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Using 2D Systems theory to Design Output Signal Based Iterative Learning Control Laws with Experimental Verification

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Abstract— **In this paper we use a 2D systems setting to develop new results on iterative learning control for linear plants, where it is well known in the subject area that a trade-off exists between speed of convergence and the response along the trials. Here we give new results by designing the control scheme using a strong form of stability for repetitive processes/2D linear systems known as stability along the pass (or trial). The resulting design computations are in terms of Linear Matrix Inequalities (LMIs) and they are also experimentally validated on a gantry robot. The control laws only use plant output information and hence the use of a state observer is avoided.**

I. INTRODUCTION

Iterative learning control (ILC) is a technique for controlling systems operating in a repetitive (or pass-to-pass) mode with the requirement that a reference trajectory $y_{ref}(t)$ defined over a finite interval $0 \leq t \leq \alpha$ is followed to a high precision. Examples of such systems include robotic manipulators that are required to repeat a given task, chemical batch processes or, more generally, the class of tracking systems.

Since the original work [1] in the mid 1980's, the general area of ILC has been the subject of intense research effort. Initial sources for the literature here are the survey papers [2] and [3]. The analysis of ILC schemes is firmly outside standard (or 1D) control theory — although it is still has a significant role to play in certain cases of practical interest. Instead, ILC must be seen (as one approach) in the context of fixed-point problems or, more precisely, repetitive processes (see the references in [4]) which are a distinct class of 2D systems where information propagation in one of the two independent directions only occurs over a finite duration.

In ILC, a major objective is to achieve convergence of the trial-to-trial error and often this has been treated as the only one that needs to be considered. In fact, it is possible that enforcing fast convergence could lead to unsatisfactory performance along the trial. In this paper, we address this problem by first showing that ILC schemes can be designed for a class of discrete linear systems by extending techniques developed for linear repetitive processes. This allows us to use the strong concept of stability along the pass (or trial) for these processes, in an ILC setting, as a possible means of dealing with poor/unacceptable transients in the along the trial dynamics. The results developed give control law design algorithms which can be implemented via LMIs. Finally, the resulting control laws are experimentally validated on a gantry robot executing a pick and place operation where the plant models used for design are obtained by frequency response tests.

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The remainder of this paper begins with a simulation study which demonstrates that it is possible for trial-to-trial error convergence to occur where the along the trial response is very poor. This is followed by analysis which shows how the design of a class of ILC laws based only on directly measured outputs can be formulated in a repetitive process setting and designed via LMIs to ensure stability along the trial. Finally, the results of experimentally validating the designs on a gantry robot system are given.

In this paper, $\Gamma \succ 0$, $\Gamma \prec 0$, are used to denote symmetric matrices which are positive definite and negative definite, respectively. The symbol $\mathrm{r}(\cdot)$ is used to denote the spectral radius of a given matrix. Finally, $(\star)$ is used to denote block entries in the symmetric Linear Matrix Inequalities (LMIs) which are the means by which the necessary computations can be completed for a given numerical example.

II. BACKGROUND

Consider the case when the plant to be controlled can be modeled as a differential linear time-invariant system with state-space model defined by $\{A_c, B_c, C_c\}$. In an ILC setting this is written as

$$\begin{array}{rcl} \dot{x}_k(t) & = & A_c x_k(t) + B_c u_k(t), 0 \leq t \leq \alpha, \\ y_k(t) & = & C_c x_k(t), \end{array} \tag{1}$$

where on trial k, $x_k(t) \in \mathbb{R}^n$ is the state vector, $y_k(t) \in \mathbb{R}^m$ is the output vector, $u_k(t) \in \mathbb{R}^r$ is the vector of control inputs, and the trial length $\alpha < \infty$. If the signal to be tracked is denoted by $y_{ref}(t)$ then $e_k(t) = y_{ref}(t) - y_k(t)$ is the error on trial k, and the most basic requirement is to force the error to converge in k. In fact, however, it is possible that trial-to-trial convergence will occur but produce along the trial performance which is far from satisfactory for many practical applications, e.g. a gantry robot whose task is to collect an object from a location, place it on a moving conveyor, and then return for the next one and so on. If, for example, the object has an open top and is filled with liquid, and/or is fragile in nature, then unwanted vibrations during the transfer time could have very detrimental effects. Hence in such cases there is also a need to control the along the trial dynamics and in this paper the method is to use a stronger form of stability theory for linear repetitive processes.

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As an example to illustrate this last point consider the case of a linear continuous-time system whose dynamics are modeled by the transfer-function

$$G(s) = \frac{(s+1)(s+5)}{(s+3)(s^2+4s+29)}, \tag{2}$$

which is to be controlled in the ILC setting using the P-type law

$$u_{k+1}(t) = u_k(t) + Le_{k+1}(t), \tag{3}$$

with, in particular, $L = 3$ which is easily shown to result in trial-to-trial error convergence. Fig. 1 shows the response of the controlled system over 50 trials when the reference signal ($y_{ref}(t)$) is a unit step function of 2 seconds duration is applied at $t = 0$. Fig. 2 shows the performance of the controlled system for the 30th trial. These responses confirm that trial-to-trial error convergence occurs but along the trial performance can be very poor. The key purpose of this paper is to develop methods which can avoid such poor performance by the use of stability theory for linear repetitive processes for which we next summarize the required results.

The unique characteristic of a repetitive, or multipass [4], process is a series of sweeps, termed passes, through a set of dynamics defined over a fixed finite duration known as the pass length. On each pass an output, termed the pass profile, is produced which acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile. This, in turn, leads to the unique control problem in that the output sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass-to-pass direction.

To introduce a formal definition, let $\alpha < +\infty$ denote the pass length (assumed constant). Then in a repetitive process the pass profile $y_k(t), 0 \leq t \leq \alpha$, generated on pass k acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile $y_{k+1}(t),\ 0 \leq t \leq \alpha,\ k \geq 0$.

Attempts to control these processes using standard (or 1D) systems theory and algorithms fail (except in a few very restrictive special cases) precisely because such an approach ignores their inherent 2D systems structure, i.e. information propagation occurs from pass-to-pass (k direction) and along a given pass (t direction) and also the initial conditions are reset before the start of each new pass. To remove these deficiencies, a rigorous stability theory has been developed [4] based on an abstract model of the dynamics in a Banach space setting which includes a very large class of processes with linear dynamics and a constant pass length as special cases, including those described by (4) below. In terms of their dynamics, it is the pass-to-pass coupling (noting again their unique feature) which is critical. This is of the form $y_{k+1} = L_\alpha y_k$, where $y_k \in E_\alpha$ (E_α a Banach space with norm $||\cdot||$) and L_α is a bounded linear operator mapping E_α into itself.

Consider now discrete linear repetitive processes described by the following state-space model over $p = 0, 1, \ldots, \alpha - 1, k \geq 1$,

$$\begin{aligned} x_k(p+1) &= A_d x_k(p) + B_d u_k(p) + B_{d0} y_{k-1}(p), \\ y_k(p) &= C_d x_k(p) + D_d u_k(p) + D_{d0} y_{k-1}(p), \end{aligned} \tag{4}$$

where $x_k(p) \in \mathbb{R}^n$, $u_k(p) \in \mathbb{R}^r$, $y_k(p) \in \mathbb{R}^m$ are the state, input and pass profile vectors respectively. To complete the process description, it is necessary to specify the initial, or boundary, conditions, i.e. the state initial vector on each pass and the initial pass profile. Here these are taken to be zero. In the next section, we show how a repetitive process setting can be used to analyze ILC schemes and, in particular, how the stability theory of these processes can be employed leading to control law design which prevents performance such as that of Fig.2 from arising.

III. ILC AS A REPETITIVE PROCESS

From this point onwards we work in the discrete domain and so assume that the process dynamics have been sampled by the zero-order hold method at a uniform rate T_s seconds to produce a discrete state-space model with matrices $\{A, B, C\}$. Also introduce

$$\begin{aligned} \eta_{k+1}(p+1) &= x_{k+1}(p) - x_k(p), \\ \Delta u_{k+1}(p) &= u_{k+1}(p) - u_k(p), \end{aligned} \tag{5}$$

and let $e_k(p) = y_{ref}(p) - y_k(p)$ denote the current trial error. Then it is possible to proceed as in [5] and use an ILC law which requires the current trial state vector $x_k(p)$ of the plant. In practical applications, this vector may not be available for measurement or, at best, only some of its entries are and hence in general an observer will be required. In this paper, we avoid the use of an observer by using the control law

$$\Delta u_{k+1}(p) = K_1 \mu_{k+1}(p+1) + K_2 \mu_{k+1}(p) + K_3 e_k(p+1), \tag{6}$$

where $\Delta u_{k+1}(p)$ represents the term to be added to the previous trial input and

$$\mu_k(p) = y_k(p-1) - y_{k-1}(p-1) = C\eta_k(p). \tag{7}$$

The extra term in the control law considered here has been added as a means, if necessary, of compensating for the effects of not assuming that the state vector is available for use in the control law.

By routine analysis, we can write (6) as

$$\Delta u_{k+1}(p-1) = K_1 C\eta_{k+1}(p) + K_2 C\eta_{k+1}(p-1) + K_3 e_k(p), \tag{8}$$

and hence on introducing

$$\tilde{\eta}_{k+1}(p+1) = \begin{bmatrix} \eta_{k+1}(p+1) \\ \eta_{k+1}(p) \end{bmatrix} \tag{9}$$

we the controlled system dynamics can be written as

$$\begin{aligned} \tilde{\eta}_{k+1}(p+1) &= \hat{A}\tilde{\eta}_{k+1}(p) + \hat{B}_0 e_k(p), \\ e_{k+1}(p) &= \hat{C}\tilde{\eta}_{k+1}(p) + \hat{D}_0 e_k(p), \end{aligned} \tag{10}$$

where

$$\begin{aligned} \hat{A} &= \begin{bmatrix} A + BK_1C & BK_2C \\ I & 0 \end{bmatrix}, \\ \hat{B}_0 &= \begin{bmatrix} BK_3 \\ 0 \end{bmatrix}, \\ \hat{C} &= \begin{bmatrix} -CA - CBK_1C & -CBK_2C \end{bmatrix}, \\ \hat{D}_0 &= (I - CBK_3), \end{aligned} \tag{11}$$

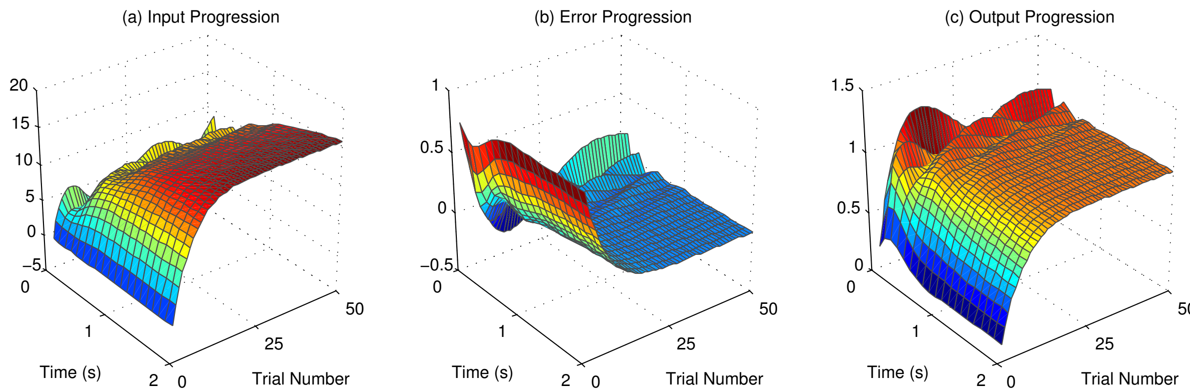


Fig. 1. Responses produced by (2) under the ILC law (3) with $L = 3$.

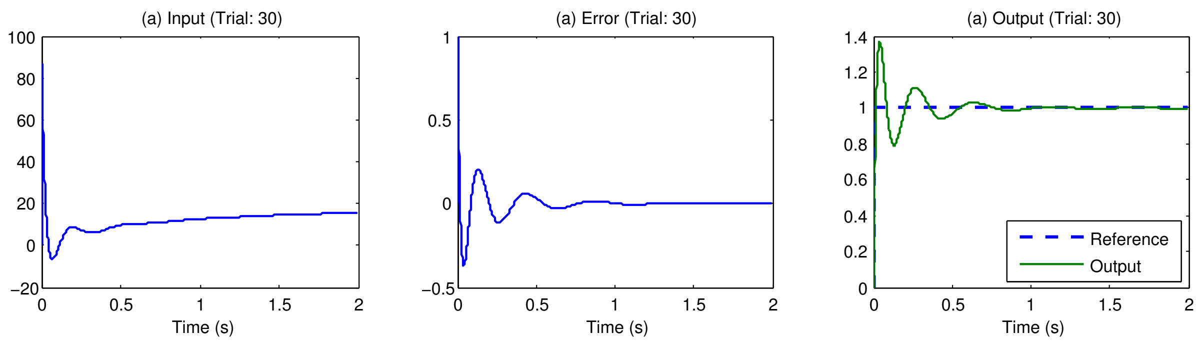


Fig. 2. Responses produced by (2) under the ILC law (3) with $L = 3$ and $k = 30$.

which is of the form (4) and hence the repetitive process stability theory can be applied to this ILC control scheme.

The stability theory for linear repetitive processes with constant pass length consists of two distinct concepts. Asymptotic stability, i.e. BIBO stability over the fixed finite pass length $\alpha > 0$, requires the existence of finite real scalars $M_\alpha > 0$ and $\lambda_\alpha \in (0, 1)$ such that $||L_\alpha^k|| \leq M_\alpha \lambda_\alpha^k$, $k \geq 0$ (where $||\cdot||$ also denotes the induced operator norm). For processes described by (4) it has been shown elsewhere (see, for example, Chapter 3 of [4]) that this property holds if, and only if, $\mathrm{r}(D_{d0}) < 1$. When applied to the ILC state-space model (10) this requires that $\mathrm{r}(\hat{D}_0) = \mathrm{r}(I - CBK_3) < 1$.

This last condition is precisely that obtained by applying 2D discrete linear systems stability theory to (10) as first proposed in [6] to ensure trial-to-trial error convergence only. Using the repetitive process setting, however, provides a means of examining what happens after a 'very large' number of trials have elapsed if this form of stability holds. The method of doing this is by the so-called limit profile for asymptotically stable linear repetitive process which we now introduce in terms of (4).

Suppose that $\mathrm{r}(D_{d0}) < 1$ for a discrete linear repetitive process described by (4). Suppose also and the input sequence applied $\{u_{k+1}\}_k$ converges strongly as $k \to \infty$ (i.e. in the sense of the norm on the underlying function space) to u_∞. Then the strong limit $y_\infty := lim_{k\to\infty} y_k$ is termed the limit profile corresponding to this input sequence and its dynamics (with $D_d = 0$ for ease of presentation) is described by

$$\begin{aligned} x_\infty(p+1) &= (A_d + B_{d0}(I - D_{d0})^{-1}C_d)x_\infty(p) \\ &+ B_d u_\infty(p), \\ y_\infty(p) &= (I - D_{d0})^{-1}C_d x_\infty(p). \end{aligned} \quad (12)$$

Note, however, that this property does not guarantee that the limit profile is stable as a 1D discrete linear system, i.e. $\mathrm{r}(A_d + B_{d0}(I - D_{d0})^{-1}C_d) < 1$ — a point which is easily illustrated by the case when $A_d = -0.5$, $B_d = 0$, $B_{d0} = 0.5 + \beta$, $C_d = 1$, $D_d = 0$, $D_{d0} = 0$ and $\beta > 0$ is a real scalar such that $|\beta| \geq 1$.

The reason why asymptotic stability does not guarantee a limit profile which is stable along the pass is due to the finite pass length. In particular, asymptotic stability is easily shown to be bounded-input bounded-output (BIBO) stability with respect to the finite and fixed pass length. Also in cases where this feature is not acceptable, the stronger concept of stability along the pass must be used. In effect, for the model (4), this requires that the BIBO stability property holds uniformly with respect to the pass length α.

For the discrete linear repetitive processes considered here, there are a wide range of stability along the pass tests but here we use an LMI based condition since, see also below, it leads immediately to algorithms for control law design – a feature which is not present in alternatives.

Theorem 1: [4] A discrete linear repetitive process described by (4) is stable along the pass if there exist matrices

$Y \succ 0$ and $Z \succ 0$ such that the following LMI holds

$$\begin{bmatrix} Z-Y & * & * \\ 0 & -Z & * \\ \hat{A}_1 Y & \hat{A}_2 Y & -Y \end{bmatrix} \prec 0, \quad (13)$$

where

$$\hat{A}_1 = \begin{bmatrix} \hat{A} & \hat{B}_0 \\ 0 & 0 \end{bmatrix}, \hat{A}_2 = \begin{bmatrix} 0 & 0 \\ \hat{C} & \hat{D}_0 \end{bmatrix}. \quad (14)$$

IV. LMI BASED ILC DESIGN

Return now to the ILC setting. Then we have the following result

Theorem 2: An ILC scheme described by (10) is stable along the trial if there exist matrices $Y \succ 0$, $Z \succ 0$, N_1, N_2 and N_3 such that the following LMI with linear constraints holds

$$\begin{bmatrix} Z-Y & * & * \\ 0 & -Z & * \\ \Omega_1 & \Omega_2 & -Y \end{bmatrix} \prec 0, \quad (15)$$
$$CY_1 = PC,$$
$$CY_2 = QC,$$

where

$$Y = \begin{bmatrix} Y_1 & 0 & 0 \\ 0 & Y_2 & 0 \\ 0 & 0 & Y_3 \end{bmatrix}, \quad (16)$$

and

$$\Omega_1 = \begin{bmatrix} AY_1 + BN_1C & BN_2C & BN_3 \\ Y_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$\Omega_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -CAY_1 - CBN_1C & -CBN_2C & Y_3 - CBN_3 \end{bmatrix}. \quad (17)$$

The matrices P and Q are additional decision variables. If the LMI with equality constraints of (15) is feasible, the control law matrices can be calculated using

$$\begin{aligned} \hat{K}_1 &= N_1 P^{-1}, \\ \hat{K}_2 &= N_2 Q^{-1}, \\ \hat{K}_3 &= N_3 Y_3^{-1}. \end{aligned} \quad (18)$$

Proof: Interpret Theorem 1 in terms of the state-space model and then set $CY_1 = PC$, $CY_2 = QC$, where P and Q are unknown matrices to obtain (after routine analysis)

$$\begin{bmatrix} Z-Y & * & * \\ 0 & -Z & * \\ \Omega_1 & \Omega_2 & -Y \end{bmatrix} \prec 0, \quad (19)$$

where

$$\Omega_1 = \begin{bmatrix} AY_1 + BK_1PC & BK_2QC & BK_3Y_3 \\ Y_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$\Omega_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -CAY_1 - CBK_1PC & -CBK_2QC & Y_3 - CBK_3Y_3 \end{bmatrix},$$
$$CY_1 = PC,$$
$$CY_2 = QC. \quad (20)$$

Substituting

$$K_1 P = N_1,\ K_2 Q = N_2,\ K_3 Y_3 = N_3, \quad (21)$$

into (20) to obtain (17). Finally, it is easy to see that (18) can be calculated from (21) and the proof is complete. ■

Finally, to apply the control law of (8) note that after simple algebraic manipulations we obtain

$$\begin{aligned} u_k(p) = \; & u_{k-1}(p) + K_1(y_k(p) - y_{k-1}(p)) \\ & + K_2(y_k(p-1) - y_{k-1}(p-1)) \\ & + K_3(y_{ref}(p+1) - y_{k-1}(p+1)). \end{aligned}$$

V. EXPERIMENTAL VERIFICATION

To experimentally verify the practical value of the described approach tests were undertaken using a multi-axis gantry robot, see Fig. 3, previously used for testing and comparing the performances of other ILC algorithms, see, for example, [7]. Each axis of the gantry robot is controlled individually and the models of all were obtained by means of frequency response tests that determined the continuous-time transfer-functions.

Fig. 3. The gantry robot

The transfer function for X-axis is

$$G(s) = \frac{13077183.4436(s+113.4)}{s(s^2+61.57s+1.125\cdot10^4)(s^2+227.9s+5.647\cdot10^4)} \cdot \frac{(s^2+30.28s+2.13\cdot10^4}{(s^2+466.1s+6.142\cdot10^5)}. \quad (22)$$

This was discretized using the zero-order hold method for a sampling time of $T_s = 0.05$ seconds. The required reference trajectory was designed to simulate a "pick and place" process and this reference signal has been used in all previous algorithm tests allowing comparison of obtained results. The X-axis component of this trajectory is shown in Fig. 4.

The result of Theorem 2 provides infinitely many solutions for the control law matrices where in the example here these are scalars. Moreover, many of the values of K_3 produced could be very small with possibly detrimental effects on along the trial performance. One means of maximizing the value of K_3 is to minimize the objective function

$$f(N_3, Y_3) = -N_3 + Y_3 \cdot h, \quad (23)$$

subject to the LMI constraints (15)–(17), where N_3 and Y_3 are as in Theorem 2 and h is a positive real scalar to be

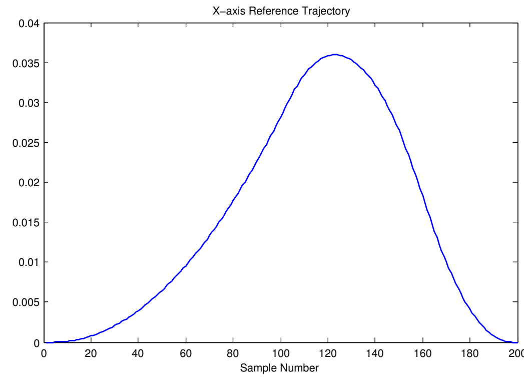


Fig. 4. The X-axis reference trajectory

selected. By this route the following set of alternative feasible control law matrices can be obtained

- $K_3 = 17.74083$,($K_1 = -326.4815$, $K_2 = 4.3525 \cdot 10^{-13}$),
- $K_3 = 47.7641$,($K_1 = -262.8253$, $K_2 = 2.87865 \cdot 10^{-11}$),
- $K_3 = 239.375$,($K_1 = -207.8933$, $K_2 = -4.7220 \cdot 10^{-7}$),

The control laws here have been designed using a sampling period of 0.05 seconds, however this is of the same order as the time constants of the gantry robot actuators and consequently performance of the ILC algorithm may be compromised. A choice of a higher sampling frequency for the design was not possible as the resulting LMIs were not feasible, however, previous experiments have shown that a sampling frequency of at least 100 Hz is required to ensure a reasonable performance. To overcome this problem 5 independent parallel control laws (operating with a 0.05 sec sampling period) were used to produce an updated control demand at 0.01 sec intervals.

Fig. 5 gives the experimental results obtained with the control laws computed above. These clearly show that different choices of feasible control law matrices do indeed influence the performance achieved. In particular, the convergence rate of the algorithm can be increased (but at the possible cause of an increase in the final error).

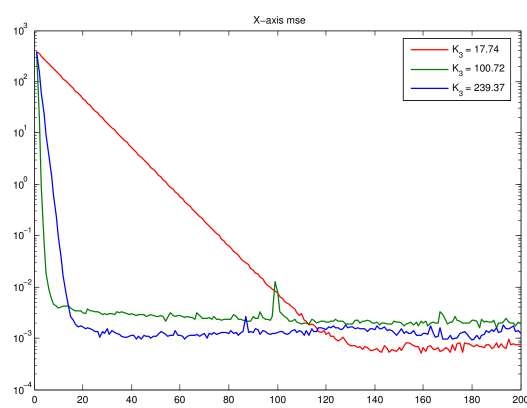


Fig. 5. Influence of the control law on the error $e_k(p)$

It has been frequently reported (see, for example, [8]) that ILC algorithms can exhibit higher frequency noise build up as the number of trials increases and tracking of the reference signal then begins to diverge (due to numerical problems in both computation and measurement). In this design, the higher-frequency component buildup was observed in some cases, resulting in vibrations which greatly increased the error $e_k(p)$ as illustrated in Fig. 6. To deal with such cases, one relatively simple option is to employ a zero-phase Chebyshev low-pass filter. Here it was decided to use a 6 th order filter with parameters tuned to obtain best performance. Here we compare the performance of two filters with different cut-off frequencies. The first has a cut-off at 15 Hz and the second 5 Hz and the filter z transfer-functions, denoted by $H_{15}(z)$ and $H_5(z)$ respectively, are given in the appendix.

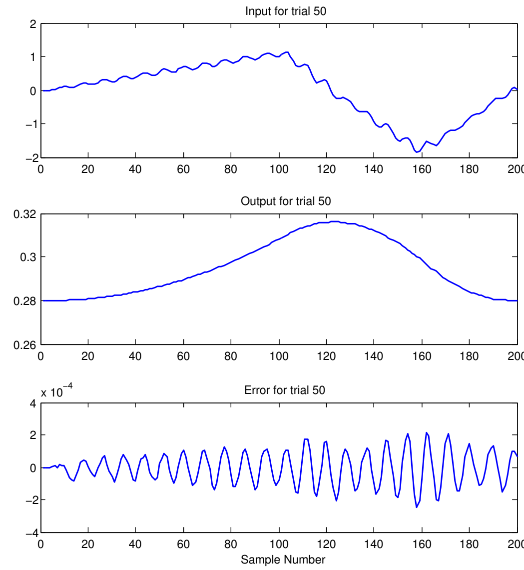
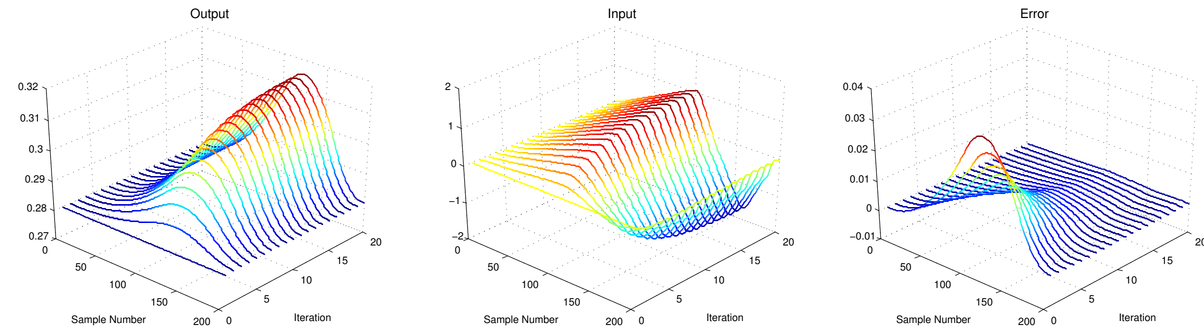


Fig. 6. The Input/Output/Error signals on the 50 th trial without filtering

Fig. 7(a) shows the learning progression of the outputs produced by the X-axis over 20 trials with Figs. 7(b) and 7(c) showing the corresponding control input and error dynamics respectively. These results demonstrate, in particular, that the new ILC design algorithm developed here is capable of preventing the undesirable along the pass dynamics, such as illustrated by Fig. 1 without requiring excessive control action. It must be stressed, however, that if a zero-phase filter is required in application to a physical example then its cut-off frequency influences the performance obtained - see Fig. 8.

VI. Conclusions

This paper has considered the design of ILC schemes using a discrete linear repetitive processes setting. This releases a stability theory for application which demands uniformly bounded along the pass (or trial) dynamics (whereas previous approaches only demand bounded dynamics over the finite pass length). Here we have shown that this approach leads to a stability condition expressed in terms of an LMI with immediate formulas for computing the control law matrices. This is a potentially powerful approach in this general area



(a) (b) (c)

Fig. 7. (a) Evolution of the output of X-axis for the first 20 trials in one experiment, (b) Evolution of the control input, (c) Evolution of the error dynamics

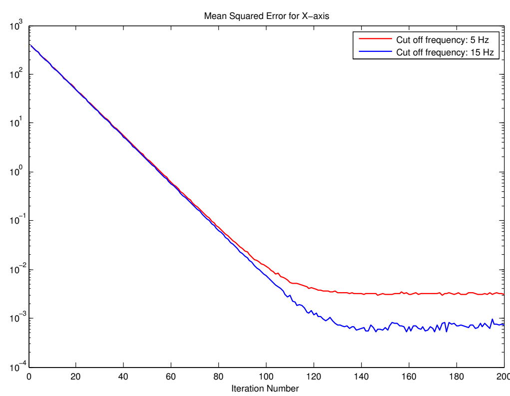


Fig. 8. Influence of the filter cut-off frequency on the error $e_k(p)$

which also makes a significant step forward in the application of repetitive process systems theory. Another particularly notable feature of the results here is that they allow control law design without access to state information together with experimental verification.

The results here establish the basic feasibility of this approach in terms of both theory and experimentation. There is a significant degree of flexibility in the resulting design algorithm and current work is undertaking a detailed investigation of how this can be fully exploited.

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VII. APPENDIX

$$
\begin{aligned}
H_{15}(z) &= \frac{a_{15}(z)}{b_{15}(z)} \\
a_{15}(z) &= 0.0020 + 0.0101z^{-1} + 0.0202z^{-2} \\
&+ 0.0202z^{-3} + 0.0101z^{-4} + 0.0020z^{-5} \\
b_{15}(z) &= 1.0000 - 3.1624z^{-1} + 4.7607z^{-2} \\
&- 4.0528z^{-3} + 1.9344z^{-4} - 0.4153z^{-5}
\end{aligned}
$$

$$
\begin{aligned}
H_5(z) &= \frac{a_5(z)}{b_5(z)} \\
a(z) &= 0.0102 \times 10^{-3} + 0.0512 \times 10^{-3}z^{-1} \\
&+ 0.1024 \times 10^{-3}z^{-1} + 0.1024 \times 10^{-3}z^{-1} \\
&+ 0.0512 \times 10^{-3}z^{-1} + 0.0102 \\
b(z) &= 1.0000 - 4.5879z^{-1} + 8.5399z^{-1} \\
&- 8.0560z^{-1} + 3.8495z^{-1} - 0.7452
\end{aligned}
$$