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Receding Horizon Rank Minimization Based Estimation with Applications to Visual Tracking

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Abstract— **This paper addresses the problem of predicting future outputs of an unknown Linear Time Invariant System based solely on past input/output data corrupted by noise, and an a-priori bound on the system order. This situation arises in many scenarios of practical interest where an explicit a-priori model of the system is not available. The main result of the paper is a simple, computationally efficient tracking algorithm that does not entail identifying first the unknown dynamics. Rather, the problem of estimating the next value of the output is recast into a rank minimization problem and solved using some recently introduced convex relaxations. The potential of the proposed approach is illustrated using as an example the problem of tracking multiple targets in video sequences in the presence of occlusion.**

I. INTRODUCTION

A situation commonly arising in many practical applications involves predicting future outputs of an unknown linear, possibly slowly time-varying, plant based on noisy past input/output observations. In the case of LTI dynamics this problem can be solved using a two-tiered approach where a suitable dynamics is identified first and then propagated using a standard filter. However, extending this approach to the case of slowly varying dynamics requires an on-line implementation–either re-identifying the plant at each instant or performing on-line model (in)validation and re-identifying only when necessary–which could be problematic given the relatively high computational complexity entailed in both processes. Finally, in addition to divergence problems that could arise from errors in estimating the dynamics, Kalman-filtered based approaches can fail (e.g. lead to unbounded error covariance) in the presence of intermittent observations [22]. This effect can be mitigated by resorting to a Receding Horizon based approach [23], but this further increases the computational complexity and does not address divergence issues due to miss-identified dynamics.

To avoid these difficulties, in this paper, motivated by earlier work on subspace identification [18], [20], rank-minimization based track matching [4] and receding horizon based estimation ([17], [16], [9], [1] and references therein), we propose a simple approach for interpolating/extrapolating trajectories of a (piecewise) linear plant that does not necessitate identifying first the dynamics of the plant. Instead, the main idea is to recast the problem into a rank minimization form, where unknown data points are estimated by minimizing on-line the rank of a Hankel matrix constructed from the past n available measurements. In turn, this rank minimization problem is solved by using the convex relaxation proposed in [6]. Since the rank of the Hankel matrix is an estimate of the order of the underlying dynamical system, intuitively this approach amounts to adding the new data in such a way that the *complete* trajectory can be explained by the same model that explains the existing measurements. As we show in the paper, in the case of noiseless measurements, under mild assumptions this approach indeed leads to the correct values for the missing data. Measurements corrupted by unknown-but-bounded noise can be easily handled by simply adding more variables and convex constraints to the optimization. Finally, since the optimization is carried out based on a sliding window, it automatically accommodates (slowly) time varying dynamics.

In the second portion of the paper we illustrate these results in a problem that has been the object of considerably attention in the computer vision community: tracking multiple objects in a sequence of frames in the presence of occlusion. During the past decade extensive research has been carried out in this area, leading to several techniques (see for instance [2], [10], [11], [21], [24], [13] and references therein). In particular, a class of dynamics based trackers has been developed that combine a-priori assumed dynamic models of the target motion with optimal filtering –(unscented) Kalman, particle– [19], [14], [15] to track in the presence of occlusion. While successful in many scenarios, a mismatch between the actual and assumed dynamics can result in divergence of the estimates, leading to tracking failure in the presence of occlusion. As shown in [3], this effect can be avoided by using the two tiered approach –identification followed by filtering– mentioned above. Further, implementing this approach on–line and updating the models as needed can, in principle, accommodate slowly time varying dynamics. However, since the entailed computational complexity of this step is not small, it may fail in the case of targets with moderately fast dynamics. Finally, identifying the dynamics of the plant requires the availability of a sufficiently long unoccluded trajectory during the training phase. On the other hand, as we illustrate with several examples, using the rank minimization based estimators proposed in this paper leads to trackers capable of handling fragmented trajectories, slowly

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varying dynamics and substantial occlusion and clutter.

II. Notation and Background Results

$\mathbf{x}$	real–valued vector.
x_k	k^{th} element of a vector $\mathbf{x}$.
$\mathbf{A}^T$	conjugate transpose of matrix $\mathbf{A}$.
$\mathbf{A} > 0$	$\mathbf{A} = \mathbf{A}^T$ is positive definite.
$\mathbf{A} < (\leq)\mathbf{B}$	$(\mathbf{A} - \mathbf{B}) < (\leq)0$
$\sigma_i(\mathbf{A})$	i^{th} singular value of matrix $\mathbf{A}$.
$\overline{\sigma}(\mathbf{A})$	maximum singular value of $\mathbf{A}$.
$\underline{\sigma}(\mathbf{A})$	minimum singular value of $\mathbf{A}$.
rank($\mathbf{A}$)	rank of matrix $\mathbf{A}$.
Tr($\mathbf{A}$)	trace of matrix $\mathbf{A}$.
card($\mathcal{I}$)	cardinality of the set $\mathcal{I}$.

III. Statement of the problem

In the sequel we consider (unknown) single input single output linear shift invariant plants with McMillan degree n:

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}\mathbf{x}_k + \mathbf{B}u_k \\ \zeta_k &= \mathbf{C}\mathbf{x}_k \\ y_k &= \mathbf{C}\mathbf{x}_k + v_k \end{aligned} \tag{1}$$

where $\mathbf{x} \in R^n$, u, ζ and y represent the states, inputs, outputs and measurements corrupted by noise v, respectively, and where the realization (A, B, C) is minimal. Alternatively, we will also represent the system by its transfer function:

$$\begin{aligned} \zeta(z) &= G(z)u(z) \\ G(z) &\doteq \frac{\sum_{i=0}^{n} b_i z^{-i}}{1 + \sum_{i=1}^{n} a_i z^{-i}} \end{aligned} \tag{2}$$

In its simplest form, the problem addressed in this paper can be stated as:

Problem 1: Given:

1) *a priori* information consisting of a set membership description of the measurement noise $v \in \mathcal{N}$ and an upper bound N of n.
2) *a posteriori* experimental information consisting of n_m input/output measurements $\{u_i, y_i\}_{i=k-n_m+1}^{k}$,

estimate the value of the ζ_i, $i = k+1-n_m, \ldots, k+1$. That is, we want to predict the next value of the output ζ and estimate its past n_m values based on the noisy measurements y_i. Variations of this problem that will be discussed later include multiple steps ahead prediction and data interpolation, e.g. the case where the goal is to estimate ζ_k, $k_\ell \leq k \leq k_u$ based on measurements y_i, $i = k_\ell - 1, \ldots, k_\ell - n_1$ and $i = k_u + 1, \ldots, k_u + n_2$, with $n_1 + n_2 = n_m$.

IV. Reducing the problem to rank minimization

Next, we show how Problem 1 can be reduced to a rank minimization form. We begin by introducing a result that provides the theoretical underpinning of the proposed algorithm.

Proposition 1: Given an input sequence $\{u_k\}$, $k = 0, \ldots$, denote by $u(z) \doteq \frac{r_u(z)}{d_u(z)}$ its corresponding z–transform, and assume that $d_u(z)$ has degree $n_u \leq n$ and that there are no pole/zero cancellations between $u(z)$ and $G(z)$. Then, given $2n_u + n + 1$ consecutive values of the input $\{u_i\}_{i=1}^{n+2n_u+1}$ and the first $2n + n_u$ values of the corresponding output $\{\zeta_i\}_{i=1}^{2n+n_u}$, the output value ζ_{2n+n_u+1} is the unique solution to the following rank minimization problem:

$$\zeta_{2n+n_u+1} = \underset{x}{argmin} \left\{ \text{rank} \left[\mathbf{H}_{(n+1,n_u)}(x) \right] \right\} \tag{3}$$

where

$$\mathbf{H}_{n+1,n_u}(x) \doteq \begin{bmatrix} \mathbf{H}_\zeta \\ \mathbf{H}_u \end{bmatrix}$$

$$\begin{aligned} \mathbf{H}_\zeta &\doteq \begin{bmatrix} \zeta_1 & \zeta_2 & \cdots & \zeta_{n+n_u+1} \\ \zeta_2 & \zeta_3 & \cdots & \zeta_{n+n_u+2} \\ \vdots & \vdots & \ddots & \vdots \\ \zeta_{n+1} & \zeta_{n+2} & \cdots & x \end{bmatrix} \\ \mathbf{H}_u &\doteq \begin{bmatrix} u_1 & u_2 & \cdots & u_{n+n_u+1} \\ u_2 & u_3 & \cdots & u_{n+n_u+2} \\ \vdots & \vdots & \ddots & \vdots \\ u_{n_u} & u_{n_u+1} & \cdots & u_{n+2n_u+1} \end{bmatrix} \end{aligned} \tag{4}$$

Proof: Consider the Hankel matrix

$$\widetilde{\mathbf{H}}_{(r,s)} \doteq \begin{bmatrix} \widetilde{\mathbf{H}}^\zeta_{(r,s)} \\ \widetilde{\mathbf{H}}^u_{(r,s)} \end{bmatrix}$$

where

$$\begin{aligned} \widetilde{\mathbf{H}}^\zeta_{(r,s)} &\doteq \begin{bmatrix} \zeta_1 & \zeta_2 & \cdots & \zeta_{r+s} \\ \zeta_2 & \zeta_3 & \cdots & \zeta_{r+s+1} \\ \vdots & \vdots & \ddots & \vdots \\ \zeta_r & \zeta_{r+1} & \cdots & \zeta_{2r+s-1} \end{bmatrix} \\ \widetilde{\mathbf{H}}^u_{(r,s)} &\doteq \begin{bmatrix} u_1 & u_2 & \cdots & u_{r+s} \\ u_2 & u_3 & \cdots & u_{r+s+1} \\ \vdots & \vdots & \ddots & \vdots \\ u_r & u_{r+1} & \cdots & u_{2r+s-1} \end{bmatrix} \end{aligned} \tag{5}$$

Note that for each k, the vector $(\zeta_k \; u_k)^T$ corresponds to the k^{th} element of the impulse response of a single input, two output system with transfer matrix

$$G(\zeta, u) \doteq \begin{bmatrix} G(z)u(z) \\ u(z) \end{bmatrix}$$

Since by assumption $G(z)$ and $u(z)$ have degrees n and n_u respectively and there are no pole/zero cancellations between G and u, it follows that $\text{rank}[\widetilde{\mathbf{H}}_{(n,n_u)}] = n + n_u$. Since for any choice of ζ_{2n+n_u+1}, $\widetilde{\mathbf{H}}_{(n,n_u)}$ is a a submatrix of $\widetilde{\mathbf{H}}_{(n+1,n_u)}$, it follows that the latter also has rank greater than or equal to $n + n_u$. Finally, since $u(z)$ has degree n_u, it follows that the first n_u rows of $\widetilde{\mathbf{H}}^u_{(n+1,n_u)}$ are linearly independent and, from Cayley–Hamilton theorem, its last $n - n_u$ rows can be written as a linear combination of these first n_u rows. Thus we have that:

$$\begin{aligned} n^o &\doteq \min_x \left\{ \text{rank}[\mathbf{H}_{(n+1,n_u)}(x)] \right\} \\ &= \min_x \left\{ \text{rank}[\widetilde{\mathbf{H}}_{(n+1,n_u)}(x)] \right\} \\ &\geq \text{rank}[\widetilde{\mathbf{H}}_{(n,n_u)}] = n + n_u \end{aligned} \tag{6}$$

To show that $n^o = n+n_u$, simply set $x^o = \zeta_{2n+n_u+1}$. From (2) we have that

$$\zeta_{n+k} + \sum_{i=1}^{n} a_i \zeta_{n+k-i} - \sum_{i=0}^{n} b_i u_{n+k-i} = 0,\ k = 1, 2, \ldots \quad (7)$$

Hence $n(x^o) \doteq \text{rank}[\widetilde{\mathbf{H}}_{n+1,n_u}(x^o)] \leq n + n_u$ since its $n+1$ row can be written as a linear combination of the other rows. The equality $n^o = n + n_u$ follows now from (6). Finally, uniqueness of the minimizer follows from the fact that since $\widetilde{\mathbf{H}}_{n,n_u}$ has rank $n + n_u$, the coefficients of this linear combination are unique. ■

Remark 1: By simply proceeding in a sequential fashion, it can be easily shown that the results above hold for the case of multiple step ahead predictions, that is, $\zeta_k,\ k = 2n+n_u+1, \ldots, 2n+n_u+n_p$ can be obtained by minimizing the rank of the associated Hankel matrix. A similar reasoning shows that missing data can be correctly interpolated via rank-minimization, provided that enough contiguous data is available so that the corresponding Hankel submatrix has full rank. If this condition fails, then Hankel rank minimization amounts to interpolating the data using the lowest order interpolant that is consistent with the available data.

V. Receding Horizon Predictors/Interpolators

The results of section IV show that both future and missing values of the data can be obtained, provided that enough data points are available, by minimizing the rank of a Hankel matrix. This suggest the following (conceptual) Receding Horizon type algorithm:

Algorithm 1: (Conceptual) Rank Minimization Based Prediction/Interpolation

Input at time k: N_h: Horizon length; $\mathcal{I}_a \subseteq [k - N_h, k]$: set of indices of available measurements (with $\text{card}(\mathcal{I}_a) \geq n$); $\mathcal{I}_e \subseteq [k - N_h, k + 1]$: set of indices of data to be estimated, with $\mathcal{I}_a \cup \mathcal{I}_e = \mathcal{I} \doteq [k - N_h, k + 1]$; input/ output data ζ_ℓ, $\ell \in \mathcal{I}_a$, u_ℓ, $\ell \in \mathcal{I}$
Output: Estimates $\hat{\zeta}_\ell$ of ζ_ℓ, $\forall \ell \in \mathcal{I}_e$

1. Let ζ^* denote the following sequence, where x are free variables: $\zeta_i^* = \begin{cases} \zeta_i \text{ if } i \in \mathcal{I}_a \\ x_i \text{ if } i \in \mathcal{I}_e \end{cases}$
and form the matrix $\mathbf{H}(x) \doteq \begin{bmatrix} \mathbf{H}_\zeta \\ \mathbf{H}_u \end{bmatrix}$ where:

$$\mathbf{H}_\zeta \doteq \begin{bmatrix} \zeta^*_{i_1} & \zeta^*_{i_2} & \cdots & \zeta^*_{i_{n+n_u+1}} \\ \zeta^*_{i_2} & \zeta^*_{i_3} & \cdots & \zeta^*_{i_{n+n_u+2}} \\ \vdots & \vdots & \ddots & \vdots \\ \zeta^*_{i_{n+1}} & \zeta^*_{i_{n+2}} & \cdots & \zeta^*_{i_{2n+n_u+1}} \end{bmatrix}$$

$$\mathbf{H}_u \doteq \begin{bmatrix} u_{i_1} & u_{i_2} & \cdots & u_{i_{n+n_u+1}} \\ u_{i_2} & u_{i_3} & \cdots & u_{i_{n+n_u+2}} \\ \vdots & \vdots & \ddots & \vdots \\ u_{i_{n_u}} & u_{i_{n_u+1}} & \cdots & u_{i_{n+2n_u+1}} \end{bmatrix}$$

2. Estimate the missing data by solving:
$\hat{\zeta}_\ell = \underset{x_i,\ i \in \mathcal{I}_e}{argmin} \{rank\,[\mathbf{H}(x)]\}$

The algorithm above is a conceptual, rather than a practical one since it takes into account neither the presence of measurement noise nor the fact that rank minimization problems are generically NP-hard. The latter issue can be solved by replacing the rank minimization by the convex relaxation introduced by Fazel *et. al.* [7], [6], [5], [8], while (bounded) noise can be taken into account by replacing ζ_k in $\mathbf{H}_\zeta$ by $\zeta_k + v_k$, subject to a constraint of the form $v_k \in \mathcal{N}$. These considerations lead to the following algorithm.

Algorithm 2: Receding Horizon Rank Minimization Based Prediction/Interpolation

Input at time k: N_h: Horizon length; $\mathcal{I}_a \subseteq [k - N_h, k]$: set of indices of available measurements (with $\text{card}(\mathcal{I}_a) \geq n$); $\mathcal{I}_e \subseteq [k - N_h, k + 1]$: set of indices of data to be estimated; with $\mathcal{I}_a \cup \mathcal{I}_e = \mathcal{I}$; input/output data $y_\ell,\ \ell \in \mathcal{I}_a,\ u_\ell,\ \ell \in \mathcal{I}$; set membership description of the noise $v \in \mathcal{N}$.
Output: Estimates $\hat{\zeta}_\ell$ of ζ_ℓ, $\forall \ell \in \mathcal{I}_e \cup \mathcal{I}_a$

1. Let ζ^* denote the following sequence:
$\zeta_i^* = \begin{cases} y_i - v_i \text{ if } i \in \mathcal{I}_a \\ x_i \text{ if } i \in \mathcal{I}_e \end{cases}$ where v, x are free variables, and form the matrix
$\mathbf{H}(x, v) \doteq \begin{bmatrix} \mathbf{H}_\zeta \\ \mathbf{H}_u \end{bmatrix}$ where

$$\mathbf{H}_\zeta \doteq \begin{bmatrix} \zeta^*_{i_1} & \zeta^*_{i_2} & \cdots & \zeta^*_{i_{n+n_u+1}} \\ \zeta^*_{i_2} & \zeta^*_{i_3} & \cdots & \zeta^*_{i_{n+n_u+2}} \\ \vdots & \vdots & \ddots & \vdots \\ \zeta^*_{i_{n+1}} & \zeta^*_{i_{n+2}} & \cdots & \zeta^*_{i_{2n+n_u+1}} \end{bmatrix}$$

$$\mathbf{H}_u \doteq \begin{bmatrix} u_{i_1} & u_{i_2} & \cdots & u_{i_{n+n_u+1}} \\ u_{i_2} & u_{i_3} & \cdots & u_{i_{n+n_u+2}} \\ \vdots & \vdots & \ddots & \vdots \\ u_{i_{n_u}} & u_{i_{n_u+1}} & \cdots & u_{i_{n+2n_u+1}} \end{bmatrix}$$

2. (approximately) minimize rank$[\mathbf{H}(x, v)]$ by solving the following convex problem in $x,\ v,\ \mathbf{R},\ \mathbf{S}$:
minimize $Tr(\mathbf{R}) + Tr(\mathbf{S})$
subject to $\begin{bmatrix} \mathbf{R} & \mathbf{H}(x) \\ \mathbf{H}(x)^T & \mathbf{S} \end{bmatrix} \geq 0$
subject to: $\{v_\ell\} \in \mathcal{N}$.
3. Estimate/predict the output ζ_ℓ from the noisy measurements y_ℓ by:
$\hat{\zeta}_i = \begin{cases} y_i - v_i \text{ if } i \in \mathcal{I}_a \text{ (estimation)} \\ x_i \text{ if } i \in \mathcal{I}_e \text{ (interpolation/prediction)} \end{cases}$

In the case where $\mathcal{N}$ admits an LMI description, then step 2 reduces to an LMI optimization problem. Examples of these descriptions are balls in ℓ_∞, e.g. $\mathcal{N} \doteq \{v\colon |v_k| \leq \epsilon\}$ or constraints on the norm of $\mathbf{H}_v$, the Hankel matrix of the noise sequence. Note that the (i,j) element of $\mathbf{H_v^T H_v}$ is given by:

$$\mathbf{H_v^T H_v}(i,j) = \sum_{r=1}^{n_v} v(i+r)v(j+r)$$

Hence, under mild ergodicity assumptions $\mathbf{H_v^T H_v}$ is an estimate of the noise covariance matrix. Thus constraints on $\|\mathbf{H}_v\|$ are (approximately) equivalent to constraints on the magnitude of the noise covariance.

Remark 2: Intuitively, the algorithm above attempts to predict the missing data by finding the lowest order system that interpolates the given data within the noise levels. Since $\mathbf{H}_y = \mathbf{H}_\zeta + \mathbf{H}_v$, it follows that $\underline{\sigma}(\mathbf{H}_\zeta) \geq \underline{\sigma}(\mathbf{H}_y) - \overline{\sigma}(\mathbf{H}_v)$. Hence, in cases where the Hankel matrix of the measured data satisfies $\underline{\sigma}(\mathbf{H}_y) \geq \overline{\sigma}(\mathbf{H}_v)$ (essentially an empirical observability/controllability condition), then the data is interpolated using a model with the same order as the (unknown) system generating the data.

VI. Application: Multiframe Tracking

In this section we illustrate the application of the proposed receding horizon filters to the problem of tracking a target in a sequence of frames, in the presence of occlusion. In all cases the measurements, –the location of n_f features of the object– were obtained using conventional feature based (e.g. color, shape, appearance) algorithms. Further, following [3], we assumed that the position of these features can be modelled as the output of an unknown (piecewise) linear time invariant system driven by a stochastic input. Finally, by absorbing, if necessary, the spectrum of this input into the unknown plant, we assumed, without loss of generality, that this input is an impulse. Thus, we used the values $n_u = 1$ and $\mathbf{H}^u = \begin{bmatrix}1 & 0\ldots0\end{bmatrix}$. For ease of reference, the tracking algorithm is summarized below:

Algorithm 3: Rank Minimization Based Tracking

Input: n_f, number of features being tracked; the measurements matrix $\mathcal{W} \in R^{2n_f \times N_w}$, where $w_{i,k} = r_k^i$ and $w_{i+1,k} = s_k^i$ are the i^{th} feature position in the k^{th} frame; length of the observation window, N_w; prediction horizon N_p; noise bound ϵ.
Output: Estimated target location $w_{i,k}$ at time $\{k \geq t\}$

1. **While** {tracking continues} {
   for all $i \in \{1, \cdots, 2n_f\}$ **do**
   Apply **Algorithm 2** on $\{w_{i,k}\}_{k=t-N_w}^{t-1}$
   to compute $\{w^*{}_{i,k}\}_{k=t}^{t+N_p-1}$.
   end for
2. Locate target around the predicted position and update $\{w_{i,t}\}$, otherwise use the value $\{w^*{}_{i,t}\}$ instead (target is occluded).
3. t=t+1
   }

Next we illustrate the use of the proposed tracking algorithm with several examples and compare the results against existing techniques.

Example 1: Trajectory Prediction. In this example we consider the problem of predicting the location of the centroid of the jumping person shown in Fig. 1, from past measurements of its coordinates, (r_k, s_k), corrupted by uncorrelated measurement noise, $\|v\| \leq 2$. In this case we used 31 past measurements of the centroid position (green dots), to predict its next 14 values (blue dots). As shown in the Figure, the rank-minimization based filter successfully predicts the location of the target. For comparison, a Kalman filter based tracker (trajectory labeled 2 in the right figure) fails due to the substantial occlusion.

Example II: Improving Robustness of a CamShift Tracker. This example illustrates the combination of receding horizon rank minimization and CamShift [12] algorithms to improve tracking robustness against occlusion. The data consist of 36 frames of a sequence where the target is not occluded, followed by 13 frames with varying degrees of occlusion. Fig. 2 shows a comparison of using Camshift alone (top) versus a combination of Camshift and receding horizon rank minimization (bottom). As shown there, while the former fails, the combination successfully tracks the target throughout the entire sequence.

Example III: Multi-target Tracking. This example compares the tracking results obtained using a Kalman filter, a combination identification via Caratheodory-Fejer/ Particle Filter (CF-PF) [3], and the proposed receding horizon rank-minimization filters (RHRMF). The goal is to track the two individuals shown in Figure 3 through the occlusion, using video-data obtained with a moving camera. In this case, as standard in the field, the Kalman filter used an assumed simple model of the dynamics, in this case constant velocity, together with the observed data, to estimate and propagate the states and estimate the positions during occlusion. The CF-PF combination used the unoccluded data to identify first the dynamics of the target, followed by the use of these dynamics in conjunction with a particle filter [15] to estimate the target position during occlusion. Finally, the receding horizon rank minimization filter was implemented using *Algorithm 3* with the values $N_w = 35$, $N_p = 6$, and $\epsilon = 2$. The results after processing are shown in the bottom portion of Figure 3. As shown there, the receding horizon filter yields the lowest prediction error. This is due to the fact that the simple model used in the Kalman filter does no completely capture the target dynamics. These dynamics are captured by the CF-based identification (since it is interpolatory). However, this approach leads to high order dynamics (the order of the central interpolator coincides with the number of data points used in the identification), necessitating the use of a model reduction step. The resulting identification error leads to the position prediction error. On the other hand, this effect is not present when using the receding horizon filter, since it automatically identifies the lowest order dynamics consistent with the experimental data record.

Example IV: Track Matching. This example illustrates the

Fig. 1. Trajectory prediction. Left: training data. Right: Rank Minimization (1) versus Kalman Filtering (2)

Fig. 2. Top: CamShift based tracking. Bottom: combination CamShift/Receding Horizon Rank Minimization using $N = 30$, $N_p = 8$, and $\epsilon = 2$.

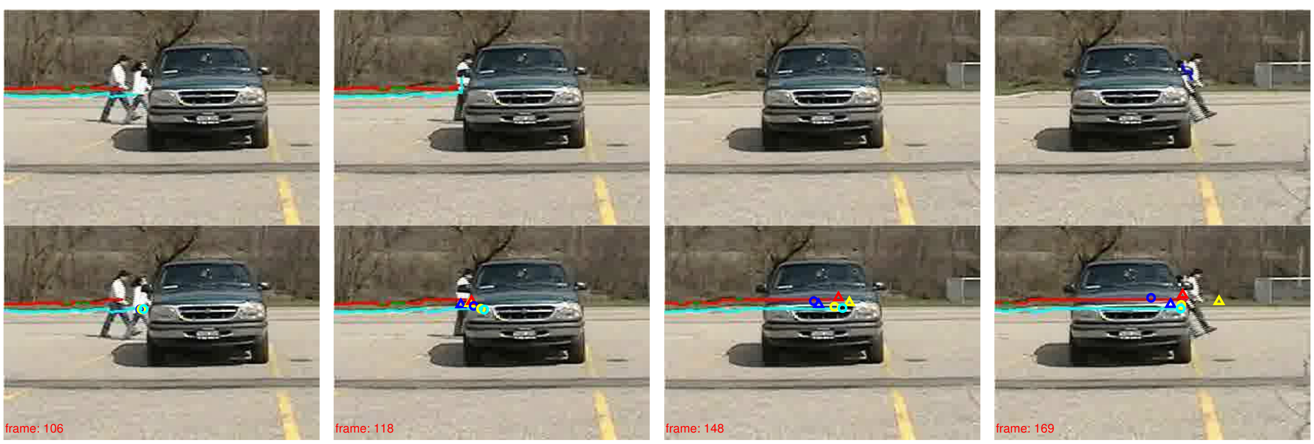

Fig. 3. Tracking two individuals through occlusion using a moving camere. (a-d) frame 106, 118, 148, and 169. 'Triangle' denotes the faster person, 'Circle' denotes the slower person. 'Blue' denotes the track predicted by a Kalman Filter, 'yellow' denotes the track predicted by the combination CF-PF, and 'red' and 'cyan' denote the tracks (one for each target) predicted by the RHRMF. Frame 169 compares the final position estimated by each method against the ground truth.

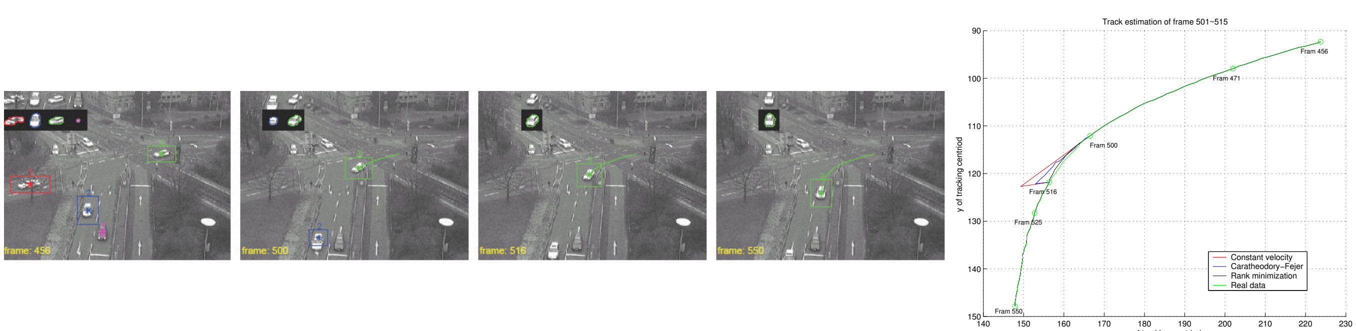

Fig. 4. Track estimation by constant velocity, CF, and rank minimization approach in frame 501∼515. Left: frames 456, 500, 516, and 550. Right: estimated versus actual position.

ability of *Algorithm 2* to interpolate missing data from existing measurements. The existing data consists of a sequence of frames taken from a video clip of a street intersection showing a turning car (see Figure 4). In order to test the algorithm against ground truth, we assumed that the data in frames 501∼515 is missing and attempted to recreate it using a Kalman filter, a combination identification/filtering and rank minimization. The first two methods used data prior to frame 501 only (frames $460 \sim 500$), since they cannot easily accommodate fragmented data, while rank minimization used both data pre-frame 501 and post-frame 515 (frames $470 \sim 500$ and $516 \sim 525$). As shown in Figure 4 (b), receding horizon rank minimization substantially outperforms the other methods.

VII. Conclusions and Further Research

Many problems of practical interest require predicting/interpolating the value of a given output based solely on existing experimental data and some minimal a-priori information about the underlying dynamical process. In this paper, motivated by some earlier work on subspace identification methods and rank-minimization based track matching we show that this problem can be solved, without explicitly identifying the dynamics of the plant by recasting it into a rank minimization form, which in turn can be relaxed to a convex optimization problem. Advantages of this approach include avoiding the need for an intermediate identification step (and the entailed identification errors) and the ability to automatically generate the lowest order interpolant consistent with the available information. Further, by carrying out the optimization on-line, based on a sliding window of data, the approach can accommodate slowly time varying dynamics. The potential of this approach to outperform existing techniques was illustrated using a non-trivial problem arising in the context of computer–vision based applications: robust multiframe tracking in the presence of clutter and occlusion. On going research seeks to extend the techniques above to classes of nonlinear dynamics, such as Hammerstein and Wiener systems. In the context of the computer vision applications discussed above, this situation arises when, in order to avoid dimensionality problems, the data is compressed using non-linear embeddings.

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