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Design of a Multivariable Implicit Self-Tuning Controller

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***Abstract*— In this paper, an implicit multivariable self-tuning controller is designed based on the Lyapunov function. The STC parameters convergence is proved when the numbers of plant input and output signals are same. The obtained result is a generalization of [2] to the multivariable case.**

I. INTRODUCTION

By merging control with identification, Self-Tuning Control (STC) potentially has numerous practical applications. A STC approach which directly estimates controller parameters from plant I/O data sequence is referred to as "Implicit Self-Tuning Control". The other approach which estimates plant parameters at first and control is designed on the basis of the obtained parameters is "Explicit Self-Tuning Control". The present paper concerns the former. Minimum-Variance Control (MVC) or Generalized Minimum-Variance Control (GMVC) laws have been used in STC systems because of its less computational load. However, even in the Single-Input Single-Output (SISO) case, the general stability proof for STC systems is known difficult due to its time-varying and nonlinear natures caused by the involved recursive parameters estimation. Some past researches have resorted to numerical analyses for evaluating the parameters convergence and stability.

It was recently clarified that the stability of an implicit self-tuning control with generalized minimum variance criterion can be proved in the standard way [2], which is based on the Lyapunov function and the ideas of discrete-time sliding-mode control [3][4]. The advanced technology like today's automobile engines tends to increase the number of controlled variables required in the systems. The multivariable STC systems have been also studied extensively since the studies [5][6]. In the present paper, we propose an implicit multivariable self-tuning control design by generalizing the result [2] to Multi-Input Multi-Output (MIMO) systems. The essence of this generalization is to introduce the different Lyapunov function for each plant output. Simulation examples are also given to evaluate the performance of the proposed design.

II. MULTIVARIABLE GENERALIZED MINIMUM VARIANCE CONTROL

Consider a m-input and m-output plant described by the vector difference equation

$$A(z^{-1})y_k = B(z^{-1})z^{-d}u_k \tag{1}$$

where u_k is the plant input, y_k is the plant output

$$u_k = \begin{bmatrix} u_{k,1} \\ \vdots \\ u_{k,m} \end{bmatrix},\ y_k = \begin{bmatrix} y_{k,1} \\ \vdots \\ y_{k,m} \end{bmatrix} \tag{2}$$

, $A(z^{-1})$ and $B(z^{-1})$ are the $m \times m$ square polynomial matrices

$$A(z^{-1}) = I + A_1 z^{-1} + \cdots + A_n z^{-n} \tag{3}$$

$$B(z^{-1}) = B_0 + B_1 z^{-1} + \cdots + B_l z^{-l} \tag{4}$$

(B_0: nonsingular)

For an actual sampling-time T, the backward-shift operator z^{-1} means $z^{-1} = e^{-sT}$ in the Laplace domain.

Consider the following controlled vector consisting of sliding-mode type variables $s_{k,i}$ $(i = 1, 2, \cdots, m)$:

$$s_{k+d} = C(z^{-1})(y_{k+d} - r_{k+d}) + Q(z^{-1})u_k \tag{5}$$

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$$s_{k+d} = \begin{bmatrix} s_{k+d,1} \\ \vdots \\ s_{k+d,m} \end{bmatrix}, \; r_{k+d} = \begin{bmatrix} r_{k+d,1} \\ \vdots \\ r_{k+d,m} \end{bmatrix} \tag{6}$$

where r_k is the reference signal vector and $C(z^{-1})$ is the polynomial matrix given by

$$C(z^{-1}) = I + C_1 z^{-1} + \cdots + C_n z^{-n} \tag{7}$$

Note that all the polynomial elements of $C(z^{-1})$ should be selected as *Schur*, i.e., all the polynomial zeros exist inside the unit desk because the error dynamics $e_k := y_k - r_k$ is specified by $C(z^{-1})$. The $Q(z^{-1})$ is additionally introduced to treat non-minimum phase systems, which is called Generalized Minimum Variance Control.

Following [2], the control u_k is designed so that the controlled vector (5) may be vanished when the deterministic system (1) is considered.

To derive the control law, consider the $m \times m$ square polynomial matrices

$$E(z^{-1}) = E_0 + E_1 z^{-1} + \cdots + E_{d-1} z^{-(d-1)} \tag{8}$$
$$F(z^{-1}) = F_0 + F_1 z^{-1} + \cdots + F_{n-1} z^{-(n-1)} \tag{9}$$

which should be designed to satisfy the *Diophantine* equation

$$C(z^{-1}) = E(z^{-1})A(z^{-1}) + z^{-d}F(z^{-1}) \tag{10}$$

Multiplying (1) by $E(z^{-1})$ from the left,

$$E(z^{-1})A(z^{-1})y_k = E(z^{-1})B(z^{-1})u_{k-d} \tag{11}$$

Substituting (11) into (5) gives

$$\begin{aligned} s_{k+d} \; = \; & E(z^{-1})B(z^{-1})u_k + F(z^{-1})y_k \\ & -C(z^{-1})r_{k+d} + Q(z^{-1})u_k \end{aligned} \tag{12}$$

Thus the control u_k achieving $s_{k+d} = 0$ is obtained as

$$u_k = G(z^{-1})^{-1}(C(z^{-1})r_{k+d} - F(z^{-1})y_k) \tag{13}$$

where $G(z^{-1})$ is a nonsingular matrix defined by

$$G(z^{-1}) = E(z^{-1})B(z^{-1}) + Q(z^{-1}) \tag{14}$$

In the present paper, the diagonal form is assumed for $Q(z^{-1})$.

$$Q(z^{-1}) = \begin{bmatrix} Q_{11}(z^{-1}) & & \\ & \ddots & \\ & & Q_{mm}(z^{-1}) \end{bmatrix} \tag{15}$$
$$Q_{ii}(z^{-1}) = q_{ii}(1 - z^{-1}) \tag{16}$$

For the closed-loop stability analysis, the elimination of u_k between (1) and (5) yields

$$\begin{aligned} &(C(z^{-1}) + Q(z^{-1})B^{-1}(z^{-1})A(z^{-1}))y_k \\ &\quad = s_k + C(z^{-1})r_k \end{aligned} \tag{17}$$

Thus the closed-loop characteristic from r_k to y_k may be evaluated by the solutions of (18).

$$\det[C(z^{-1}) + Q(z^{-1})B^{-1}(z^{-1})A(z^{-1})] = 0 \tag{18}$$

The polynomial matrix $Q(z^{-1})$ should be designed so that all the solutions of (18) exist inside the unit disk. However it is not generally easy in MIMO cases to find $Q(z^{-1})$ (and $C(z^{-1})$) which stabilize the closed loop.

III. MULTIVARIABLE IMPLICIT SELF-TUNING CONTROL

Figure 1 is the considered block diagram, where the control parts $F(z^{-1})$ and $G(z^{-1})$ are recursively and directly estimated from plant I/O data sequence. A generalization of the STC study [2] into a multivariable case will be presented in this section.

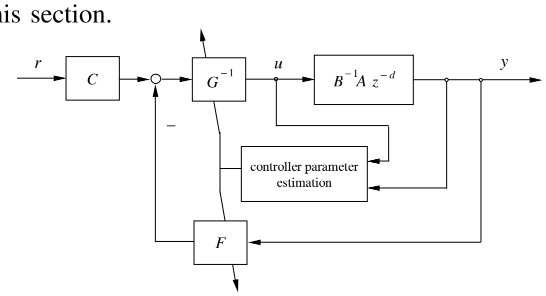

Fig. 1. Implicit Self-tuning Control

Based on (13), we may obtain

$$u_k = -\hat{G}(z^{-1})^{-1}(\hat{F}(z^{-1})y_k - C(z^{-1})r_{k+d}) \tag{19}$$

where $\hat{F}(z^{-1})$ and $\hat{G}(z^{-1})$ are the estimates of $F(z^{-1})$ and $G(z^{-1})$, respectively. Defining the estimation error matrices $\tilde{F}(z^{-1})$, $\tilde{G}(z^{-1})$ and using (12), we have

$$s_{k+d} = \tilde{G}(z^{-1})u_k + \tilde{F}(z^{-1})y_k \tag{20}$$

where

$$\tilde{F}(z^{-1}) = \begin{bmatrix} \tilde{F}_{11}(z^{-1}) & \cdots & \tilde{F}_{m1}(z^{-1}) \\ \vdots & & \vdots \\ \tilde{F}_{1m}(z^{-1}) & \cdots & \tilde{F}_{mm}(z^{-1}) \end{bmatrix} \tag{21}$$

$$\tilde{G}(z^{-1}) = \begin{bmatrix} \tilde{G}_{11}(z^{-1}) & \cdots & \tilde{G}_{m1}(z^{-1}) \\ \vdots & & \vdots \\ \tilde{G}_{1m}(z^{-1}) & \cdots & \tilde{G}_{mm}(z^{-1}) \end{bmatrix} \tag{22}$$

$$\tilde{F}_{ij}(z^{-1}) = \tilde{f}^0_{ij} + \cdots + \tilde{f}^{n-1}_{ij} z^{-(n-1)} \tag{23}$$
$$= (f^0_{ij} - \hat{f}^0_{ij}) + \cdots + (f^{n-1}_{ij} - \hat{f}^{n-1}_{ij}) z^{-(n-1)} \tag{24}$$
$$\tilde{G}_{ij}(z^{-1}) = \tilde{g}^0_{ij} + \cdots + \tilde{g}^{l+d-1}_{ij} z^{-(l+d-1)} \tag{25}$$
$$= (g^0_{ij} - \hat{g}^0_{ij}) + \cdots + (g^{l+d-1}_{ij} - \hat{g}^{l+d-1}_{ij}) z^{-(l+d-1)} \tag{26}$$

Notice that the symbol x of $\tilde{f}^x_{ij}$ and $\tilde{g}^x_{ij}$ is not the power of a number but the suffix.

By defining

$$\tilde{G}_{i*}(z^{-1}) = \left[\; \tilde{G}_{i1}(z^{-1}) \; \cdots \; \tilde{G}_{im}(z^{-1}) \; \right] \tag{27}$$
$$\tilde{F}_{i*}(z^{-1}) = \left[\; \tilde{F}_{i1}(z^{-1}) \; \cdots \; \tilde{F}_{im}(z^{-1}) \; \right] \tag{28}$$

, (20) is written as

$$s_{k+d,i} = \tilde{G}_{i*}(z^{-1})u_k + \tilde{F}_{i*}(z^{-1})y_k \;\; (i = 1, 2, \cdots, m) \tag{29}$$

For the parameter estimation, (29) may be written in the following form:

$$s_{k+d,i} = \varphi_k^T \tilde{\theta}_{k+d,i*} \tag{30}$$

where φ_k is the plant I/O data vector, and $\tilde{\theta}_{k,i*}$ is the concatenated error vector

$$\tilde{\theta}_{k,i*} = \hat{\theta}_{k,i*} - \theta_{i*} \tag{31}$$

for the controller parameter errors $\tilde{F}_{i*}$ and $\tilde{G}_{i*}$.

For (30), consider the Lyapunov function candidate

$$V_{k,i} = \frac{1}{2}s_{k,i}^2 + \frac{1}{2}\tilde{\theta}_{k,i*}^T P_{k,i}^{-1} \tilde{\theta}_{k,i*} \tag{32}$$

where $P_{k,i}^{-1}$ is a given weight on the convergence of estimated parameters. The Lyapunov function for the considered multivariable case may be conveniently written as

$$V_k = \frac{1}{2}s_k^T s_k + \frac{1}{2}\tilde{\theta}_k^T P_k^{-1} \tilde{\theta}_k \tag{33}$$

where

$$P_k^{-1} = \begin{bmatrix} P_{k,1}^{-1} & & \\ & \ddots & \\ & & P_{k,m}^{-1} \end{bmatrix}, \ \tilde{\theta}_k = \begin{bmatrix} \tilde{\theta}_{k,1*} \\ \vdots \\ \tilde{\theta}_{k,m*} \end{bmatrix} \tag{34}$$

That is, (33) implies

$$V_k = \sum_{i=1}^{m} V_{k,i} \tag{35}$$

For each $V_{k,i}$ $(i = 1, 2, \cdots, m)$, the following parameter estimation alrorithm is considered:

$$\hat{\theta}_{k,i*} = \hat{\theta}_{k-1,i*} + K_{k,i}(s_{k,i} + Cr_{k,i} - \varphi_{k-d}^T \hat{\theta}_{k-1,i*}) \tag{36}$$

$$K_{k,i} = P_{k-1,i}\varphi_{k-d}(\mu_i + \varphi_{k-d}^T P_{k-1,i}\varphi_{k-d})^{-1} \tag{37}$$

$$\begin{aligned} P_{k,i} = \mu^{-1}(&P_{k-1,i} - P_{k-1,i}\varphi_{k-d} \\ &\times(\mu_i + \varphi_{k-d}^T P_{k-1,i}\varphi_{k-d})^{-1}\varphi_{k-d}^T P_{k-1,i}) \end{aligned} \tag{38}$$

where μ_i $(0 < \mu_i \leq 1)$ is the forgetting factor. Following [2], $V_k \leq 0$ can be shown as follows:

The difference of $V_{k,i}$ is given by

$$\Delta V_{k,i} = V_{k,i} - \mu_i V_{k-1,i} \tag{39}$$

$$\begin{aligned} &= \frac{1}{2}s_{k,i}^2 + \frac{1}{2}\tilde{\theta}_{k,i*}^T P_{k,i}^{-1}\tilde{\theta}_{k,i*} - \frac{1}{2}\mu_i s_{k-1,i}^2 \\ &\quad -\frac{1}{2}\mu_i \tilde{\theta}_{k-1,i*}^T P_{k-1,i}^{-1}\tilde{\theta}_{k-1,i*} \end{aligned} \tag{40}$$

By adding $0 = \mu_i\tilde{\theta}_{k,i*}^T P_{k-1,i}^{-1}\tilde{\theta}_{k,i*} - \frac{1}{2}\mu_i\tilde{\theta}_{k,i*}^T P_{k-1,i}^{-1}\tilde{\theta}_{k,i*} - \frac{1}{2}\mu_i\tilde{\theta}_{k,i*}^T P_{k-1,i}^{-1}\tilde{\theta}_{k,i*}$ and $0 = \frac{1}{2}\mu_i\tilde{\theta}_{k,i*}^T P_{k-1,i}^{-1}\tilde{\theta}_{k-1,i*} + \frac{1}{2}\mu_i\tilde{\theta}_{k-1,i*}^T P_{k-1,i}^{-1}\tilde{\theta}_{k,i*} - \mu_i\tilde{\theta}_{k,i*}^T P_{k-1,i}^{-1}\tilde{\theta}_{k-1,i*}$, (40) is rewritten as

$$\begin{aligned} &\Delta V_{k,i} \\ &= s_{k,i}^2 - \frac{1}{2}s_{k,i}^2 - \frac{1}{2}\mu_i s_{k-1,i}^2 + \frac{1}{2}\tilde{\theta}_{k,i*}^T(P_{k,i}^{-1} - \mu_i P_{k-1,i}^{-1})\tilde{\theta}_{k,i*} \\ &\quad +\mu_i\tilde{\theta}_{k,i*}^T P_{k-1,i}^{-1}\tilde{\theta}_{k,i*} - \mu_i\tilde{\theta}_{k,i*}^T P_{k-1,i}^{-1}\tilde{\theta}_{k-1,i*} \\ &\quad -\frac{1}{2}\mu_i(\tilde{\theta}_{k,i*} - \tilde{\theta}_{k-1,i*})^T P_{k-1,i}^{-1}(\tilde{\theta}_{k,i*} - \tilde{\theta}_{k-1,i*}) \end{aligned} \tag{41}$$

Substituting $s_{k,i} = \varphi_{k-d}^T\tilde{\theta}_{k,i*}$ and rearranging the equation,

$$\begin{aligned} \Delta V_{k,i} &= -\frac{1}{2}\mu_i s_{k-1,i}^2 \\ &\quad -\frac{1}{2}\mu_i(\tilde{\theta}_{k,i*} - \tilde{\theta}_{k-1,i*})^T P_{k-1,i}^{-1}(\tilde{\theta}_{k,i*} - \tilde{\theta}_{k-1,i*}) \\ &\quad +\frac{1}{2}\tilde{\theta}_{k,i*}^T(P_{k,i}^{-1} - \mu_i P_{k-1,i}^{-1} - \varphi_{k-d}\varphi_{k-d}^T)\tilde{\theta}_{k,i*} \\ &\quad +\mu_i\tilde{\theta}_{k,i*}^T P_{k-1,1}^{-1}(\tilde{\theta}_{k,i*} - \tilde{\theta}_{k-1,i*} \\ &\qquad +\mu_i^{-1}P_{k-1,i}\varphi_{k-d}\varphi_{k-d}^T\tilde{\theta}_{k,i*}) \end{aligned} \tag{42}$$

By the matrix inversion lemma, (38) is rewritten as

$$P_{k,i} = (\mu_i P_{k-1,i}^{-1} + \varphi_{k-d}\varphi_{k-d}^T)^{-1} \tag{43}$$

, which implies the third term in (42) is eliminated. On the other hand, Substituting (37) and (5) into (36) gives

$$\begin{aligned} \hat{\theta}_{k,i*} &= \hat{\theta}_{k-1,i*} + P_{k-1,i}\varphi_{k-d}(\mu_i + \varphi_{k-d}^T P_{k-1,i}\varphi_{k-d})^{-1} \\ &\quad \times(Cy_k + Qu_{k-d} - \varphi_{k-d}^T\hat{\theta}_{k-1,i*}) \end{aligned} \tag{44}$$

With the equality $P_{k-1,i}\varphi_{k-d}(\mu_i + \varphi_{k-d}^T P_{k-1,i}\varphi_{k-d})^{-1} = (\mu_i + P_{k-1,i}\varphi_{k-d}\varphi_{k-d}^T)^{-1}P_{k-1,i}\varphi_{k-d}$, we have

$$\begin{aligned} \tilde{\theta}_{k,i*} &= \tilde{\theta}_{k-1,i*} - P_{k-1,i}\varphi_{k-d}(\mu_i + \varphi_{k-d}^T P_{k-1,i}\varphi_{k-d})^{-1} \\ &\quad \times\varphi_{k-d}^T\tilde{\theta}_{k-1,i*} \end{aligned} \tag{45}$$

$$\begin{aligned} &= \tilde{\theta}_{k-1,i*} - (\mu_i + P_{k-1,i}\varphi_{k-d}\varphi_{k-d}^T)^{-1} \\ &\quad \times P_{k-1,i}\varphi_{k-d}\varphi_{k-d}^T\tilde{\theta}_{k-1,i*} \end{aligned} \tag{46}$$

Multiplying (46) by $\mu_i + P_{k-1,i}\varphi_{k-d}\varphi_{k-d}^T$ from the left and rearranging the equation yield

$$\mu_i\tilde{\theta}_{k,i*} + P_{k-1,i}\varphi_{k-d}\varphi_{k-d}^T\tilde{\theta}_{k,i*} = \mu_i\tilde{\theta}_{k-1,i*} \tag{47}$$

, which tells that the fourth term in (42) vanishes.

Therefore, it is found that

$$\Delta V_{k,i} \leq 0 \tag{48}$$

Taking the summation with respect to i yields

$$\Delta V_k = \sum_{i=1}^{m}\Delta V_{k,i} \leq 0 \tag{49}$$

Thus the convergence of MIMO-STC parameters can be proved by the Multi-Input Single-Output (MISO) formulation described above.

IV. Example

A. Minimum Phase System

As a design example, the following 2-input 2-output plant is considered:

$$y_k = A_1 y_{k-1} + B_0 u_{k-1} \tag{50}$$

$$A_1 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \ B_0 = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \tag{51}$$

The controller parts $\hat{F}$ and $\hat{G}$ to be estimated are of the forms

$$\hat{F} = \begin{bmatrix} \hat{f}_{11} & \hat{f}_{12} \\ \hat{f}_{21} & \hat{f}_{22} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{52}$$

$$\hat{G} = \begin{bmatrix} \hat{g}_{11} & \hat{g}_{12} \\ \hat{g}_{21} & \hat{g}_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \tag{53}$$

Define the Lyapunov function

$$V_k = V_{k,1} + V_{k,2} \tag{54}$$

$$V_{k,1} = \frac{1}{2}s_{k,1}^2 + \frac{1}{2}\tilde{\theta}_{k,1*}^T P_{k,1}^{-1}\tilde{\theta}_{k,1*} \tag{55}$$

$$V_{k,2} = \frac{1}{2}s_{k,2}^2 + \frac{1}{2}\tilde{\theta}_{k,2*}^T P_{k,2}^{-1}\tilde{\theta}_{k,2*} \tag{56}$$

where

$$s_{k,1} = \varphi_{k-1}^T\tilde{\theta}_{k,1*} \tag{57}$$

$$s_{k,2} = \varphi_{k-1}^T\tilde{\theta}_{k,2*} \tag{58}$$

$$\begin{aligned} \varphi_{k-1}^T &= \begin{bmatrix} y_{k-1,1} & y_{k-1,2} & u_{k-1,1} & u_{k-1,2} \end{bmatrix} \\ \tilde{\theta}_{k,1*}^T &= \left[f_{11} - \hat{f}_{k,11}, f_{12} - \hat{f}_{k,12}, g_{11} - \hat{g}_{k,11}, g_{12} - \hat{g}_{k,12}\right] \\ \tilde{\theta}_{k,2*}^T &= \left[f_{21} - \hat{f}_{k,21}, f_{22} - \hat{f}_{k,22}, g_{21} - \hat{g}_{k,21}, g_{22} - \hat{g}_{k,22}\right] \end{aligned} \tag{59}$$

In this case, the parameter estimation laws are given as follow:
For $V_{k,1}$,

$$\begin{aligned} \hat{\theta}_{k,1*} &= \hat{\theta}_{k-1,1*} \\ &\quad + K_{k,1}(y_{k,1} + Q_{11}u_{k,1} - \varphi_{k-1}^T\hat{\theta}_{k-1,1*}) \end{aligned} \tag{60}$$

$$K_{k,1} = P_{k-1,1}\varphi_{k-1}(\mu_1 + \varphi_{k-1}^T P_{k-1,1}\varphi_{k-1})^{-1} \tag{61}$$

$$\begin{aligned} P_{k,1} &= \mu_1^{-1}(P_{k-1,1} - P_{k-1,1}\varphi_{k-1} \\ &\quad \times(\mu_1 + \varphi_{k-1}^T P_{k-1,1}\varphi_{k-1})^{-1}\varphi_{k-1}^T P_{k-1,1}) \end{aligned} \tag{62}$$

For $V_{k,2}$,

$$\begin{aligned} \hat{\theta}_{k,2*} &= \hat{\theta}_{k-1,2*} \\ &\quad + K_{k,2}(y_{k,2} + Q_{22}u_{k,2} - \varphi_{k-1}^T\hat{\theta}_{k-1,2*}) \end{aligned} \tag{63}$$

$$K_{k,2} = P_{k-1,2}\varphi_{k-1}(\mu_2 + \varphi_{k-1}^T P_{k-1,2}\varphi_{k-1})^{-1} \tag{64}$$

$$\begin{aligned} P_{k,2} &= \mu_2^{-1}(P_{k-1,2} - P_{k-1,2}\varphi_{k-1} \\ &\quad \times(\mu_2 + \varphi_{k-1}^T P_{k-1,2}\varphi_{k-1})^{-1}\varphi_{k-1}^T P_{k-1,2}) \end{aligned} \tag{65}$$

where the sizes of the associated vectors and matrices are $s_k(1\times 2)$, $y_k(1\times 2)$, $u_k(1\times 2)$, $\varphi_k(4\times 1)$ and $\hat{\theta}_{k,i*}(4\times 1)$, $K_{k,i}(4\times 1)$, $P_{k,i}(4\times 4)$ for $i = 1, 2$.

The following model parameters are assumed for simulations:

$$A_1 = \begin{bmatrix} 0.8 & 0.1 \\ 0.5 & 0.2 \end{bmatrix},\ B_0 = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.1 \end{bmatrix} \tag{66}$$

From the root of $\det[I - A_1 z^{-1}]$, this system is stable. The STC design parameters are chosen as follows:

Schur polynomial matrix:

$$C(z^{-1}) = \begin{bmatrix} 1 + c_{11}z^{-1} & 0 \\ 0 & 1 + c_{22}z^{-1} \end{bmatrix} \tag{67}$$

$$c_{11} = c_{22} = 0 \tag{68}$$

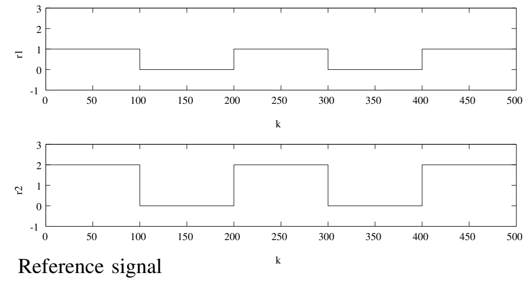

Fig. 2. Reference signal

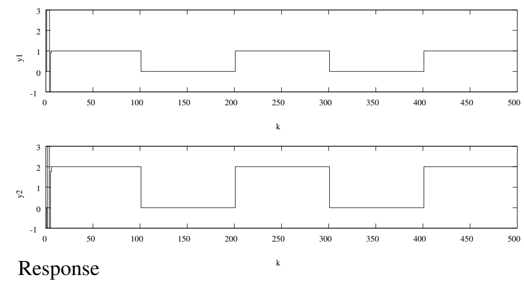

Fig. 3. Response

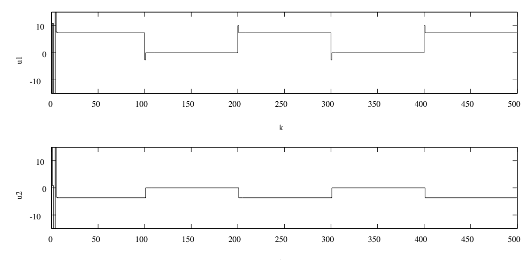

Fig. 4. Control input

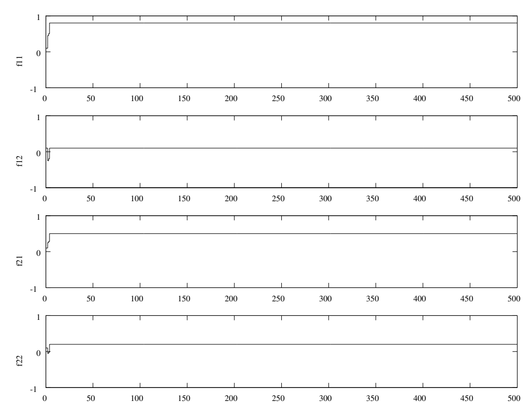

Fig. 5. Controller parameter F

Fig. 6. Controller parameter G

Initial estimates of controller parameters:

$$\begin{bmatrix} \hat{f}_{0,11} & \hat{f}_{0,21} \\ \hat{f}_{0,12} & \hat{f}_{0,22} \\ \hat{g}_{0,11} & \hat{g}_{0,21} \\ \hat{g}_{0,12} & \hat{g}_{0,22} \end{bmatrix} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \\ 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix} \quad (69)$$

Initial weights in recursive estimation:

$$P_{0,1} = \text{diag}(0.01,\ 0.01,\ 0.01,\ 0.01) \quad (70)$$

$$P_{0,2} = \text{diag}(0.01,\ 0.01,\ 0.01,\ 0.01) \quad (71)$$

Forgetting factors:

$$\mu_1 = 1.0,\ \mu_2 = 1.0 \quad (72)$$

A simulation result is given in Figs. 2 to 6. Although there are some overshoots in the beginning of control, the both outputs arrive at the given setpoints.

B. Nonminimum-Phase System

As a next, consider the system

$$y_k = A_1 y_{k-1} + B_0 u_{k-1} + B_1 u_{k-2} \quad (73)$$

This gives

$$A(z^{-1}) y_k = B(z^{-1}) z^{-1} u_k \quad (74)$$

$$A(z^{-1}) = I - A_1 z^{-1} \quad (75)$$

$$= \begin{bmatrix} 1 - a_{11} z^{-1} & -a_{12} \\ -a_{21} & 1 - a_{22} z^{-1} \end{bmatrix} \quad (76)$$

$$B(z^{-1}) = B_0 + B_1 z^{-1} \quad (77)$$

$$= \begin{bmatrix} b_{11} + z^{-1} & b_{12} \\ b_{21} & 1 + b_{22} + z^{-1} \end{bmatrix} \quad (78)$$

The following model parameters are assumed to have the unstable zeros:

$$A_1 = \begin{bmatrix} 0.8 & 0.1 \\ 0.5 & 0.2 \end{bmatrix}, \quad (79)$$

$$B_0 = \begin{bmatrix} 0.2 & 1.0 \\ 0.25 & 0.2 \end{bmatrix},\ B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (80)$$

The simulation is carried out in the following setting: Initial estimates:

$$\begin{bmatrix} \hat{f}_{0,11} & \hat{f}_{0,21} \\ \hat{f}_{0,12} & \hat{f}_{0,22} \\ \hat{g}_{0,11} & \hat{g}_{0,21} \\ \hat{g}_{0,12} & \hat{g}_{0,22} \\ \hat{g}_{1,11} & \hat{g}_{1,21} \\ \hat{g}_{1,12} & \hat{g}_{1,22} \end{bmatrix} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \\ 0.1 & 0.1 \\ 0.1 & 0.1 \\ 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix} \quad (81)$$

Initial weights in estimation:

$$P_{0,1} = \text{diag}(0.1,\ 0.1,\ 0.1,\ 0.1,\ 0.1,\ 0.1) \quad (82)$$

$$P_{0,2} = \text{diag}(0.1,\ 0.1,\ 0.1,\ 0.1,\ 0.1,\ 0.1) \quad (83)$$

From $\det[A(z^{-1})] = 0$, all the open-loop poles are inside the unit disk.

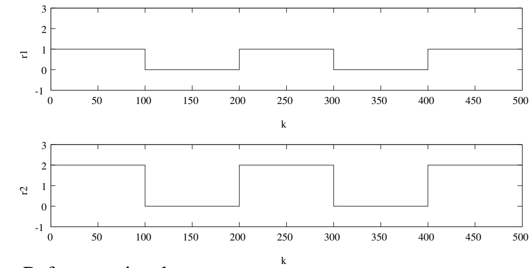

Fig. 7. Reference signal

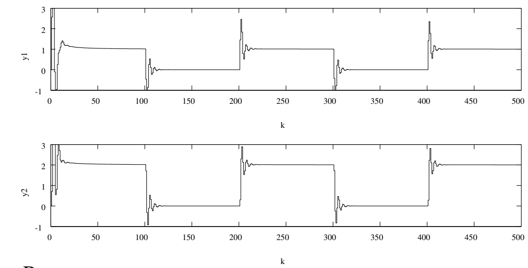

Fig. 8. Response

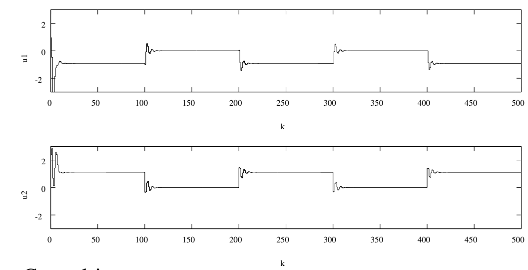

Fig. 9. Control input

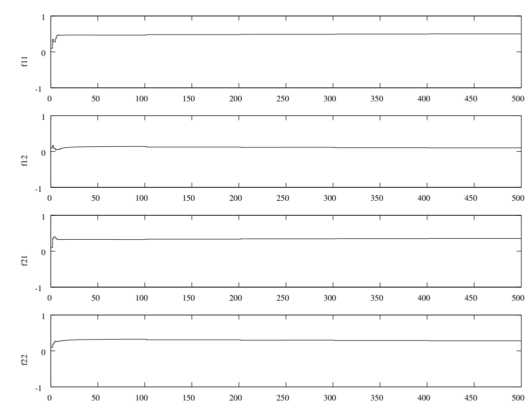

Fig. 10. Controller parameter F

This model is nonminimum-phase since $\det[B(z^{-1})] = 0$ has the solutions outside the unit disk. The same setpoints as the previous simulation are used and STC design parameters are set to

$$Q(z^{-1}) = \begin{bmatrix} 0.5(1 - z^{-1}) & 0 \\ 0 & 0.5(1 - z^{-1}) \end{bmatrix} \quad (84)$$

Note that $Q(z^{-1}) = 0$ makes the considered system unstable. As in Figs. 7 to 11, the control result in the minimum-phase MIMO case has the stable performance.

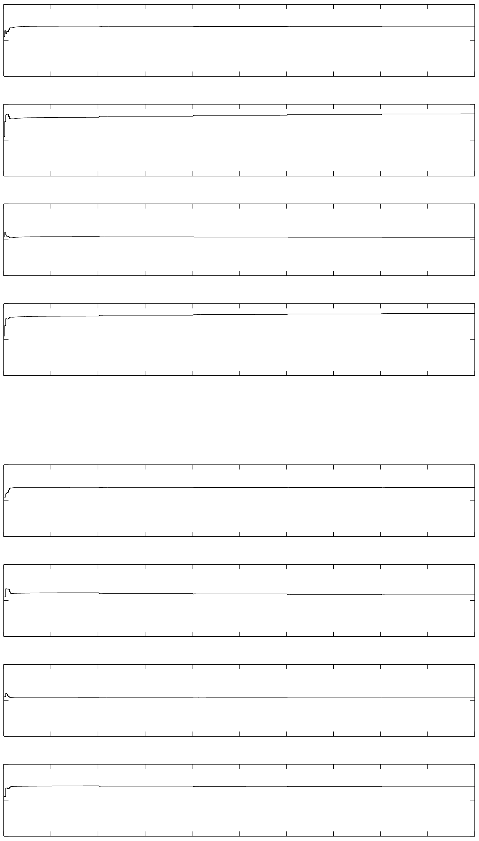

Fig. 11. Controller parameter G

V. Conclusion

A STC design scheme was presented in this paper. The MIMO-STC parameters convergence was proved based on the Lyapunov function. In the framework of closed-loop stability, all the closed-loop poles are not always stable in the considered MIMO case, even if the actual control response is stable. We shall investigate this point further.

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