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Guidance of a moving collocated actuator/sensor for improved control of distributed parameter systems

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Abstract— **This paper proposes a scheme for the guidance of a moving collocated sensor/actuator pair for the performance enhancement of a class of spatially distributed processes. It is assumed that a spatially moving source forces the process state to deviate from its equilibrium throughout the spatial domain. Such a configuration minimizes the control effectiveness of a fixed-in-space sensor/actuator pair. To minimize the effects of the moving source on the state regulation, it is assumed that a locally distributed sensor, capable of moving within the spatial domain with a prescribed velocity can also provide locally distributed actuator. Such a pair utilizes locally distributed state information in order to generate the control signal, which takes the form of a locally distributed static output feedback. At a given time interval, the maximum deviation of the spatially localized state from the equilibrium is used to guide the sensor/actuator pair to the next position and at the same time provide a simplified static output feedback controller in order to improve performance. As an extension, a time varying scalar gain is proposed which is updated adaptively, according to the distributed state deviation. Extensive simulation studies applied to a diffusion-advection system are included to verify the effectiveness of a such a moving actuator/sensor pair.**

I. Introduction

Many engineering applications consider the use of sensor and actuator networks to provide efficient and effective monitoring and control of processes. In particular, the use of mobile sensors and actuators has been receiving attention as it brings forth an added dimension to the efficient use of sensing and actuating devices as regards to reduction in power consumption, improved performance and efficient monitoring. The use of mobile (moving) actuators is abundant in many industrial applications as for example the automotive and thermal manufacturing industries. Many actuating devices are attached to robotic manipulators and provide improved performance with reduced operating and production costs.

More recently, there is additional interest in the use of mobile sensor (and actuator) networks for coverage, detection and containment [1] for environmental applications (such as fire monitoring and oil spill detection at high seas) and defense/military applications (such as border patrolling, predator detection and avoidance). In the above cases mobile agents (terrain robots, underwater vehicles, UAVs) are utilized to sense and monitor within a spatial domain of interest. When they also have actuating capabilities, then these mobile agents move at a particular point within the spatial domain and provide a control signal with the goal of addressing and improving certain control objectives.

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In the majority of cases, such mobile sensing and actuating devices are not integrated with the spatial process at which they are interacting. Earlier work on the use of moving/scanning/scheduled sensors and actuators in processes governed by partial differential equations that represented many physical systems were considered in [2], [3], [4], [5], [6]. Notable exception is the work by Butkovskiy [7], [8], [9] which considered moving sensors and actuators with a spatially pointwise distribution (footprint). Closer to the work under consideration, is the earlier work [10] where a group of closely placed pointwise sensors were used to provide an "almost" locally distributed state information, and a single pointwise actuator, placed in the middle of the group of sensors, was used to dispense actuation using static output feedback.

Therefore, an integrating framework that encompasses the motion of sensing and/or actuating devices along with the process at which they monitor and control is warranted and constitutes the main point of this work. This work in fact fills one of the many existing gaps between theory and applications.

The remainder of this paper is as follows: We shall first extend the result from [11], [12] to a class of systems with locally distributed state measurements, i.e. the sensor can provide distributed state information over a small region of the spatial domain. At the same time, a collocated actuating device is assumed to provide locally distributed actuation. The problem under consideration, which utilizes such a moving sensor/actuator pair in order to address the effects of a moving source is described in Section II. In Section III we present the guidance policy of the moving sensor/actuator pair and provide a simple static output feedback in order to minimize the control architecture complexity. An adaptive version of the constant scalar feedback is also provided. Extensive simulation studies examining the control performance of a moving and a fixed-in-space sensor/actuator pair are presented in Section IV and conclusions follow in Section V.

II. Problem formulation

We consider the control of

$$\begin{aligned}\frac{\partial x(t,\xi)}{\partial t} &= a_1\frac{\partial^2 x(t,\xi)}{\partial \xi^2} - a_2\frac{\partial x(t,\xi)}{\partial \xi} - a_3 x(t,\xi) \\ &\quad + b_1(t,\xi) + b_2(\xi;\xi_a(t))u(t),\ x(0,\xi) = x_0(\xi),\end{aligned} \tag{1}$$

with Dirichlet boundary conditions $x(t,0) = x(t,\ell) = 0$. The function $b_1(t,\xi)$ denotes the moving disturbance and $b_2(\xi;\xi_a(t))$ denotes the spatial distribution of the moving actuating device, while $u(t)$ denotes the associated control

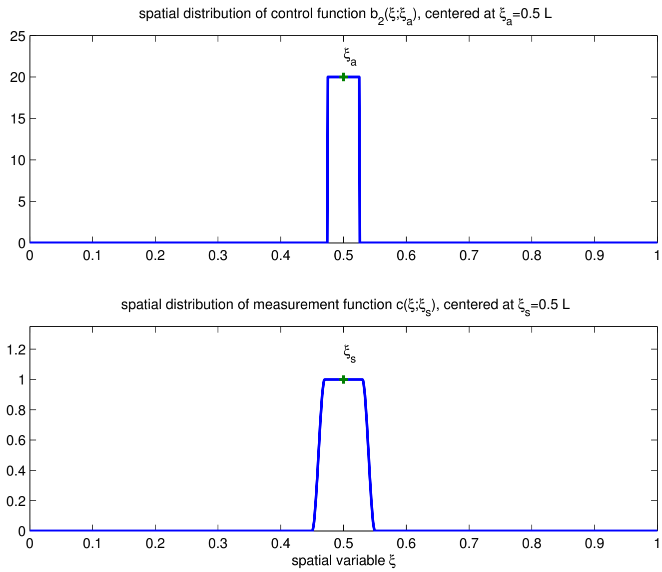

Fig. 1. Spatial distributions of input and measurement functions.

signal. It is assumed that a moving sensor can be used to provide locally distributed measurements of the state $x(t,\xi)$

$$y(t,\xi;\xi_s(t)) = c(\xi;\xi_s(t))x(t,\xi) \tag{2}$$

which essentially provides locally distributed measurements of the state over the sensor range (footprint). The function $c(\xi;\xi_s(t))$ denotes the spatial distribution of the sensing device and its time dependence describes the time variation of its location; in fact, it is the centroid $\xi_s(t)$ of the sensor distribution that is changing in time. The proposed scheme considers the minimization of the effects of the moving source $b_1(t,\xi)$ via the use of a moving collocated actuator/sensor pair. This then simplifies the structure of the control architecture to that of a static output feedback. A standing assumption of the collocated actuator/sensor pair is that the spatial distribution of the actuating device, denoted here by $b_2(\xi;\xi_a)$, has the same footprint as the spatial distribution of the sensing device, denoted in (2) by $c(\xi;\xi_s)$. A representative spatial distribution of the actuator and sensor whose centroid is at $\xi_a = \xi_s = \ell/2$ is depicted in Figure 1. Specifically, the distributed measurements from the sensor are assumed to be available over the spatial interval $[\xi_s - \Delta\xi, \xi_s + \Delta\xi]$, where the sensor footprint has length equal to twice the one-half spatial support of the actuating device i.e. equal to $2\Delta\xi$. A smoothened distribution of a polynomial function describing the spatial distribution of the sensing device was used for the numerical study reported in Section IV and is given by

$$c(\xi;\xi_s) = \begin{cases} 1 & \text{if } \xi \in [\xi_s - 0.6\Delta\xi, \xi_s + 0.6\Delta\xi] \\ 1 - 3\xi_l^2 - 2\xi_l^3 & \text{if } \xi \in [\xi_s - \Delta\xi, \xi_s - 0.6\Delta\xi] \\ 1 - 3\xi_r^2 + 2\xi_r^3 & \text{if } \xi \in [\xi_s + 0.6\Delta\xi, \xi_s + \Delta\xi] \\ 0 & \text{otherwise} \end{cases}$$

where $\xi_r = \frac{\xi - \xi_s - 0.6\Delta\xi}{0.4\Delta\xi}$ and $\xi_l = \frac{\xi - \xi_s + 0.6\Delta\xi}{0.4\Delta\xi}$. Similarly, the spatial distribution of the collocated actuating devices was taken to be an approximation of the spatial delta function, as approximated by the box function

$$b_2(\xi;\xi_s) = \begin{cases} \frac{1}{2\Delta\xi} & \text{if } \xi \in [\xi_s - \Delta\xi, \xi_s + \Delta\xi] \\ 0 & \text{otherwise} \end{cases}$$

Remark 1: In order for the proposed collocated output feedback controller to be implementable, one requires that the footprint (spatial domain of definition, $\mathcal{D}(b_2) = [\xi_s - \Delta\xi, \xi_s + \Delta\xi]$) of the actuating device be inside the spatial domain of the sensing device; this means $\mathcal{D}(b_2) \subset \mathcal{D}(c)$. In this work, it will be assumed that the actuator spatial footprint is equal to the sensor spatial footprint.

Using the above assumption of collocated centroids ($\xi_a = \xi_s$) of the sensing and actuating devices, the process and its measured output equations (1), (2) can now be written as

$$\begin{aligned} \frac{\partial x(t,\xi)}{\partial t} &= a_1 \frac{\partial^2 x(t,\xi)}{\partial \xi^2} - a_2 \frac{\partial x(t,\xi)}{\partial \xi} - a_3 x(t,\xi) \\ &\quad + b_1(t,\xi) + b_2(\xi;\xi_s(t))u(t), \; x(0,\xi) = x_0(\xi) \\ y(t,\xi;\xi_s(t)) &= c(\xi;\xi_s(t))x(t,\xi). \end{aligned} \tag{3}$$

The problem under consideration can now be stated:
Problem statement: *Given the spatially distributed system* (3) *with a moving disturbance and a collocated moving actuator/sensor pair, find a trajectory of the sensor centroid and an associated control law that would minimize the effects of the moving disturbance on the distributed state* $x(t,\xi)$.

III. Guidance of Moving Sensor/Actuator Pair and Control Delivery

One way to simplify the control architecture and minimize the associated computational costs is to consider a static output feedback of the form $u = -ky$, which would then make the control signal a locally distributed signal dependent on the sensor location $u = u(t,\xi;\xi_s(t))$. The advantage of that is the ability to implement the control law in real time. In such a case there will be no need to implement a real-time state estimator with its associated computationally expensive control gains. Now, to incorporate velocity constraints of the moving sensor, we view the above feedback system-plus-sensor guidance as a hybrid system wherein the change in the sensor position within the spatial domain occurs at discrete time instances and the distance it can move over a prescribed time interval is bounded by velocity constraints. Therefore, the time interval $[t_0, t_f]$ is divided into n equidistant subintervals $[t_0, t_1, t_2 \ldots, t_f]$, with $t_{k+1} = t_k + \Delta t$. In a given time interval $[t_k, t_k + \Delta t]$, the sensor is constrained to move at most within a distance $\pm\Delta\xi$ from the current position of its centroid $\xi_s(t_k)$. Therefore, the maximum average speed is bounded by $\upsilon_{av} = \frac{\Delta\xi}{\Delta t}$.

The proposed sensor position motion can be summarized as moving the sensor from the current position $\xi_s(t_k)$ to the next position $\xi_s(t_{k+1}) \in [\xi_s(t_k) - \Delta\xi, \xi_s(t_k) + \Delta\xi]$ by using the maximum deviation of the measured state $c(\xi;\xi_s(t_k))x(t,\xi)$

from the equilibrium. Figure 2 depicts a scenario of the proposed sensor/actuator motion with the current sensor position $\xi_s(t_k)$ and next position $\xi_s(t_{k+1})$. Basically, it finds the maximum of the spatially distributed output over the sensor range (footprint) at the current position $\xi_s(t_k)$ and moves the sensor to that maximum. Analytically, the sensor motion is given by

$$\xi_s(t_{k+1}) = \arg \max_{\xi_s(t_k)-\Delta\xi \le \xi \le \xi_s(t_k)+\Delta\xi} \left| c(\xi;\xi_s(t_k))\, x(t,\xi) \right| \quad (4)$$

The simplified static output feedback control law takes the form

$$u(t,\xi;\xi_s(t)) = -k\, y(t,\xi;\xi_s(t)) = -k\, c(\xi;\xi_s(t))\, x(t,\xi) \quad (5)$$

where the static scalar gain $k > 0$ may be chosen arbitrarily. Alternatively, one may also employ optimization techniques to find the gain k for each new position of the collocated actuator/sensor pair, but that would increase the computational load and design complexity. Another avenue is to employ adaptive methods in order to obtain an update law for a time varying gain. Such a case will be presented below when the stability of the closed loop system is examined.

The algorithm summarizing the main features of the proposed guidance of the moving sensor/actuator pair with static output feedback is now presented.

Algorithm 1: sensor/actuator guidance based on maximum state deviation using static output feedback

1) using velocity considerations υ_{av} and sensor specifications $(\Delta\xi)$, find the smallest time interval Δt which takes into consideration data processing delays and dwell time [13], [14]
2) (initialization) first consider the interval $[t_0, t_0+\Delta t)$
3) place the sensor at an initial location $\xi_s(t_0)$ that maximizes observability of the associated pair $(\mathcal{A}, \mathcal{C}(\xi_s(t_0)))$ representing the process and output operators in the abstract formulation of (3)
4) implement the static locally distributed control law
$$u(t,\xi;\xi_s(t_0)) = -k\, c(\xi;\xi_s(t_0))\, x(t,\xi), \quad t \in [t_0, t_0+\Delta t]$$
5) find location of maximum state deviation from equilibrium over the span of the sensing device (sensor footprint) at current centroid location $\xi_s(t_0)$
$$\xi_s(t_1) = \arg \max_{\xi_s(t_0)-\Delta\xi \le \xi \le \xi_s(t_0)+\Delta\xi} \left| c(\xi;\xi_s(t_0)) x(t,\xi) \right|$$
6) move sensor at new position $\xi_s(t_1)$ and implement control law
$$u(t,\xi;\xi_s(t_1)) = -k\, c(\xi;\xi_s(t_1)) x(t,\xi), \quad t \in [t_1, t_1+\Delta t]$$
7) consider next time subinterval by setting $t_{k+1} = t_k + \Delta t$, and perform search in step 5 over current sensor span $[\xi_s(t_k)-\Delta\xi, \xi_s(t_k)+\Delta\xi]$ and repeat step 6 for new time interval.

The well-posedness of the combined sensor/actuator motion (4) and static output feedback (5) applied to the system (3) can be argued within the context of switched infinite dimensional systems as detailed in [12], [14]. Similarly, the stability of the resulting closed loop system can be made along the same lines, using Lyapunov theory arguments.

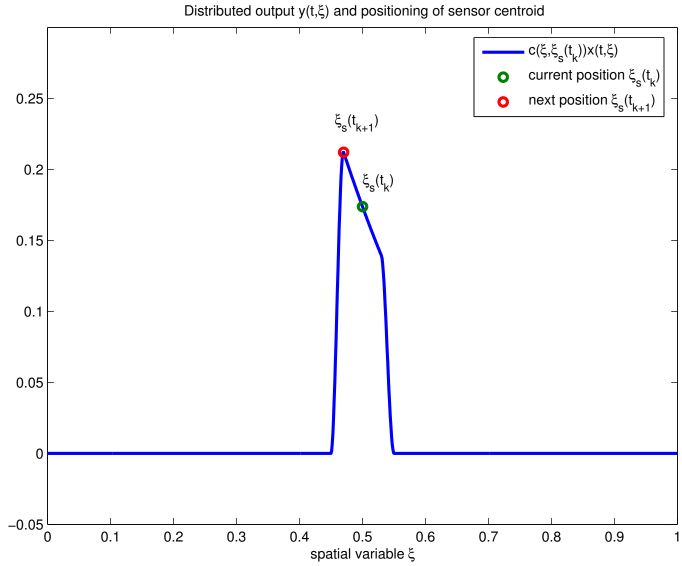

Fig. 2. Guidance of moving sensor from centroid position $\xi_s(t_k)$ to centroid position $\xi_s(t_{k+1})$ using spatially localized state.

One may also propose an adaptive gain in the control law (5) in the form of

$$\begin{aligned} u(t,\xi;\xi_s(t)) &= -k(t)\, c(\xi;\xi_s(t))\, x(t,\xi) \\ \dot{k}(t) &= \gamma \int_0^\ell y^2(t,\xi;\xi_s(t))\, d\xi, \;\; k(0) = k_0 > 0. \end{aligned} \quad (6)$$

where $\gamma > 0$ is the adaptive gain [15]. The associated algorithm in this case is summarized below.

Algorithm 2: sensor/actuator guidance based on maximum state deviation using adaptive output feedback

1) (initialization) consider the interval $[t_0, t_0+\Delta t)$
2) place the sensor at an initial location $\xi_s(t_0)$ that maximizes observability of the associated pair $(\mathcal{A}, \mathcal{C}(\xi_s(t_0)))$ representing the process and output operators in the abstract formulation of (3)
3) implement the adaptive control law
$$\left.\begin{aligned} u(t,\xi;\xi_s(t_0)) &= -k(t)\, c(\xi;\xi_s(t_0)) x(t,\xi), \\ \dot{k}(t) &= \gamma \int_0^\ell y^2(t,\xi;\xi_s(t_0))\, d\xi, \end{aligned}\right. \quad t \in [t_0, t_0+\Delta t]$$
4) find location of maximum state deviation from equilibrium over the span of sensor at current centroid location $\xi_s(t_0)$
$$\xi_s(t_1) = \arg \max_{\xi_s(t_0)-\Delta\xi \le \xi \le \xi_s(t_0)+\Delta\xi} \left| c(\xi,\xi_s(t_0)) x(t,\xi) \right|$$
5) move sensor at new position $\xi_s(t_1)$ and implement control law
$$\left.\begin{aligned} u(t,\xi;\xi_s(t_1)) &= -k(t)\, c(\xi;\xi_s(t_1))\, x(t,\xi), \\ \dot{k}(t) &= \gamma \int_0^\ell y^2(t,\xi;\xi_s(t_1))\, d\xi, \end{aligned}\right. \quad t \in [t_1, t_1+\Delta t]$$

6) consider next time subinterval by setting $t_{k+1} = t_k + \Delta t$, and perform search in step 4 over current sensor span $[\xi_s(t_k) - \Delta\xi, \xi_s(t_k) + \Delta\xi]$ and repeat step 5 for new time interval.

Remark 2: The proposed adaptation is implementable, in the sense that it requires the $L_2(0,\ell)$ norm of the available distributed output signal $y(t,\xi;\xi_s)$, i.e.

$$\dot{k}(t) = \gamma |y(t,\cdot)|^2_{L_2(0,\ell)}, \quad k(0) = k_0 > 0.$$

The stability of the proposed adaptive law as presented in Algorithm 2, is now summarized in the Lemma below.

Lemma 1: Consider the spatially distributed system (3) with a moving source $b_1(t,\xi) \in L_\infty(0,\infty; L_2([0,\ell]))$ having a locally distributed sensor/actuator pair capable of moving within the spatial domain with a prescribed velocity υ_{av}. Assume that the footprint of the collocated actuating device is equal to the footprint of the sensing device and additionally

$$b_2(\xi;\xi_s) \geq \beta c(\xi;\xi_s) > 0, \ \forall \xi_s \in [\Delta\xi, \ell - \Delta\xi], \ \beta > 0. \quad (7)$$

The proposed sensor guidance scheme (4) along with the adaptive control law (6) results in a stable closed loop system.

Proof: Consider the following Lyapunov-like function

$$V = \frac{1}{2}\int_0^\ell x^2(t,\xi)\,d\xi + \frac{\beta}{2\gamma}k^2(t). \quad (8)$$

Its derivative along the trajectories of (3), (6) is

$$\begin{aligned}
\dot{V} &= \int_0^\ell \dot{x}(t,\xi)x(t,\xi)\,d\xi + \frac{\beta}{\gamma}\dot{k}(t)k(t) \\
&= \int_0^\ell \left(a_1 x_{\xi\xi}(t,\xi) - a_2 x_\xi(t,\xi) - a_3 x(t,\xi)\right) x(t,\xi)\,d\xi \\
&\quad + \int_0^\ell b_1(\xi,t)x(t,\xi)\,d\xi \\
&\quad - k(t)\int_0^\ell b_2(\xi;\xi_s(t))c(\xi;\xi_s(t))x^2(t,\xi)\,d\xi + \frac{\beta}{\gamma}\dot{k}(t)k(t) \\
&= -\int_0^\ell a_1 x_\xi^2(t,\xi)\,d\xi - \int_0^\ell a_2 x_\xi(t,\xi)x(t,\xi)\,d\xi \\
&\quad - \int_0^\ell a_3 x^2(t,\xi)\,d\xi + \int_0^\ell b_1(\xi,t)x(t,\xi)\,d\xi \\
&\quad - k(t)\int_0^\ell b_2(\xi;\xi_s(t))c(\xi;\xi_s(t))x^2(t,\xi)\,d\xi + \frac{\beta}{\gamma}\dot{k}(t)k(t)
\end{aligned}$$

Using the coercivity of the elliptic operator in (3) we have

$$\begin{aligned}
-\int_0^\ell a_1 x_\xi^2(t,\xi)\,d\xi - \int_0^\ell a_2 x_\xi(t,\xi)x(t,\xi)\,d\xi - \int_0^\ell a_3 x^2(t,\xi)\,d\xi \\
\leq -c_1\int_0^\ell x_\xi^2(t,\xi)\,d\xi - c_2\int_0^\ell x^2(t,\xi)\,d\xi
\end{aligned}$$

for some positive constants c_1, c_2. Additionally, the assumption $b_2(\xi;\xi_s) \geq \beta c(\xi;\xi_s)$ in (7) results in the last term

$$\begin{aligned}
&-k(t)\int_0^\ell b_2(\xi;\xi_s(t))c(\xi;\xi_s(t))x^2(t,\xi)\,d\xi + \frac{\beta}{\gamma}\dot{k}(t)k(t) \\
&\leq -k(t)\beta\int_0^\ell c(\xi;\xi_s(t))c(\xi;\xi_s(t))x^2(t,\xi)\,d\xi + \frac{\beta}{\gamma}\dot{k}(t)k(t) \\
&= -k(t)\beta\int_0^\ell \left(c(\xi;\xi_s(t))x(t,\xi)\right)^2\,d\xi + \frac{\beta}{\gamma}\dot{k}(t)k(t) \\
&= \beta\gamma k(t)\left(\dot{k}(t) - \gamma\int_0^\ell y^2(\xi;\xi_s(t))\,d\xi\right).
\end{aligned}$$

Finally, using the square-integrability assumption of the moving source term along with the identity $2ab \leq \mu a^2 + \frac{1}{\mu}b^2$, we have

$$\begin{aligned}
\int_0^\ell b_1(\xi,t)x(t,\xi)\,d\xi &\leq \left|\int_0^\ell b_1(\xi,t)x(t,\xi)\,d\xi\right| \\
&\leq \frac{1}{2\mu}\int_0^\ell b_1^2(\xi,t)\,d\xi + \frac{\mu}{2}\int_0^\ell x^2(\xi,t)\,d\xi \\
&\leq \frac{1}{2\mu}|b_1|^2_{L_\infty(0,\infty,L_2(0,\ell))} + \frac{\mu}{2}\int_0^\ell x^2(\xi,t)\,d\xi
\end{aligned}$$

Using the above, the expression for $\dot{V}$ now reduces to

$$\begin{aligned}
\dot{V} \leq\ & -c_1\int_0^\ell x_\xi^2(t,\xi)\,d\xi - c_2\int_0^\ell x^2(t,\xi)\,d\xi \\
& + \frac{1}{2\mu}|b_1|^2_{L_\infty(0,\infty,L_2(0,\ell))} + \frac{\mu}{2}\int_0^\ell x^2(\xi,t)\,d\xi \\
& + \beta\gamma k(t)\left(\dot{k}(t) - \gamma\int_0^\ell y^2(\xi;\xi_s(t))\,d\xi\right)
\end{aligned}$$

The proposed adaptation law eliminates the last term in the inequality above. However, it was implicitly assumed that the adaptive gain $k(t)$ will always be positive, otherwise the positivity of b_2 and c alone cannot result in

$$\begin{aligned}
&-k(t)\int_0^\ell b_2(\xi;\xi_s(t))c(\xi;\xi_s(t))x^2(t,\xi)\,d\xi \\
&\quad \leq -k(t)\beta\int_0^\ell c(\xi;\xi_s(t))c(\xi;\xi_s(t))x^2(t,\xi)\,d\xi.
\end{aligned}$$

Fortunately, there is no reason to resort to adaptive modification methods such as projection methods [16] to ensure that $k(t) > 0$ for all $t > 0$. When the initial choice of $k(0) = k_0$ is made positive, then following Remark 2, one has that $k(t) > 0$ for all $t > 0$. Still, projection methods may have to be employed to ensure that the adaptively updated gain $k(t)$ does not grow unbounded. Using the embedding of $H_0^1(0,\ell) \hookrightarrow L_2(0,\ell)$ [17] allows one to bound the norm of the gradient $x_\xi(t,\xi)$ by the norm of the state $x(t,\xi)$ in the sense

$$\int_0^\ell x^2(t,\xi)\,d\xi \leq \kappa^2\int_0^\ell x_\xi^2(t,\xi)\,d\xi,$$

where κ is the embedding constant, and hence

$$-\int_0^\ell x_\xi^2(t,\xi)\,d\xi \leq -\frac{1}{\kappa^2}\int_0^\ell x^2(t,\xi)\,d\xi.$$

Therefore

$$\dot{V} \le -\left(\frac{c_1}{\kappa^2} + c_2 - \frac{\mu}{2}\right)\int_0^{\ell} x^2(t,\xi)\,d\xi + \frac{1}{2\mu}|b_1|^2_{L_\infty(0,\infty,L_2(0,\ell))}.$$

The choice of $\mu = c_1/\kappa^2 + c_2$ provides

$$\dot{V} \le -\mu\int_0^{\ell} x^2(t,\xi)\,d\xi + \frac{1}{2\mu}|b_1|^2_{L_\infty(0,\infty,L_2(0,\ell))}.$$

The remaining arguments leading to closed loop stability are rather standard and omitted, see for example [18]. In essence, it applies the infinite dimensional analogue to Barbălat's [19] to the above hybrid system. ∎

IV. Results

The PDE in (1) was simulated using 80 linear elements [20] in $\Omega = [0,1]$ and an initial condition $x(0,\xi) = \sin(\pi\xi)e^{-7\xi^2}$. The coefficients of the elliptic operator were $a_1 = 0.005, a_2 = 0.15, a_3 = 0.003$. The moving source was taken as

$$b_1(t,\xi) = 10^{-5}\left(0.3\cos(\tfrac{9\pi t}{t_f}) + 0.5\right)\times \left(H(\xi+\xi_c(t)+\Delta\xi) - H(\xi-\xi_c(t)-\Delta\xi)\right)$$

where $\xi_c(t)$ denotes the centroid of the moving source and $\Delta\xi = \ell/20$ denotes the one-half of the spatial support of the spatial distribution of the moving source. The same spatial support $\Delta\xi$ was used for the moving sensing and actuating devices. The closed loop system was simulated in the time interval $[0,4]$ with a maximum velocity $\upsilon_{av} = 5$, thus resulting in $n = \frac{t_f \upsilon_{av}}{\Delta\xi} = 400$ subintervals and therefore $\Delta t = \frac{t_f}{n} = 0.01$, i.e. the switching times at which the sensor/actuator pair was being moved was occurring every $\Delta t = 0.01$ time units. The moving source, as described above, had its own centroid moving with a lower speed of $\upsilon_{source} = 0.1125$.

Both proposed algorithms were implemented, the first with a fixed gain of $k = 100$ and the second with $k(0) = 100$ and an adaptive gain $\gamma = 2\times 10^5$. Figure 3 depicts the state $L_2(0,\ell)$ norm for the open loop case, the case of a collocated sensor/actuator pair fixed at $\xi_s = \ell/2$ with a fixed gain $k = 100$, and the case of a collocated moving sensor/actuator pair and a fixed gain $k = 100$. The effects of the moving sensor/actuator pair on the performance improvement are overwhelmingly encouraging, when compared to the case of a fixed-in-space sensor/actuator pair.

The pointwise convergence of the state to zero with a moving sensor/actuator pair is also evident in Figure 4, which depicts the state distribution at four different time instances. The distributed state (green dotted line) converges to zero much faster when the sensor/actuator pair is allowed to move. Similarly, the spatial distribution of the state at the time $t = 1$ is depicted in Figure 5, where one may also observe that pointwise convergence can be achieved sooner when a moving sensor/actuator pair is utilized. Figure 6 depicts the trajectory of the centroid of the sensor/actuator pair for the fixed ($\xi_s = \ell/2$) and moving cases.

Finally, the effects of an adaptive gain vs the static output feedback gain on the cumulative L_2 norm are presented in

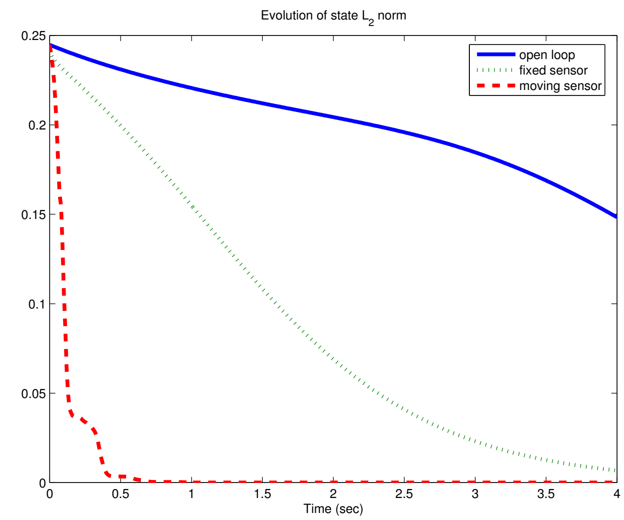

Fig. 3. Evolution of spatial L_2 norm.

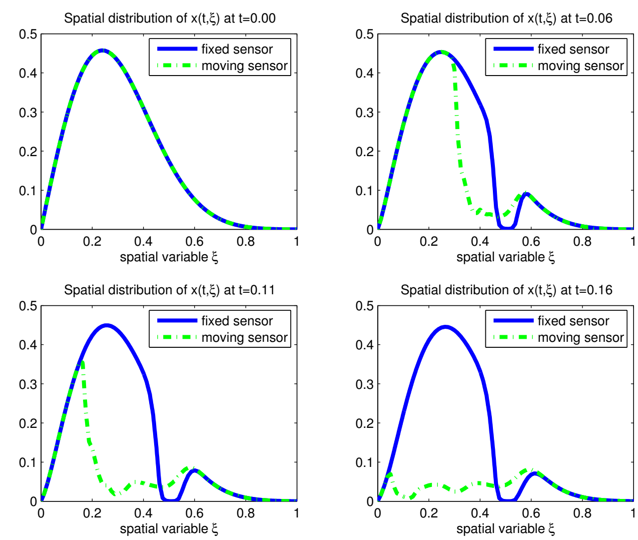

Fig. 4. Closed loop state vs spatial variable at different time instances.

Table I, where an appreciable reduction on the state norm is observed, for both the fixed and moving sensor cases.

V. Conclusions

This paper examined the positive effects of a moving sensor/actuator pair on the state regulation for a process governed by a 1-D diffusion-advection partial differential equation. It was assumed that an unknown source was moving within the spatial domain thereby forcing the state to deviate from its equilibrium.

The moving sensor/actuator pair utilized a static output feedback control policy as a means to minimize the effects of the moving source. The combined sensor/actuator motion and closed loop system was viewed as an infinite dimensional hybrid system, where the sensor/actuator was moved to a new position at the beginning of a given time interval. The duration of such an interval was dictated by speed considerations of the mobile sensor, stability under

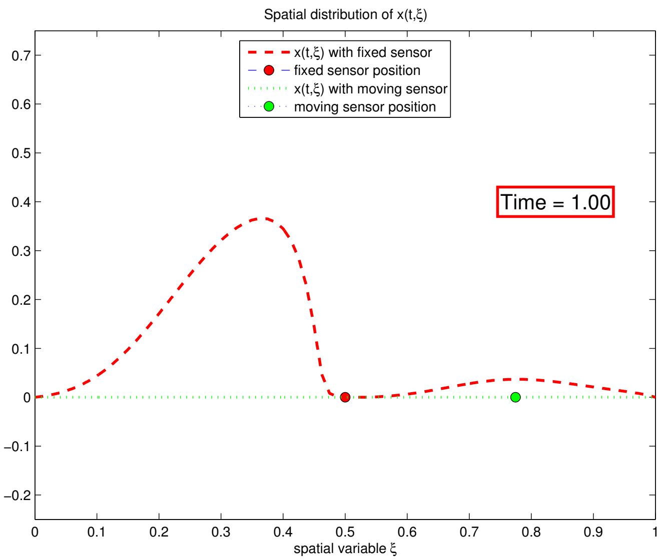

Fig. 5. Spatial distribution of state at time $t = 1$ sec.

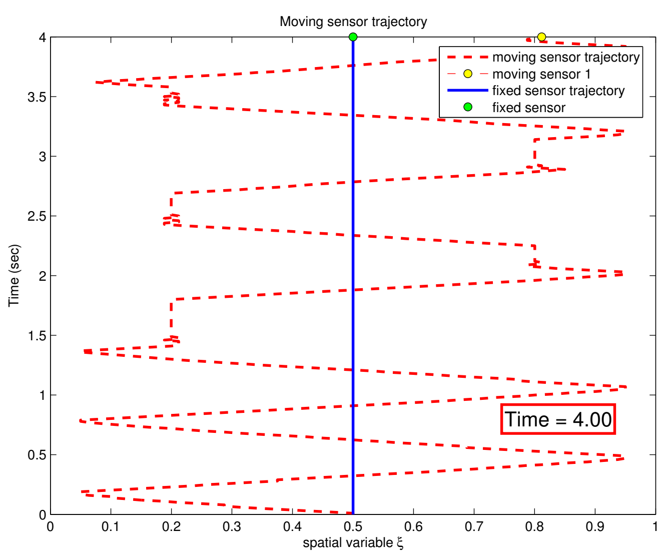

Fig. 6. Sensor/actuator trajectory.

TABLE I

$L_2(0,4;L_2(0,\ell))$ STATE NORM

case	fixed gain	adaptive gain
open loop	0.4066	0.4066
fixed sensor/actuator pair	0.2342	0.2316
moving sensor/actuator pair	0.0654	0.0612

switching (dwell time), the physical span of the sensing device (footprint) and processing time. At a given time, the maximum deviation of the process state from its equilibrium over the span (footprint) of the current sensor position of the sensing device was chosen as the next sensor position. Both a fixed static gain and an adaptive analogue were proposed.

Extensive simulation studies revealed that a moving sensor/actuator pair capable of providing locally distributed measurements and dispensing locally distributed control action at the spatial range of the sensor, can significantly minimize the effects of a moving source on the process state.

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