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Simultaneous Updating of a Model and a Controller Based on the Data-Driven Fictitious Controller

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***Abstract*— In this paper, we provide a data-driven approach for a simultaneous updating of a mathematical model of a plant and a controller with respect to a desired response of the closed loop. Our basic strategy is to adapt the *fictitious controller*, which has been proposed by the authors, described by the nominal model and the initial controller with the tunable parameter to the one-shot experimental data *directly*. As a result, such a controller yields both a plant model and a desired controller *simultaneously*. In order to give the validity of the proposed method, we also show an experimental result.**

I. INTRODUCTION

It is no doubtful that a mathematical model of a plant plays a central role in the synthesis of a control system. At the same time, as stated in [17], the modeling and the synthesis of a controller should not be separated for the achievement of the desired closed loop performance and they should be performed interactively. That is to say, we should consider the interplay between the used model and the controller in the closed loop. In this point, one of the rational approaches to this issue is to model the uncertainty with respect to robust control theory. There are nice references on this issue, e.g., cf. [3], [14] and so on. Of course, in this case, various methods for closed loop system identifications (cf. e.g., [4], [12], and so on) are also useful. Another approach is to perform a modeling and a synthesis of a controller simultaneously as studied in [1]. Our standpoint to this issue is categorized as the latter.

Instead of using a mathematical model, a synthesis of a controller with the direct use of the data is also a rational and effective way. Because, the trajectories with which the dynamics evolves, i.e., the data, have fruitful information of a dynamical system. From such a standpoint, there are some studies on the controller synthesis of the direct use of the data, [2], [5], [6], [11], [15], [19] and so on. These studies are also so-called *data-driven approaches*. It is also natural to consider that the closed loop also has a lot of information on the relationship between the controller designed by using the model and the actual plant. Thus, it is expected that the direct use of the closed loop data enables us to obtain the controller that refines the performance of the closed loop and the mathematical model that reflects the dynamics of the plant under the closed loop.

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From these backgrounds and the expectations, we provide a data-driven approach for a simultaneous updating of a mathematical model of a plant and a controller with respect to a desired response of the closed loop. Our basic strategy is to adapt the *fictitious controller*, which has been proposed by the authors (cf.[8], in these reference we used the fictitious controller for the purpose of *only* the identification), described by the nominal model and the initial controller with the tunable parameter to the one-shot experimental data *directly* with respect to the fictitious reference (cf. [15]). As a result, such a controller yields both a plant model and a desired controller *simultaneously*. The adaptation of the fictitious controllers is basically performed by a nonlinear optimization with respect to the tunable parameter. In order to give the validity of the proposed method, we also show an experimental result.

II. PRELIMINARIES

Let $\mathbb{R}^n$ denote the sets of real vectors of size n. Let $\mathbb{R}^{\mathbb{Z}}$ denote the set of time series. Let I_n and 0_n denote the unit and the zero square matrix of size n, respectively. For $w \in (\mathbb{R})^{\mathbb{Z}}$ and $a, b \in \mathbb{Z}$ such that $a \leq b$, $w_{[a,b]}$ denotes the finite time part of w in the time interval $[a, b]$. We regard $w_{[a,b]}$ as an element of $\mathbb{R}^{[b-a+1]}$. Let q denote the shift operator defined by $qw_t := w_{t+1}$ for a time series $w \in (\mathbb{R})^{\mathbb{Z}}$. In the case of $w_{[a,b]}$, we regard $(qw_{[a,b]})_b = 0$.

Consider a single-input single-output, linear, time-invariant system in discrete time described by the transfer function $G(q)$. We denote the i-th Markov parameter of $G(q)$ with $G_{[i]}$. Let $u_{[0,N]}$ and $y_{[0,N]}$ denote the input and output data, respectively, obtained in the interval $[0, N]$. The output y_t of an operator $G(q)$ with respect to the input $u_{[0,t]}$ is written by the form of $y_t = \sum_{k=0}^{t} G_{[k]} u_{t-k}$. The output data obtained in the finite time interval $y_{[0,N]} \in \mathbb{R}^{N+1}$ is regarded as the range of the following Toeplitz matrix operator with respect to $u_{[0,N]}$:

$$\begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_N \end{pmatrix} = \begin{pmatrix} G_{[0]} & 0 & \cdots & 0 \\ G_{[1]} & G_{[0]} & \cdots & 0 \\ \vdots & \ddots & \ddots & \\ G_{[N]} & \cdots & G_{[1]} & G_{[0]} \end{pmatrix} \begin{pmatrix} u_0 \\ u_1 \\ \vdots \\ u_N \end{pmatrix}, \quad (1)$$

or equivalently,

$$\begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_N \end{pmatrix} = \begin{pmatrix} u_0 & 0 & \cdots & 0 \\ u_1 & u_0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \\ u_N & \cdots & u_1 & u_0 \end{pmatrix} \begin{pmatrix} G_{[0]} \\ G_{[1]} \\ \vdots \\ G_{[N]} \end{pmatrix}. \quad (2)$$

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We denote the Toeplitz operator of Markov parameters $G_{[i]}$ for $i = 0 \cdots N$ as

$$\mathcal{T}^G_{[0,N]} := \begin{pmatrix} G_{[0]} & 0 & \cdots & 0 \\ G_{[1]} & G_{[0]} & \cdots & 0 \\ \vdots & \ddots & \ddots & \\ G_{[N]} & \cdots & G_{[1]} & G_{[0]} \end{pmatrix} \in \mathbb{R}^{(N+1)\times(N+1)}. \quad (3)$$

Similarly, we denote the Toeplitz matrix consisting of truncated time series $w_{[0,N]}$ as

$$\mathcal{T}^w_{[0,N]} := \begin{pmatrix} w_0 & 0 & \cdots & 0 \\ w_1 & w_0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \\ w_N & \cdots & w_1 & w_0 \end{pmatrix} \in \mathbb{R}^{(N+1)\times(N+1)}. \quad (4)$$

We also prepare the following vector expression of Markov parameters $G_{[i]}$ for $i = 0 \cdots N$

$$[G(q)]_{[0,N]} := \begin{pmatrix} G_{[0]} & G_{[1]} & \cdots & G_{[N]} \end{pmatrix}^{\mathrm{T}} \in \mathbb{R}^{N+1}. \quad (5)$$

By using these notations, it is easy to see that

$$(G(q)u)_{[0,N]} = \mathcal{T}^G_{[0,N]} u_{[0,N]}. \quad (6)$$

Moreover, Eq.(1) and Eq.(2) are described by

$$y_{[0,N]} = \mathcal{T}^G_{[0,N]} u_{[0,N]} = \mathcal{T}^u_{[0,N]} [G(q)]_{[0,N]}. \quad (7)$$

In Eq.(1), the invertibility of $G(q)$ is equivalent to the nonsingularity of the Toeplitz matrix because of $g_0 \neq 0$. For proper rational transfer functions $G(q)$ and $H(q)$, it follows from the well-known commutative property of the product of Toeplitz matrices that

$$\mathcal{T}^{HG}_{[0,N]} = \mathcal{T}^H_{[0,N]} \mathcal{T}^G_{[0,N]} = \mathcal{T}^G_{[0,N]} \mathcal{T}^H_{[0,N]} \quad (8)$$

holds. Moreover, it is trivial that $\mathcal{T}^H_{[0,N]} + \mathcal{T}^G_{[0,N]} = \mathcal{T}^{H+G}_{[0,N]}$ also holds. In the case of the invertible $G(q)$, i.e., $G(q)^{-1}$ is also proper, we see that

$$\mathcal{T}^{G^{-1}}_{[0,N]} = (\mathcal{T}^G_{[0,N]})^{-1} \quad (9)$$

also holds. In the case where G is strictly proper, we can not write $u_{[0,N]} = \mathcal{T}^{1/G} y_{[0,N]}$. However, by using another proper rational function such that H/G is proper, we obtain the following lemma.

Lemma 1: For two proper rational functions $H(q)$ and $G(q)$, assume that H/G is also proper. Let y be the output of G with respect to u. Then, $\mathcal{T}^H_{[0,N]} u_{[0,N]} = \mathcal{T}^{H/G}_{[0,N]} y_{[0,N]}$. holds. □

The proof is straight forward from the direct computation by using the properties of Toeplitz operators.

Finally, throughout this paper, we often omit the notation of 'q' from '$G(q)$' if it clearly follows from the context that this is a rational function with respect to q.

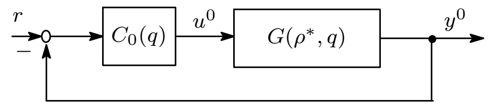

Fig. 1. A closed loop system

III. Problem formulation

Consider a conventional one-degree of freedom control system illustrated in Fig.1. We assume that a system is a linear, time-invariant, discrete time system described by

$$G(\rho, q) = \frac{\sum_{i=0}^{n_N} \rho_i^N q^i}{\sum_{i=0}^{n_D} \rho_i^D q^i} \quad (10)$$

with unknown parameter $\rho := [\rho^{N^{\mathrm{T}}} \ \rho^{D^{\mathrm{T}}}]^{\mathrm{T}} \in \mathbb{R}^n$ where $\rho^N := [\rho_0^N \ \rho_1^N \ \cdots \ \rho_{n_N}^N]^{\mathrm{T}} \in \mathbb{R}^{n_N}$ and $\rho^D := [\rho_0^D \ \rho_1^D \ \cdots \ \rho_{n_D}^D]^{\mathrm{T}} \in \mathbb{R}^{n_D}$, $n = n_D + n_N$ with $n_N < n_D$ (this means G is strictly proper). That is, the coefficients of the denominator and the numerator are unknown parameters. Let ρ^* be the unknown actual parameter of the plant. We denote $G(\rho^*, q)$

$$G^*(q) := G(\rho^*, q) \quad (11)$$

as the actual transfer function. Moreover, we have the nominal transfer function of this model

$$G_{\mathrm{n}}(q) := G(\bar{\rho}, q) \quad (12)$$

with the known parameter $\bar{\rho} \in \mathbb{R}^n$. We assume that $G_{\mathrm{n}} \neq G^*$. Define

$$T(C, G) := \frac{GC}{1 + GC}, \ S(C, G) := 1 - T(C, G). \quad (13)$$

For example, The transfer function from the reference to the output in the closed loop in Fig.1 is described by $T(C_0, G^*)$

The initial controller C_0 is designed by using G_{n} so as to achieve the desired output y_{d} described by

$$y_{\mathrm{d}} := T_{\mathrm{d}} r := T(C_0, G_{\mathrm{n}}) r \quad (14)$$

with respect to the reference signal r. We also define $S_{\mathrm{d}} := 1 - T_{\mathrm{d}}$. Without loss of generality, we assume that T_{d} is strictly proper and the rational degree of T_d is greater than or equal to that of G^* in this paper. The former standard assumption also implies that S_{d} is invertible. The latter assumption is also commonly imposed in the design of a control system that achieves the desired response. C_0 is also designed so as to stabilize the closed loop including G_{n}. Moreover, C_0 is assumed to be invertible.

Under these settings, we implement the controller C_0 and perform an experiment in the closed loop. We then obtain only the input and the output experimental data in this feedback system, $u^0_{[0,N]} = (C_0 S(C_0, G^*) r)_{[0,N]}$ and $y^0_{[0,N]} = (T(C_0, G^*) r)_{[0,N]}$, respectively. From the assumption $G_{\mathrm{n}} \neq G^*$, we also see that $y_{\mathrm{d}\,[0,N]} \neq y_{0\,[0,N]}$.

Then, the purpose of this paper is to solve the following problem.

Problem 1: In addition to the above setting and assumption, assume also that C_0 stabilizes G^* and obtain the data

$u_{[0,N]}$ and $y_{[0,N]}$ of the actual closed loop $T(C_0, G^*)$. Then, find the controller and the parameter ρ such that the first N Markov parameters of a model $G(\rho)$ are close to those of G^* so as to minimize

$$J_{\text{model}}(\rho) := \left\|[G(\rho)]_{[0,N]} - [G^*]_{[0,N]}\right\|_2 \tag{15}$$

and

$$J_{\text{controller}}(C) := \left\|(y_{\text{d}} - T(G^*, C)r)_{[0,N]}\right\|_2 \tag{16}$$

simultaneously, based on the direct use of the actual data $u_{[0,N]}$ and $y_{[0,N]}$ together with G_{n} and C_0. □

Besides the initial nominal model G_{n} and the initial controller, this problem requires only the actual data $u_{[0,N]}$ and $y_{[0,N]}$ without re-modeling of a plant. Thus, we can regard such an approach to this problem as *a data-driven approach to simultaneous updating of a model and a controller*.

IV. Main result

A. Simultaneous updating of a model and a controller

Firstly, we define "*fictitious controller* " $\tilde{C}(\rho, q)$ described by

$$\tilde{C}(\rho, q) = \frac{G_{\text{n}}(q)}{G(\rho, q)} C_0(q) \tag{17}$$

by using the plant model $G(\rho)$ taking the form of Eq.(10) with an unknown variable parameter ρ, the nominal model G_{n} and the initial controller C_0. Since $G(\rho)$ and G_{n} have the same structures, the assumption on the invertibility of C_0 guarantees that of $\tilde{C}(\rho)$.

Then, by using the experimental data $u^0_{[0,N]}$ and $y^0_{[0,N]}$, we compute the fictitious reference (cf. [15])

$$\tilde{r}(\rho)_{[0,N]} \quad = \quad \mathcal{T}^{(\tilde{C}(\rho)^{-1})}_{[0,N]} u^0_{[0,N]} + y^0_{[0,N]} \tag{18}$$

which was introduced within the unfalsified control framework. By noting that $y^0_{[0,N]} = (G^* u^0)_{[0,N]}$ and using Eq.(18), we see that $y^0_{[0,N]} = (G^*\tilde{C}(\rho)(\tilde{r}(\rho) - y))_{[0,N]}$ which is equivalent to $y^0_{[0,N]} = (T(\tilde{C}(\rho), G^*)\tilde{r}(\rho))_{[0,N]}$. This means that y^0 is not only the output of the actual closed loop with the controller C_0 but also the output of the "fictitious closed loop" $T(\tilde{C}(\rho), G^*)$ with the fictitious controller $\tilde{C}(\rho)$ with respect to the fictitious reference $\tilde{r}(\rho)$.

We then provide the following theorem.

Theorem 1: Together with the above assumptions, we assume that $u^0_0 \neq 0$, $y^0_0 \neq 0$ and $y^0_d \neq 0$. Then the following statements are equivalent for a certain parameter $\tilde{\rho} \in \mathbb{R}^n$ such that $\tilde{\rho}^D_{nD} \neq 0$ and $\tilde{\rho}^N_{nN} \neq 0$.

(a). $\lim_{\rho\to\tilde{\rho}} J_{\text{model}}(\rho) = 0$
(b). $\lim_{\rho\to\tilde{\rho}} J_{\text{controller}}(\tilde{C}(\tilde{\rho})) = 0$
(c). $\lim_{\rho\to\tilde{\rho}} \sum_{t=0}^{N} \left[\left(y^0 - T_{\text{d}}\tilde{r}(\rho)\right)_t\right]^2 = 0$

Proof: :Here we give the sketch of the proof.

(a)⇔(c): Firstly, we show that the $J_{\text{model}}(\tilde{\rho}) = 0$ is equivalent to $\sum_{t=0}^{N}[\left(y^0 - T_{\text{d}}\tilde{r}(\tilde{\rho})\right)_t]^2 = 0$. Together with the definition of the fictitious controller described by Eq.(17), we see

$$\begin{aligned}&(y^0_{[0,N]} - T_d(q)\tilde{r}(\tilde{\rho}))_{[0,N]} \\ &= \mathcal{T}^{S_{\text{d}}}_{[0,N]}\left(y^0_{[0,N]} - \mathcal{T}^{u^0}_{[0,N]}[G(\tilde{\rho})]_{[0,N]}\right).\end{aligned} \tag{19}$$

In the above computation, we have used the properties on Toeplitz matrices described by Eq.(8) and Eq.(9). Note also that the above computation on Toeplitz operators has never violated the invertibility, e.g, we have never used and introduced the $(\mathcal{T}^{G}_{[0,N]})^{-1}$ for strictly proper G, and so on. Moreover, y^0 is the actual output of G^* with respect to the actual input u^0, so

$$y^0_{[0,N]} = \mathcal{T}^{G^*}_{[0,N]}\mathcal{T}^{u^0}_{[0,N]} = \mathcal{T}^{u^0}_{[0,N]}[G^*]_{[0,N]} \tag{20}$$

holds. Due to $u^0_0 \neq 0$, $\mathcal{T}^{u^0}_{[0,N]}$ is nonsingular. Thus, by using Eq.(19) and Eq.(20) together with the nonsingularity of $\mathcal{T}^{S_{\text{d}}}_{[0,N]}$, we see that $y^0_{[0,N]} - T_{\text{d}}(q)\tilde{r}(\tilde{\rho})_{[0,N]} = 0 \Leftrightarrow [G(\tilde{\rho})]_{[0,N]} = [G(\rho^*)]_{[0,N]}$. Hence, this also implies that $\sum_{t=0}^{N}[(y^0 - T_{\text{d}}(q)\tilde{r}(\tilde{\rho}))_t]^2 = 0 \Leftrightarrow J_{\text{model}}(\tilde{\rho}) = 0$. By using the standard convergence technique, we also see that the condition (a) is equivalent to c.

(b)⇒(a): Note that $S(G^*, \tilde{C}(\tilde{\rho}))$ is invertible due to the assumption that G is strictly proper. Perform the following computation

$$\begin{aligned}&(y_{\text{d}} - T(G^*, \tilde{C}(\tilde{\rho}))r)_{[0,N]} = \left(\mathcal{T}^{T_{\text{d}}}_{[0,N]} - \mathcal{T}^{T(G^*,\tilde{C}(\tilde{\rho}))}_{[0,N]}\right) r_{[0,N]} \\ &= \mathcal{T}^{S(G^*,\tilde{C}(\tilde{\rho}))}_{[0,N]}\left((\mathcal{T}^{S(G^*,\tilde{C}(\tilde{\rho}))}_{[0,N]})^{-1}\mathcal{T}^{T_{\text{d}}}_{[0,N]} - \mathcal{T}^{G^*\tilde{C}(\tilde{\rho})}_{[0,N]}\right) r_{[0,N]}.\end{aligned} \tag{21}$$

Due to the nonsingularity of $\mathcal{T}^{S(G^*,\tilde{C}(\tilde{\rho}))}_{[0,N]}$ in Eq.(21), we focus on

$$\begin{aligned}&\left((\mathcal{T}^{S(G^*,\tilde{C}(\tilde{\rho}))}_{[0,N]})^{-1}\mathcal{T}^{T_{\text{d}}}_{[0,N]} - \mathcal{T}^{G^*\tilde{C}(\tilde{\rho})}_{[0,N]}\right) r_{[0,N]} \\ &= \left(I_{N+1} - \mathcal{T}^{\frac{G^*}{G(\tilde{\rho})}}_{[0,N]}\right) y_{\text{d}\,[0,N]}.\end{aligned} \tag{22}$$

In the above computation, note that the rational degree of $G(\tilde{\rho})$ and that of G^* are the same due to $\tilde{\rho}^D_{n_D} \neq 0$ and $\tilde{\rho}^N_{n_N} \neq 0$, which implies the invertibility of $G^*/G(\tilde{\rho})$. From the above computation, we see that $y_{\text{d}\,[0,N]} - (T(\tilde{C}(\tilde{\rho}), G^*)r)_{[0,N]} = 0$ is equivalent to

$$\left(I_{N+1} - \mathcal{T}^{\frac{G^*}{G(\tilde{\rho})}}_{[0,N]}\right) y_{\text{d}\,[0,N]} = 0. \tag{23}$$

From the property of the Toeplitz operator, we also see that Eq.(23) leads to the matrix form described by

$$\left(I_{N+1} - \mathcal{T}^{\frac{G^*}{G(\tilde{\rho})}}_{[0,N]}\right) \mathcal{T}^{y_{\text{d}}}_{[0,N]} = 0_{N+1}. \tag{24}$$

Premultiplying Eq.(24) by $\mathcal{T}^{G(\tilde{\rho})}_{[0,N]}$ yields

$$\left(\mathcal{T}^{G(\tilde{\rho})}_{[0,N]} - \mathcal{T}^{G^*}_{[0,N]}\right) \mathcal{T}^{y_{\text{d}}}_{[0,N]} = 0_{N+1}. \tag{25}$$

Note also that the above computations have also never used the inverse of the Toeplitz operator of a strictly proper rational function. Together with the nonsingularity of $\mathcal{T}^{y_{\text{d}}}_{[0,N]}$, Eq.(25) implies that the first N Markov parameters of G^*

and $G(\tilde{\rho})$ are the same. Thus, $J_{\rm controller}(\tilde{C}(\tilde{\rho})) = 0 \Rightarrow J_{\rm model}(\tilde{\rho}) = 0$. As for the argument of the limit, the standard convergence technique also enables us to see (b) $\Rightarrow$ (a).

(c)$\Rightarrow$(b): By using Lemma 1 and so on, we see

$$(y^0_{[0,N]} - T_{\rm d}(q)\tilde{r}(\tilde{\rho}))_{[0,N]} = \left(I_{N+1} - \mathcal{T}^{T_{\rm d}\frac{1+\tilde{C}(\tilde{\rho})G^*}{\tilde{C}(\tilde{\rho})G^*}}\right) y^0_{[0,N]} \quad (26)$$

In the above computation, we have used that $T_{\rm d}(1 + \tilde{C}(\tilde{\rho})G^*)/\tilde{C}(\tilde{\rho})G^*$ is proper due to the invertibility of $\tilde{C}(\tilde{\rho})$ and the assumption on the rational degrees of $T_{\rm d}$ and G^*. Premultiplying Eq.(26) by $\mathcal{T}^{T(\tilde{C}(\tilde{\rho}),G^*)}_{[0,N]}$ yields

$$\begin{aligned}&\mathcal{T}^{T(\tilde{C}(\tilde{\rho}),G^*)}_{[0,N]}(y^0_{[0,N]} - T_{\rm d}(q)\tilde{r}(\tilde{\rho}))_{[0,N]} \\ &= \mathcal{T}^{y^0}_{[0,N]}\left([T(\tilde{C}(\tilde{\rho}), G^*)]_{[0,N]} - [T_{\rm d}]_{[0,N]}.\right)\end{aligned} \quad (27)$$

Thus, we see that $(y^0_{[0,N]} - T_{\rm d}(q)\tilde{r}(\tilde{\rho}))_{[0,N]} \Rightarrow [T(\tilde{C}(\tilde{\rho}), G^*)]_{[0,N]} - [T_{\rm d}]_{[0,N]} = 0$, which also implies that $\sum_{t=0}^N[(y^0_{[0,N]} - T_{\rm d}(q)\tilde{r}(\tilde{\rho}))_t]^2 = 0 \Rightarrow J_{\rm controller}(\tilde{\rho}) = 0$. As for the argument of the limit, the standard convergence technique also enables us to see (c) $\Rightarrow$ (b). ■

In this theorem, the condition (a) and the condition (b) are related to our purpose while the condition (c) is related to the actual computation by using the data with the fictitious controller directly. If the complete minimization of the cost function

$$J(\rho) := \sum_{t=0}^N[(y^0 - T_{\rm d}(q)(\tilde{C}(\rho, q)^{-1}u^0 + y^0))_t]^2 \quad (28)$$

is achieved, then the first N Markov parameters of the actual plant G^* are obtained as those of $G(\tilde{\rho})$ and *simultaneously* the implementation of the fictitious controller $\tilde{C}(\tilde{\rho})$ as the actual controller instead of C_0 yields the desired output $y_{\rm d}$ in the closed loop. In this sense, *Theorem 1 connects the updating of a controller for the achievement of the desired specification and the updating of a model for obtaining the actual plant that unfalsifies the experimental data, simultaneously*.

B. The gap between the minimization of J and the minimizations of both $J_{\rm controller}$ and $J_{\rm model}$

In Theorem 1, the minimization of the cost function $J(\rho)$ in Eq.(28) is assumed to be arrived at zero. However, there are many case in which it is impossible to achieve such a complete minimization. In this subsection, we discuss the gap between the minimization of $J(\rho)$, and both the minimizations of $J_{\rm model}(\rho)$ and $J_{\rm controller}(\rho)$

We first focus on the relationship between $J(\rho)$ and $J_{\rm model}(\rho)$. Let $J_{\min} \neq 0$ be the minimized value of $J(\rho)$ and $\hat{\rho} := \arg\min_\rho J(\rho)$. From Eq.(19) and Eq.(20), we see

$$J_{\min} = \left\|\mathcal{T}^{S_{\rm d}}_{[0,N]}\mathcal{T}^{u^0}_{[0,N]}\left([G(\rho^*)]_{[0,N]} - [G(\hat{\rho})]_{[0,N]}\right)\right\|_2^2 \quad (29)$$

holds. By defining the maximal singular value of the matrix $\mathcal{T}^{S_{\rm d}}_{[0,N]}\mathcal{T}^{u^0}_{[0,N]}$ as $\lambda^{S_{\rm d}u^0}_{\max}$, a simple calculation with Eq.(29) yields

$$\frac{J_{\min}}{\lambda^{S_{\rm d}u^0}_{\max}} \le \left\|\left([G^*]_{[0,N]} - [G(\hat{\rho})]_{[0,N]}\right)\right\|_2^2. \quad (30)$$

Similarly, define the minimal singular value of the matrix $\mathcal{T}^{S_{\rm d}}_{[0,N]}\mathcal{T}^{u^0}_{[0,N]})$ as $\lambda^{S_{\rm d}u^0}_{\min}$. Then, together with Eq.(30), we see

$$\frac{J_{\min}}{\lambda^{S_{\rm d}u^0}_{\max}} \le \left\|\left([G^*]_{[0,N]} - [G(\hat{\rho})]_{[0,N]}\right)\right\|_2^2 \le \frac{J_{\min}}{\lambda^{S_{\rm d}u^0}_{\min}}. \quad (31)$$

Eq.(31) gives the gap between N Markov parameters of the updated model $G(\hat{\rho})$ of a plant by using the minimization of $J(\rho)$ and those of the actual (unknown) G^*. In this sense, Eq.(31) determines the accuracy of the obtained optimal parameter $\hat{\rho}$ with respect to the accuracy of a model.

Next, we consider the relationship between $J(\rho)$ and $J_{\rm controller}$. Similarly to the above computations, we see

$$\begin{aligned}J_{\min} &= \sum_{t=0}^N[(y^0 - T_{\rm d}(q)\tilde{r}(\hat{\rho}))_t]^2 \\ &= \left\|\mathcal{T}^{y^0}_{[0,N]}[1 - T_{\rm d}/T(G^*, \tilde{C}(\hat{\rho}))]_{[0,N]}\right\|_2^2\end{aligned} \quad (32)$$

holds. Denote the maximal and the minimal singular values of $\mathcal{T}^{y^0}_{[0,N]}$ with $\gamma^{y^0}_{\max}$ and $\gamma^{y^0}_{\min}$, respectively. (we here assume that $\mathcal{T}^{y^0}_{[0,N]}$ is nonsingular in addition to the previous assumptions). Then, similarly to Eq.(31), we obtain

$$\frac{J_{\min}}{\gamma^{y^0}_{\max}} \le \left\|[1 - T_{\rm d}/T(G^*, \tilde{C}(\hat{\rho}))]_{[0,N]}\right\|_2^2 \le \frac{J_{\min}}{\gamma^{y^0}_{\min}} \quad (33)$$

Eq.(33) also gives the gap between the desired transfer function T_d and the transfer function consisting of G^* with the updated controller $\tilde{C}(\hat{\rho})$. We also see that the gap with respect to the performance on the response of the closed loop is determined by $T_{\rm d}$ (i,e, $G_{\rm n}$ and $C_{\rm n}$) and the initial output data.

C. Algorithm

Here, we summarize the proposed method as follows.

0 We have already been with a nominal model of a plant $G_{\rm n}(q)$ and a controller $C_0(q)$ (this controller was designed by using $G_{\rm n}$) so as to be assumed that this yields the desired response of the closed loop $y_{\rm d} = T_{\rm d}r$ where $T_{\rm d} = \frac{C_0G_{\rm n}}{1+C_0G_{\rm n}}$.

1 Perform one-shot closed loop experiment, and obtain the finite time data $u^0_{[0,N]}$ and $y_{[0,N]}$, respectively.

2 Set the fictitious controller $\tilde{C}(\rho)$ described by Eq.(17) and the fictitious reference $\tilde{r}(\rho)$ described by (18).

3 Minimize the cost function $J(\rho)$ in described by Eq.(28) by using a nonlinear optimization and obtain the parameter $\hat{\rho} := \arg\min_\rho J(\rho)$.

4-1 The updating of a model:A model of a plant is obtained as $G(\hat{\rho})$ in the sense that $[G(\hat{\rho})]_{[N]}$ is close to $[G(\rho^*)]_{[N]}$.

4-2 The updating of a controller: Implement the fictitious controller $\tilde{C}(\hat{\rho})$ and perform the closed loop experiment again. The output of $T(\tilde{C}(\hat{\rho}), G^*)$ is closer to $T_{\rm d}$ than that of the initial closed loop $T(C_0, G^*)$.

Remark 1: The nice reference [16] discussed *how* conventional closed loop system identification and controller tuning method are unified. On the other hand, the issue of this paper is not an unification of the controller parameter tuning and closed loop system identification but one of the applications of the controller tuning to the simultaneous updating of a controller and a model of a plant. Thus, the issue treated in this paper differs from that in [16]. However, it is expected that there might be deep relationship the proposed method and the observations in [16]. This is one of the future interesting studies. □

Remark 2: The real measured data u^0 and y^0 include the noise. If it is difficult to neglect the effect of noise, we repeat the experiment with respect to *the same controller* $C_0(q)$ *twice* under the assumption that the noises in the different experiments are uncorrelated each other. This technique and the assumption are also taken by IFT or VRFT (cf.[6] and [2]). We denote the first experimental data with $\{y_n^{0(1)} := y^{0(1)} + n_y^{(1)},\ u_n^{0(1)} = u^{0(1)} + n_u^{(1)}\}$ and the second experimental data with $\{y_n^{0(2)} := y^{0(2)} + n_y^{(2)}$, $u_n^{0(2)} = u^{0(2)} + n_u^{(2)}\}$, respectively. Here, $n_y^{(i)}$ and $n_u^{(i)}$ denotes the noise in the i-th experiment on the input and the output, respectively. $y^{0(i)}$ and $u^{0(i)}$ denotes the pure signal we require in this method. The experiment is performed in the closed loop, the correlation between e.g., $n_y^{(1)}$ and $u_n^{0(1)}$ can not be neglected. However, the two experiments are performed in the different time, it is possible to assume that $n_y^{(i)}$ and $n_u^{(i)}$ in the first experiment have no correlation with $n_y^{(j)}$, $n_u^{(j)}$, $y_n^{0(j)}$ and $u_n^{0(j)}$, where $i, j = (1,2)$ or $(2,1)$. Thus, by modifying Eq.(28) as

$$\tilde{J}_n(\rho) = \left({y_n}^{0(1)}_{[0,N]} - T_{\mathrm{nom}}(q)\tilde{r}(\rho)^1_{[0,N]}\right)^{\mathrm{T}} \times \left({y_n}^{0(2)}_{[0,N]} - T_{\mathrm{d}}(q)\tilde{r}(\rho)^2_{[0,N]}\right) \tag{34}$$

(where, $\tilde{r}^i(\rho)_{[0,N]} := \tilde{C}(\rho)^{-1}{u_n}^{0i\prime}_{[0,N]} + {y_n}^{0i\prime}_{[0,N]},\ i = 1,2$), we can approximate the cost function so as to eliminate the effect of the noise. □

V. Experimental Result

In this section, we give an experimental result to show the validity of our approach. The system we address here is described by Fig.2. The cart is attached to the belt which

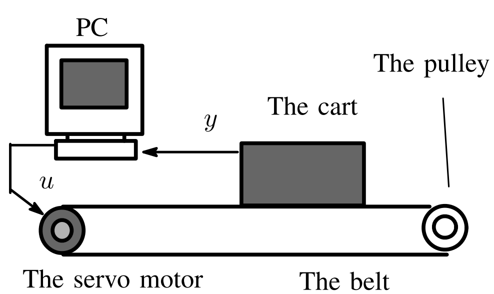

Fig. 2. The cart system

is moving by the rolling of the servo motor. The location y (output) from the initial position of the cart is measured by the potentiometer attached in the pulley and is send to the personal computer (PC). And the servo motor is driven by the voltage u (input) from PC. The sampling time of this

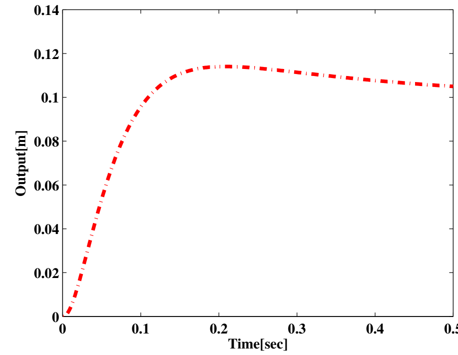

Fig. 3. The desired response y_d

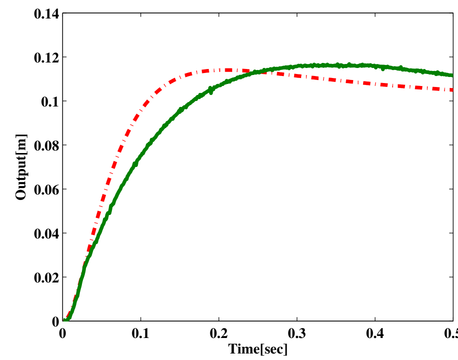

Fig. 4. The initial output y^0 (The dotted line:y_d, The real line:y^0)

control system is 0.001[sec]. The structure of the transfer function of this system is described by

$$G(\rho, q) = \frac{\rho_1 q + \rho_2}{q^2 + \rho_3 q^1 + \rho_4}\ ,\ \rho = [\rho_1\ \rho_2\ \rho_3\ \rho_4] \in \mathbb{R}^4$$

in discrete time. The nominal plant $G_{\mathrm{n}}(q)$ we used for the initial design is described by using the nominal parameter $\bar{\rho} := [2.031 \times 10^{-4}\ 2.031 \times 10^{-4}\ -1.9320\ 0.9324]$. The initial controller for the desired response is designed by $C_0(q) = \frac{0.3.005q - 2.995}{q-1}$ based on the nominal model $G_{\mathrm{n}}(q)$. In Fig.3, the desired output, i.e., the output of $T_{\mathrm{d}}(q) = \frac{G_{\mathrm{n}}C_0}{1+G_{\mathrm{n}}C_0}$, is shown. Under these settings, we perform the first initial experiment by using C_0. The output y^0 of this initial experiment is shown in Fig.4. Naturally, as shown in Fig.4, the output data of the actual closed loop is different from the desired output. The most crucial reasons are as follows. One is the fact that the nominal model has uncertainties. The other is the fact that the controller is designed based on such a nominal model.

We then apply our proposed method in order to a more accurate model and a more desired controller, simultaneously, with using only the initial experimental data. Define the fictitious controller $\tilde{C}(\rho)$ described by Eq.(17) with the nominal model $G_{\mathrm{n}}(q)$, the initial controller $C_0(q)$, and the model with unknown parameter $G(\rho)$. Next, define the fictitious reference described by Eq.(18) with the fictitious controller and the experimental input $u^0_{[0,N]}$ and output data

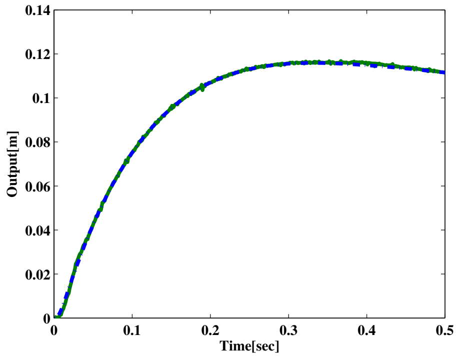

Fig. 5. The modeling result (The real line:y^0, The dotted line:the simulation with the obtained (updated) model $G(\hat{\rho})$)

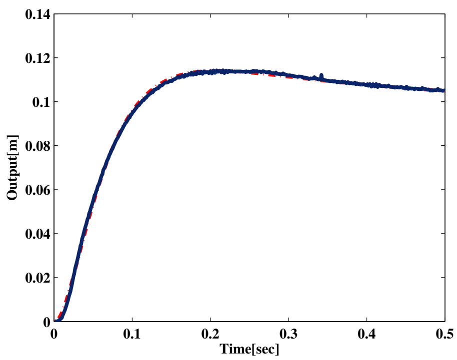

Fig. 6. The control result (The dotted line:y_d, The real line : the experimental result with the updated controller $\tilde{C}(\hat{\rho})$

$y^0_{[0,N]}$. By minimizing the cost function $J(\rho)$ described by Eq.(28) via a nonlinear optimization (here we apply the Gauss Newton method), as a result, we obtain the parameter $\hat{\rho}$ that minimizes $J_1(\rho)$ where $\hat{\rho} = [3.2504 \times 10^{-4}\ 3.2504 \times 10^{-4}\ \ -1.8327\ 0.8326]$. In order to show the validity of this obtained parameter $\hat{\rho}$, the simulation with $G(\hat{\rho})$ and the actual output y^0 are illustrated in Fig.5. From this figure, we see that $G(\hat{\rho})$ has been updated so as to reflect the dynamics more closely than $G_{\rm n}$ (y^0 and the simulation with the obtained (updated) model $G(\hat{\rho})$ are almost the same). Simultaneously, in order to see whether the controller $\tilde{C}(\hat{\rho})$ yields the desired closed loop property with respect to the output response, we implement it to the actual closed loop, and we perform the experiment. The result is shown in Fig.6. From this figure, we see $\tilde{C}(\hat{\rho})$ achieves the desired specification ($y_{\rm d}$ and the experimental result with the updated controller $\tilde{C}(\hat{\rho})$ are almost the same). From these results, we see that $\hat{\rho}$ which is the result of the minimization of $J(\rho)$ yields both a more accurate model of a plant and a more a useful controller simultaneously with only the initial experimental data. [1]

[1]Here, $J(\hat{\rho})$ is 5.93401×10^{-8} which is very small, so it is possible to regard that the minimization of $J(\rho)$ is almost completely achieved.

VI. CONCLUDING REMARKS

In this paper, we have provided a new approach for a simultaneous updating of a mathematical model of a plant and a controller with respect to a desired response. The proposed method requires only adaptation of the *fictitious controller* described by the nominal model and the initial controller with tunable parameter, to the one-shot experimental data *directly* with respect to the fictitious reference. As a result, such a controller with the desired parameter yields both a plant model and a desired controller simultaneously. In order to give the validity of our approach, we have also shown an experimental result.

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