaptive fuzzy fault tolerant control will be studied in this paper.

A novel robust adaptive fuzzy control scheme for a class of nonlinear systems against both lock-in-place and loss of effectiveness faults of actuators without resorting to FDD mechanism is presented in this paper. The controlled system cannot be feedback linearizable and the nonlinear system functions are unknown, besides there are external disturbances enter into system. With a special control structure inspired by [15], the fault-tolerant controller can be designed in a systematic backstepping procedure, where fuzzy logic systems are employed to approximate the unknown part of the ideal virtue or real control in each step. Additional control effort is designed to deal with the influences of the external

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disturbances by the help of a hyperbolic tangent function. With the proposed control scheme, the following control objects can be achieved though there actuator faults and large uncertainties in system: 1) all signals of the closed-loop system are guaranteed semi-globally uniformly ultimately bounded; 2) the control performances can be shaped as desired by properly choosing the design parameters. Besides, this approach avoided perfectly the controller singularity problem which may happen in some existing adaptive fuzzy control. The corresponding nonlinear systems without external disturbances are considered in another paper of ours.

This paper is organized as follows. The problem formulation and preliminaries are presented in Section II. Then, a systematic backstepping procedure for synthesis of the adaptive fuzzy fault tolerant controller along with some analysis is given in Section III. In Section IV, a simulation example demonstrates the effectiveness of the proposed scheme. Finally, Section V concludes the paper.

II. Problem formulation and preliminaries

Consider the following nonlinear system with m inputs:

$$\begin{array}{ll} \dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + d_i(t) & 1 \le i \le n-1 \\ \dot{x}_n = f_n(\bar{x}_n) + \bar{g}_n^T(\bar{x}_n)u + d_n(t) & n \ge 2 \\ y = x_1 & \end{array} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \cdots, x_i]^T \in R_i$, $i = 1, \cdots, n$, $u = [u_1, u_2, \cdots, u_m]^T \in R_m$ is the input vector whose component may fail during the system operation, $y \in R$ is the system output, $\bar{g}_n(\bar{x}_n) = [g_{n1}(\bar{x}_n), g_{n2}, (\bar{x}_n), \cdots, g_{nm}(\bar{x}_n)]^T \in R^m$, f_i, g_i, f_n and g_{nj} for $i = 1, \cdots, n-1$, $j = 1, \cdots, m$ are unknown continuous nonlinear functions, besides, g_i, g_{nj} are smooth. d_i $i = 1, 2, \cdots, n$ are bounded external disturbances. The state variables x_i are measurable and the reference output y_m is sufficiently derivable. This is a multi-input-single-output (MISO) system with all the inputs contributed to a common control object like stabilizing the closed-loop system, tracking a reference signal of the system output or both. There are many such systems in our real life. The provided approach is also effective for multi-input multi-output systems. We only consider a simple case to simplify the presentation. The actuator faults considered in this paper are lock-in-place and loss of effectiveness which are modeled as follows respectively.

Lock-in-place model:

$$u_j(t) = \bar{u}_j \quad t \ge t_j, \quad j \in \{j_1, j_2 \cdots, j_p\} \subset \{1, 2, \cdots, m\} \tag{2}$$

Loss of effectiveness model:

$$u_i(t) = \rho_i u_i(t) \quad t \ge t_i \quad i \in \overline{\{j_1, j_2, \cdots, j_p\}} \cap \{1, 2, \cdots, m\}$$

$$\rho_i \in [\underline{\rho}_i, 1], \quad 0 < \underline{\rho}_i \le 1 \tag{3}$$

where $\bar{u}_j$ is the constant value where the actuator stuck at, t_j and t_i are the time when some faults take place. ρ_i is the still active proportion of the actuator after loss of effectiveness, $\underline{\rho}_i$ is the lower bound of ρ_i. When $\underline{\rho}_i$ is 1, the respective actuator is normal (that is completely active). In fact, lock-in-place fault of actuator brings some disturbance into system if $u_j \neq 0$ besides complete loss of effectiveness. So, taking the actuator faults (2) and (3)into account, the input vector u(t) can be written as

$$u(t) = \rho v(t) + \sigma(\bar{u} - \rho v(t)) \tag{4}$$

where $v(t) = [v_1(t), v_2(t), \cdots, v_m(t)]^T$ is the applied control inputs vector, $\bar{u} = [\bar{u}_1, \bar{u}_2, \cdots, \bar{u}_m]^T$ with $\bar{u}_j$, $j = 1, 2, \cdots, m$, being a constant value, and

$$\begin{array}{l} \rho = diag\{\rho_1, \rho_2, \cdots, \rho_m\} \\ \sigma = diag\{\sigma_1, \sigma_2, \cdots, \sigma_m\} \\ \sigma_i = \begin{cases} 1 & if\ the\ ith\ actuator\ fails\ as\ (2)\ i.e.,\ u_i = \bar{u}_i \\ 0 & otherwise \end{cases} \end{array} \tag{5}$$

The control objective of this paper is to design a state feedback adaptive control law for the system (1) with the actuator faults (2) and (3) to ensure that all signals in the closed-loop system are bounded and the output $y(t)$ can track the given reference signal $y_m(t)$ as closely as possible, though there are large uncertainties in the controlled system concerned about the unknown nonlinear system functions, the pattern of actuator faults, the values of the faults and the instants at which the faults take place. From the faults model (2) and (3), it is reasonable that there is at least one actuator still active for the control purpose, however, it can lose some effectiveness only if $\rho_i \ge \underline{\rho}_i$. Of course there can be more actuators active partly or completely. For the accomplishment of control task, the proposed control design needs the following assumptions.

Assumption 1: The system (1) is so constructed that for any up to $m-1$ actuators fail as (2) and the remaining active proportion of the others meet $\rho \in [\underline{\rho}_i, 1]$, the resulted system can still be controllable.

Assumption 2: There exist some constants $g_{i0} > 0$ such that $|g_i(\cdot)| \ge g_{i0}, \forall \bar{x}_i \in \Omega_i \subset R^i$, $i = 1, 2, \cdots, n-1$ and $g_{nj0} > 0$ such that $|g_{nj}(\cdot)| \ge g_{nj0}$, $\forall \bar{x}_n \in \Omega_n \subset R^n$, $j = 1, 2, \cdots, m$.

We can see from Assumption 2 that the smooth functions $g_i(\cdot)$ are strictly either positive or negative. Without loss of generality, it can be assumed that $g_i(\cdot) \ge g_{i0}$, $\forall \bar{x}_i \in \Omega_i \subset R^i$, $i = 1, 2, \cdots, n-1$ and $g_{nj}(\cdot) \ge g_{nj0}$, $\forall \bar{x}_n \in \Omega_n \subset R^n$, $j = 1, 2, \cdots, m$. Ω_i is a sufficient large compact set in R_i where $\bar{x}_i$ is included.

Assumption 3: There exist constants $g_{id} > 0$ such that $|\dot{g}_i(\cdot)| \le g_{id}$, $\forall \bar{x}_i \in \Omega_i \subset R^i$, $i = 1, 2, \cdots, n-1$ and $g_{njd} > 0$ such that $|\dot{g}_{nj}(\cdot)| \le g_{njd}$, $\forall \bar{x}_n \in \Omega_n \subset R^n$, $j = 1, 2, \cdots, m$.

Assumption 4: The external disturbances are bounded, that is, $|d_i(t)| \le \bar{d}_i$ where $\bar{d}_i$ $i = 1, 2, \cdots, n$ are known constants.

The designed control scheme employs fuzzy logic systems to approximate the unknown part of the virtue controller in each step for it had been proved that fuzzy logic systems are universal approximators [1]. We can design adaptive fuzzy control scheme with the help of the following lemma.

Lemma 1: For any given real continuous function $F(x)$, on a compact set $\Omega \subseteq R^n$, there exits a fuzzy logic system

$Y(x) = \theta^T \xi(x)$ such that $\forall \varepsilon > 0$,

$$\left|F(x) - \theta^T \xi(x)\right| < \varepsilon \tag{6}$$

where $\theta = (\theta_1, \theta_2, \cdots, \theta_M)^T$ is the estimate parameter vector, and $\xi(x) = (\xi_1(x), \xi_2(x), \cdots, \xi_M(x))^T$ is the vector of fuzzy basis functions, M is the number of fuzzy rules.

The optimal parameter vector θ^* is defined as:

$$\theta^* = \arg\min_{\theta\in R^n}\{\sup_{x\in\Omega}\left|F(x) - \theta^T \xi(x)\right|\} \tag{7}$$

which makes the fuzzy logic system approximate the unknown function closest. The fuzzy membership functions are usually chosen as the Guassian functions, which are located on a regular grid that contains the subset of interest of the state space.

III. Controller Synthesis procedure and stability analysis

Because in this single output system, all the inputs are contributed to stabilize the closed-loop system and make the output track a reference signal, we can choose a special control structure as

$$v_j = b_j(\bar{x}_n)u_0 \tag{8}$$

where $0 < \underline{b}_j \le b_j(\bar{x}_n) \le \bar{b}_j$, $\forall \bar{x}_n \in \Omega_n \subset R^n$, $j = 1, 2, \cdots, m$, $\underline{b}_j$ and $\bar{b}_j$ are the lower and upper bounds of $b_j(\bar{x}_n)$ respectively. u_0 is the designed adaptive fuzzy controller by the following backstepping procedure to accommodate uncertain actuator faults for nonlinear system (1).

Step 1: Let $x_{1d} = y_m$, $e_1 = x_1 - x_{1d}$, we have

$$\begin{aligned}\dot{e}_1 &= f_1(x_1) + g_1(x_1)x_2 - \dot{x}_{1d} + d_1(t)\\ &= g_1(x_1)[g_1^{-1}(x_1)f_1(x_1) + x_2 - g_1^{-1}(x_1)\dot{x}_{1d}] + d_1(t)\end{aligned} \tag{9}$$

If the external disturbance $d_1(t) = 0$, virtual controller can be assumed to be x_{2d}^* as follows

$$x_{2d}^* = -g_1^{-1}(x_1)f_1(x_1) + g_1^{-1}(x_1)\dot{x}_{1d} - k_1e_1 \tag{10}$$

$k_1 > 0$ is a constant. Take (10) into (9), we can get $\dot{e}_1 = -g_1(x_1)k_1e_1$, and there is a Lyapunov function $V_1 = \frac{1}{2}e_1^2$ that from (9) $\dot{V}_1 = -g_1(x_1)k_1e_1^2 \le -g_{10}k_1e_1^2 \le 0$. So we can see that e_1 is asymptotically stable. Unfortunately, the nonlinear functions $f_1(x_1)$ and $g_1(x_1)$ are all unknown, and $d_1(t) \ne 0$, we can not get the virtual controller. So fuzzy logic system is used to approximate the unknown nonlinear part of (10), from Lemma 1, we have

$$-g_1^{-1}(x_1)f_1(x_1) + g_1^{-1}(x_1) + g_1^{-1}(x_1)\dot{x}_{1d} = \theta_1^{*T}\xi_1(x_1) + \omega_1$$

with $|\omega_1| \le \varepsilon_1$. And there also needs additional control to deal with $d_1(t)$, then the applied virtual control is designed as

$$x_{2d} = \alpha_1(x_1 \mid \theta_1) = \theta_1^T\xi_1(x_1) - k_1e_1 - \delta_1 tanh\frac{e_1\delta_1}{\eta_1} \tag{11}$$

where $tanh$ the hyperbolic tangent function, $\delta_1 = \varepsilon_1 + \frac{\bar{d}_1}{g_{10}}$. Let $e_2 = x_2 - x_{2d}$, and rewritten (9)

$$\begin{aligned}\dot{e}_1 &= f_1(x_1) + g_1x_1e_2 + g_1(x_1)(x_{2d} - x_{2d}^*) + g_1(x_1)x_{2d}^* - \dot{x}_{1d}\\ &= g_1(x_1)(e_2 + \tilde{\theta}_1^T\xi_1(x_1) - \omega_1 - k_1e_1 - \delta_1 tanh\tfrac{e_1\delta_1}{\eta_1}) + d_1\end{aligned} \tag{12}$$

where θ_1 is the estimate of θ_1^* and $\tilde{\theta}_1 = \theta_1 - \theta_1^*$. Design the parameter updating law as

$$\dot{\theta}_1 = -\gamma_1e_1\xi_1(x_1) - r_1\theta_1 \tag{13}$$

Then consider Lyapunov function candidate

$$V_1 = \frac{1}{2g_1(x_1)}e_1^2 + \frac{1}{2\gamma_1}\tilde{\theta}_1^T\tilde{\theta}_1 \tag{14}$$

The derivative of V_1 is

$$\begin{aligned}\dot{V}_1 &= \tfrac{1}{g_1(x_1)}e_1\dot{e}_1 - \tfrac{\dot{g}_1(x_1)}{2g_1^2(x_1)}e_1^2 + \tfrac{1}{\gamma_1}\tilde{\theta}_1^T\dot{\theta}_1\\ &= e_1e_2 - k_1e_1^2 - \tfrac{\dot{g}_1(x_1)}{2g_1^2(x_1)}e_1^2 - e_1\omega_1 + e_1\tfrac{d_1}{g_1(x_1)}\\ &\quad - e_1\delta_1 tanh\tfrac{e_1\delta_1}{\eta_1} + \tilde{\theta}_1^T(\xi_1(x_1)e_1 + \tfrac{1}{\gamma_1}\dot{\theta}_1)\\ &\le e_1e_2 - (k_1 + \tfrac{\dot{g}_1(x_1)}{2g_1^2(x_1)})e_1^2 + |e_1\delta_1| - e_1\delta_1 tanh\tfrac{e_1\delta_1}{\eta_1}\\ &\quad - \tfrac{r_1}{\gamma_1}\tilde{\theta}_1^T\theta_1\end{aligned} \tag{15}$$

where $\dot{\tilde{\theta}}_1 = \dot{\theta}_1$ is considered. $\gamma_1 > 0$, $r_1 > 0$ are design constants. From the fact that $0 \le |q| - qtanh(\frac{q}{\varepsilon}) \le \kappa\varepsilon$ with $\kappa = e^{-(\kappa+1)}$ (i.e.$\kappa \approx 0.2785$) for any $\varepsilon > 0$ and any $q \in R$, and choosing k_1 such that $k_1^* = k_1 - \frac{g_{1d}}{2g_{10}^2} \ge \frac{c}{2g_{10}}$ for a given positive constant c, (15) can be expressed as

$$\dot{V}_1 \le e_1e_2 - k_1^*e_1^2 - \tfrac{r_1}{2\gamma_1}\tilde{\theta}_1^T\tilde{\theta}_1 + \tfrac{r_1}{2\gamma_1}\theta_1^{*T}\theta_1^* + \kappa\eta_1 \tag{16}$$

where $-\frac{r_1}{\gamma_1}\tilde{\theta}_1^T\theta_1 = -\frac{r_1}{\gamma_1}\tilde{\theta}_1^T(\tilde{\theta}_1 + \theta_1^*) \le -\frac{r_1}{2\gamma_1}\tilde{\theta}_1^T\tilde{\theta}_1 + \frac{r_1}{2\gamma_1}\theta_1^{*T}\theta_1^*$ is taken into account.

Step 2: This step is to make the error e_2 as small as possible.

$$\begin{aligned}\dot{e}_2 &= \dot{x}_2 - \dot{x}_{2d}\\ &= f_2(\bar{x}_2) + g_2(\bar{x}_2)x_3 - \dot{x}_{2d} + d_2(t)\end{aligned} \tag{17}$$

Similarly, if $d_2(t) = 0$, the virtual controller in this step is assumed to be

$$x_{3d}^* = -g_2^{-1}(\bar{x}_2)f_2(\bar{x}_2) + g_2^{-1}(\bar{x}_2)\dot{x}_{2d} - e_1 - k_2e_2 \tag{18}$$

The applied virtual control is

$$x_{3d} = \alpha_2(\bar{x}_2|\theta_2) = \theta_2^T\xi_2(\bar{x}_2) - e_1 - k_2e_2 - \delta_2 tanh\frac{e_2\delta_2}{\eta_2} \tag{19}$$

where $k_2 > 0$, $\delta_2 = \varepsilon_2 + \frac{\bar{d}_2}{g_{20}}$ with ε_2 is the upper bound of the fuzzy logic approximation error for $-g_2^{-1}(\bar{x}_2)f_2(\bar{x}_2) + g_2^{-1}(\bar{x}_2)\dot{x}_{2d}$. Define $e_3 = x_3 - x_{3d}$, (17) can be rewritten as

$$\begin{aligned}\dot{e}_2 = g_2(x_2)(&e_3 + \tilde{\theta}_2^T\xi_2(\bar{x}_2) - \omega_2 - k_2e_2\\ &- e_1 - \delta_2 tanh\tfrac{e_2\delta_2}{\eta_2}) + d_2\end{aligned} \tag{20}$$

Let

$$\dot{\theta}_2 = -\gamma_2e_2\xi_2(\bar{x}_2) - r_2\theta_2 \tag{21}$$

Consider the Lyapunov function candidate as

$$V_2 = V_1 + \frac{1}{2g_2(\bar{x}_2)}e_2^2 + \frac{1}{2\gamma_2}\tilde{\theta}_2^T\tilde{\theta}_2 \tag{22}$$

The derivative of (22) has the form of

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + \tfrac{1}{g_2(\bar{x}_2)} e_2\dot{e}_2 - \tfrac{\dot{g}_2(\bar{x}_2)}{2g_2^2(\bar{x}_2)} e_2^2 + \tfrac{1}{\gamma_2}\tilde{\theta}_2^T\dot{\theta}_2 \\ &= \dot{V}_1 - e_1e_2 + e_2e_3 - k_2e_2^2 - \tfrac{\dot{g}_2(\bar{x}_2)}{2g_2^2(\bar{x}_2)} e_2^2 - e_2\omega_2 + e_2\tfrac{d_2}{g_2(\bar{x}_2)} \\ &\quad - e_2\delta_2 tanh\tfrac{e_2\delta_2}{\eta_2} + \tilde{\theta}_2^T(\xi_2(\bar{x}_2)e_2 + \tfrac{1}{\gamma_2}\dot{\theta}_2)\end{aligned} \tag{23}$$

where (16) and (21) are used, and $\gamma_2 > 0$, $r_2 > 0$. Choose k_2 so that $k_2^* = k_2 - \frac{g_{2d}}{2g_{20}^2} \geq \frac{c}{2g_{20}}$ for c, the following inequality can be obtained

$$\dot{V}_2 \leq e_2e_3 - \sum_{i=1}^{2}(k_i^*e_i^2 + \frac{r_i}{2\gamma_i}\tilde{\theta}_i^T\tilde{\theta}_i) + \sum_{i=1}^{2}(\frac{r_i}{2\gamma_i}\theta_i^{*T}\theta_i^* + \kappa\eta_i) \tag{24}$$

Step i: ith $(2 \leq i \leq n-1)$ step is to make $e_i = x_i - x_{id}$ as small as possible with $x_{(i+1)d}$. Conduct the similar procedure, we can get the following expressions

$$x_{(i+1)d}^* = -g_i^{-1}(\bar{x}_i)f_i(\bar{x}_i) + g_i^{-1}(\bar{x}_i)\dot{x}_{id} - e_{i-1} - k_ie_i \tag{25}$$

$$x_{(i+1)d} = \alpha_i(\bar{x}_i|\theta_i) = \theta_i^T\xi_i(\bar{x}_i) - e_{i-1} - k_ie_i - \delta_i tanh\frac{e_i\delta_i}{\eta_i} \tag{26}$$

where $k_i > 0$, $\delta_i = \varepsilon_i + \frac{\bar{d}_i}{g_{i0}}$ with ε_i is the upper bound of the fuzzy logic approximation error for $-g_i^{-1}(\bar{x}_i)f_i(\bar{x}_i) + g_i^{-1}(\bar{x}_i)\dot{x}_{id}$.

$$\dot{\theta}_i = -\gamma_ie_i\xi_i(\bar{x}_i) - r_i\theta_i \tag{27}$$

Choose Lyapunov function in this step as

$$V_i = V_{i-1} + \frac{1}{2g_i(\bar{x}_i)}e_i^2 + \frac{1}{2\gamma_i}\tilde{\theta}_i^T\tilde{\theta}_i \tag{28}$$

It can be deduced that

$$\dot{V}_i \leq e_ie_{i+1} - \sum_{l=1}^{i}(k_l^*e_l^2 + \frac{r_l}{2\gamma_l}\tilde{\theta}_l^T\tilde{\theta}_l) + \sum_{l=1}^{i}(\frac{r_l}{2\gamma_l}\theta_l^{*T}\theta_l^* + \kappa\eta_l) \tag{29}$$

where $e_{i+1} = x_{i+1} - x_{(i+1)d}$, $\gamma_l > 0$, $r_l > 0$, k_l is selected to meet $k_l^* = k_l - \frac{g_{ld}}{2g_{l0}^2} \geq \frac{c}{2g_{l0}}$ for c.

Step n: Suppose p actuators stuck at some unknown places at time t_j, that is, $u_j(t) = \bar{u}_j$, $j = j_1, j_2, \cdots, j_p$, $0 \leq p \leq m-1$, and the others may lose effectiveness or be normal, during (t_j, t_{j+1}), there will be no actuator fault. With the control signal and the parameter updating law

$$u_0 = \alpha_n(\bar{x}_n|\theta_n) = \theta_n^T\xi_n(\bar{x}_n) - e_{n-1} - k_ne_n - \delta_n tanh\frac{e_n\delta_n}{\eta_n} \tag{30}$$

$$\dot{\theta}_n = -\gamma_ne_n\xi_n(\bar{x}_n) - r_n\theta_n \tag{31}$$

with $\gamma_n > 0$ and $r_n > 0$ are design constants, and k_n is selected so $k_n^* = k_n - \frac{\sum_{j\neq j_1\cdots j_p}\bar{\rho}_jg_{njd}\bar{b}_j}{2(\sum_{j\neq j_1\cdots j_p}\underline{\rho}_jg_{nj0}\underline{b}_j)^2} \geq \frac{c}{2\sum_{j\neq j_1\cdots j_p}\underline{\rho}_jg_{nj0}\underline{b}_j}$ for c. Then the following theorem is ready to presented.

Theorem 1: The adaptive fuzzy control structure (8) consisting of the state feedback law (30) and the parameter updating laws (13), (21), (27) and (31) ensures the following properties of system (1) which subject to actuator faults (2) and (3), if the initial conditions are bounded.

1) All signals in the closed-loop system remain bounded;

2) the output tracking error $e = y(t) - y_m(t)$ converges to a small neighborhood around zero by appropriately choosing the design parameters.

Proof: With the input vector (4) and the control structure (8), the derivative of $e_n = x_n - x_{nd}$ can be written as

$$\begin{aligned}\dot{e}_n &= f_n(\bar{x}_n) + \sum_{j\neq j_1\cdots j_p}\rho_jg_{nj}(\bar{x}_n)v + \sum_{j=j_1\cdots j_p}\rho_jg_{nj}(\bar{x}_n)\bar{u}_j \\ &\quad - \dot{x}_{nd} + d_n \\ &= \sum_{j\neq j_1\cdots j_p}\rho_jg_{nj}b_j[(\sum_{j\neq j_1\cdots j_p}\rho_jg_{nj}b_j)^{-1}f_n + u_0 + (\sum_{j\neq j_1\cdots j_p} \\ &\quad \rho_jg_{nj}b_j)^{-1}\sum_{j=j_1\cdots j_p}g_{nj}\bar{u}_j - (\sum_{j\neq j_1\cdots j_p}\rho_jg_{nj}b_j)^{-1}\dot{x}_{nd}] + d_n\end{aligned} \tag{32}$$

where g_{nj}, f_n and b_j are short for $g_{nj}(\bar{x}_n)$, $f_n(\bar{x}_n)$ and $b_j(\bar{x}_n)$ respectively. Because $g_{nj} > 0$ and $b_j > 0$, $\forall\bar{x}_n \in \Omega_n \subset R^n$, it is obvious that $(\sum_{j\neq j_1\cdots j_p}\rho_jg_{nj}b_j)^{-1} > 0$. The idealized controller when neglect $d_n(t)$ is

$$\begin{aligned}u_0^* &= -(\sum_{j\neq j_1\cdots j_p}\rho_jg_{nj}b_j)^{-1}f_n - (\sum_{j\neq j_1\cdots j_p}\rho_jg_{nj}b_j)^{-1}\sum_{j=j_1\cdots j_p} \\ &\quad g_{nj}\bar{u}_j + (\sum_{j\neq j_1\cdots j_p}\rho_jg_{nj}b_j)^{-1}\dot{x}_{nd} - e_{n-1} - k_ne_n\end{aligned} \tag{33}$$

Since f_n, g_{nj}, $d_n(t)$, ρ_j and $\bar{u}_j$ are all unknown, the controller (30) is employed, then (34) can be rewritten as

$$\begin{aligned}\dot{e}_n &= \sum_{j\neq j_1\cdots j_p}\rho_jg_{nj}b_j(\tilde{\theta}_n^T\xi_n(\bar{x}_n) - \omega_n - k_ne_n \\ &\quad - e_{n-1} - \delta_n tanh\tfrac{e_n\delta_n}{\eta_n}) + d_n\end{aligned} \tag{34}$$

where $k_n > 0$, $\delta_n = \varepsilon_n + \frac{\bar{d}_n}{\min_{j=1,\cdots,m}\{\underline{\rho}_jg_{nj0}\underline{b}_j\}}$ with ε_n is the upper bound of the fuzzy logic approximation error for $-(\sum_{j\neq j_1\cdots j_p}\rho_jg_{nj}b_j)^{-1}f_n - (\sum_{j\neq j_1\cdots j_p}\rho_jg_{nj}b_j)^{-1}\sum_{j=j_1\cdots j_p}g_{nj}\bar{u}_j + (\sum_{j\neq j_1\cdots j_p}\rho_jg_{nj}b_j)^{-1}\dot{x}_{nd}$.

Consider the Lyapunov function candidate as

$$V = V_{n-1} + \frac{1}{2\sum_{j\neq j_1\cdots j_p}\rho_jg_{nj}b_j}e_n^2 + \frac{1}{2\gamma_n}\tilde{\theta}_n^T\tilde{\theta}_n \tag{35}$$

Differentiating V one can obtain

$$\begin{aligned}\dot{V} &= \dot{V}_{n-1} + \frac{e_n\dot{e}_n}{\sum_{j\neq j_1\cdots j_p}\rho_jg_{nj}b_j} - \frac{\sum_{j\neq j_1\cdots j_p}\rho_j\dot{g}_{nj}b_je_n^2}{2(\sum_{j\neq j_1\cdots j_p}\rho_jg_{nj}b_j)^2} + \frac{1}{\gamma_n}\tilde{\theta}_n^T\dot{\theta}_n \\ &= \dot{V}_{n-1} - e_{n-1}e_n - k_ne_n^2 - \frac{\sum_{j\neq j_1\cdots j_p}\rho_j\dot{g}_{nj}b_je_n^2}{2(\sum_{j\neq j_1\cdots j_p}\rho_jg_{nj}b_j)^2} - e_n\omega_n \\ &\quad - e_n\delta_n tanh\tfrac{e_n\delta_n}{\eta_n} + \tilde{\theta}_n^T(\xi(\bar{x}_n)e_n + \tfrac{1}{\gamma_n}\dot{\theta}_n)\end{aligned} \tag{36}$$

where $\dot{\tilde{\theta}}_n = \dot{\theta}_n$ is considered.

From (31), and by substituting $\dot{V}_{n-1}$ by (29) with $i = n-1$ in (36), $\dot{V}$ can be expressed as

$$\dot{V} \leq -\sum_{i=1}^{n}(k_i^* e_i^2 + \frac{r_i}{2\gamma_i}\tilde{\theta}_i^T\tilde{\theta}_i) + \sum_{i=1}^{n}(\frac{r_i}{2\gamma_i}\theta_i^{*T}\theta_i^* + \kappa\eta_i) \quad (37)$$

From the definition of k_i^* and by choosing $r_i > c$ for $i = 1,2,\cdots,n$, we can get the following expression.

$$\dot{V} \leq -cV + b \quad (38)$$

with $b = \sum_{i=1}^{n}(\frac{r_i}{2\gamma_i}\theta_i^{*T}\theta_i^* + \kappa\eta_i)$. Then it can be obtained that

$$V \leq (V(0) - \frac{b}{c})e^{-ct} + \frac{b}{c} \quad (39)$$

Then e_i, θ_i $(i = 1,2,\cdots,n)$ are bounded and belong to the compact set $\Omega = \{(e_i,\theta_i)|V \leq (\frac{b}{c})\}$. Furthermore, $e_1^2 \leq V_1 \leq (V(0) - \frac{b}{c})e^{-ct} + \frac{b}{c}$, which implies that $\lim_{t\to\infty} e_1^2 \leq \frac{b}{c}$. Some actuator faults will occur at time t_{j+1}, then V will change its value because the change of θ_n^*, however, since V is bounded in (t_j, t_{j+1}), and its changed value is finite, so the initial value of V in the interval (t_{j+1}, t_{j+2}) is bounded, then from the above analysis, V is also bounded in (t_{j+1}, t_{j+2}). Because the actuators subject to only finite faults during operation, so it can be concluded that $V(t)$ is always bounded. Thus, Theorem 1 has been proved.

IV. Simulation Example

A simple example is given to show the effectiveness of the proposed adaptive fuzzy backstepping fault tolerant control structure in this paper. The controlled system is

$$\begin{aligned}\dot{x}_1 &= 0.5x_1 + (1 + 0.1x_1^2)x_2 + d_1(t)\\ \dot{x}_2 &= x_1x_2 + (2 + cos(x_1))u_1 + (2 + sin(x_1))u_2 + d_2(t)\\ y &= x_1\end{aligned} \quad (40)$$

where x_1 and x_2 are states, and y is the output of the system, respectively. The external disturbances are chosen as $d_1(t) = 0.1sint$, $d_2(t) = d(t)$ for simulation, where $d(t)$ is a square wave with the amplitude 0.05 and the period 2. The initial conditions is $x(0) = [1,0]^T$ and the desired reference signal of the system is $y_m(t) = sin(t)$.

Selecting fuzzy membership functions as $\mu_{F_i^1}(x_i) = 1/(1 + exp(5(x_i + 2)))$, $\mu_{F_i^2}(x_i) = exp(-(x_i + 1.5)^2)$, $\mu_{F_i^3}(x_i) = exp(-(x_i + 0.5)^2)$, $\mu_{F_i^4}(x_i) = exp(-x_i^2)$, $\mu_{F_i^5}(x_i) = exp(-(x_i - 0.5)^2)$, $\mu_{F_i^6}(x_i) = exp(-(x_i - 1.5)^2)$, $\mu_{F_i^7}(x_i) = 1/(1 + exp(-5(x_i - 2)))$.

Defining fuzzy basis functions

$$\xi_1^j(x_1) = \frac{\mu_{F_1^j}(x_1)}{\sum_{j=1}^{7}\mu_{F_1^j}(x_1)}, \qquad \xi_2^l(\bar{x}_2) = \frac{\mu_{F_1^i}(x_1)\mu_{F_2^j}(x_2)}{\sum_{i=1}^{7}\sum_{j=1}^{7}\mu_{F_1^i}(x_1)\mu_{F_2^j}(x_2)}$$

$i = 1,2,\cdots,7$, $j = 1,2,\cdots,7$

$\xi_1(x_1) = [\xi_1^1(x_1), \xi_1^2(x_1), \cdots, \xi_1^7(x_1),]$, $\xi_2(\bar{x}_2) = [\xi_2^1(\bar{x}_2), \xi_2^2(\bar{x}_2), \cdots, \xi_2^{49}(\bar{x}_2)]$, then the fuzzy logic systems $\theta_1^T\xi_1(x_1)$ and $\theta_2^T\xi_2(\bar{x}_2)$ can be obtained for constructing the adaptive fuzzy controller (30).

The design parameters $k_i = 25$, $\gamma_i = 0.2$, $r_i = 5$, for $i = 1,2$ and $b_1 = 0.1$, $b_2 = 0.2$. The initial parameters of the fuzzy approximate systems are $\theta_1 = 0_{7\times1}$ and $\theta_2 = 0_{1\times49}$. The actuator faults introduced for simulation are $u_1 = 2$ when $t \geq 4$, and $u_2 = 0.6u_2$ for $t \geq 12$. Here we consider a case that the both types of actuator faults will occur during operation. It is obviously easier to deal with just one type of them in the system as long as Assumption 1 is satisfied. The simulation results are given in Fig. 1-Fig. 3, where Fig. 1 shows the output of the system tracking the reference signal y_m closely and smoothly, Fig. 2 shows the curve of the state variable x_2, and from Fig. 3 we can see the control signals of the system. Simulation results of applying the proposed control scheme to system (40) show that good tracking performance is obtained though there are large uncertainties in nonlinear system functions and actuator faults.

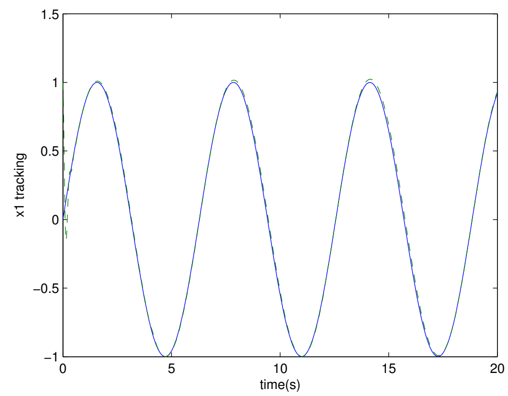

Fig. 1. The curves of x_1 (dash) and y_m (solid)

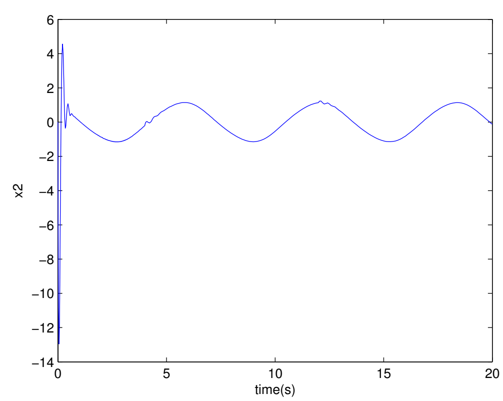

Fig. 2. The curve of x_2

V. Conclusion

A novel robust adaptive fuzzy controller is proposed by using backstepping design technique in this paper to accommodate actuator faults as well as external disturbances for unknown nonlinear systems which cannot be feedback linearized. The considered systems are multi-input single

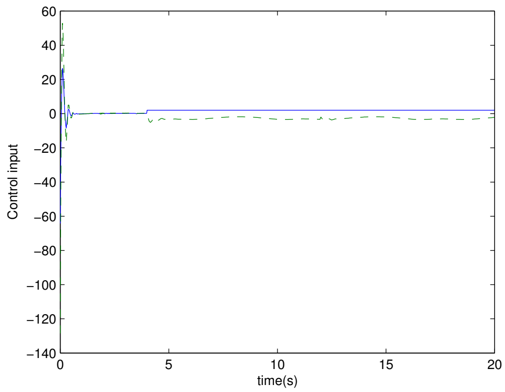

Fig. 3. The curve of u

output and subject to both lock-in-place and loss of effectiveness actuator faults. With all the input signals are contributed to the same control objective, a special control structure is employed. It has been proved in theory and demonstrated in simulation results that the designed control scheme can guarantee that all signals in the closed-loop system are remain bounded and tracking error between the system output and the reference signal converges to a small neighborhood around zero, though the nonlinear system functions and the information about the faults are all unknown. Desired control performance can be obtained by appropriately choosing the design parameters despite there are external disturbances, and the controller singularity problem is avoided perfectly. This control approach can also be applied to MIMo systems.

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