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Model-based Adaptive Friction Compensation for Accurate Position Control

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***Abstract*— An adaptive friction compensator for position control is proposed using the generalized Maxwell-slip (GMS) friction model, with a new, linearly-parameterized Stribeck function. It employs a polynomial equation that is linear-in-the-parameter to approximate the nonlinear Stribeck effect in the GMS model, and simplifies the design of the adaptive friction compensator. The proposed compensator has a switching structure to accomodate for the hybrid nature of the GMS model, and contains a robustifying term to account for unmodelled dynamics. The stability of the proposed adaptive algorithm is analyzed and its stability conditions are clarified. The validity and effectiveness of the proposed, linearly-parameterized friction compensator is verified by simulations for the positional control of an inertia system under the influence of dynamic friction.**

I. Introduction

It is widely recognized that most mechanical systems involving two or more contact surfaces with relative motion, would experience to varying degrees some form of frictional effects. The presence of dynamic friction in such industrial applications as robotic manipulators, hydraulic systems, precision engineering, and so forth, can lead to significant tracking error, or even instability. However, the task of controller design is greatly complicated by nonlinearities of the surface contact mechanics, structural and parametric uncertainties.

Currently, friction compensation schemes are divided into non-model and model-based methods. Studies have shown that simple PD or PID controllers suffer significant performance degradation due to the nonlinear characteristics of friction, which can lead to hunting behaviors and instability [1]. Black-box methods employing neural networks or fuzzy logic for friction compensation have also been widely researched [2], [3]. In comparison, the potential of model-based adaptive friction compensation has been demonstrated by several researchers. These efforts include the modeling and compensation of Coulomb friction [4], [5], a control scheme for dynamic, linear friction [6], and nonlinear static mapping of the Stribeck effect [7]. These methods provide powerful arguments for the use of adaptive control in friction compensation, but do not combine it with a sufficiently complex and accurate dynamic friction model.

Therefore, this paper develops a robust adaptive compensation scheme using the generalized Maxwell-slip (GMS) friction model, which has been proposed as a more accurate representation of the friction phenomenon than the LuGre model [8]. The GMS model consists of parallel elementary blocks and separates frictional mechanism into two regimes: sticking and slipping. This results in a hybrid system, with two separate models. The GMS model yields results that correspond to experimental observation, while maintaining a simpler structure than the generic friction model [9]. Offline identification algorithms of the GMS model using Nelder-Mead simplex [10] and particle swarm optimization (PSO) [11] have been presented. However, designing an adaptive controller using the GMS model can be difficult due to its switching nature and also the nonlinear Stribeck effect.

The main novelty of this paper is the proposal of a polynomial Stribeck function that is readily applicable to the GMS friction model. The validity of using a polynomial approximation function to describe the Stribeck effect has been investigated in previous works [12]. By using the polynomial approximation function, the development of adaptive control laws are simplified, as friction models can be linearly-parameterized. This study specifically addresses the problem of robustness with respect to unmodeled dynamics by introducing a sliding-mode based smooth adaptive robustifying term into the control law [13]. Stability analysis is presented to show the robustness of the algorithm, provided that a bound on the unmodeled terms is known to exist. The validity of the proposed robust adaptive control algorithm based upon the GMS friction model is demonstrated by simulation results.

II. Problem Statement

The objective of this study is the control of a mass acting under the influence of friction forces. Consider the following state-space representation of a simple mass system:

$$m\dot{\boldsymbol{x}}_p = \boldsymbol{A}_p\boldsymbol{x}_p + \boldsymbol{b}_p^T\left(u - F_f\right) \tag{1}$$

where:

$$\boldsymbol{x}_p = \begin{bmatrix} x & \dot{x} \end{bmatrix}^T, \quad \boldsymbol{A}_p = \begin{bmatrix} 0 & m \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{b}_p = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$$

Here, m is the mass, while x and $\dot{x}$ are the mass position and velocity, respectively. u is the input force and F_f represents friction. To describe the effects of friction, this study employs the generalized Maxwell-slip (GMS) friction model [**?**], [8]. The GMS model is an asperity-based description of the friction phenomenon. It consists of parallel connections of elementary blocks, shown in Figure 1, and expressed by:

$$F_f = \sum_{i=1}^{N} F_i \tag{2}$$

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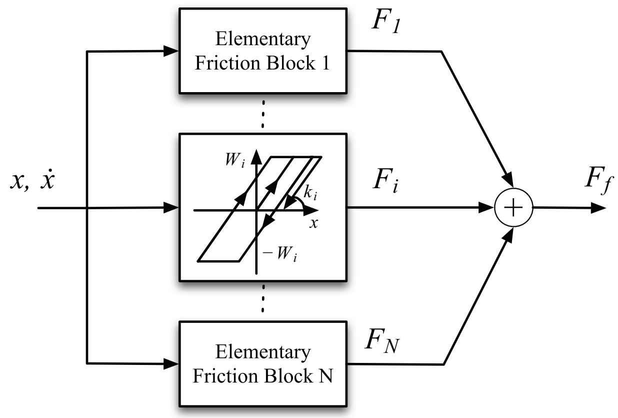

Fig. 1. Parallel connection of N elementary blocks in the GMS model.

Here, N represents the number of elementary blocks employed by the GMS model, and viscous friction is neglected. Each elementary block is governed by a set of two dynamical equations, depending on whether it is in a sticking or slipping state. The sticking state contains a Maxwell-slip equation to describe hysteresis and other presliding characteristcs. The slipping state equation results in frictional lag and the Stribeck effect. Mathematically, this is expressed as:

- If the elementary block is sticking, the differential equation is given by:

$$\dot{F}_i = k_i \dot{x} \tag{3}$$

  and the elementary block remains sticking until $|F_i| > \alpha_i s(\dot{x}) = W_i$.

- If the elementary block is slipping, the differential equation is given by:

$$\dot{F}_i = C\left(\alpha_i sgn(\dot{x}) - \frac{F_i}{s(\dot{x})}\right) \tag{4}$$

  and the elementary block remains slipping until the velocity goes through zero.

Here, C is a constant term introduced by the GMS model to directly account for frictional lag dynamics, and $\sum \alpha_i = 1$. $s(\dot{x})$ describes the Stribeck effect, which is generally expressed by the following function:

$$s(\dot{x}) = F_C + (F_S - F_C)\, e^{-\left(\frac{|\dot{x}|}{V_S}\right)^{\sigma_S}} \tag{5}$$

where F_C is the Coulomb friction parameter, F_S represents static friction, V_S is the Stribeck velocity, and σ_S is a shaping factor.

Considering only the slipping state of friction under constant velocity, the steady-state equation for each elementary block reduces to:

$$F_{i,ss} = \alpha_i s(\dot{x}) sgn(\dot{x}) \tag{6}$$

Defining $\delta_{i,D} = F_i - F_{i,ss}$, the frictional force equation in the slipping regime becomes:

$$\begin{aligned} F_i &= F_{i,ss} + \delta_{i,D} \\ &= \alpha_i s(\dot{x}) sgn(\dot{x}) + \delta_{i,D} \end{aligned} \tag{7}$$

Analysis of the above equation reveals that the friction force is comprised of two terms: a static term corresponding to the Stribeck effect; and a dynamic term $\delta_{i,D}$ that acts as a perturbation.

A Lyapunov argument can be used to show that, given bounds on the parameter values, then the dynamic term $\delta_{i,D}$ is bounded. This is formally stated in Lemma 1.

LEMMA 1 *Assuming that the system parameters are bounded, the dynamic perturbations in the slipping state of each elementary block in the GMS friction model are also bounded.*

Proof: Define a candidate Lyapunov function as:

$$V_i = \frac{1}{2} F_i^2 \tag{8}$$

Then the derivative of (8) along the frictional dynamics is given as:

$$\begin{aligned} \dot{V}_i &= F_i \dot{F}_i \\ &= F_i C\left(\alpha_i sgn(\dot{x}) - \frac{F_i}{s(\dot{x})}\right) \\ &= C\,|F_i|\, sgn(F_i) sgn(\dot{x})\left(\alpha_i - \frac{|F_i|}{s(\dot{x})}\right) \end{aligned} \tag{9}$$

In the slipping state, it is noted that the sign of F_i and $\dot{x}$ are always the same and are different from zero. Therefore, $\dot{V}_i$ is negative definite if:

$$|F_i| \leq \alpha_i s(\dot{x}) = |F_{i,ss}| \leq \alpha_i F_S \tag{10}$$

From (7) and (10), it is clear that since F_i and $F_{i,ss}$ are bounded, the perturbation term $\delta_{i,D}$ must also be bounded. □

III. LINEARLY-PARAMETERIZED GMS MODEL

The GMS model described in the previous section employs a Stribeck function that contains nonlinear parameterization. While each term in (5) has a physical meaning, the task of designing an adaptive friction compensator for the resulting nonlinear friction model becomes complicated due to the presence of nonlinearity, which results in control issues such as stability, robustness and convergence. Therefore, this study proposes a new approximator function for the Stribeck effect that is linearly-parameterized and has a polynomial form:

$$s(\dot{x}) = s^*(\dot{x}, n) + \delta_S \tag{11}$$

where:

$$\begin{aligned} s^*(\dot{x}, n) &= \beta_1 + \beta_2\,|\dot{x}| + \ldots + \beta_n\,|\dot{x}|^{n-1} \\ &= \sum_{i=1}^{n} \beta_i\,|\dot{x}|^{i-1} \end{aligned} \tag{12}$$

Here, $s^*(\dot{x}, n)$ is the proposed linearly-parameterized approximator function, δ_S is the approximation error, and n is the order of the approximator function. A bound on δ_S on any closed and bounded interval $\Omega_{\dot{x}} = [\dot{x}_{\min}, \dot{x}_{\max}]$ exists and can be expressed as:

$$\sup_{\Omega_{\dot{x}}} |\delta_S| \leq \Delta_S \tag{13}$$

The main contribution of this study is that, using this new Stribeck equation, the GMS friction model is linearly-parameterized, allowing for the applications of linear adaptive control theories for compensation.

Employing the linearly-parameterized Stribeck function, each elementary block of the GMS model becomes:

- In the sticking state, the friction force is given as:

$$F_i = \theta_{i,stick}\omega_{stick} \tag{14}$$

where:

$$\theta_{i,stick} = k_i$$
$$\omega_{stick} = \int_{t_0}^{t} \dot{x}(\tau)d\tau$$

and remains sticking until $|F_i| > \alpha_i(s^*(\dot{x}, n) + \delta_S)$.

- In the slipping state, the friction force is described by:

$$F_i = \boldsymbol{\theta}_{i,slip}^T \boldsymbol{\omega}_{slip} + \alpha_i sgn(\dot{x})\delta_S + \delta_{i,D} \tag{15}$$

where:

$$\begin{aligned}\boldsymbol{\theta}_{i,slip} &= \begin{bmatrix}\alpha_i\beta_1 & \alpha_i\beta_2 & \dots & \alpha_i\beta_n\end{bmatrix}^T \\ &= \begin{bmatrix}\theta_{i,1} & \theta_{i,2} & \dots & \theta_{i,n}\end{bmatrix}^T \\ \boldsymbol{\omega}_{slip} &= sgn(\dot{x}) \cdot \begin{bmatrix}1 & |\dot{x}| & \dots & |\dot{x}|^{n-1}\end{bmatrix}^T\end{aligned}$$

and remains slipping until the velocity goes through zero.

This linearly-parameterized friction model is used to construct a suitable adaptive controller for compensation.

To express the two regimes of the GMS model in a unified framework, define the indicator function $\chi[X]$ of the event X as:

$$\chi[X] = \begin{cases} 1 & \text{if } X \text{ is true} \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

This allows the expression of the GMS model as:

$$F_f = \boldsymbol{\theta}^T\boldsymbol{\omega} + \sum_{i=1}^{N} \chi_{i,slip}\left(\alpha_i sgn(\dot{x})\delta_S + \delta_{i,D}\right) \tag{17}$$

where:

$$\begin{aligned}\boldsymbol{\theta} = \big[&\theta_{1,stick} \quad \dots \quad \theta_{N,stick} \\ &\boldsymbol{\theta}_{1,slip}^T \quad \dots \quad \boldsymbol{\theta}_{N,slip}^T\big]^T \\ \boldsymbol{\omega} = \big[&\chi_{1,stick}\omega_{stick} \quad \dots \quad \chi_{N,stick}\omega_{stick} \\ &\chi_{1,slip}\boldsymbol{\omega}_{slip}^T \quad \dots \quad \chi_{N,slip}\boldsymbol{\omega}_{slip}^T\big]^T \\ \chi_{i,stick} &= \chi\left[F_i \text{ is sticking}\right] \\ \chi_{i,slip} &= \chi\left[F_i \text{ is slipping}\right]\end{aligned}$$

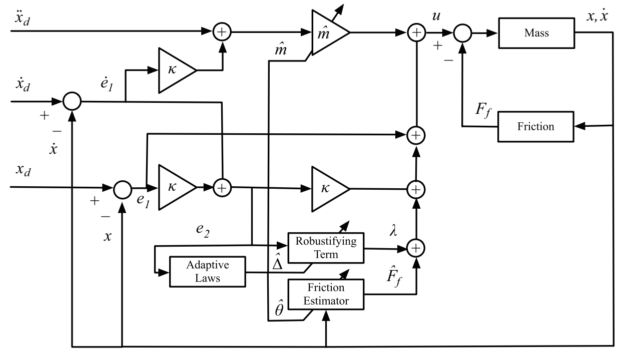

Fig. 2. Adaptive friction compensator for position control.

Notice that $\chi_{i,stick}$ and $\chi_{i,slip}$ are mutually exclusive events that indicate the current state of each elementary block in the GMS model. That is, each elementary block must either be sticking or slipping, but cannot be both, at any given time.

IV. Adaptive Friction Compensator

The structure of the adaptive friction compensator for positional control is shown in Figure 2. The system is given as a mass acting under the influence of friction as described by the GMS model. The control objective is the positional tracking of a desired trajectory defined by x_d, that is assumed to be designed such that $\dot{x}_d$ and $\ddot{x}_d$ exist and are bounded. A position tracking error is stated as:

$$e_1 = x_d - x \tag{18}$$

The following filtered tracking error is defined to facilitate the subsequent design and analysis:

$$e_2 = \dot{e}_1 + \kappa e_1 \tag{19}$$

The proposed control law is give as:

$$u = \hat{m}\ddot{x}_d + \kappa e_2 + e_1 + \hat{m}\kappa\dot{e}_1 + \hat{F}_f + \lambda \tag{20}$$

where $\hat{m}$ is the estimated value of the mass, and λ is a robustifying term to be defined later. The friction force estimate, $\hat{F}_f$, is defined as:

$$\hat{F}_f = \hat{\boldsymbol{\theta}}^T\hat{\boldsymbol{\omega}} \tag{21}$$

where:

$$\begin{aligned}\hat{\boldsymbol{\theta}} = \big[&\hat{\theta}_{1,stick} \quad \dots \quad \hat{\theta}_{N,stick} \\ &\hat{\boldsymbol{\theta}}_{1,slip}^T \quad \dots \quad \hat{\boldsymbol{\theta}}_{N,slip}^T\big]^T \\ \hat{\boldsymbol{\omega}} = \big[&\hat{\chi}_{1,stick}\omega_{stick} \quad \dots \quad \hat{\chi}_{N,stick}\omega_{stick} \\ &\hat{\chi}_{1,slip}\boldsymbol{\omega}_{slip}^T \quad \dots \quad \hat{\chi}_{N,slip}\boldsymbol{\omega}_{slip}^T\big]^T \\ \hat{\chi}_{i,stick} &= \chi\left[\hat{F}_i \text{ is sticking}\right] \\ \hat{\chi}_{i,slip} &= \chi\left[\hat{F}_i \text{ is slipping}\right]\end{aligned}$$

Here $\hat{\boldsymbol{\theta}}$ are the estimates of the parameters of the linearly-parameterized friction model. It is noted that the true GMS friction model can thus be expressed in terms of $\hat{\boldsymbol{\omega}}$ as:

$$F_f = \boldsymbol{\theta}^T\hat{\boldsymbol{\omega}} + \delta \tag{22}$$

where:

$$\delta = \delta_{sw} + \sum_{i=1}^{N} \left[\chi_{i,slip}\left(\alpha_i sgn(\dot{x})\delta_S + \delta_{i,D}\right)\right]$$
$$\delta_{sw} = \boldsymbol{\theta}^T\left(\boldsymbol{\omega} - \hat{\boldsymbol{\omega}}\right) \tag{23}$$

Here, δ is an uncertainty term that arises from the switching error between the true GMS model and friction compensator, δ_{sw}, the approximation error δ_S, and dynamic perturbation terms $\delta_{i,D}$. It can be shown that Δ is a bounded term. That is:

$$\sup_{\Omega_{\dot{x}}} |\delta| \leq \Delta \tag{24}$$

Setting the robustifying term λ as:

$$\lambda = \hat{\Delta}\eta_\Delta \tanh\left((a+bt)e_2\right) \tag{25}$$

where $\hat{\Delta}$ is the estimate of Δ, a and b are user-defined positive constants, $\kappa_\Delta > 1$. The parameter estimation errors are defined as:

$$\tilde{m} = \hat{m} - m \tag{26}$$
$$\tilde{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta} \tag{27}$$
$$\tilde{\Delta} = \hat{\Delta} - \Delta \tag{28}$$

The adaptive laws are established according to:

$$\dot{\tilde{m}} = \dot{\hat{m}} = \gamma_m\left(\ddot{x}_d + \kappa\dot{e}_1\right)e_2 - \sigma_m\gamma_m\hat{m} \tag{29}$$
$$\dot{\tilde{\boldsymbol{\theta}}} = \dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\Gamma}_\theta\hat{\boldsymbol{\omega}}e_2 - \sigma_\theta\boldsymbol{\Gamma}_\theta\hat{\boldsymbol{\theta}} \tag{30}$$
$$\dot{\tilde{\Delta}} = \dot{\hat{\Delta}} = \gamma_\Delta|e_2| - \sigma_\Delta\gamma_\Delta\hat{\Delta} \tag{31}$$

where $\boldsymbol{\Gamma}_\theta$, γ_m, γ_Δ, σ_m, σ_θ and σ_Δ are positive. The main stability result of the proposed method is now presented.

THEOREM 1 *Consider the mass system acting under the influence of friction as given in (1) and assume that (24) holds, but is unknown. The control signal (20) together with the adaptive laws (29), (30), and (31), guarantees that Lyapunov function defined as:*

$$V = \frac{1}{2}e_1^2 + \frac{1}{2}me_2^2 + \frac{1}{2}\gamma_m^{-1}\tilde{m}^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}^T\boldsymbol{\Gamma}_\theta^{-1}\tilde{\boldsymbol{\theta}} + \frac{1}{2}\gamma_\epsilon^{-1}\tilde{\Delta}^2 \tag{32}$$

is uniformly bounded and converges to a small neighborhood of the origin. The same property holds for the error signals e_1, e_2, $\tilde{m}$, $\tilde{\boldsymbol{\theta}}$, and $\tilde{\Delta}$.

Proof: By examining (18) and (19), it is noted that:

$$\dot{e}_1 = -\kappa e_1 + e_2 \tag{33}$$
$$\dot{e}_2 = \ddot{e}_1 + \kappa\dot{e}_1 \tag{34}$$

Taking the derivative of the candidate Lyapunov function defined by (32) and using (1) and (20):

$$\begin{aligned}\dot{V} &= e_1\dot{e}_1 + me_2\dot{e}_2 + \gamma_m^{-1}\tilde{m}\dot{\tilde{m}} + \tilde{\boldsymbol{\theta}}^T\ \boldsymbol{\Gamma}_\theta^{-1}\dot{\tilde{\boldsymbol{\theta}}} \\ &\quad + \gamma_\Delta^{-1}\tilde{\Delta}\dot{\tilde{\Delta}} \\ &= -\kappa e_1^2 + e_1e_2 + e_2\left(m\ddot{x}_d - u + F_f + m\kappa\dot{e}_1\right) \\ &\quad + \gamma_m^{-1}\tilde{m}\dot{\tilde{m}} + \tilde{\boldsymbol{\theta}}^T\ \boldsymbol{\Gamma}_\theta^{-1}\dot{\tilde{\boldsymbol{\theta}}} + \gamma_\Delta^{-1}\tilde{\Delta}\dot{\tilde{\Delta}} \\ &= -\kappa e_1^2 - \kappa e_2^2 + \tilde{m}\left(\gamma_m^{-1}\dot{\tilde{m}} - \left(\ddot{x}_d + \kappa\dot{e}_1\right)e_2\right) \\ &\quad + \tilde{\boldsymbol{\theta}}^T\left(\boldsymbol{\Gamma}_\theta^{-1}\dot{\tilde{\boldsymbol{\theta}}} - \hat{\boldsymbol{\omega}}e_2\right) + \Lambda\end{aligned} \tag{35}$$

where:

$$\Lambda = \gamma_\Delta^{-1}\tilde{\Delta}\dot{\tilde{\Delta}} + \delta e_2 - \lambda e_2 \tag{36}$$

Substituting the adaptive laws (29) and (30):

$$\begin{aligned}\dot{V} &= -\kappa e_1^2 - \kappa e_2^2 - \sigma_m\tilde{m}\hat{m} - \sigma_\theta\tilde{\boldsymbol{\theta}}^T\hat{\boldsymbol{\theta}} + \Lambda \\ &\leq -\kappa e_1^2 - \kappa e_2^2 - \frac{\sigma_m}{2}\tilde{m}^2 - \frac{\sigma_\theta}{2}\tilde{\boldsymbol{\theta}}^T\tilde{\boldsymbol{\theta}} + \frac{\sigma_m}{2}m^2 \\ &\quad + \frac{\sigma_\theta}{2}\boldsymbol{\theta}^T\boldsymbol{\theta} + \Lambda\end{aligned} \tag{37}$$

Using (25) and the adaptive law (31):

$$\begin{aligned}\Lambda &= \gamma_\Delta^{-1}\tilde{\Delta}\dot{\tilde{\Delta}} + \delta e_2 - \hat{\Delta}\eta_\Delta\tanh\left((a+bt)e_2\right)e_2 \\ &= \tilde{\Delta}|e_2| + \delta e_2 - \hat{\Delta}\eta_\Delta\tanh\left((a+bt)|e_2|\right)|e_2| \\ &\quad - \sigma_\Delta\tilde{\Delta}\hat{\Delta} \\ &\leq \tilde{\Delta}|e_2| + \Delta|e_2| - \hat{\Delta}\eta_\Delta\tanh\left((a+bt)|e_2|\right)|e_2| \\ &\quad - \frac{\sigma_\Delta}{2}\tilde{\Delta}^2 + \frac{\sigma_\Delta}{2}\Delta^2 \\ &= \hat{\Delta}|e_2|\left(1 - \eta_\Delta\tanh\left((a+bt)|e_2|\right)\right) \\ &\quad - \frac{\sigma_\Delta}{2}\tilde{\Delta}^2 + \frac{\sigma_\Delta}{2}\Delta^2\end{aligned} \tag{38}$$

Note that:

$$1 - \eta_\Delta\tanh\left((a+bt)|e_2|\right) \leq 0 \tag{39}$$

if and only if:

$$|e_2| \geq \nu \tag{40}$$

where:

$$\nu = \frac{1}{a+bt}\ln\left(\frac{\eta_\Delta + 1}{\eta_\Delta - 1}\right) \tag{41}$$

By examining (41), it is clear that as $t \to \infty$, $\nu \to 0$ when $\kappa_\Delta > 1$. Therefore, (40) is satisfied and:

$$\dot{V} \leq -cV + \lambda \tag{42}$$

where:

$$c = \min\left\{2\kappa, \frac{2\kappa}{m}, \sigma_m\gamma_m, \frac{\sigma_\theta}{\lambda_{\max}(\boldsymbol{\Gamma}_\theta^{-1})}, \sigma_\Delta\gamma_\Delta\right\} \tag{43}$$
$$\lambda = \frac{\sigma_m}{2}m^2 + \frac{\sigma_\theta}{2}\boldsymbol{\theta}^T\boldsymbol{\theta} + \frac{\sigma_\Delta}{2}\Delta^2 \tag{44}$$

As $\lambda/c > 0$, (42) results in:

$$0 \leq V(t) \leq \lambda/c + \left(V(0) - \lambda/c\right)e^{-ct} \tag{45}$$

Therefore all error signals e_1, e_2, $\tilde{m}$, $\tilde{\boldsymbol{\theta}}$ and $\tilde{\Delta}$ are uniformly bounded and converge to a small neighborhood of the origin. □

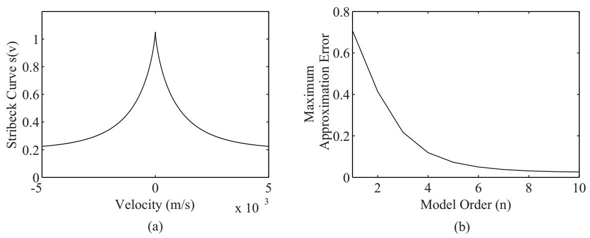

Fig. 3. (a) The Stribeck curve; and (b) approximation error of the Stribeck curve for various model order.

TABLE I

INITIAL, CONVERGED AND LS/TRUE PARAMETER VALUES.

Parameter	Initial	Converged	LS/True
β_1	0.3	0.8845	0.9200
β_2	0	-482.2020	-516.1794
β_3	0	1.31×10^5	1.37×10^5
β_4	0	-1.24×10^7	-1.24×10^7
k_1 [N/m]	1.50×10^4	0.86×10^4	1.00×10^4
k_2 [N/m]	1.05×10^4	0.66×10^4	0.70×10^4
k_3 [N/m]	0.75×10^4	0.45×10^4	0.50×10^4
k_4 [N/m]	0.45×10^4	0.32×10^4	0.30×10^4

V. SIMULATION RESULTS

Simulation results are presented to illustrate the validity of the proposed linearly-parameterized GMS model in compensating for frictional dynamics. First, an analysis is conducted on the accuracy of the polynomial Stribeck function in describing the Stribeck effect. The effectiveness of the proposed adaptive friction compensator is then demonstrated by examining the tracking performance for position trajectories.

A. Determination of Stribeck Approximation Function

The nonlinear Stribeck function is assumed to be described by (5), with the following parameters:

$$
\begin{aligned}
&F_S = 1.05\ [\mathrm{N}], \quad F_C = 0.2\ [\mathrm{N}]\\
&V_S = 98\times10^{-5}\ [\mathrm{m/s}], \quad \sigma_S = 0.78
\end{aligned} \tag{46}
$$

The approximation of this function by the proposed polynomial Stribeck equation is accomplished by using off-line, least-square, curve fitting technique. The identification process was conducted for various orders of the linearly-parameterized Stribeck function to determine a model order that provides good trade-off between modeling accuracy and minimal parameters. This will be used to construct the adaptive friction compensator.

Figure 3(a) illustrates the Stribeck effect. The model order for the linearly-parameterized function is varied from 1 to 10. The resulting modeling error is shown in Figure 3(b). Based upon these results, it is determined that $n = 4$ provides a good approximation of the nonlinear Stribeck function while maintaining a small amount of parameters. From this observation, the friction compensator proposed in this reserach is constructed using a linearly-parameterized Stribeck function of order 4.

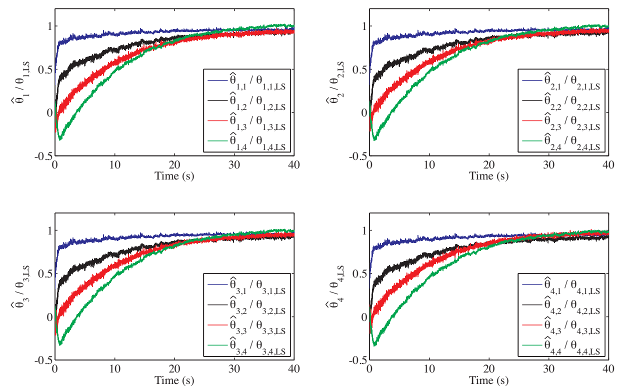

Fig. 4. Convergence of linearly-parameterized Stribeck function coefficients.

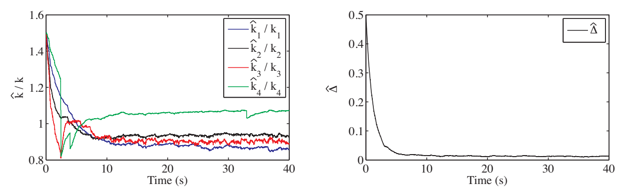

Fig. 5. Convergence of Maxwell-slip parameters and $\hat{\Delta}$.

B. Adaptive Controller Performance

The effectiveness of the proposed adaptive controllers is illustrated for two position trajectory signals. It is assumed that the upper bound on the number of elementary blocks is known from offline system identification procedures, such as PSO [11]. This study assumes the frictional dynamics to be governed by four elementary blocks. Experimental investigations have determined that a GMS model constructed with four elementary blocks is sufficient to accurately capture frictional characteristics under realistic conditions [8], [10]. The results indicate that increasing the number of elementary block does not serve to improve the modeling accuracy. The system parameters are given as:

$$
\begin{aligned}
&m = 1\ [\mathrm{kg}], \quad C = 24\ [\mathrm{N/s}]\\
&k_1 = 1.0\times10^4\ [\mathrm{N/m}], \quad k_2 = 0.7\times10^4\ [\mathrm{N/m}]\\
&k_3 = 0.5\times10^4\ [\mathrm{N/m}], \quad k_4 = 0.3\times10^4\ [\mathrm{N/m}]\\
&\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.25
\end{aligned} \tag{47}
$$

For the Stribeck effect described by the parameters given in (46), the coefficients of the proposed polynomial Stribeck function with a model order of four is determined as outlined above, and is given in Table I.

The gains κ and κ_Δ are chosen as:

$$
\kappa = 200, \quad \kappa_\Delta = 2000 \tag{48}
$$

The results of the proposed adaptive controller is compared to a PD controller with the following gains:

$$
k_P = 40000, \quad k_D = 200 \tag{49}
$$

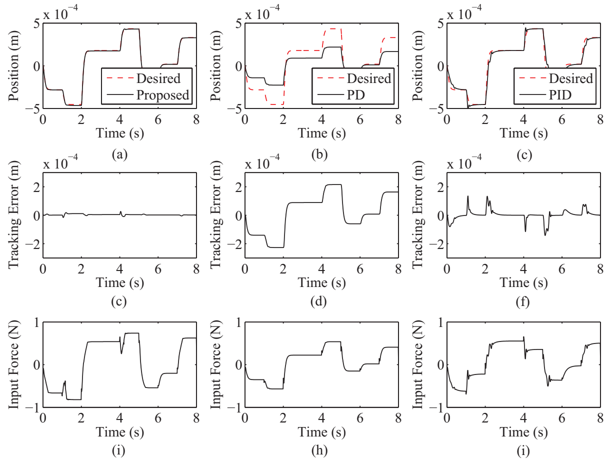

Fig. 6. Comparison of position tracking performance for random step trajectory of proposed controller after convergence, PD and PID controllers.

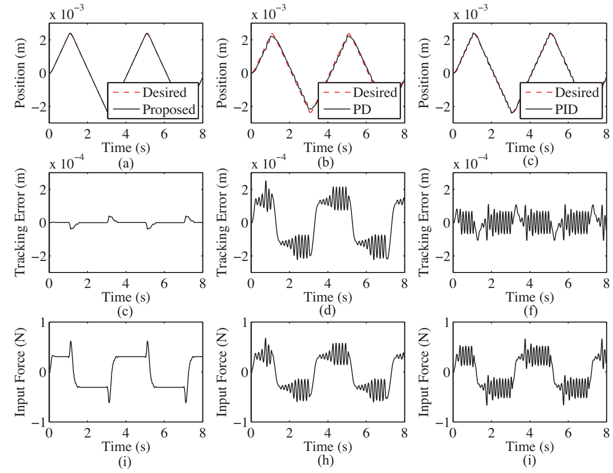

Fig. 7. Comparison of position tracking performance for ramp trajectory of proposed controller after convergence, PD and PID controllers.

Another result using a conventional PID controller is also presented, with the gains set as:

$$k_P = 40000, \ \ k_I = 400000, \ \ k_D = 200 \tag{50}$$

The convergence of the polynomial Stribeck parameters and the Maxwell-slip parameters, when the reference trajectory satisfies the persistently exciting condition, are shown in Figures 4 and 5. In this section, the performance of the proposed adaptive controller is evaluated for the trajectory tracking of a random step and ramp position signal. Using the converged estimates, trajectory tracking performances of the proposed adaptive controller, conventional PD and PID controllers are compared in Figures 6 and 7. It is noted that Figure 6 shows the effectiveness of the proposed adaptive algorithm after convergence in achieving trajectory tracking of a random step signal. The PD controller, however, exhibits considerable steady-state tracking error. The PID controller eliminates this steady-state error for a step signal. However, by examining Figure 7, it is seen that both the PD and PID controllers yield unacceptable tracking results for a ramp trajectory. In particular, the tendencies of these controllers to cause limit-cycling due to the stick-slip effect and integral wind-up is evident. This is compared to the proposed adaptive friction compensator, which effectively achieves position tracking for both random step and ramp trajectories despite modeling uncertainties.

VI. Conclusion

An adaptive friction compensator is proposed using polynomial approximation of the Stribeck effect. The structure is based upon the GMS friction model, which becomes linearly-parameterized, allowing for the straight-forward design of adaptive laws. Stability and robustness conditions with respect to unmodeled dynamics is guaranteed by introducing a robustifying term into the control signal. The friction force is accurately estimated and compensated for by the adaptive controller, and allows for trajectory tracking of velocity and position signals.

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