Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

A New Neuroadaptive Control Architecture for Nonlinear Uncertain Dynamical Systems: Beyond σ- and e-Modifications

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***Abstract*— Neural networks are a viable pararadigm for adaptive system identification and control. This paper develops a new neuroadaptive control architecture for nonlinear uncertain dynamical systems. The proposed framework involves a novel controller architecture involving additional terms in the update laws that are constructed using a moving window of the integrated system uncertainty. These terms can be used to identify the ideal system parameters as well as effectively suppress system uncertainty. A linear parameterization of the system uncertainty is considered and state feedback neuroadaptive controllers are developed.**

I. Introduction

To improve robustness of adaptive and neuroadaptive controllers several controller architectures have been proposed in the literature. These include the σ- and e-modification architectures used to keep the system parameter estimates from growing without bound in the face of system uncertainty [1], [2]. In this paper, a new neuroadaptive control architecture for nonlinear uncertain dynamical systems is developed. Specifically, the proposed framework involves a new and novel controller architecture involving additional terms, or *Q-modification terms*, in the update laws that are constructed using a moving window of the integrated system uncertainty. The Q-modification terms can be used to identify the ideal system parameters which can be used in the adaptive law. In addition, these terms effectively suppress system uncertainty. Even though the proposed approach is reminiscent to the composite adaptive control framework discussed in [3], the Q-modification framework does not involve filtered versions of the control input and system state in the update laws. Rather, the update laws involve auxiliary terms predicated on an estimate of the unknown neural network weights which in turn are characterized by an auxiliary equation involving the integrated error dynamics over a moving time interval. For a scalar linearly parameterized uncertainty structure, these ideas were first explored in [4]. In this paper, we extend the results in [4] to vector uncertainty structures with linear parameterizations. Finally, due to space limitations, all the proofs are omitted from the paper. The proofs along with extensions to nonlinear uncertainty parameterizations and output feedback are given in [5]

II. Adaptive Control with a Q-Modification Architecture

In this section, we present the notion of the Q-modification architecture in adaptive control. Specifically, consider the adaptive control problem with error dynamics given by

$$\dot{e}(t) = Ae(t) + b[\Delta(t) - \nu_{\rm ad}(t)], \quad e(0) = e_0, \quad t \geq 0, \quad (1)$$

where $e(t) \in \mathbb{R}^n$, $t \geq 0$, is the error signal, $\Delta(t) \in \mathbb{R}$, $t \geq 0$, is the system uncertainty, $\nu_{\rm ad}(t)$ is the adaptive signal whose purpose is to suppress the effect of the system uncertainty, $A \in \mathbb{R}^{n\times n}$ is a known Hurwitz matrix, and $b = [0, \ldots, 0, 1]^{\rm T} \in \mathbb{R}^n$. For simplicity of exposition, in this section we consider the case where the system uncertainty $\Delta(t)$, $t \geq 0$, is a scalar function with a perfect parametrization in terms of a constant *unknown* vector $W \in \mathbb{R}^N$ and an *available* vector of continuous basis functions $\theta(t) = [\theta_1(t), \ldots, \theta_N(t)]^{\rm T} \in \mathbb{R}^N$ such that $\theta_i(t)$, $i = 1, \ldots, N$, are bounded for all $t \geq 0$. In particular,

$$\Delta(t) = W^{\rm T}\theta(t), \quad t \geq 0. \quad (2)$$

The parametrization given by (2) suggests an adaptive control signal $\nu_{\rm ad}(t)$, $t \geq 0$, of the form

$$\nu_{\rm ad}(t) = \hat{W}^{\rm T}(t)\theta(t), \quad (3)$$

where $\hat{W}(t) \in \mathbb{R}^N$, $t \geq 0$, is a vector of the adaptive weights. Hence, the dynamics in (1) can be rewritten as

$$\dot{e}(t) = Ae(t) + b[W - \hat{W}(t)]^{\rm T}\theta(t), \;\; e(0) = e_0, \;\; t \geq 0. \quad (4)$$

The update law for $\hat{W}(t)$, $t \geq 0$, can be derived using standard Lyaupunov analysis by considering the Lyapunov function candidate

$$V(e, \tilde{W}) = \frac{1}{2}e^{\rm T}Pe + \frac{1}{2}\tilde{W}^{\rm T}\Gamma^{-1}\tilde{W}, \quad (5)$$

where $\tilde{W} \triangleq W - \hat{W}$, $\Gamma = \Gamma^{\rm T} > 0$, and $P > 0$ satisfies

$$0 = A^{\rm T}P + PA + R,$$

where $R = R^{\rm T} > 0$. Note that $V(0,0) = 0$ and $V(e,\tilde{W}) > 0$ for all $(e, \tilde{W}) \neq (0, 0)$.

Now, differentiating (5) along the trajectories of (4) yields

$$\dot{V}(e(t), \tilde{W}(t)) = -\frac{1}{2}e^{\rm T}(t)Re(t) + e^{\rm T}(t)Pb\tilde{W}^{\rm T}(t)\theta(t) - \tilde{W}^{\rm T}(t)\Gamma^{-1}\dot{\hat{W}}(t), \quad t \geq 0.$$

The standard choice of the update law is given by

$$\dot{\hat{W}}(t) = \Gamma e^{\rm T}(t)Pb\theta(t), \quad \hat{W}(0) = \hat{W}_0, \quad t \geq 0, \quad (6)$$

so that

$$\dot{V}(e(t), \tilde{W}(t)) = -\frac{1}{2}e^{\rm T}(t)Re(t) \leq 0, \quad t \geq 0, \quad (7)$$

which guarantees that the error signal $e(t)$, $t \geq 0$, and weight error $\tilde{W}(t)$, $t \geq 0$, are Lyapunov stable, and hence, are bounded for all $t \geq 0$. Since $\theta(t)$ is bounded for all $t \geq 0$, it follows from Barbalat's lemma [6] that $e(t)$ converges to zero asymptotically.

This research was supported in part by Air Force Office of Scientific Research under Grant FA9550-06-1-0240 and the National Science Foundation under Grant ECS-0601311.

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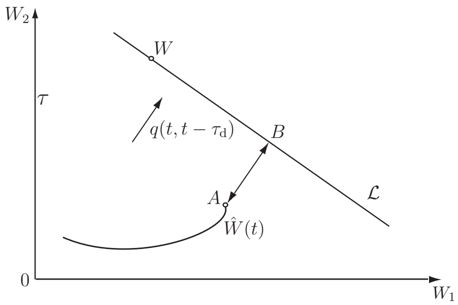

Fig. 1. Visualization of Q-modification term.

The above analysis outlines the salient features of the classical adaptive control architecture. To improve the robustness properties of the adaptive controller (3) and (6) a σ-modification term of the form $\sigma(\hat{W}-W^0)$, where $\sigma>0$ and W^0 is an approximation of the actual system parameters, can be included to the update law (6) to keep the adaptive weight (i.e., parameter estimate) $\hat{W}$ from growing without bound in the face of the system uncertainty. However, in this case, when the error $e(t)$ is small, $\dot{\hat{W}}(t)$ is dominated by $\sigma(\hat{W}-W^0)$ which causes $\hat{W}$ to be driven to W^0. If W^0 is not a good approximation of the actual system parameters W, then the system error can increase. To circumvent this problem, an e-modification term of the form $\varepsilon(e)(\hat{W}-W^0)$ (with $\varepsilon(e)=\sigma\|e\|$) can be included to the update law (6) in place of the σ-modification term. In both cases, however, the modification terms are predicated on W^0 involving a best guess for some $W\in\mathbb{R}^N$.

Next, we present a new and novel modification term that goes beyond the aforementioned modifications. Specifically, consider the error dynamics given by (4) and integrate (4) over a moving time interval $[t_{\rm d},t]$, $t\geq 0$, where $t_{\rm d}\triangleq\max\{0,\,t-\tau_{\rm d}\}$ and $\tau_d>0$ is a design parameter. Premultiplying (4) by $b^{\rm T}$ and rearranging terms yields

$$W^{\rm T}q(t,t-\tau_{\rm d})=c(t,t-\tau_{\rm d}),\quad t\geq 0,\quad \tau_{\rm d}>0, \tag{8}$$

where

$$c(t,t-\tau_{\rm d})\triangleq b^{\rm T}\left[e(t)-e(t-\tau_{\rm d})-\int_{t_{\rm d}}^{t}Ae(s){\rm d}s\right]+\int_{t_{\rm d}}^{t}\hat{W}^{\rm T}(s)\theta(s){\rm d}s,\quad t\geq 0,\quad \tau_{\rm d}>0, \tag{9}$$

and

$$q(t,t-\tau_{\rm d})\triangleq\int_{t_{\rm d}}^{t}\theta(s){\rm d}s,\quad t\geq 0,\quad \tau_{\rm d}>0. \tag{10}$$

Hence, although the vector W is *unknown*, W satisfies the *linear* equation (8). Geometrically, (8) characterizes a hyperplane in $\mathbb{R}^N$. For example, in the case where $N=2$, the hyperplane (8) is described by a line $\mathcal{L}$ with $q(t,t-\tau_{\rm d})$ being a normal vector to $\mathcal{L}$ as shown in Figure 1. Note that the distance from point A to point B shown in Figure 1, which is the shortest distance from the weight estimate $\hat{W}(t)$ to hyperplane $\mathcal{L}$ defined by (8), is given by $c(t,t-\tau_{\rm d})-\hat{W}^{\rm T}(t)q(t,t-\tau_{\rm d})$.

Next, define the error

$$\rho(\hat{W}(t),\,q(t,t-\tau_{\rm d}),\,c(t,t-\tau_{\rm d}))\triangleq\frac{1}{2}\left[\hat{W}^{\rm T}(t)q(t,t-\tau_{\rm d})-c(t,t-\tau_{\rm d})\right]^2,\quad t\geq 0, \tag{11}$$

and note that the gradient of $\rho(\hat{W}(t),\,q(t,t-\tau_{\rm d}),\,c(t,t-\tau_{\rm d}))$, $t\geq 0$, with respect to $\hat{W}(t)$, $t\geq 0$, is given by

$$\frac{\partial\rho(\hat{W}(t),\,q(t,t-\tau_{\rm d}),\,c(t,t-\tau_{\rm d}))}{\partial\hat{W}(t)}=\left[\hat{W}^{\rm T}(t)q(t,t-\tau_{\rm d})-c(t,t-\tau_{\rm d})\right]q(t,t-\tau_{\rm d}).$$

Now, consider the modified update law for the adaptive weights $\hat{W}(t)$, $t\geq 0$, given by

$$\dot{\hat{W}}(t)=\Gamma\left(e^{\rm T}(t)Pb\theta(t)+k\,Q(t)\right),\quad \hat{W}(0)=\hat{W}_0,\quad t\geq 0, \tag{12}$$

where $k>0$ and

$$Q(t)\triangleq-\left[\hat{W}^{\rm T}(t)q(t,t-\tau_{\rm d})-c(t,t-\tau_{\rm d})\right]q(t,t-\tau_{\rm d}),\quad t\geq 0.$$

In contrast to (6), the update law given by (12) contains the additional term $Q(t)$, $t\geq 0$, based on the gradient of $\rho(\hat{W}(t),\,q(t,t-\tau_{\rm d}),\,c(t,t-\tau_{\rm d}))$ with respect to $\hat{W}(t)$, $t\geq 0$. We call $Q(t)$, $t\geq 0$, a *Q-modification* term. Note that for every $t\geq 0$ the vector $Q(t)$ is directed opposite to the gradient $\frac{\partial\rho(\hat{W}(t),\,q(t,t-\tau_{\rm d}),\,c(t,t-\tau_{\rm d}))}{\partial\hat{W}(t)}$ and parallel to $q(t,t-\tau_{\rm d})$, which is a vector normal to the hyperplane defined by (8). Hence, $Q(t)$, $t\geq 0$, introduces a component in the update law (12) that drives the trajectory $\hat{W}(t)$, $t\geq 0$, in such a way so that the error given by (11) is minimized.

Note that $Q(t)$, $t\geq 0$, is zero only if $\hat{W}(t)$, $t\geq 0$, satisfies

$$\hat{W}(t)^{\rm T}q(t,t-\tau_{\rm d})=c(t,t-\tau_{\rm d}),\quad t\geq 0, \tag{13}$$

that is, the weight estimates $\hat{W}(t)$, $t\geq 0$, lie on the hyperplane defined by (8). If the weight estimates $\hat{W}(t)$, $t\geq 0$, do not satisfy (13), then $Q(t)$, $t\geq 0$, drives the trajectory $\hat{W}(t)$, $t\geq 0$, to the hyperplane defined by (8). Hence, the Q-modification term drives the trajectory of the weight estimates to the hyperplane characterized by (8) where the ideal weights W lie. As shown below, under a condition of persistent excitation, the Q-modification term also ensures the convergence of the weight estimates to the ideal weights.

Next, we establish stability guarantees of the adaptive law (3) with (12).

Theorem 2.1: Consider the uncertain dynamical system given by (4). The adaptive feedback control law (3) with update law given by (12) guarantees that the solution $(e(t),\hat{W}(t))\equiv(0,W)$ of the closed-loop system given by (4) and (12) is Lyapunov stable and $e(t)\to 0$ as $t\to\infty$ for all $e_0\in\mathbb{R}^n$ and $\hat{W}_0\in\mathbb{R}^n$.

Remark 2.1: The Q-modification term can be used to identify the ideal weights which can be used in the adaptive law. In this sense, the Q-modification architecture is reminiscent to the composite adaptation technique [3] and the combined direct and indirect adaptation technique [7]. However, the Q-modification technique markedly differs from these approaches in the manner by which the identification error is minimized.

If N time intervals $[t_i-\tau_d,\,t_i]$, $i=1,\,\ldots,\,N$, can be recorded such that the corresponding vectors $q(t_i,t_i-\tau_d)$, $i=1,\,\ldots,\,N$, given by (10) are linearly independent and

$$W^{\rm T}q(t_i,t_i-\tau_d)=c(t_i,t_i-\tau_d),\;\; t_i\geq\tau_{\rm d},\;\; i=1,\,\ldots,\,N,$$

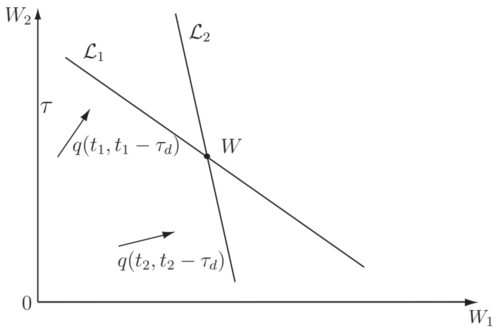

Fig. 2. Weight identification using Q-modification architecture.

where $c(t_i, t_i - \tau_d)$, $i = 1, \ldots, N$, are given by (9), then W can be identified *exactly* by solving linear equation

$$MW = c, \tag{14}$$

where

$$M = \begin{bmatrix} q^{\mathrm{T}}(t_1, t_1 - \tau_d) \\ \vdots \\ q^{\mathrm{T}}(t_N, t_N - \tau_d) \end{bmatrix}, \quad c = \begin{bmatrix} c(t_1, t_1 - \tau_d) \\ \vdots \\ c(t_N, t_N - \tau_d) \end{bmatrix}. \tag{15}$$

In the case where $N = 2$, Figure 2 shows the ideal weight W is identified as the intersection of the two hyperplanes $\mathcal{L}_1$ and $\mathcal{L}_2$ characterized by the linearly independent normal (to $\mathcal{L}_1$ and $\mathcal{L}_2$) vectors given by $q(t_1, t_1 - \tau_d)$ and $q(t_2, t_2 - \tau_d)$, respectively.

If the ideal weights can be identified, then no further adaptation is needed. In this case, we can drive the trajectory $\hat{W}(t)$, $t \geq 0$, to the point W satisfying (14) and setting $\hat{W}(t) = W$ for all $t \geq T$, where $T > \max_{i=1,\ldots,N}\{t_i\}$, so that the uncertainty $\Delta(t)$ in (1) is completely canceled by the adaptive signal $\nu_{\mathrm{ad}}(t)$ for all $t \geq T$. This, of course, corresponds to an ideal situation. Although for simple problems it may be possible to identify the ideal weights using the technique discussed above, for most problems it is difficult to find N vectors $q(t_i, t_i - \tau_d)$, $i = 1, \ldots, N$, such that the matrix M given by (15) is nonsingular and well conditioned. Hence, for such problems, we can use a moving time window to obtain information about W satisfying (8) and use this information in the adaptive law (12).

The Q-modification technique described above involves the integration of the system uncertainty. To see this, note that (8) can be rewritten as

$$\int_{t-\tau_{\mathrm{d}}}^{t} \Delta(s)\mathrm{d}s = c(t, t - \tau_{\mathrm{d}}), \quad t \geq 0,$$

where the integration is performed over a moving time window of fixed length $[t - \tau_{\mathrm{d}}, t]$, $t \geq 0$. When the system uncertainty can be perfectly parameterized as in (2), integration over the time interval $[0, t]$, $t \geq 0$, can be used instead of integration over a moving time window of fixed length. Since perfect system uncertainty parametrization eliminates approximation errors, integration over the time interval $[0, t]$, $t \geq 0$, does not introduce any distortion of the information of unknown weights W given by (8). However, in most practical problems, system uncertainty cannot be perfectly parameterized. In this case, neural networks can be used to approximate uncertain nonlinear continuous functions over a compact domain with a bounded error [1].

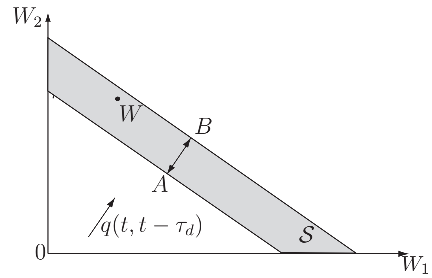

Fig. 3. Visualization of Q-modification with modeling errors.

In particular, let $\Delta(t)$, $t \geq 0$, be given by

$$\Delta(t) = W^{\mathrm{T}}\theta(t) + \varepsilon(t), \quad t \geq 0,$$

where $\varepsilon : [0, \infty] \to \mathbb{R}$ is the modeling error such that $|\varepsilon(t)| \leq \varepsilon^*$, $\varepsilon^* > 0$, for all $t \geq 0$. In this case, integration of the system uncertainty over the time interval $[0, t]$ gives

$$W^{\mathrm{T}}q(t, 0) = c(t, 0) + \int_0^t \varepsilon(s)\mathrm{d}s, \quad t \geq 0, \tag{16}$$

where the term $\int_0^t \varepsilon(s)\mathrm{d}s$ can become very large over time. Hence, (16) cannot be used effectively in the update law (12) with the appropriate modifications. Alternatively, if the system uncertainty is integrated over a moving time window $[t - \tau_{\mathrm{d}}, t]$, $t \geq 0$, then the unknown weights W satisfy

$$W^{\mathrm{T}}q(t, t - \tau_{\mathrm{d}}) = c(t, t - \tau_{\mathrm{d}}) + \int_{t-\tau_{\mathrm{d}}}^{t} \varepsilon(s)\mathrm{d}s, \quad t \geq 0, \tag{17}$$

where the term $\int_{t-\tau_{\mathrm{d}}}^{t} \varepsilon(s)\mathrm{d}s$ is bounded by $\varepsilon^*\tau_{\mathrm{d}}$. By choosing τ_{d}, one can guarantee that $\varepsilon^*\tau_{\mathrm{d}}$ is sufficiently small. Note that (17) defines a collection of parallel hyperplanes in $\mathbb{R}^N$, or a *boundary layer*, where the ideal weights lie. Figure 3 shows such a collection of hyperplanes $\mathcal{S}$ for the case where $N = 2$. Note that in Figure 3 the width of the boundary layer, that is, the distance between points A and B, is $2\tau_{\mathrm{d}}\varepsilon^*$. In the next section we consider the case of nonperfect parametrizations of the system uncertainty and show how the Q-modification technique can be used to develop static and dynamic neuroadaptive controllers using (17).

As elucidated above, the Q-modification technique is based on a gradient minimization of the error defined by (11). However, there are other error measures based on the integral of the system uncertainty that can be used. For example, define the *accumulated* error

$$\kappa(t, \hat{W}(t), q(\cdot, 0), c(\cdot, 0)) \triangleq \frac{1}{2}\int_0^t \left[\hat{W}^{\mathrm{T}}(t)q(s, 0) - c(s, 0)\right]^2 \mathrm{d}s, \quad t \geq 0.$$

The gradient of this error with respect to $\hat{W}(t)$, $t \geq 0$, is given by

$$\frac{\partial\kappa(t, \hat{W}(t), q(\cdot, 0), c(\cdot, 0))}{\partial\hat{W}(t)} = L(t, q(\cdot, 0))\hat{W}(t) - h(t, q(\cdot, 0), c(\cdot, 0)), \quad t \geq 0,$$

where

$$L(t, q(\cdot, 0)) \triangleq \int_0^t q(s, 0)q^{\mathrm{T}}(s, 0)\mathrm{d}s, \quad t \geq 0,$$

$$h(t, q(\cdot, 0), c(\cdot, 0)) \triangleq \int_0^t c(s, 0)q(s, 0)\mathrm{d}s.$$

For the statement of the next result define $\hat{L}(t) \triangleq L(t, q(\cdot,0))$, $t \geq 0$, and $\hat{h}(t) \triangleq h(t, q(\cdot,0), c(\cdot,0))$, $t \geq 0$, and consider the update law

$$\dot{\hat{W}}(t) = \Gamma\left[e^{\mathrm{T}}(t)Pb\theta(t) + k\left(\hat{h}(t) - \hat{L}(t)\hat{W}(t)\right)\right], \quad \hat{W}(0) = \hat{W}_0, \quad t \geq 0, \quad (18)$$

where $\Gamma = \Gamma^{\mathrm{T}} > 0$ and $k > 0$. Furthermore, let $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of a Hermitian matrix, respectively.

Theorem 2.2: Consider the linear uncertain dynamical system given by (4). The adaptive feedback control law (3) with update law given by (18) guarantees that the solution $(e(t), \hat{W}(t)) \equiv (0, W)$ of the closed-loop system given by (4) and (18) is Lyapunov stable and $e(t) \to 0$ as $t \to \infty$ for all $e_0 \in \mathbb{R}^n$ and $\hat{W}_0 \in \mathbb{R}^n$. Moreover, if $q(t,0)$, $t \geq 0$, is persistently excited, that is, there exists $T > 0$ such that

$$\int_t^{t+T} q(s,0)q^{\mathrm{T}}(s,0)\mathrm{d}s \geq \alpha I_N, \quad t \geq 0,$$

where I_N is the $N \times N$ identity matrix and $\alpha > 0$, then $e(t) \to 0$ and $\hat{W}(t) \to W$ exponentially as $t \to \infty$ with degree not less than

$$K = \frac{\min\{\lambda_{\min}(R),\, 2k\alpha\}}{\max\{\lambda_{\max}(P), \lambda_{\min}(\Gamma)\}}. \quad (19)$$

Next, we highlight another feature of the Q-modification technique that is useful in addressing uncertainty cancelation or suppression. Specifically, suppose that the weight estimates $\hat{W}(t)$ satisfy (13) for some $t \geq 0$ and the vector $\theta(t)$ is parallel to $q(t, t-\tau_{\mathrm{d}})$, that is, there exists $k > 0$ such that $\theta(t) = k\, q(t, t-\tau_{\mathrm{d}})$. In this case, the uncertainty $\Delta(t)$ is perfectly canceled by the adaptive signal $\nu_{\mathrm{ad}}(t)$. Using (8), it follows that

$$\begin{aligned}\Delta(t) - \nu_{\mathrm{ad}}(t) &= k\,(W - \hat{W}(t))^{\mathrm{T}} q(t, t-\tau_{\mathrm{d}}) \\ &= c(t, t-\tau_{\mathrm{d}}) - c(t, t-\tau_{\mathrm{d}}) \\ &= 0, \qquad t \geq 0.\end{aligned}$$

Since $\theta_i(t)$, $i = 1, \ldots, N$, are bounded continuous functions for all $t \geq 0$, it follows from the mean value theorem [6] that, for every $i \in \{1, \ldots, N\}$ and interval $[t_{\mathrm{d}}, t]$, $t \geq 0$, there exists $\bar{s}_i \in [t_{\mathrm{d}}, t]$ such that

$$q_i(t, t-\tau_{\mathrm{d}}) = \int_{t_{\mathrm{d}}}^{t} \theta_i(s)\mathrm{d}s = \theta_i(\bar{s}_i)\tau_{\mathrm{d}}, \quad t \geq 0.$$

Hence, for all $t \geq 0$ and each $i \in \{1, \ldots, N\}$,

$$q_i(t, t-\tau_{\mathrm{d}}) = \theta_i(t)\tau_{\mathrm{d}} + \varepsilon_i(t, \tau_{\mathrm{d}}),$$

where $\varepsilon_i(t, \tau_{\mathrm{d}}) \triangleq \tau_{\mathrm{d}}(\theta_i(\bar{s}_i) - \theta_i(t))$, or, in vector form,

$$q(t, t-\tau_{\mathrm{d}}) = \tau_{\mathrm{d}}\theta(t) + \varepsilon(t, \tau_{\mathrm{d}}), \quad t \geq 0,$$

where $\varepsilon(t, \tau_{\mathrm{d}}) \triangleq [\varepsilon_1(t, \tau_{\mathrm{d}}), \ldots, \varepsilon_N(t, \tau_{\mathrm{d}})]^{\mathrm{T}}$.

If $\hat{W}(t)$, $t \geq 0$, satisfies (13), then

$$\begin{aligned}&|\Delta(t) - \nu_{\mathrm{ad}}(t)| \\ &= \left|W^{\mathrm{T}}\theta(t) - \hat{W}(t)^{\mathrm{T}}\theta(t)\right| \\ &= \left|\frac{1}{\tau_{\mathrm{d}}}\tilde{W}^{\mathrm{T}}(t)q(t, t-\tau_{\mathrm{d}}) - \frac{1}{\tau_{\mathrm{d}}}\tilde{W}^{\mathrm{T}}(t)\varepsilon(t, \tau_{\mathrm{d}})\right| \\ &= \left|-\frac{1}{\tau_{\mathrm{d}}}\tilde{W}^{\mathrm{T}}(t)\varepsilon(t, \tau_{\mathrm{d}})\right| \\ &\leq \frac{1}{\tau_{\mathrm{d}}}\|\tilde{W}^{\mathrm{T}}(t)\|\|\varepsilon(t, \tau_{\mathrm{d}})\|, \quad t \geq 0. \quad (20)\end{aligned}$$

Now, if τ_{d} is chosen such that $\frac{1}{\tau_{\mathrm{d}}}\|\varepsilon(t, \tau_{\mathrm{d}})\|$ is sufficiently small, then it follows from (20) that $|\Delta(t) - \nu_{\mathrm{ad}}(t)|$ can be made sufficiently small regardless of the magnitude of $\|\tilde{W}(t)\|$, $t \geq 0$. Hence, the Q-modification technique, which ensures that $\hat{W}(t)$, $t \geq 0$, satisfies (13), guarantees system uncertainty suppression. Finally, note that since $\frac{1}{\tau_{\mathrm{d}}}\varepsilon(t, \tau_{\mathrm{d}}) = [\,\theta_1(\bar{s}_1) - \theta_1(t), \ldots, \theta_N(\bar{s}_N) - \theta_N(t)\,]^{\mathrm{T}}$, a choice of τ_{d} can depend on the time rate of change of $\theta(t)$.

III. Neuroadaptive Full-State Feedback Control for Nonlinear Uncertain Dynamical Systems

In this section, we consider the problem of characterizing neuroadaptive full-state feedback control laws for nonlinear uncertain dynamical systems to achieve reference model trajectory tracking. Specifically, consider the controlled nonlinear uncertain dynamical system $\mathcal{G}$ given by

$$\dot{x}(t) = A_0 x(t) + B\Lambda\left[G(x(t))u(t) + f(x(t), \hat{u}(t)) + Ax(t)\right], \quad x(0) = x_0, \quad t \geq 0, \quad (21)$$

where $x(t) \in \mathbb{R}^n$, $t \geq 0$, is the state vector, $u(t) \in \mathbb{R}^m$, $t \geq 0$, is the control input, $\hat{u}(t) \triangleq [\,u(t-\tau), u(t-2\tau), \ldots, u(t-p\tau)]$ is a vector of p-delayed values of the control input with $p \geq 1$ and $\tau > 0$ given, $A_0 \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are known matrices, $\Lambda \in \mathbb{R}^{m\times m}$ is an *unknown* positive-definite matrix, and $G : \mathbb{R}^n \to \mathbb{R}^{m\times m}$ is a known input matrix function such that $\det G(x) \neq 0$ for all $x \in \mathbb{R}^n$, $f : \mathbb{R}^n \times \mathbb{R}^{mp} \to \mathbb{R}^m$ is Lipschitz continuous on $\mathbb{R}^n \times \mathbb{R}^{mp}$ but otherwise *unknown*, and $A \in \mathbb{R}^{m\times n}$ is *unknown*. Furthermore, we assume that $x(t)$, $t \geq 0$, is available for feedback and the control input $u(\cdot)$ in (21) is restricted to the class of *admissible controls* consisting of measurable functions such that $u(t) \in \mathbb{R}^m$, $t \geq 0$.

In order to achieve trajectory tracking, we construct the reference system $\mathcal{G}_{\mathrm{ref}}$ given by

$$\dot{x}_{\mathrm{ref}}(t) = A_{\mathrm{ref}}x_{\mathrm{ref}}(t) + B_{\mathrm{ref}}r(t), \; x_{\mathrm{ref}}(0) = x_{\mathrm{ref}_0}, \; t \geq 0, \quad (22)$$

where $x_{\mathrm{ref}}(t) \in \mathbb{R}^n$, $t \geq 0$, is the reference state vector, $r(t) \in \mathbb{R}^r$, $t \geq 0$, is a bounded piecewise continuous reference input, $A_{\mathrm{ref}} \in \mathbb{R}^{n\times n}$ is Hurwitz, and $B_{\mathrm{ref}} \in \mathbb{R}^{n\times r}$. The goal here is to develop an adaptive control signal $u(t)$, $t \geq 0$, that guarantees that $\|x(t) - x_{\mathrm{ref}}(t)\| < \gamma$, $t \geq T$, where $\|\cdot\|$ denotes the Euclidean vector norm and $\gamma > 0$ is sufficiently small.

Consider the control law given by

$$u(t) = G^{-1}(x(t))(u_{\mathrm{n}}(t) + u_{\mathrm{ad}}(t)), \quad t \geq 0, \quad (23)$$

where $u_{\mathrm{n}}(t)$, $t \geq 0$, and $u_{\mathrm{ad}}(t)$, $t \geq 0$, are defined below. Using the parameterization $\Lambda = \hat{\Lambda} + \Delta\Lambda$, where $\hat{\Lambda} \in \mathbb{R}^{m\times m}$ is a known positive-definite matrix and $\Delta\Lambda \in \mathbb{R}^{m\times m}$ is an unknown symmetric matrix such that $\hat{\Lambda} + \Delta\Lambda$ is positive definite, the dynamics in (21) can be rewritten as

$$\dot{x}(t) = A_0 x(t) + B\hat{\Lambda}u_{\mathrm{n}}(t) + B\left[\hat{\Lambda}u_{\mathrm{ad}}(t) + \Lambda Ax(t) + \Lambda f(x(t), \hat{u}(t)) + \Delta\Lambda u_{\mathrm{n}}(t) + \Delta\Lambda u_{\mathrm{ad}}(t)\right], \quad x(0) = x_0, \quad t \geq 0. \quad (24)$$

The following matching conditions are needed for the main results of this section.

Assumption 3.1: There exist $K_{\mathrm{x}} \in \mathbb{R}^{m\times n}$ and $K_{\mathrm{r}} \in \mathbb{R}^{m\times r}$ such that $A_0 + B\hat{\Lambda}K_{\mathrm{x}} = A_{\mathrm{ref}}$ and $B\hat{\Lambda}K_{\mathrm{r}} = B_{\mathrm{ref}}$.

Now, let $u_{\rm n}(t)$, $t \geq 0$, in (23) be given by

$$u_{\rm n}(t) = K_{\rm x}x(t) + K_{\rm r}r(t), \quad t \geq 0. \quad (25)$$

In this case, the system dynamics (24) can be rewritten as

$$\dot{x}(t) = A_{\rm ref}x(t) + B_{\rm ref}r(t) + B\Big[\hat{\Lambda}u_{\rm ad}(t) + \Lambda Ax(t) + \Lambda f(x(t),\hat{u}(t)) + \Delta\Lambda u_{\rm n}(t) + \Delta\Lambda u_{\rm ad}(t)\Big], \quad x(0) = x_0, \quad t \geq 0. \quad (26)$$

Defining the tracking error $e(t) \triangleq x(t) - x_{\rm ref}(t)$, $t \geq 0$, the error dynamics is given by

$$\dot{e}(t) = A_{\rm ref}e(t) + B\Big[\hat{\Lambda}u_{\rm ad}(t) + \Lambda f(x(t),\hat{u}(t)) + \Lambda Ax(t) + \Delta\Lambda u_{\rm n}(t) + \Delta\Lambda u_{\rm ad}(t)\Big], \quad e(0) = e_0, \ t \geq 0, \quad (27)$$

where $e_0 \triangleq x_0 - x_{{\rm ref}_0}$. We assume that the function $f(x,\hat{u})$ can be approximated over a compact set $\mathcal{D}_x \times \mathcal{D}_{\hat{u}}$ by a linear in parameters neural network up to a desired accuracy. In this case, there exists $\hat{\varepsilon} : \mathbb{R}^n \times \mathbb{R}^{mp} \to \mathbb{R}^m$ such that $\|\hat{\varepsilon}(x,\hat{u})\| < \hat{\varepsilon}^*$ for all $(x,\hat{u}) \in \mathcal{D}_x \times \mathcal{D}_{\hat{u}}$, where $\varepsilon^* > 0$, and

$$f(x,\hat{u}) = W_f^{\rm T}\hat{\sigma}(x,\hat{u}) + \hat{\varepsilon}(x,\hat{u}), \ (x,\hat{u}) \in \mathcal{D}_x \times \mathcal{D}_{\hat{u}},$$

where $W_f \in \mathbb{R}^{s\times m}$ is an optimal *unknown* (constant) weight that minimizes the approximation error over $\mathcal{D}_x \times \mathcal{D}_{\hat{u}}$, $\hat{\sigma} : \mathbb{R}^n \times \mathbb{R}^{mp} \to \mathbb{R}^s$ is a vector of basis functions such that each component of $\hat{\sigma}(\cdot,\cdot)$ takes values between 0 and 1, and $\hat{\varepsilon}(\cdot,\cdot)$ is the modeling error. Note that s denotes the total number of basis functions or, equivalently, the number of nodes of the neural network.

Since $f(\cdot,\cdot)$ is continuous on $\mathbb{R}^n \times \mathbb{R}^{mp}$, we can choose $\hat{\sigma}(\cdot,\cdot)$ from a linear space $\mathcal{X}$ of continuous functions that forms an algebra and separates points in $\mathcal{D}_x \times \mathcal{D}_{\hat{u}}$. In this case, it follows from the Stone-Weierstrass theorem [8, p. 212] that $\mathcal{X}$ is a dense subset of the set of continuous functions on $\mathcal{D}_x \times \mathcal{D}_{\hat{u}}$. Now, as is the case in the standard neuroadaptive control literature [1], we can construct a signal involving the estimates of the optimal weights and basis functions as our adaptive control signal.

Next, define $W_1 \triangleq W_f\Lambda$, $W_2 \triangleq A^{\rm T}\Lambda$, and $W_3 \triangleq \Delta\Lambda^{\rm T}$, and let $u_{\rm ad}(t)$, $t \geq 0$, in (23) be given by

$$u_{\rm ad}(t) = -\Big[\hat{\Lambda} + \hat{W}_3^{\rm T}(t)\Big]^{-1}\Big[\hat{W}_1^{\rm T}(t)\hat{\sigma}(x(t),\hat{u}(t)) + \hat{W}_2^{\rm T}(t)x(t) + \hat{W}_3^{\rm T}(t)u_{\rm n}(t)\Big], \quad (28)$$

where $\hat{W}_1(t) \in \mathbb{R}^{s\times m}$, $t \geq 0$, $\hat{W}_2(t) \in \mathbb{R}^{n\times m}$, $t \geq 0$, and $\hat{W}_3(t) \in \mathbb{R}^{m\times m}$, $t \geq 0$, are update weights. It will be shown later (see Remark 3.1) that the adaptive weight $\hat{W}_3(t)$ is such that $\Big[\hat{\Lambda} + \hat{W}_3^{\rm T}(t)\Big]^{-1}$ exists for all $t \geq 0$. Next, define $W \triangleq \big[W_1^{\rm T} \ W_2^{\rm T} \ W_3^{\rm T}\big]^{\rm T} \in \mathbb{R}^{(s+n+m)\times m}$, $\hat{W}(t) \triangleq \Big[\hat{W}_1^{\rm T}(t) \ \hat{W}_2^{\rm T}(t) \ \hat{W}_3^{\rm T}(t)\Big]^{\rm T} \in \mathbb{R}^{(s+n+m)\times m}$, $t \geq 0$, and $\tilde{W}(t) \triangleq W - \hat{W}(t)$, and note that, using (28), the error dynamics (27) can be rewritten as

$$\dot{e}(t) = A_{\rm ref}e(t) + B\tilde{W}^{\rm T}(t)\sigma(x(t),\hat{u}(t),v(t)) + \varepsilon(x(t),\hat{u}(t)), \quad e(0) = e_0, \quad t \geq 0, \quad (29)$$

where $\sigma(x,\hat{u},v) \triangleq \big[\hat{\sigma}^{\rm T}(x,\hat{u}), \ x^{\rm T}, \ v^{\rm T}\big]^{\rm T}$, $v \triangleq u_{\rm n} + u_{\rm ad}$, and $\varepsilon(x,\hat{u}) \triangleq B\Lambda\hat{\varepsilon}(x,\hat{u})$.

Next, we develop a neuroadaptive control architecture which involves additional terms in the update laws that are predicated on auxiliary terms involving an estimate of the unknown weights W_1, W_2, and W_3. In particular, by integrating the error dynamics (29) over the moving time interval $[t_{\rm d}, t]$, where $t_{\rm d} = \max\{0, t - \tau_{\rm d}\}$ and $\tau_{\rm d} > 0$ is a design parameter, we obtain

$$BW^{\rm T}q(t,t-\tau_{\rm d}) = c(t,t-\tau_{\rm d}) + \delta(t,t-\tau_{\rm d}), \ t \geq 0, \quad (30)$$

where

$$q(t,t-\tau_{\rm d}) \triangleq \int_{t_{\rm d}}^{t}\sigma(x(\xi),\hat{u}(\xi),v(\xi)){\rm d}\xi,$$

$$c(t,t-\tau_{\rm d}) \triangleq e(t) - e(t_{\rm d}) - \int_{t_{\rm d}}^{t}A_{\rm ref}e(\xi){\rm d}\xi + \int_{t_{\rm d}}^{t}\hat{W}^{\rm T}(t)\sigma(x(\xi),\hat{u}(\xi),v(\xi)){\rm d}\xi,$$

$$\delta(t,t-\tau_{\rm d}) \triangleq \int_{t_{\rm d}}^{t}\varepsilon(x(\xi),\hat{u}(\xi)){\rm d}\xi.$$

Note that $q(t,t-\tau_{\rm d})$ and $c(t,t-\tau_{\rm d})$ are computable, whereas $\delta(t,t-\tau_{\rm d})$ is *unknown*.

Next, choose $\tau_{\rm d}$ such that $\|q(t,t-\tau_{\rm d})\| \leq q_{\max}$ and $\|c(t,t-\tau_{\rm d})\| \leq c_{\max}$ for all $t \geq 0$. Now, using (30) it follows that for every $k > 0$ and $\Gamma = \Gamma^{\rm T} > 0$,

$$\begin{aligned}
&{\rm tr}\,\tilde{W}^{\rm T}(t)\Gamma^{-1}\Big[k\,\Gamma q(t,t-\tau_{\rm d})\Big(B\hat{W}^{\rm T}(t)q(t,t-\tau_{\rm d}) - c(t,t-\tau_{\rm d})\Big)^{\rm T}B\Big] \\
&= k\,{\rm tr}\Big[B\tilde{W}^{\rm T}(t)q(t,t-\tau_{\rm d})\Big(B\hat{W}^{\rm T}(t)q(t,t-\tau_{\rm d}) - c(t,t-\tau_{\rm d})\Big)^{\rm T}\Big] \\
&= -k\,\|B\hat{W}^{\rm T}(t)q(t,t-\tau_{\rm d}) - c(t,t-\tau_{\rm d})\|^2 + k\Big(B\hat{W}^{\rm T}(t)q(t,t-\tau_{\rm d}) - c(t,t-\tau_{\rm d})\Big)^{\rm T}\delta(t,t-\tau_{\rm d}) \\
&\leq -k\,\|B\hat{W}^{\rm T}(t)q(t,t-\tau_{\rm d}) - c(t,t-\tau_{\rm d})\|^2 + k(\|B\|'\hat{W}_{\max}q_{\max} + c_{\max})\|B\Lambda\|'\hat{\varepsilon}^*\tau_{\rm d}, \ t \geq 0, \quad (31)
\end{aligned}$$

where $\|\cdot\|' : \mathbb{R}^{n\times m} \to \mathbb{R}$ is the matrix norm induced by the vector norms $\|\cdot\|'' : \mathbb{R}^n \to \mathbb{R}$ and $\|\cdot\|''' : \mathbb{R}^m \to \mathbb{R}$, and $\hat{W}_{\max}$ is a norm bound imposed on $\hat{W}(t)$, $t \geq 0$. Next, define the Q-modification term $Q(t)$ by

$$Q(t) = \begin{bmatrix} Q_1(t) \\ Q_2(t) \\ Q_3(t) \end{bmatrix} \triangleq q(t,t-\tau_{\rm d})\Big[B\hat{W}^{\rm T}(t)q(t,t-\tau_{\rm d}) - c(t,t-\tau_{\rm d})\Big]^{\rm T}B, \quad t \geq 0, \quad (32)$$

where for $t \geq 0$, $Q(t) \in \mathbb{R}^{(s+n+m)\times m}$, $Q_1(t) \in \mathbb{R}^{s\times m}$, $Q_2(t) \in \mathbb{R}^{n\times m}$, and $Q_3(t) \in \mathbb{R}^{m\times m}$.

For the statement of next result, define the projection operator ${\rm Proj}(\tilde{W}, Y)$ given by

$${\rm Proj}(\tilde{W},Y) \triangleq \begin{cases} Y, & \text{if } \mu(\tilde{W}) < 0, \\ Y, & \text{if } \mu(\tilde{W}) \geq 0 \text{ and } \mu'(\tilde{W})Y \leq 0, \\ Y - \frac{\mu'^{T}(\tilde{W})\mu'(\tilde{W})Y}{\mu'(\tilde{W})\mu'^{T}(\tilde{W})}\mu(\tilde{W}), & \text{otherwise}, \end{cases}$$

where $\tilde{W} \in \mathbb{R}^{s\times m}$, $Y \in \mathbb{R}^{n\times m}$, $\mu(\tilde{W}) \triangleq \frac{{\rm tr}\,\tilde{W}^{\rm T}\tilde{W} - \tilde{w}_{\max}^2}{\varepsilon_{\tilde{W}}}$, $\tilde{w}_{\max} \in \mathbb{R}$ is the norm bound imposed on $\tilde{W}$, and $\varepsilon_{\tilde{W}} > 0$.

Consider the feedback controller (23) with $u_{\rm n}(t)$ and $u_{\rm ad}(t)$ given by (25) and (28), respectively, and update laws given by

$$\dot{\hat{W}}_1(t) = \Gamma_1 \mathrm{Proj}[\hat{W}_1(t), \hat{\sigma}(x(t), \hat{u}(t))e^{\rm T}(t)PB - k\,h(\hat{W}(t))Q_1(t)],\ \hat{W}_1(0) = \hat{W}_{10},\ t \geq 0, \quad (33)$$

$$\dot{\hat{W}}_2(t) = \Gamma_2 \mathrm{Proj}[\hat{W}_2(t), x(t)e^{\rm T}(t)PB - k\,h(\hat{W}(t))Q_2(t)], \quad \hat{W}_2(0) = \hat{W}_{20}, \quad (34)$$

$$\dot{\hat{W}}_3(t) = \Gamma_3 \mathrm{Proj}[\hat{W}_3(t), v(t)e^{\rm T}(t)PB - k\,h(\hat{W}(t))Q_3(t)], \quad \hat{W}_3(0) = \hat{W}_{30}, \quad (35)$$

where $\Gamma_1 \in \mathbb{R}^{s\times s}$, $\Gamma_2 \in \mathbb{R}^{n\times n}$, and $\Gamma_3 \in \mathbb{R}^{m\times m}$ are positive-definite matrices, $P \in \mathbb{R}^{n\times n}$ is a positive-definite solution of the Lyapunov equation

$$0 = A_{\rm ref}^{\rm T}P + PA_{\rm ref} + R, \quad (36)$$

where $R > 0$, $k > 0$, $Q_1(t)$, $Q_2(t)$, $Q_3(t)$, $t \geq 0$, are given by (32), and $h : \mathbb{R}^{(s+n+m)\times m} \to \mathbb{R}$ is a bounded nonnegative function taking values between 0 and 1 such that if $\mathrm{tr}\,\hat{W}_i^{\rm T}(t)\hat{W}_i(t) = \hat{w}_{i\,\max}^2$, for $i = 1, 2$, or 3, then $h(\hat{W}(t)) = 0$, where $\hat{w}_{i\,\max}^2$ are the norm bounds imposed on $\hat{W}_i(t)$, $i = 1, 2, 3$, $t \geq 0$.

Theorem 3.1: Consider the nonlinear uncertain dynamical system $\mathcal{G}$ given by (21) with $u(t)$, $t \geq 0$, given by (23) and reference model given by (22) with tracking error dynamics given by (29). Assume Assumption 3.1 holds. Then there exists a compact positively invariant set $\mathcal{D}_\alpha \subset \mathbb{R}^n \times \mathbb{R}^{s\times m} \times \mathbb{R}^{n\times m} \times \mathbb{R}^{m\times m}$ such that $(0, W_1, W_2, W_3) \in \mathcal{D}_\alpha$, where $W_1 \in \mathbb{R}^{s\times m}$, $W_2 \in \mathbb{R}^{n\times m}$, and $W_2 \in \mathbb{R}^{m\times m}$, and the solution $(e(t), \hat{W}_1(t), \hat{W}_2(t), \hat{W}_3(t))$, $t \geq 0$, of the closed-loop system given by (29) and (33)–(35) is ultimately bounded for all $(e(0), \hat{W}_1(0), \hat{W}_2(0), \hat{W}_3(0)) \in \mathcal{D}_\alpha$ with ultimate bound $\|e(t)\| < \gamma$, $t \geq T$, where

$$\gamma > \Big[(\rho + \sqrt{\rho^2+\nu})^2 + \lambda_{\max}(\Gamma_1^{-1})\hat{w}_{1\max}^2 + \lambda_{\max}(\Gamma_2^{-1})\hat{w}_{2\max}^2 + \lambda_{\max}(\Gamma_3^{-1})\hat{w}_{3\max}^2\Big]^{\frac{1}{2}}, \quad (37)$$

$$\rho \triangleq \lambda_{\min}^{-1}(R)\|PB\Lambda\|'\hat{\varepsilon}^*, \quad (38)$$

$$\nu \triangleq 2k\lambda_{\min}^{-1}(R)\,(\|B\|'\hat{W}_{\max}q_{\max} + c_{\max})\|B\Lambda\|'\hat{\varepsilon}^*\tau_{\rm d}, \quad (39)$$

$\hat{w}_{i\max}$, $i = 1, 2, 3$, are norm bounds imposed on $\hat{W}_i$, and $P \in \mathbb{R}^{n\times n}$ is the positive-definite solution of the Lyapunov equation (36).

Remark 3.1: Note that since $e(t)$, $t \geq 0$, and $x_{\rm ref}(t)$, $t \geq 0$, are bounded, it follows that $x(t)$, $t \geq 0$, is bounded, and hence, $u_{\rm n}(t)$, $t \geq 0$, given by (25) is bounded. Furthermore, since $\hat{W}_3(t)$ is bounded for all $t \geq 0$, it is always possible to choose $\hat{\Lambda}$ and $\hat{w}_{3\max}^2$ so that $\left[\hat{\Lambda} + \hat{W}_3^{\rm T}(t)\right]^{-1}$ exists and is bounded for all $t \geq 0$. This follows from the fact that for any two square matrices A and B, $\det(A + B) \neq 0$ if and only if there exists $\alpha > 0$ such that $\sigma_{\min}(A) > \alpha$ and $\sigma_{\max}(B) \leq \alpha$. Hence, it follows that for $A = \hat{\Lambda}$ and $B = \hat{W}_3^{\rm T}(t)$, $t \in [0,\infty)$, $\left[\hat{\Lambda} + \hat{W}_3^{\rm T}(t)\right]^{-1}$ exists for all $t \geq 0$ if $\hat{w}_{3\max}^2$ is sufficiently small. Hence, the adaptive signal $u_{\rm ad}(t)$, $t \geq 0$, given by (28) is bounded. Since $u_{\rm n}(t)$, $t \geq 0$, and $u_{\rm ad}(t)$, $t \geq 0$ are bounded, and $\det G(x) \neq 0$ for all $x \in \mathbb{R}^n$, it follows that control input $u(t)$, $t \geq 0$, given by (23) is bounded for all $t \geq 0$.

Remark 3.2: The Q-modification term defined by (32) is similar to the modification terms appearing in the update laws for composite adaptive control discussed in [3]. The key difference, however, is that the two approaches use different signals. Specifically, in the proposed Q-modification framework, the additional terms appearing in the update laws are constructed using a moving window of the integrated system uncertainty, whereas in composite adaptive control the update laws involve filtered versions of the control input and the system state.

Remark 3.3: It is straightforward to show that the Q-modification framework can be incorporated within a radial basis function neural network-based adaptive controller and combined with the robust adaptive control laws discussed in [2], such as σ- or e-modifications.

Remark 3.4: Note that the Q-modification terms in the update laws (33)-(35) drive the trajectories of the neural network weights to a collection of hyperplanes characterized by (30) involving the unknown neural network weights. It can be shown that in the case where $\hat{\varepsilon}(x, \hat{u}) \equiv 0$ and $\sigma(x, \hat{u}, u)$ is persistently excited, that is,

$$\int_t^{t+T} \sigma(x(s), \hat{u}(s), v(s))\sigma^{\rm T}(x(s), \hat{u}(s), v(s))\mathrm{d}s \geq \alpha I_{s+n+m}, \qquad t \geq 0,$$

where $\alpha > 0$, the neural network weight estimates $\hat{W}(t)$ converge to the ideal weights W.

Remark 3.5: Finally, it is important to note that the Q-modification terms appearing in (33)-(35) are different from the e- and σ-modification terms presented in the literature [2].

IV. Conclusion

In this paper we developed a new neuroadaptive control architecture for nonlinear uncertain systems. The proposed framework involves a novel controller architecture involving additional terms in the update laws that can identify ideal system weights and effectively suppress system uncertainty. Extensions of the Q-modification technique to general nonlinear dynamical systems with nonlinear uncertainty parameterizations and output feedback are addressed in [5].

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