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Quasi-decentralized State Estimation and Control of Process Systems Over Communication Networks

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Abstract— **This paper develops a quasi-decentralized state estimation and control architecture for plants with limited state measurements and distributed, interconnected units that exchange information over a shared communication network. The objective is to stabilize the plant while minimizing network resource utilization and communication costs. The networked control architecture is composed of a family of local control systems that transmit their data in a discrete (on/off) fashion over the network. Each control system includes a state observer that generates estimates of the local state variables from the measured outputs. The estimates are used to implement the local feedback control law and are also shared over the network with the control systems of the interconnected units to account for the interactions between the units. To reduce the exchange of information over the network as much as possible without sacrificing stability, dynamic models of the interconnected units are embedded in the local control system of each unit to provide it with an estimate of the evolution of its neighbors when data are not transmitted through the network. The state of each model is then updated using the state estimate generated by the observer of the corresponding unit and transmitted over the network when communication is re-established. By formulating the networked closed-loop plant as a switched system, an explicit characterization of the maximum allowable update period (i.e., minimum cross communication frequency) between the distributed control systems is obtained in terms of plant-model mismatch, controller and observer design parameters. It is shown that the lack of full state measurements imposes limitations on the maximum allowable update period even if the models used to recreate the plant units' dynamics are accurate. The results are illustrated using a chemical process example and compared with other networked control strategies. The comparison shows that the minimum communication frequency required using quasi-decentralized control is less than what is required by a centralized control architecture indicating that the former is more robust with respect to communication suspension.**

I. INTRODUCTION

Traditionally, control of plants with multiple geographically-distributed interconnected units has been studied within either the centralized or decentralized control frameworks. In centralized control, all measurements are collected and sent to a central unit for processing, and the resultant control commands are then sent back to the plant. In decentralized control, on the other hand, the plant is decomposed into a number of simpler subsystems (typically based on functional and/or time-scale differences of the unit operations) with interconnections, and a number of local controllers are connected to each subsystem with no signal transfer taking place between different local controllers. Significant research work has explored in depth the benefits and limitations of centralized and decentralized controllers as well as possible ways of overcoming some of their limitations (e.g., see [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11] and the references therein). Other examples of recent works on control of process networks include the analysis and stabilization of process networks based on passivity and concepts from thermodynamics ([12], [13]), the development of agent-based systems to control reactor networks ([14], [15]), and the analysis and control of integrated process networks using time-scale decomposition and singular perturbations ([16], [17]).

An approach that provides a compromise between the complexity of traditional centralized control schemes, on the one hand, and the performance limitations of decentralized control approaches on the other, is quasi-decentralized control. The term quasi-decentralized control refers to a situation in which most signals used for control are collected and processed locally – although some signals (the total number of which is kept to a minimum) still need to be transferred between local units and controllers to adequately account for the interactions between the different units and minimize the propagation of disturbances and process upsets from one unit to another. One of the key problems that need to be addressed in the design of quasi-decentralized control systems is the coordination between the control and communication tasks and how to account for possible limitations of the communication medium in the formulation and solution of the control problem.

The importance of this problem stems from the increased reliance in the process industries in recent years on sensor and control systems that are accessed over communication networks rather than dedicated links (e.g., [18], [19], [20]), which is motivated in part by the substantial savings in installation and maintenance time and costs as well as the flexibility and enhanced fault-tolerance capabilities of networked control systems. Also, as the trend towards augmenting dedicated control networks with low-cost wireless sensor and actuator networks in the process industry continues to take hold in order to achieve high-density sensing and actuation (e.g., [21], [20]), the need to account for communication costs in the controller design framework becomes apparent. In this context, communication limitations arise both from the disruptions caused by interference in the

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field and/or environmental impact, as well as the inherent constraints on the power, computation and communication capabilities of the wireless devices.

The design of a quasi-decentralized control strategy that enforces the desired closed-loop objectives with minimal cross communication between the component subsystems is an appealing goal since it reduces reliance on the communication medium and helps save on communication costs. This is an important consideration particularly when the communication medium is a (potentially unreliable) wireless sensor network where conserving network resources is key to prolonging the service life of the network. Beyond saving on communication costs, the study of this problem provides an assessment of the robustness of a given networked control system, and allows designers to identify the fundamental limits on the tolerance of a given networked control system to communication suspension.

While the emerging paradigm of control over networks (e.g., see [22], [23], [24], [25], [26], [27], [28], [29] for some results and references in this area) provides a natural framework to address the issues of control and communication integration, the majority of research studies on networked control systems have focused mainly on single-unit processes using a centralized control architecture. A centralized control structure implemented over a network, however, is not always the best choice for control of a large-scale plant. By comparison, results on networked control of multi-unit plants with tightly interconnected units have been limited. In an effort to address this problem, we developed in [30] a quasi-decentralized networked control architecture that enforces close-loop stability with minimal cross communication between the constituent subsystems. The main idea was to embed in the local control system of each unit a set of dynamic models that provide the local controller with estimates of the states of the neighboring units, in order to be used when state information is not transmitted over the network. Both the control and communication laws in this case were derived under the assumption that the full state of each unit is available for measurement.

In many practical applications, direct measurements of the full state are seldom available. The lack of full state measurements has important implications that need to be accounted for both at the local control level, and the plant-wide communication level. Motivated by these considerations, we develop in this work a quasi-decentralized output feedback control architecture for multi-unit plants with limited state measurements and tightly interconnected units that exchange information over a shared communication network. We address the problem of designing an integrated state estimation, control and communication policy that requires minimal communication between the units without sacrificing closed-loop stability. To this end, we embed in the local control system of each unit a set of dynamic models that provide an approximation of the interactions between the given unit and its neighbors in the plant when communication is suspended over the network. To deal with the lack of full state measurements, an appropriate state observer is included in the local control system of each unit to generate estimates of the local state variables from the measured outputs. The estimates are used to implement the local state feedback controllers and are also transmitted over the plant-wide communication network to update the state of the corresponding model embedded in the interconnected subsystems when communication is re-established.

The rest of the paper is organized as follows. Following some preliminaries in Section II, the networked quasi-decentralized control structure is presented in Section III. The closed-loop system is then cast as a switched system in Section IV and its stability properties are analyzed leading to an explicit characterization of the maximum allowable update period (i.e., minimum communication frequency) between each control system and the control systems of its neighboring units in terms of the accuracy of the models and the choice of control laws and state observers. The proposed framework is illustrated in Section V using an example of chemical reactors with recycle. Finally, concluding remarks are given in Section VI.

II. Preliminaries

We consider a large-scale distributed plant composed of n interconnected processing units and represented by the following state-space description:

$$\begin{aligned}
\dot{x}_1 &= A_1x_1 + B_1u_1 + \sum_{j=2}^{n} A_{1j}x_j, & y_1 &= C_1x_1 \\
\dot{x}_2 &= A_2x_2 + B_2u_2 + \sum_{j=1, j\neq 2}^{n} A_{2j}x_j, & y_2 &= C_2x_2 \\
&\vdots & &\vdots \\
\dot{x}_n &= A_nx_n + B_nu_n + \sum_{j=1}^{n-1} A_{nj}x_j, & y_n &= C_nx_n
\end{aligned} \tag{1}$$

where $x_i := [x_i^{(1)} \ x_i^{(2)} \ \cdots \ x_i^{(p_i)}]^T \in \mathbb{R}^{p_i}$ denotes the vector of process state variables associated with the i-th processing unit, p_i is the number of state variables in the i-th unit, $y_i := [y_i^{(1)} \ y_i^{(2)} \ \cdots \ y_i^{(q_i)}]^T \in \mathbb{R}^{q_i}$ and $u_i := [u_i^{(1)} \ u_i^{(2)} \ \cdots \ u_i^{(r_i)}]^T \in \mathbb{R}^{r_i}$ denote the vector of measured outputs and manipulated inputs associated with the i-th processing unit, respectively. x^T denotes the transpose of a column vector x, A_i, B_i, A_{ij} and C_i are constant matrices. The interconnection term $A_{ij}x_j$, where $i \neq j$, describes how the dynamics of the i-th unit are influenced by the j-th unit in the plant. Note from the summation notation in Eq.1 that each processing unit can in general be connected to all the other units in the plant. Note also that even though each subsystem is referred to as a unit for simplicity, each subsystem can comprise a collection of unit operations depending on how the plant is decomposed. Our main objective is to devise an integrated control and communication strategy that stabilizes the individual units (and the overall plant) at the origin while simultaneously accounting for (1) the presence of a communication network (instead of ideal dedicated links) that is shared by the various subsystems to exchange information, and (2) the availability of measurements of only a few state variables within each unit. In the next section, we describe the design procedure of a quasi-decentralized control strategy that meets these objective by relying on a collection of process models and state observers that compensate for the lack of the needed information.

III. Model-based quasi-decentralized estimation and control over networks

A. Distributed output feedback controller synthesis

To realize the desired quasi-decentralized control structure, the first step is to synthesize for each unit a controller that stabilizes the states at the origin in the absence of communication constraints (i.e., when the control systems are connected via ideal point-to-point connections). Specifically, we consider control laws of the form:

$$u_i(x) = K_i x_i + \sum_{j=1, j\neq i}^{n} K_{ij} x_j, \tag{2}$$

where $K_i x_i$ is the local feedback component responsible for stabilizing the i-th subsystem in the absence of interconnections, and $K_{ij} x_j$ is a "feedforward" component that compensates for the effect of the j-th neighboring subsystem on the dynamics of the i-th unit. Note that a choice of $K_{ij} = O$ reduces the control strategy to a fully decentralized one where only measurements of the process variables of the i-th unit are collected and processed with no signal transfer taking place across the network. Note also that the implementation of the control law of Eq.2 requires the availability of state measurements from both the local subsystem being controlled and the connected units, which are seldom available in practice. Considering this, a state observer is designed for each local controller to generate estimates of the local state variables from the local measured outputs, and is combined with the state feedback law of Eq.2 to yield an output feedback controller of the form:

$$\begin{aligned} u_i &= K_i \bar{x}_i + \sum_{j=1, j\neq i}^{n} K_{ij} \bar{x}_j \\ \dot{\bar{x}}_i &= (A_i - L_i C_i)\bar{x}_i + \sum_{j=1, j\neq i}^{n} A_{ij}\bar{x}_j + B_i u_i + L_i y_i, \end{aligned} \tag{3}$$

where $\bar{x}_i$ is the observer state estimate for the i-th subsystem and L_i is the observer gain (chosen so that $A_i - L_i C_i$ is Hurwitz). Notice that the observer generating $\bar{x}_i$ from y_i resides in the local control system of the i-th subsystem, while the observer generating $\bar{x}_j$ is located in the j-th unit (i.e., on the other side of the shared network). Based on this, and without loss of generality, we will consider in the remainder of this paper the case when $\bar{x}_i$ is available to the local controller of unit i continuously, while $\bar{x}_j$ is available only when the information is transmitted over the network.

Remark 1: The requirement that the controller and observer gains be stabilizing in the absence of network constraints (i.e., when there is continuous transmission of measurements between the units) is a desirable feature in the sense that it decouples the control and communication design tasks from one another and offers the designer the flexibility to choose the desired control law independent of the characteristics of the communication medium deployed. However, this requirement is not necessary for stability of the networked closed-loop system.

B. Design of communication logic: a model-based scheme

To reduce the transfer of information (in this case $\bar{x}_j$) between the local control systems without sacrificing closed-loop stability, a dynamic model of each connected unit is included in the control system of the i-th unit to provide it with an estimate of the evolution of the states of those units when data are not sent over the network. This allows the control systems of the neighboring units to send their data at discrete time instants and not continuously. "Feedforward" from one unit to another is then performed by updating the state of each model using the observer-generated estimates of the corresponding unit transmitted at discrete time instances. In-between consecutive transmission times, the control action for each unit relies on a collection of models that are embedded in the local control system and are running for a certain period of time. A schematic of this model-based control architecture is shown in Figure 1.

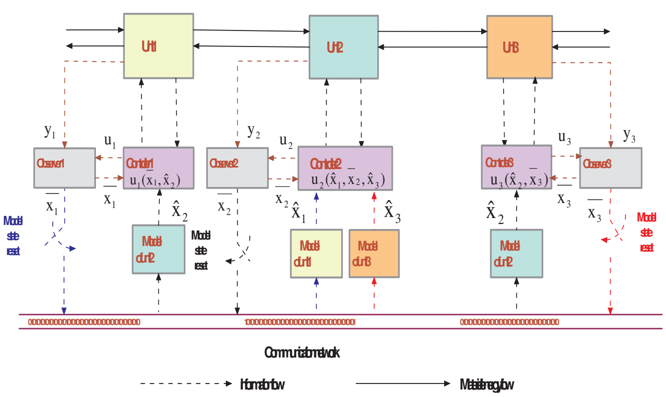

Fig. 1. Quasi-decentralized networked state estimation and control architecture.

Within this architecture, the control law for each unit is implemented as follows:

$$\begin{aligned} u_i(t) &= K_i \bar{x}_i(t) + \sum_{j=1, j\neq i}^{n} K_{ij} \widehat{x}_j^i(t), \;\; i = 1, 2, \cdots, n \\ \dot{\bar{x}}_i(t) &= (A_i - L_i C_i)\bar{x}_i(t) + \sum_{j=1, j\neq i}^{n} A_{ij} \widehat{x}_j^i(t) \\ &+ \; B_i u_i(t) + L_i y_i(t) \\ \dot{\widehat{x}}_j^i(t) &= \widehat{A}_j \widehat{x}_j^i(t) + \widehat{B}_j \widehat{u}_j^i(t) + \widehat{A}_{ji} \bar{x}_i(t) \\ &+ \sum_{l=1, l\neq i, l\neq j}^{n} \widehat{A}_{jl} \widehat{x}_l^i(t), \;\; t \in (t_k, t_{k+1}) \\ \widehat{u}_j^i(t) &= K_j \widehat{x}_j^i(t) + K_{ji} \bar{x}_i(t) + \sum_{l=1, l\neq i, l\neq j}^{n} K_{jl} \widehat{x}_l^i(t), \\ &\text{for} \;\; t \in (t_k, t_{k+1}) \\ \widehat{x}_j^i(t_k) &= \bar{x}_j(t_k), \;\; j = 1, \cdots, n, j \neq i, \; k = 0, 1, 2, \cdots \end{aligned} \tag{4}$$

where $\bar{x}_i$ is the estimate of x_i generated by the local observer, $\widehat{x}_j^i$ is the estimate of x_j provided by a dynamic model of j-th unit involving $\widehat{A}_j$, $\widehat{B}_j$, $\widehat{A}_{jl}$, which are constant matrices. The fact that $\bar{x}_i$ appears directly in the model of the j-th unit follows from (1) the structure of the plant whereby the i-th unit feeds material and energy back into unit j and (2) the fact that the observer-generated estimates of x_i are assumed to be available continuously to the local control system of the i-th unit. In the case when data of $\bar{x}_i$ are not available continuously (i.e., if they are exchanged over the shared network instead), then a model of the i-th unit will have to be added to estimate the evolution of x_i for the times that the data are not available. Note also that the models used by the i-th controller to recreate the behavior of the neighboring units do not necessarily match the actual dynamics of those processes, i.e., in general $\widehat{A}_j \neq A_j$, $\widehat{B}_j \neq B_j$, $\widehat{A}_{jl} \neq A_{jl}$. Furthermore, a choice of $\widehat{A}_j = O$, $\widehat{B}_j = O$, $\widehat{A}_{jl} = O$ corresponds to the special case where in between consecutive transmission times, the corresponding model acts as a zero-order hold by keeping the last available estimates from neighboring units until the next ones are available from the network.

Remark 2: An important feature that distinguishes the control structure of Fig.1 from centralized networked control structures (e.g., [24]) is that the communication medium in Fig.1 is shared by multiple interconnected units and connects the sensor/observer suite of each unit with the control systems of the other units. The network therefore serves as a "feedforward" path over which updates from the neighboring units are transmitted, i.e., the information communicated is used only to update the feedforward component of each controller and not the entire control law. In a centralized networked control system, on the other hand, the communication network is inserted between the sensors/observer on one side and a single controller on the other, and thus provides a feedback path through which updates of the entire plant state information are transmitted and used by the controller. An implication of this is that, whereas in the centralized structure no information from the plant will be available to the controller during intervals of communication suspension (and thus the control action will be based solely on the model forecasts during those time periods), some plant information in the form of local measurements/state estimates will be available to each controller in the quasi-decentralized control structure and the control action will be based on a combination of the data received from the local sensors/observer suite as well as the embedded models' forecasts. The continuous availability of (at least) partial measurements from the plant units enhances the robustness of the plant to communication suspensions which helps reduce network resource utilization further (see Section V for a demonstration of this point).

IV. Closed-loop stability analysis

A key parameter in the analysis of the control law of Eq.4 is the update period $h := t_{k+1} - t_k$, which determines the frequency at which a given unit receives observer estimates from the other units through the network to update the corresponding model state. To simplify the analysis, we consider the case when the update periods are constant and the same for all the units, i.e., we require that all units communicate their information concurrently every h seconds. This assumes that the control systems of all the units are given access to the network and can successfully transmit their data simultaneously. Extensions to the case where the different units transmit their data at different rates and the case when the update period is time-varying (or stochastic) are the subject of other research work.

A. A switched system formulation

The successful implementation of the proposed quasi-decentralized output feedback control architecture requires characterizing the maximum allowable update period (equivalently, the minimum transmission frequency) between the controller of one unit and the state observers of its neighboring units, which is the time between information exchanges. To this end, we define the following estimation errors as:

$$e_j^i = \begin{cases} \bar{x}_j - \widehat{x}_j^i, & j \neq i, \\ 0, & j = i \end{cases}, \quad i, j = 1, 2, \cdots, n \tag{5}$$

where e_j^i represents the difference between the observer estimate of the j-th unit provided by its local state observer and the model estimate used in the local control system of the i-th unit. Note that since the observer estimates of x_i, $\bar{x}_i$, are assumed to be available to the local control system of the i-th unit at all times, we always have $e_i^i = 0$. Introducing the augmented vectors $\mathbf{e}_j := [(e_j^1)^T \ (e_j^2)^T \ \cdots \ (e_j^n)^T]^T$, $\mathbf{e} := [\mathbf{e}_1^T \ \mathbf{e}_2^T \ \cdots \ \mathbf{e}_n^T]^T$, $\mathbf{x} := [x_1^T \ x_2^T \ \cdots \ x_n^T]^T$, $\bar{\mathbf{x}} := [\bar{x}_1^T \ \bar{x}_2^T \ \cdots \ \bar{x}_n^T]^T$, it can be shown that the overall closed-loop plant of Eq.1 and Eq.4 can be formulated as a hybrid (switched) system of the following form:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \Lambda_{11}\mathbf{x}(t) + \Lambda_{12}\bar{\mathbf{x}}(t) + \Lambda_{13}\mathbf{e}(t) \\ \dot{\bar{\mathbf{x}}}(t) &= \Lambda_{21}\mathbf{x}(t) + \Lambda_{22}\bar{\mathbf{x}}(t) + \Lambda_{23}\mathbf{e}(t) \\ \dot{\mathbf{e}}(t) &= \Lambda_{31}\mathbf{x}(t) + \Lambda_{32}\bar{\mathbf{x}}(t) + \Lambda_{33}\mathbf{e}(t), \quad t \in (t_k, t_{k+1}) \\ \mathbf{e}(t_k) &= \mathbf{0}, \quad k = 0, 1, 2, \cdots, \end{aligned} \tag{6}$$

where the process and observer states evolve continuously in time and the estimation errors are reset to zero at each transmission instance since the state of each model in each unit is updated every h seconds. Referring to Eq.6, Λ_{11}, Λ_{12}, Λ_{13}, Λ_{21}, Λ_{22}, Λ_{23}, Λ_{31}, Λ_{32}, and Λ_{33} are $m \times m$, $m \times m$, $m \times mn$, $m \times m$, $m \times m$, $m \times mn$, $mn \times m$, $mn \times m$, and $mn \times mn$ constant matrices, respectively, where $m = \sum_{i=1}^{n} p_i$ and p_i is the dimension of the i-th state vector. These matrices are linear combinations of A_i, B_i, A_{ij}, $\widehat{A}_i$, $\widehat{B}_i$, $\widehat{A}_{ij}$, K_i, K_{ij}, L_i which are the matrices used to describe the dynamics, the models, the control laws, and the local state observers of the different units. The explicit forms of these matrices are omitted for brevity but can be obtained by substituting Eq.4 into Eq.1 (see the simulation study in Section V for the explicit forms of these matrices in the case of a two-unit plant).

Defining the augmented state $\xi(t) := [\mathbf{x}^T(t)\ \bar{\mathbf{x}}^T(t)\ \mathbf{e}^T(t)]^T$, we can re-write the closed-loop dynamics of the overall plant as:

$$\dot{\xi}(t) = \Lambda\xi(t), t \in [t_k, t_{k+1}),\ \xi(t_k) = [\mathbf{x}^T(t_k)\ \bar{\mathbf{x}}^T(t_k)\ \mathbf{0}]^T, \tag{7}$$

where $k = 1, 2, \cdots$, and $\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \end{bmatrix}$.

B. Characterizing the maximum allowable update period

Following [24], it can be shown that the system described by Eq.7 with initial condition $\xi(t_0) = [\mathbf{x}^T(t_0)\ \bar{\mathbf{x}}^T(t_0)\ \mathbf{0}]^T = \xi_0$ has the following response:

$$\xi(t) = e^{\Lambda(t-t_k)}\left(I_o e^{\Lambda h} I_o\right)^k \xi_0, \text{ for } t \in [t_k,\ t_{k+1}) \tag{8}$$

with $t_{k+1} - t_k = h$, where $I_o = \begin{bmatrix} I_{m\times m} & O_{m\times m} & O_{m\times mn} \\ O_{m\times m} & I_{m\times m} & O_{m\times mn} \\ O_{mn\times m} & O_{mn\times m} & O_{mn\times mn} \end{bmatrix}$, $I_{m\times m}$ is the $m \times m$ identity matrix and $O_{m\times mn}$ is the $m \times mn$ zero matrix. Specifically, for $t \in [t_k,\ t_{k+1})$, the plant response is given by:

$$\xi(t) = \begin{bmatrix} \mathbf{x}(t) \\ \bar{\mathbf{x}}(t) \\ \mathbf{e}(t) \end{bmatrix} = e^{\Lambda(t-t_k)} \begin{bmatrix} \mathbf{x}(t_k) \\ \bar{\mathbf{x}}(t_k) \\ \mathbf{0} \end{bmatrix} = e^{\Lambda(t-t_k)}\xi(t_k) \tag{9}$$

Note that at times t_k, $k = 1, 2, \cdots$, $\xi(t_k) = [\mathbf{x}^T(t_k)\ \bar{\mathbf{x}}^T(t_k)\ \mathbf{0}]^T$, i.e., the error $\mathbf{e}(t)$ is reset to zero. This can be represented by writing $\xi(t_k) = I_o\xi(t_k^-)$, where I_o as defined earlier and $\xi(t_k^-) = \left[\mathbf{x}^T(t_k^-)\ \bar{\mathbf{x}}^T(t_k^-)\ \mathbf{e}^T(t_k^-)\right]^T$. Using Eq.9 to calculate $\xi(t_k^-)$ yields $\xi(t_k) = I_o e^{\Lambda h}\xi(t_{k-1})$. Therefore, given that at time $t = t_0$, $\xi(t_0) = \xi_0$ is the initial condition, we have $\xi(t) = e^{\Lambda(t-t_k)}(I_o e^{\Lambda h})^k \xi_0 = e^{\Lambda(t-t_k)}(I_o e^{\Lambda h} I_o)^k \xi_0$.

Having characterized the overall closed-loop response in terms of the update period, one can finally show (e.g., see [24]) that a necessary and sufficient condition for the zero solution of the system of Eq.7, $\xi = [\mathbf{x}^T\ \bar{\mathbf{x}}^T\ \mathbf{e}^T]^T = [\mathbf{0}\ \mathbf{0}\ \mathbf{0}]^T$, to be globally exponentially stable is to have the eigenvalues of the matrix $M(h) = I_o e^{\Lambda h} I_o$ strictly inside the unit circle.

Owing to the dependence of the closed-loop matrix Λ on the matrices of the compensating models, the minimum stabilizing communication frequency is parameterized by the degree of mismatch between the dynamics of the units and the models used to describe them. This is intuitively expected given that if the compensating models describe the behavior of the connected units exactly, then the maximum allowable period for measurement updates to any unit can be arbitrarily large since there will be no need to communicate measurements in this case. Given bounds on the size of the uncertainty, it is then possible to use the above stability criteria to determine the range of stabilizing update periods that can be used. Alternatively, if the update period is fixed by the characteristics of the communication medium, it is possible to use the stability criteria to determine the maximum size of tolerable process-model mismatch (see the simulation study in Section V for a demonstration of this point). Similarly, since the maximum update period is dependent also on the choice of the control laws (both the feedback and feedforward components) and state observers for the various units, this dependence can serve as a criterion for comparing different controllers, as well as state observers in terms of their robustness with respect to communication suspension (i.e., which ones require data updates less frequently than others). In fact, for a fixed update period, one can use the stability criterion to design the controller and observer gains that will ensure stability of the networked closed-loop system. In this sense, the controller gains need not be designed a priori (i.e., under continuous communication) or independent of the communication logic (see Remark 1). These interdependencies are illustrated and analyzed in the simulation study presented in the next section.

V. Simulation study: Application to chemical reactors with recycle

In this section, we present a simulation study that demonstrates the application of the developed quasi-decentralized output feedback control system design methodology to a plant composed of interconnected units with recycle. To this end, we consider a plant composed of two well-mixed, non-isothermal continuous stirred-tank reactors (CSTRs) with interconnections, where three parallel irreversible elementary exothermic reactions of the form $A \xrightarrow{k_1} B$, $A \xrightarrow{k_2} U$ and $A \xrightarrow{k_3} R$ take place, where A is the reactant species, B is the desired product and U, R are undesired byproducts. The feed to CSTR 1 consists of two streams, one containing fresh A at flow rate F_0, molar concentration C_{A0} and temperature T_0, and another containing recycled A from the second reactor at flow rate F_r, molar concentration C_{A2} and temperature T_2. The feed to CSTR 2 consists of the output of CSTR 1, and an additional fresh stream feeding pure A at flow rate F_3, molar concentration C_{A03}, and temperature T_{03}. The output of CSTR 2 is passed through a separator that removes the products and recycles unreacted A to CSTR 1. Due to the non-isothermal nature of the reactions, a jacket is used to remove/provide heat to both reactors. Under standard modeling assumptions, a plant model of the following form can be derived:

$$\begin{aligned}
\dot{T}_1 &= \frac{F_0}{V_1}(T_0 - T_1) + \frac{F_r}{V_1}(T_2 - T_1) + \sum_{i=1}^{3} G_i(T_1)C_{A1} + \frac{Q_1}{\rho c_p V_1} \\
\dot{C}_{A1} &= \frac{F_0}{V_1}(C_{A0} - C_{A1}) + \frac{F_r}{V_1}(C_{A2} - C_{A1}) - \sum_{i=1}^{3} R_i(T_1)C_{A1} \\
\dot{T}_2 &= \frac{F_1}{V_2}(T_1 - T_2) + \frac{F_3}{V_2}(T_{03} - T_2) + \sum_{i=1}^{3} G_i(T_2)C_{A2} + \frac{Q_2}{\rho c_p V_2} \\
\dot{C}_{A2} &= \frac{F_1}{V_2}(C_{A1} - C_{A2}) + \frac{F_3}{V_2}(C_{A03} - C_{A2}) - \sum_{i=1}^{3} R_i(T_2)C_{A2}
\end{aligned} \tag{10}$$

where $R_i(T_j) = k_{i0} \exp\left(\frac{-E_i}{RT_j}\right)$, $G_i(T_j) = \frac{(-\Delta H_i)}{\rho c_p} R_i(T_j)$, for $j = 1, 2$. T_j, C_{Aj}, Q_j, and V_j denote the temperature of the reactor, the concentration of A, the rate of heat input to the reactor, and the reactor volume, respectively, with subscript 1 denoting CSTR 1. ΔH_i, k_i, E_i, $i = 1, 2, 3$, denote the enthalpies, pre–exponential constants and activation energies of the three reactions, respectively, c_p and ρ denote the heat capacity and density of fluid in the reactor. Using typical values for the process parameters (see [30]), the plant with $Q_1 = Q_2 = 0$, $C_{A0} = C_{A0}^s, C_{A03} = C_{A03}^s$ and recycle rate $r = 0.5$, has three steady states: two locally asymptotically stable and one unstable at $(T_1^s, C_{A1}^s, T_2^s, C_{A2}^s) = (457.9\ K, 1.77\ kmol/m^3, 415.5\ K, 1.75\ kmol/m^3)$.

The control objective is to stabilize the plant at the (open-loop) unstable steady-state. Operation at this point is typically sought to avoid high temperatures, while simultaneously achieving reasonable conversion. The manipulated variables for the first reactor are chosen to be Q_1 and C_{A0}, while Q_2 and C_{A03} are used as manipulated variables for the second reactor. Only the temperatures of the two reactors are assumed to be directly measured. Linearizing the plant equations around the unstable steady state yields the following system to which the quasi-decentralized output feedback control architecture is applied:

$$\begin{array}{rcll} \dot{x}_1 &=& A_1x_1 + B_1u_1 + A_{12}x_2, & y_1 \ = \ C_1x_1 \\ \dot{x}_2 &=& A_2x_2 + B_2u_2 + A_{21}x_1, & y_2 \ = \ C_2x_2 \end{array} \quad (11)$$

where x_i, u_i and y_i are the (dimensionless) state, manipulated input and measured output vectors for the i-th unit, respectively, A_i, B_i, A_{ij} and C_i are constant matrices.

Following the proposed methodology, a stabilizing output feedback controller of the form $u_i = K_i\bar{x}_i + K_{ij}\bar{x}_j$, $j \neq i$, $\dot{\bar{x}}_i = (A_i - L_iC_i)\bar{x}_i + L_iy_i + A_{ij}\bar{x}_j + B_iu_i$ is initially designed for each reactor, where K_1 and K_2, were selected by placing the eigenvalues of both $\bar{A}_1$ and $\bar{A}_2$ at -5 and -1, respectively, where $\bar{A}_i = A_i + B_iK_i$, $i = 1, 2$, while K_{12} and K_{21} were chosen to force $\bar{A}_{12} = \bar{A}_{21} = O$, where $\bar{A}_{ij} = A_{ij} + B_iK_{ij}$, and L_1 and L_2, were selected by placing the eigenvalues of both $A_1 - L_1C_1$ and $A_2 - L_2C_2$ at -50 and -25, respectively. It was verified that, when the estimates are communicated continuously between the two units, the output feedback controllers successfully stabilize the plant at the desired steady state.

However, since the observer estimate from the neighboring reactor can be received only through the network, and in order to reduce utilization of network resources, a model of the form $\dot{\widehat{x}}_j = \widetilde{A}_j\widehat{x}_j + \widetilde{A}_{ji}\bar{x}_i$, where $\widetilde{A}_j = \widehat{A}_j + \widehat{B}_jK_j$, $\widetilde{A}_{ji} = \widehat{A}_{ji} + \widehat{B}_jK_{ji}$, and $\widehat{A}_j$, $\widehat{B}_j$, $\widehat{A}_{ji}$ are estimates of A_j, B_j and A_{ji}, respectively, is embedded in the local control system of the i-th unit to provide it with an estimate of $\bar{x}_j$. Correspondingly, the local state observer in the i-th unit takes the form $\dot{\bar{x}}_i = (A_i - L_iC_i)\bar{x}_i + L_iy_i + A_{12}\widehat{x}_j + B_iu_i$ and the control law is implemented as $u_i = K_i\bar{x}_i + K_{ij}\widehat{x}_j$. The model state is used by the local controller so long as no data from the neighboring units are transmitted over the network, but is updated using the observer estimate provided by the local state observer of the other reactor whenever it becomes available from the network. Our objective is to determine the largest update period that guarantees plant stability. Following some algebraic manipulations, it can be shown that Λ consists of the following sub-matrices:

$$\Lambda_{11} = \begin{bmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{bmatrix}, \ \Lambda_{12} = \begin{bmatrix} B_1K_1 & B_1K_{12} \\ B_2K_{21} & B_2K_2 \end{bmatrix}$$

$$\Lambda_{13} = \begin{bmatrix} O & O & -B_1K_{12} & O \\ O & -B_2K_{21} & O & O \end{bmatrix}, \Lambda_{21} = \begin{bmatrix} L_1C_1 & O \\ O & L_2C_2 \end{bmatrix}$$

$$\Lambda_{22} = \begin{bmatrix} H_1 & \bar{A}_{12} \\ \bar{A}_{21} & H_2 \end{bmatrix}, \ \Lambda_{23} = \begin{bmatrix} O & O & -\bar{A}_{12} & O \\ O & -\bar{A}_{21} & O & O \end{bmatrix}$$

$$\Lambda_{31} = \begin{bmatrix} O & O \\ L_1C_1 & O \\ O & L_2C_2 \\ O & O \end{bmatrix}, \ \Lambda_{32} = \begin{bmatrix} O & O \\ H_1 - \tilde{A}_1 & \bar{A}_{12} - \tilde{A}_{12} \\ \bar{A}_{21} - \tilde{A}_{21} & H_2 - \tilde{A}_2 \\ O & O \end{bmatrix}$$

$$\Lambda_{33} = \begin{bmatrix} O & O & O & O \\ O & \tilde{A}_1 & -\tilde{A}_{12} & O \\ O & -\tilde{A}_{21} & \tilde{A}_2 & O \\ O & O & O & O \end{bmatrix}$$

where $H_i = \bar{A}_i - L_iC_i$. By examining the above expressions and from the fact that $M(h) = I_s e^{\Lambda h} I_s$, it can be seen that the eigenvalues of M depend on the mismatch between the models and the reactors, the controller and observer gains, and the update period. In the remainder of this section, we will investigate the interplays between these parameters. Since closed-loop stability of the linearized plant requires all eigenvalues of M to lie within the unit circle, it is sufficient to consider only the maximum eigenvalue magnitude, denoted λ_{max}, in the following analysis.

A. Dependence of update period on plant-model mismatch

To investigate the effect of model uncertainty, we consider parametric uncertainty in the enthalpy of the first reaction and define $\delta_1 = (\Delta H_1^m - \Delta H_1)/\Delta H_1$, where ΔH_1^m is the nominal value used in the models, as a measure of model accuracy. Fig.2(a) is a contour plot showing the dependence of λ_{max} on both δ_1 and the update period. In this plot, the area enclosed by the unit contour lines represents the stability region of the linearized plant. As expected, the range of tolerable parametric uncertainty shrinks as the update period is increased. Notice that unlike the full state feedback case (shown in Fig.2(b)), the update period under output feedback cannot be chosen arbitrarily large without loss of stability even when an exact model is used (i.e., with $\delta_1 = 0$) and that there is a finite maximum value for h that can be used. This can be explained by the fact that in the output feedback architecture, the model states are updated using observer-generated estimates (which contain errors) rather than exact state information. In this case, the stability condition is to have $h \leq 0.043$ hr; and for $h = 0.043$ hr the test matrix M has one eigenvalue with unit magnitude. This is further confirmed by the closed–loop temperature profile in Fig.2(d) where the linearized plant is stable for $h = 0.04$ hr, marginally stable for $h = 0.043$ hr, and unstable for

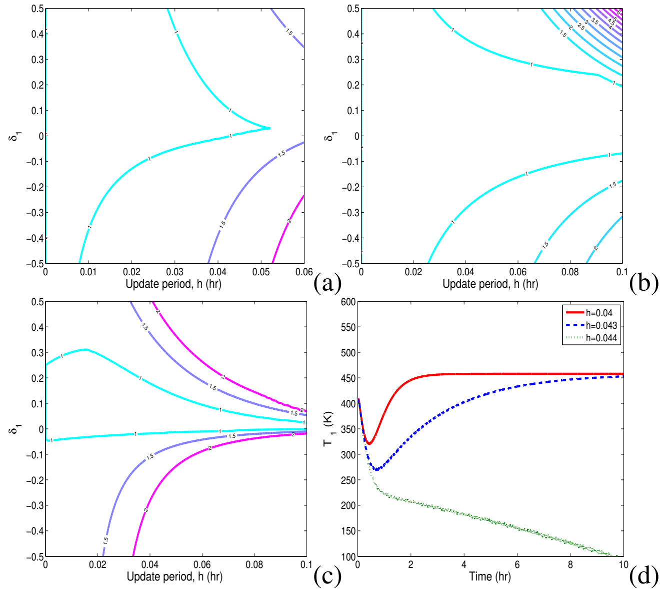

Fig. 2. Dependence of $\lambda_{\max}$ on the update period and model uncertainty under quasi-decentralized output feedback (a) and state feedback (b) control architectures, and under a centralized output feedback control architecture (c). Plot (d) shows the closed-loop profiles of T_1 under the quasi-decentralized output feedback control architecture using a perfect model for different update periods.

$h = 0.044$ hr (only a plot of T_1 is shown due to space limitations; T_2 exhibits similar tendencies).

For comparison, Fig.2(c) shows the dependence of λ_{max} on both δ_1 and h under a centralized output feedback controller structure designed for the whole plant and applied over a communication network (using the same controller and observer gain matrices results in [24]). It can be seen that the stability region of the quasi-decentralized networked control system is larger than its counterpart under centralized control. For large values of δ_1, the quasi-decentralized networked control scheme allows the use of a larger range of update periods without loss of stability. Also, for a fixed h, the quasi-decentralized networked controller can tolerate a wider range of model uncertainty. This result indicates that by distributing the control and estimation tasks across the various units, further reduction in network resource utilization – over what is obtained using a centralized structure – is possible (see Remark 3).

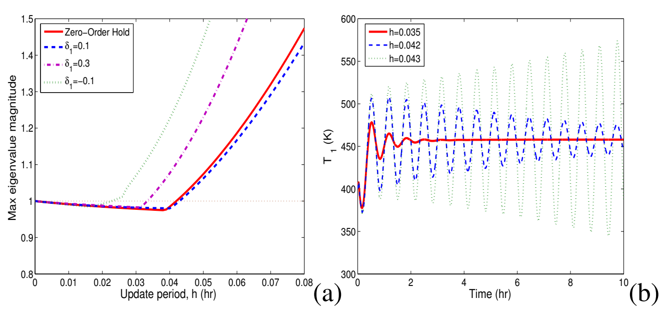

Fig. 3. (a) Dependence of $\lambda_{\max}$ on the update period under different models. Plot (b) shows the closed-loop profiles of T_1 under a zero-order hold scheme for different update periods.

Fig.3(a) shows the maximum eigenvalue magnitude versus the update period for different values of δ_1. For comparison, included in this plot also is the case when a zero-order hold scheme is used. In this case, the local controller of each reactor holds the last estimate received from its neighbor until the next time an estimate is transmitted and received from the communication network. This corresponds to using models of the form $\dot{\hat{x}}_j = \tilde{A}_j \hat{x}_j + \tilde{A}_{ji} x_i$ with $\tilde{A}_j = O$ and $\tilde{A}_{ji} = O$. The solid line in Fig.3(a) shows that the condition for stability in this case is to have $h \leq 0.042$ hr. This is further confirmed by the closed–loop temperature profiles shown in Fig.3(b). It is clear that a model-based scheme with relatively accurate models can yield update periods larger than the zero-order hold scheme. In the case of large plant–model mismatch, however, the zero-order hold scheme outperforms its model-based counterpart.

B. Impact of observer design on closed-loop stability

In this part, we investigate the effect of varying the observer gains on the maximum tolerable process-model mismatch for a fixed update period. To this end, we fix the update period at 0.04 hr, and consider varying the local observer gain L_1 first. Different values of L_1 can be used to place the two eigenvalues of the matrix $A_1 - L_1 C_1$ at different locations. For simplicity, we fix one of the poles at -50 and vary the other one, which we denote by λ_{12}. Fig.4 shows the dependence of $\lambda_{\max}$ on δ_1 and λ_{12} (i.e., on L_1). In the contour plot (a), the stability region for the system is the region enclosed by the unit contour lines. Note that as λ_{12} becomes more negative, the size of tolerable model uncertainty increases. Plot (b) in this figure shows the dependence of $\lambda_{\max}$ on λ_{12} for different values of δ_1 and for the zero-order hold scheme. The predictions of Fig.4(a) are confirmed by the closed–loop state and manipulated input profiles in Fig.4(c)-(d) which show that the linearized plant is stable when we select a point inside the unit contour zone ($\delta_1 = 0.1, \lambda_1 = -25$), and unstable when the point is barely outside the unit contour zone ($\delta_1 = 0.1, \lambda_1 = -20.8$). Similar analysis can be performed by varying the observer gain of the second reactor, L_2.

VI. Concluding Remarks

In this work, we presented a methodology for the design of quasi-decentralized output feedback controllers for plants with distributed interconnected units and limited state measurements. The approach is based on a hierarchical architecture in which each unit in the plant has a local control system with its sensors, state observers and actuators connected to the local controller through a dedicated communication network, and the local control systems in turn communicate with one another through a shared communication network. To achieve closed-loop stability with minimal cross-communication between the units, each control system relies on a set of models of its neighboring units to recreate the states of those units when direct information of their values are not available. The

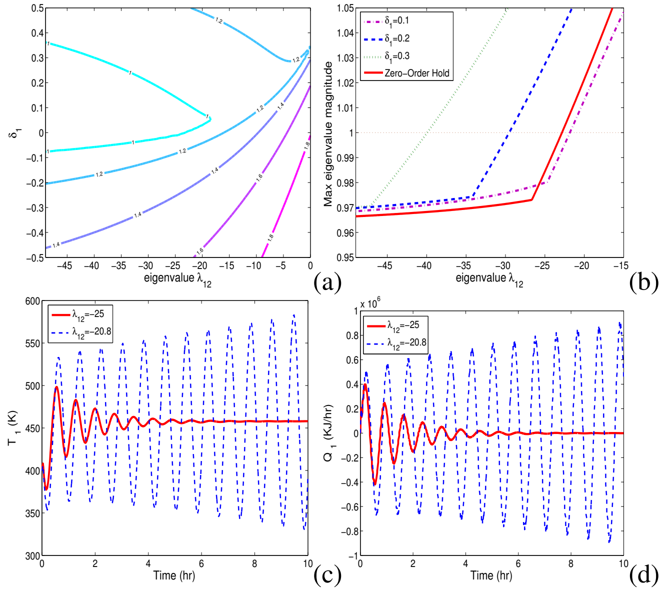

Fig. 4. Plots (a)-(b): Dependence of $\lambda_{\max}$ on model uncertainty and the observer gain L_1 under a fixed update period of $h = 0.04$ hr. The solid line in (b) corresponds to the case when a zero-order hold scheme is used. Plots (c)-(d): Closed-loop state and manipulated input profiles under the quasi-decentralized output feedback control strategy with $\delta_1 = 0.1$ and different update periods.

models are updated at discrete time instances to compensate for modeling errors with the observer estimates provided by local state observers. An explicit characterization of the maximum allowable update period in terms of model uncertainty, controller and observer design parameters was obtained. The analysis was facilitated by the linear structure of the plants considered which allowed obtaining both necessary and sufficient conditions for the stability by applying results from the networked control systems literature. The developed quasi-decentralized control strategy was illustrated using a simulation example involving chemical reactors with recycle. Finally, we note that in addition to applying the results to the linearized plant, the quasi-decentralized control structure has also been successfully implemented on the original nonlinear plant of Eq.10. The results (not shown here) indicate that, for a given update period predicted by the linear analysis, stability can be achieved for sufficiently small initial conditions. The design of a quasi-decentralized control structure for nonlinear plants and the characterization of the critical update period in the nonlinear case are topics under current investigation.

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