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On observer-based estimation enhancement by parametric amplification in a weak force measurement device

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Abstract— **This paper focuses on the problem of force estimation performances in some EFM-like device (*Electric Force Microscope*). In this context it is shown how those performances can be significantly improved w.r.t. noise measurement by combining techniques in the spirit of the so-called 'parametric amplification' in particular investigated in physics, with observer techniques as they are developed in the control community. The minimal force which can be estimated in this way is expressed in terms of estimation rate and measurement noise variance.**

Keywords: measurement noise, observer, EFM, force measurement, parametric amplification.

I. INTRODUCTION

Measuring smaller and smaller signals means coping with noises of higher and higher effects (see e.g. [1], [2]). This is typically what happens in the area of so-called *Scanning Probe Microscopy* [3], where appropriate conditions are looked for, aiming at higher performances (measurement in vacuum, at low temperature...). In this context, it has been emphasized a few years ago how some appropriate excitation of the measurement device, known as *parametric amplification* can yield better measurement performances, in particular w.r.t. the sensor noise [4], [5].

On the other hand, it has also been shown how those measurement problems can be efficiently tackled via a system approach relying on state space formalism, allowing to get a better insight into the phenomena and to provide solutions in terms of *state observers* (as in [6] for instance). In particular it has been shown how those observer techniques (such as *Kalman-like* [7] or *high-gain* [8] designs) can yield very efficient noise filtering in the presence of 'minimal excitation' (only aiming at observability) [6], [9].

In the present paper, as a continuation of those previous studies, it is shown how combining observer techniques with techniques of amplification can give better performances of the observer w.r.t. sensor noise, making it possible to reduce the estimation time for a given accuracy - or conversely, to increase the accuracy for a given estimation time. In that respect, the present study can be compared with that of [10] where 'optimal' parametric amplification is investigated with some observer approach, except that in the present paper, we choose some more specific amplification context, and analytically show how it can be tuned w.r.t. some pre-specified performances, in terms of estimation time and accuracy. As a result, we get some formal characterization of the smallest force which can be estimated, given an estimation time and a sensor noise variance.

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This study is based on a measurement device typical of *EFM* (Electric Force Microscopy) [5] firstly presented in section II. The considered amplification principle is then described in section III, together with a full analysis on its effect on observer performances. This analysis is illustrated via simulations in section IV.

Some conclusions and perspectives finally end the paper in section V.

II. CONSIDERED MEASUREMENT DEVICE

We will here consider a measurement device following the principle of the so-called *Electric Force Microscope*, namely depicted by figure 1 below: in short, it is made of a micro-cantilever with a mass m, a stiffness k and subject to friction of coefficient f, whose position x is optically monitored and is supposed to be sensitive to the effect of some force F_D to be measured. A specific feature of the EFM within the SPM techniques is the application of a voltage $V(t)$ between the substrate under study and the cantilever, with the cantilever position being itself mechanically excited. In the present study, this excitation will be assimilated to the effect of F_D, as in [4], the purpose being as in [4] to study the effect of amplification, but here in the context of observer design.

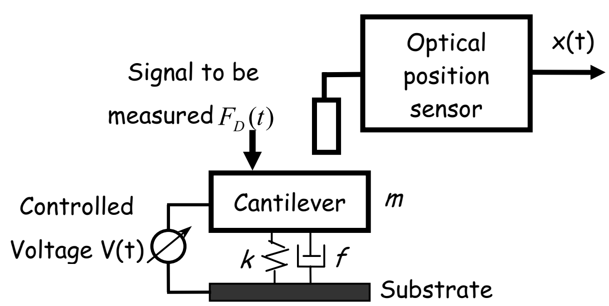

Fig. 1. Measurement device principle.

The resulting overall dynamics can then be classically described by a second order equation of the form:

$$m\ddot{x}(t) + f\dot{x}(t) + kx(t) = F_D(t) + F_{elec}(V, x) \quad (1)$$

where F_{elec} is the electrostatic force due to V between the cantilever and the substrate, which can be written as:

$$F_{elec}(V,x) = \frac{\varepsilon_0 S \cdot V^2(t)}{(D - x(t))^2} \quad (2)$$

with S the surface between the two plates of the capacitor, D their distance in the absence of any deflection, and ε_0 the permittivity in vacuum.

In practice, the directly measured variable y will be assumed to be the position x, delivered by an optical sensor up to some sensor noise ν assumed to be Gaussian white noise. Namely the actual measurement takes the following form:

$$y(t) = x(t) + \nu(t) \quad (3)$$

Notice that in practice there are various types of additional noises that can affect the measurement accuracy, such as the thermal noise which directly disturbs equation (1). It was shown in [9] how such noises can be filtered out, together with ν, by appropriate observer design, but at the expense of the estimation time. The purpose here is to show how the *sensor noise* can be attenuated in shorter time by appropriate *excitation* of the device, following the parametric amplification idea of [4]. It was indeed already underlined in this previous work that such an amplification method is not efficient w.r.t. other noises - such as thermal noise, since it amplifies them together with the signal of interest. For this reason, we will only consider sensor noise in this study (the other ones can anyway be filtered out by the observer, but with no improved performance with the excitation we will consider).

III. CONSIDERED AMPLIFICATION TECHNIQUE AND OBSERVER DESIGN

The problem here considered is thus that of 'measuring' a force F_D which may be very small w.r.t. noises affecting the measurement device here described by equations (1)-(3): in other words, the problem is that of reconstructing F_D from direct measurement of x.

In [4] in particular, it has been shown how this reconstruction can be improved in spite of measurement noises ν within the usual measurement framework in microscopy, by using some specific excitation known as 'parametric amplification': in short, a parameter is periodically modified in the system by some appropriate periodic excitation, so that the effect of some signal to be measured be significantly amplified in the motion x w.r.t. noise ν. More precisely, for system (1), if F_D is of the form $F_0 \cos(w_0 t + \phi)$, such an amplification can be achieved by choosing $V(t) = V_0 + V_p \sin(w_p t)$. This indeed yields a dynamical behaviour which can be (approximately) described by the modified equation:

$$m\ddot{x}(t) + f\dot{x}(t) + (k - \delta k(t))x(t) = F_D(t) \quad (4)$$

where $\delta k(t)$ depends on the excitation. The appropriate choice for such an excitation (V_p, w_p) in this 'pure' amplification context has been fully studied in [4], [5], [11].

Some optimal choice of w_p in a context of position and velocity reconstruction by state observer has also been investigated in [10].

In the present paper, the purpose is to study the improvement obtained by such an amplification-like approach, when using an observer to directly reconstruct the force F_D. To that end, we will consider some 'static' version of the parametric amplification previously recalled, for which a full analysis will then be provided.

The idea indeed is that force reconstruction can be achieved via observer techniques by considering a state space representation including the force as a state variable (as in [6], [9] for instance, following the typical state extension approach of the Extended Kalman Filter). When considering a constant force for instance, the model will be based on $z_1 := x$, $z_2 := \dot{x}$ and $z_3 := F_D$ as state variables, yielding:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= -\frac{k}{m}z_1 - \frac{f}{m}z_2 + \frac{z_3}{m} + \frac{\varepsilon_0 S \cdot V^2(t)}{m(D - z_1)^2} \\ \dot{z}_3 &= 0 \\ y &= z_1 + \nu \end{aligned} \quad (5)$$

Now let us consider a first order approximation around some equilibrium $z_1 = z_{1e}$, $z_2 = 0$ corresponding to a voltage $V(t) = V_0$ when $F_D = 0$. We get:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= -\frac{(k - k_1 V_0^2)}{m}z_1 - \frac{f}{m}z_2 + \frac{z_3}{m} \\ \dot{z}_3 &= 0 \\ y &= z_1 + \nu \end{aligned} \quad (6)$$

with $k_1 = \dfrac{2\varepsilon_0 S}{(D - z_{1e})^3}$.

From this, it appears that V_0 allows us to tune coefficient $k - k_1 V_0^2$ (in a similar way as $k - \delta k(t)$ is somehow tuned in the classical periodic case (4)) - and can even cancel it (with V_0 s.t. $z_{1e} = \frac{D}{3}$). It is also clear that $k - k_1 V_0^2$ is the inverse DC gain between z_3 and y: it is thus possible to arbitrarily tune this gain via V_0 and in particular make it arbitrarily large. More precisely, it can be checked that given any desired gain G, choosing V_0 such that:

$$\begin{aligned} V_0 &= \sqrt{\tfrac{k x_e}{\varepsilon_0 S}}(D - x_e), \text{ with} \\ x_e &= \tfrac{(kG-1)D}{3kG-1} \end{aligned} \quad (7)$$

makes $k - k_1 V_0^2$ to be equal to $\frac{1}{G}$ for the system (6) obtained by linearizing (5) around the equilibrium $z_1 = x_e, z_2 = 0, V = V_0$. Of course for any finite strictly positive gain G, this linearized model is stable, and thus the original nonlinear one is locally stable by standard Lyapunov arguments [12], namely the analysis carried out on the basis of model (6) will be valid for forces (z_3) 'small enough'.

In the sequel, we will call this gain G 'amplification gain'. Intuitively, increasing this gain will improve the signal-to-noise ratio in the measured output, and thus result in improving estimation performances. But noting that increasing G in (6) also results in making the system slower, one can also expect that the best gain is not necessarily the largest one.

The effect of this amplification gain on observer performances when used for a purpose of force (z_3) reconstruction can actually be formulated via the following result:

Proposition *3.1:* Given a system in observable form:

$$\begin{aligned} \dot{\xi} &= \begin{pmatrix} -a_1 & 1 & 0 \\ -a_2 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \xi = A\xi \\ y &= \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \xi + \nu = C\xi + \nu, \end{aligned} \tag{8}$$

where ν is a gaussian white noise with variance W,

and given any observer:

$$\dot{\hat{\xi}} = A\hat{\xi} - K(C\hat{\xi} - y)$$

for a convergence rate arbitrarily chosen by $K = \begin{pmatrix} k_1 & k_2 & k_3 \end{pmatrix}^T$,

then the third component e_3 of the estimation error $e := \hat{\xi} - \xi$ decays to zero in means with a rate given by K, and its variance $v_3(t) := E[e_3(t)^2]$ satisfies:

- v_3 asymptotically goes to:

$$\begin{aligned} v_{3\infty} = & \frac{k_3 W}{2[(k_1 + a_1)(k_2 + a_2) - k_3]} \times \\ & \times [k_3 a_1^2 + k_2^2(k_1 + a_1) + (a_2 - k_2)[(k_1 + a_1)(k_2 + a_2) - k_3]] \end{aligned} \tag{9}$$

- $v_{3\infty}$ decreases with a_2, as long as:

$$a_2 \geq \frac{k_3}{k_1 + a_1} =: a_{20} \tag{10}$$

- $v_{3\infty}$ admits a minimum for $a_2 = a_{20}$ of the form:

$$v_{3\infty 0} := \frac{k_3^2}{2k_2(k_1 + a_1)}(k_2 + a_1^2)W \tag{11}$$

•

Proof: The proof follows from the form of the estimation error equation:

$$\dot{e} = (A - KC)e + K\nu$$

From this indeed, the expectation $E[e]$ of e classically goes to zero according to the dynamics fixed by $A - KC$, and its variance $M(t) = E[e(t)e^T(t)]$ satisfies [13]:

$$\dot{M}(t) = M(t)(A - KC)^T + (A - KC)M(t) + KWK^T.$$

By stability of $A - KC$ it admits a stationary solution given by:

$$0 = M(A - KC)^T + (A - KC)M + KWK^T.$$

Expression (9) for the variance of e_3 is obtained by directly solving this equation (notice that the denominator in $v_{3\infty}$ is strictly positive whenever $A - KC$ is stable, by simple application of Routh stability criterion for instance, i.e. it cannot be zero).

Condition (10) then follows by studying the evolution of $v_{3\infty}$ w.r.t. a_2: it can indeed be checked that the derivative of $v_{3\infty}$ w.r.t. a_2 is positive as long as (10) holds (notice, when computing this derivative, that in $v_{3\infty}$, $k_1 + a_1$, $k_2 + a_2$, k_3 only depend on the chosen rate of convergence, and not on a_2).

Finally, (11) directly follows from (9) and (10). ■

This result can be interpreted as follows:

For system (8), the amplification gain (between ξ_3 - third component of ξ - and y) is $\frac{1}{a_2}$. Then property (10) of proposition 3.1 says that for a given convergence rate (in means), the achieved variance on the estimation of ξ_3 is indeed reduced when increasing the amplification gain, but only up to $\frac{1}{a_{20}}$. This means that amplification indeed improves the observer performances for ξ_3 reconstruction, only up to a certain gain.

Expression (11) then gives the corresponding minimal variance which can be achieved: from it, it can be checked that this minimal value grows when the observer is made faster (by noting that changing poles p_i of $A - KC$ into λp_i roughly turns into changing k_i's into $\lambda^i k_i$'s).

Now coming back to the original problem of force reconstruction, it can be noticed that system (6) can be turned into (8) by :

$$\begin{aligned} \xi_1 &= z_1 \\ \xi_2 &= z_2 + \tfrac{f}{m} z_1 \\ \xi_3 &= \tfrac{z_3}{m} \\ a_1 &= \tfrac{f}{m} \\ a_2 &= \tfrac{1}{mG} \end{aligned}$$

where G is the considered tunable amplification gain.

From this, and proposition 3.1, we get the 'optimal gain' for force reconstruction given by:

$$G_{opt} = \frac{1}{m a_{20}}, \tag{12}$$

and the corresponding achieved minimal variance:

$$v_{opt} = m^2 v_{3\infty 0}. \tag{13}$$

This actually corresponds to the minimal variance which can be achieved *for a fixed convergence rate*: namely it gives a characterization of the minimal force which can be detected given a sensor noise variance, *taking dynamics into account*: this makes it a dynamical version of the minimal detectable force analysis proposed in [4] for instance. It also recalls how reducing even more this minimal detectable force can be obtained by making the observer slower.

IV. SIMULATION VALIDATION

Let us consider here as an example a device similar to the one presented in [4], namely with numerical values for model (1)-(2) given by:

$$m = 0.22e-12kg,\ f = 4.7e-11Nsm^{-1},\ k = 1Nm^{-1},$$

$$S = 3.4e-8m^2,\ D = 20e-6m.$$

Let us also consider the estimation of nanoN forces (typically $F_D = 1e - 9N$ in the simulations), and a sensor noise significantly larger than the cantilever motion induced by such a force (see figure 2 for the x response to F_D when $V_0 = 0$, and the corresponding simulated measurement y).

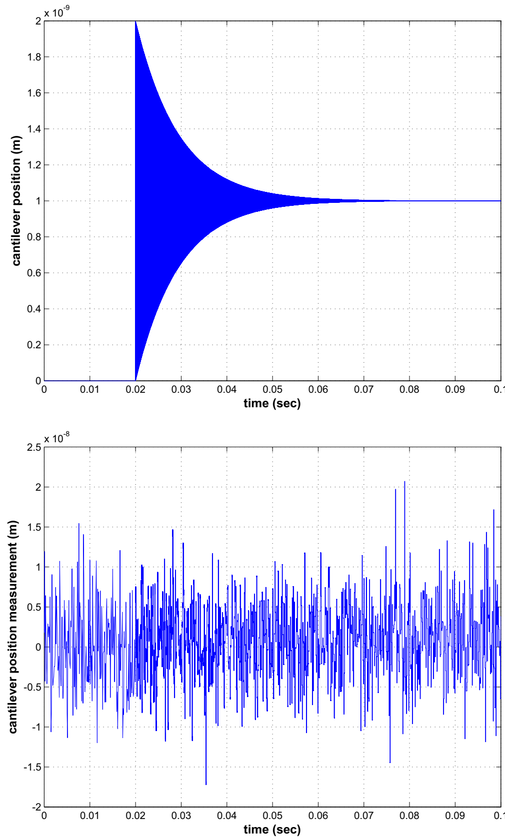

Fig. 2. Response to force F_D in position x (top) and measurement y (bottom)

For this example, we will illustrate how with an observer corresponding to a given convergence rate in means (a given set of poles for $A - KC$) the estimation accuracy for F_D can indeed be improved by gain amplification.

All simulations have been performed with the nonlinear model (5).
The observer gain has been chosen so as to achieve a convergence rate of about $1ms$.

In this context, simulation results are presented for two different tuned gains: a first one with no amplification ($G = 1$, corresponding to $V_0 = 0V$), and a second one with a large one ($G = 10$, corresponding to $V_0 = 62.5V$). The respective estimation results and errors for F_D can be seen on figures 3 and 4.

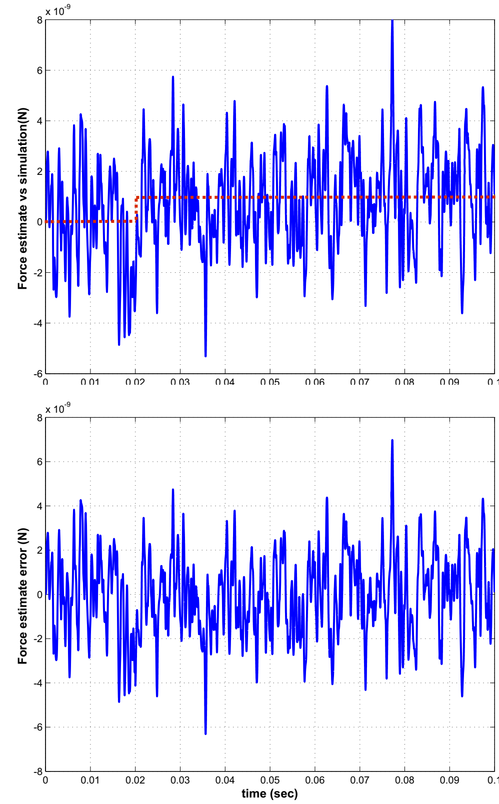

Fig. 3. Estimation (top) and error (bottom) for $F_D = 1e-9$ at $t = 0.02s$ and $G = 1$

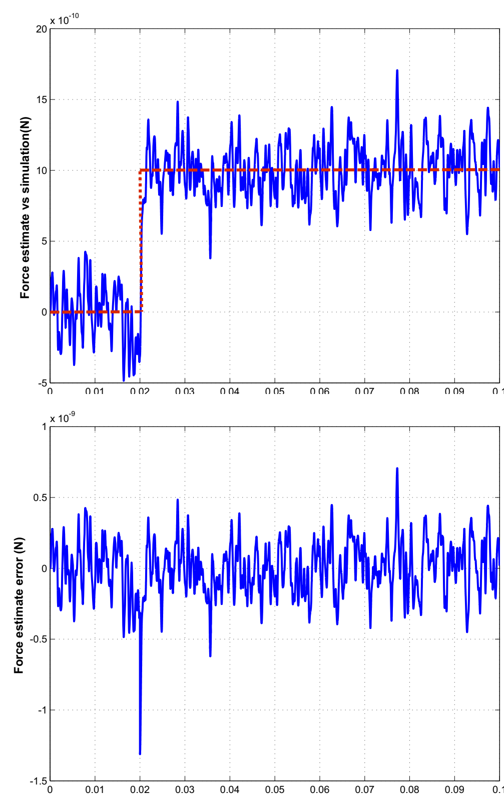

Fig. 4. Estimation (top) and error (bottom) for $F_D = 1e-9$ at $t = 0.02s$ and $G = 10$

From those figures, the results are consistent with the analysis of previous section: for a given convergence rate, the estimation accuracy can indeed be improved by increasing the amplification gain. Furthermore, this is obtained for quite reasonable values of the voltage V_0 (they can even become smaller depending on the device numerical characteristics).

The variations of V_0 w.r.t. the desired gain G, as well as the corresponding achieved variance, are reported for this example on figure 5 below (for $G \in [1, 10]$).

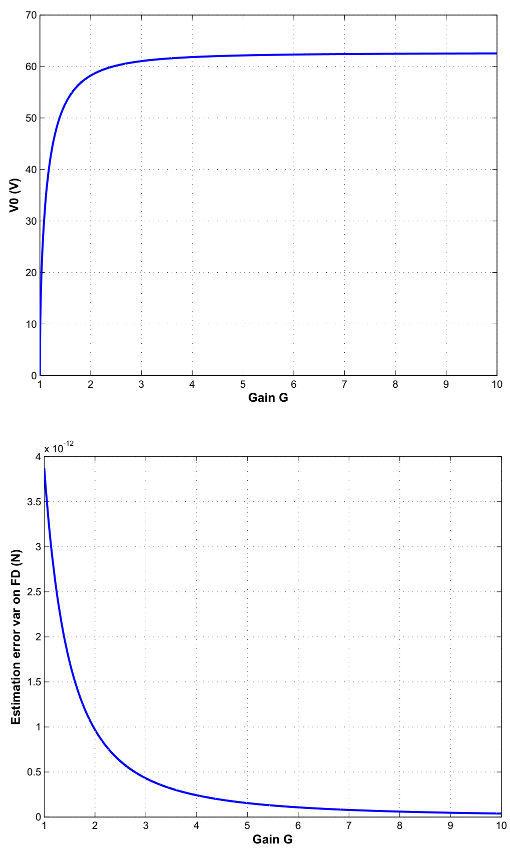

Fig. 5. V_0 (top) and achieved asymptotic variance on F_D (bottom) vs amplification gain G

From this, it is clear that increasing the gain of only a few units significantly improves the accuracy, while the voltage is only to be increased from 0 up to an asymptotic value of about $62V$ (consistent with expression (7) from which V_0 approaches $\frac{2D}{3}\sqrt{\frac{kD}{3\varepsilon_0 S}} = 62.66V$ as G goes to infinity). Theoretically, the optimal gain and variance given by (12)-(13) here correspond to a much larger value of the gain equal to $1.95e5$, for which the variance reaches $8.11e-22N^2$. A zoom of the variance of figure 5 in those values of gain is given in figure 6, where it can be seen how an optimal variance is indeed reached, and after the optimal gain, the variance becomes worse again (however in practice, this effect is not really significant in the estimation results).

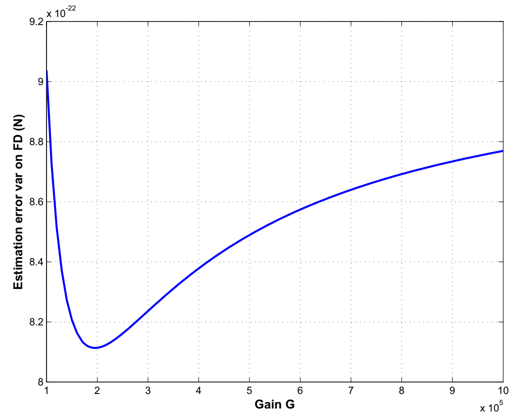

Fig. 6. Achieved asymptotic variance on F_D vs amplification gain G near optimality

All those results are in very good accordance with the analysis of previous section. As an extension, it can finally be checked how classical parametric amplification also improves observer performances in a similar fashion, when estimating some *harmonic* force F_D on the basis of an extended system as in the analysis of section III: simulation results in that respect are presented on figures 7 and 8 (estimation errors) for two different magnitudes of the excitation voltage, while a full analysis in this configuration is left for further studies (typically in this case, the excitation frequency has also an effect [4], [10], [5]).

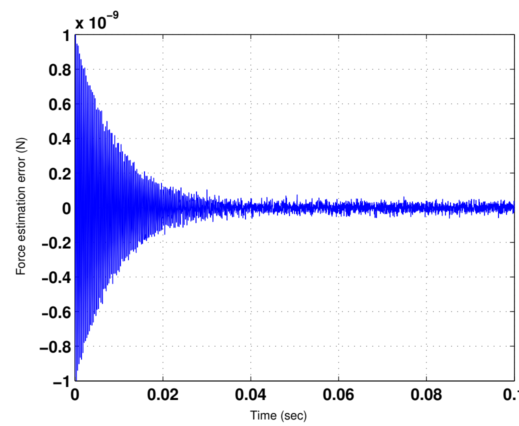

Fig. 7. Estimation error for some harmonic F_D with $V(t) = 10 + 0.26sin(2w_0t + \frac{\pi}{2})$

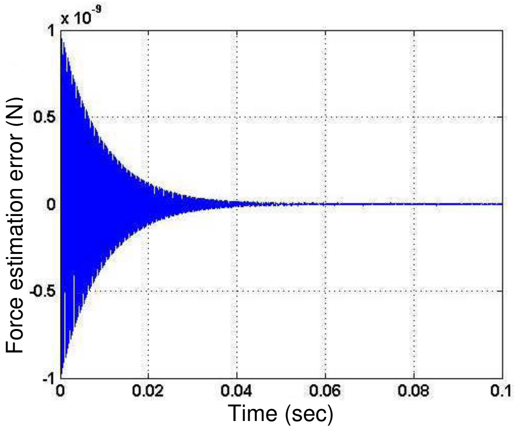

Fig. 8. Estimation error for some harmonic F_D with $V(t) = 10 + 2.6sin(2w_0t + \frac{\pi}{2})$

V. CONCLUSIONS AND FUTURE WORKS

In this work, it has been analytically shown how some appropriate amplification gain in a system can improve the estimation performances of an observer w.r.t. measurement noise. This has been emphasized in the context of weak force measurement where noises are of significant effect. The analysis has only been carried out w.r.t. sensor noise since the proposed excitation cannot improve the results w.r.t. other noises. But it can of course be combined with usual filtering properties of observers in that respect. This will be part of future studies. Extending this analysis to the more classical parametric amplification case will also be part of further developments, as well as including robustness study.

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