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Evolutionary game dynamics with migration for hybrid power control in wireless communications

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Abstract— **We propose an evolutionary game dynamics with migration for hybrid population games with many local interactions at the same time. Each local interaction concerns a random number of interacting players. The strategies of a player have two components. Specifically, each player chooses both (i) the region or subpopulation and (ii) an action among a finite set of secondary pure strategies in each region. We assume that when updating a strategy, a player can change only the secondary strategies associate to the region at a time. We investigate what impact this restriction has on the population dynamics. We apply this model to the integrated power control and base station assignment problem in a multi-cell in code division multiple access (CDMA) wireless data networks with large number of mobiles. We show that global neutrally evolutionary stable strategies are stationary points of *hybrid* mean dynamics called dynamics with *multicomponent strategies* under the positive correlation conditions. We give some convergence results of our hybrid model in stable population games and potential population games.**

Index Terms— **evolutionary game dynamics, power control.**

I. Introduction

Power control in wireless networks has became an important research area. Since the technology in the current state cannot provide batteries which have small weight and large energy capacity, the design of tools and algorithms for efficient power control is crucial. For a comprehensive survey of recent results on power control in wireless networks an interested reader can consult e.g., [9] and the references therein. Power control protocols based on game theory have been designed for already ten years starting with the pioneering work [3], [5]. Non-cooperative games provide a convenient framework for decentralization and distributed decision making in those applications, where as cooperative approaches of game theory have allowed to handle issues concerning fairness in power allocation [2]. Most applications of game theory to power control consider mobile terminals as players of the same type and study strategic one-shot games with a fixed number of players. Here, we consider a population game with many simultaneous local interactions, where the game is played infinitely under some self-organizing process called ”hybrid dynamic” and where each interaction concerns a random number of players [11]. We develop a game dynamic with migration [8] in a *hybrid evolutionary game model* in code division multiple access (CDMA) wireless data networks.

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A hybrid non-cooperative game model for wireless communication has been studied by Alpcan and Başar in [1] as an extension of the classical non-cooperative power control game formulation in CDMA system in which mobiles were considered to be connected to the closest base station (BS).

We consider in this paper an evolutionary game model and study a *hybrid* dynamic for updating actions along with its convergence to an equilibrium point. We apply this model to the integrated power control and base station assignment problem as in such that each mobile's action space consists of the choice a power level and a base station in some region which corresponds to a cell. The first advantage to use evolutionary framework to model this problem is that evolutionary game dynamics give naturally an algorithm for equilibrium point. Evolutionary Game theory describes the evolution with some dynamic process and involve strategic interaction over time in large populations of users.

The paper is structured as follows. We first provide in the next section the model with different second strategies set in each region and we develop the *hybrid* mean dynamics. After that we study the power control game in multi-cell CDMA and we give some numerical investigations. We give some convergence results in potential games and *stable population games*[4] with constraints.

II. Model and notations

We consider a population game model with *multicomponent strategies*. The population game consists of

- A large number of players.
- Each member of the population has a multicomponent strategy: first strategy (region) and secondary strategy.
- The players with the same first strategy have to choose their secondary strategies in a finite set and compete in a local non-cooperative game with either a finite random number of interacting players or with all the players in the region. The set of strategies available to players in all the population is $\mathcal{S}$ and has typical elements $(r, a), (\bar{r}, b)$. Let $\mathcal{A}^r = \{p_r^1, \ldots, p_r^{n^r}\}$ the pure secondary strategies set of the region r and, $\mathcal{S}^r = \{(r, a),\ a \in \mathcal{A}^r\}$ the pure strategies of a $r-$player. Then $\mathcal{S} = \cup_r \mathcal{S}^r$. Denote by l^r the mass of the region r, l the total mass of the population,

$$\mathcal{X}^r = \{\ x^r \in \mathbb{R}^{|\mathcal{A}^r|},\ x_a^r \geq 0, \sum_{a \in \mathcal{A}^r} x_a^r = l^r\}$$

the set of mixed secondary strategies in the region r and,

$$\mathcal{X} = \{ x \in \mathbb{R}^{\sum_r |\mathcal{A}^r|},\ x_a^r \geq 0, \sum_r \sum_{a \in \mathcal{A}^r} x_a^r = \sum_r l^r = l\}$$

the set of all mixed strategies. Without loss of generality, we normalize the total mass of the population l to one. We call *state of the population* an element of $\mathcal{X}$ and, *state of the region* r, an element of $\mathcal{X}^r$.

- Each player from each local interaction receives some payoff. By $F_a^r(x)$ we denote the expected payoff (reward-loss) of the second strategy $a \in \mathcal{A}^r$ in the region r when its state is x^r and the state of the other regions is x^{-r}. If every user takes a part in some region in a non-cooperative game between $K^r + 1$ users where K^r is a random variable over $\{0, 1, 2, \ldots\}$, then the payoff $F_a^r(x)$ can be expressed as

$$F_a^r(x) = \sum_{k \geq 0} \mathbb{P}(K^r = k) F_a^{r,k}(x)$$

where $F_a^{r,k}(x)$ is the expected payoff of the strategy a obtained in the local interaction between k users in the region r when the state is x. We denote by $F: \mathcal{X} \rightarrow \mathbb{R}^{\sum_r |\mathcal{A}^r|}$, the payoff function of all the population.

In addition to the study of the equilibrium of the game we shall consider also some evolutionary dynamics and study its convergence properties. We shall assume that players revise their strategies and use the strategies with higher payoffs. The system evolves under some evolutionary game dynamic process that describes the change of strategies (inflow and outflow) in the population.

A. Definitions

Global Nash Equilibrium (GNE) We say that $x = (x_a^r)_{r,a}$ is a GNE if for all deviation strategy $mut = (mut_a^r)_{r,a}$,

$$\sum_{r,a} (x_a^r - mut_a^r) F_a^r(x) \geq 0.$$

Wardrop equilibrium When the number of opponents is very large (possibly infinity), the corresponding Nash equilibrium is also called Wardrop equilibrium. A state x is a Wardrop equilibrium if for any region r, all strategies being used by the members of the region r yield the same payoff, the payoff that would be obtained by members which chose the region r is lower for all strategies not used by players with the first action r.

$$\forall r,\ \forall a,\ x_a^r > 0 \Longrightarrow\ F_a^r(x) = \bar{F}^r = constant,$$

$$x_a^r = 0 \Rightarrow F_a^r(x) \leq F_{a'}^{r'},\ \forall\ a', r' \mid x_{a'}^{r'} > 0$$

The relation between the two definitions is given by the following equivalences (see also the interaction between several populations model in [10]):

$$x \text{ is GNE} \iff x \in \arg\max_{mut} \{\textstyle\sum_r \sum_a mut_a^r F_a^r(x)\} \quad (1)$$
$$\iff [x_a^r > 0 \Longrightarrow\ F_a^r(x) \geq F_{a'}^{r'}(x),\ \forall r', a'] \quad (2)$$

Global evolutionary stable strategy (ESS) We use here the notion of ESS defined in a multipopulation game model but in our case the fitness of the region r can be independent of the state of the other regions and users migrate to the regions with higher fitness. We say that $x = (x_a^r)_{r,a}$ is a *global* evolutionary stable strategy if for each deviation strategy called "mutations" $mut = (mut_a^r)_{r,a} \neq x$, there exists some $\epsilon_{mut} > 0$ such that $\forall \epsilon \in (0, \epsilon_{mut})$,

$$\sum_{r,a} (x_a^r - mut_a^r) F_a^r(\epsilon\ mut + (1-\epsilon)x) > 0. \quad (3)$$

If the inequality (3) is non-strict the corresponding equilibrium is called *Neutrally ESS*.

Choice Constrained Equilibrium A strategy x is a *choice constrained equilibrium*(CCE) if for all (r, a) such that $x_a^r > 0$ one has

$$F_a^r(x) = \max_{\bar{r},\ b \in \mathcal{A}^{\bar{r}}} F_b^{\bar{r}}(x). \quad (4)$$

Note that a global ESS is a *neutrally ESS* which is a GNE. But a GNE can not be a global ESS. A CCE is a collection of constrained local Nash equilibria.

B. General Game Dynamics with Migration

We study a game-theoretic dynamics of a large population of users with *migration*. Users move to a region in which the average payoff of their power control strategy is higher. If user from region $\bar{r}$ chooses a base station from region r we will say that *the user migrate to region* r. Migration between regions could help the population to evolve towards a stationary point which is related to a Wardrop equilibrium.

1) Strategies with migration as replicators: We introduce here the replicator dynamics with migration which describes the evolution in the population of the various strategies. In the replicator dynamics, the share of a strategy in the population grows at a rate equal to the difference between the payoff of that strategy and the average payoff of the population under the migration constraints. More precisely, let (x_t) be the state of the population at time t. Thus we have $\sum_r \sum_a x_{a,t}^r = l$ and $x_{a,t}^r \geq 0$ where l is the mass of the population and $x_{a,t}^r$ represents the fraction of players playing a strategy (r, a) in period t. This total mass can be normalized to one.

We describe by approximating from stochastic influence on the change in frequency of actions, the replication dynamics. We will describe more general class of evolutionary game dynamics in next subsection. Suppose that in every period Δt, each player learns with probability $\alpha \Delta t > 0$ the expected payoff to the other opponent players and changes to the other's strategy if he perceives that the other's payoff is higher. However the information about the difference in the expected payoffs of the strategies is imperfect, so the larger the difference in the payoffs, the more likely the player is to perceive it, and change. Specially, we assume that the probability that a player using (r, a) will shift to $(\bar{r}, b)$ in some neighboring set $\mathcal{N}_{(r,a)}$ is given by

$$x_{a,t+\Delta t}^r = \begin{cases} \mu[F_{a,t}^r(x_t) - F_{b,t}^{\bar{r}}(x_t)] & \text{if } F_{a,t}^r(x_t) > F_{b,t}^{\bar{r}}(x_t) \\ 0 & \text{if } F_{a,t}^r(x_t) \leq F_{b,t}^{\bar{r}}(x_t) \end{cases}$$

where μ sufficiently small that $x^r_{a,t} \leq 1$ holds $\forall\ (\bar{r}, b), (r, a)$. The expected fraction $\mathbb{E}x^r_{a,t+\Delta t}$ of the population using (r, a) in period $t + \Delta t$ is given by

$$x^r_{a,t} - \alpha\Delta t x^r_{a,t} \sum_{(\bar{r},b)\in\mathcal{I}} x^{\bar{r}}_{b,t}\mu[F^{\bar{r}}_{b,t}(x_t) - F^r_{a,t}(x_t)]$$
$$+ \sum_{(\bar{r},b)\in\mathcal{N}_{(r,a)}\setminus\mathcal{I}} \alpha\Delta t x^r_{a,t} x^{\bar{r}}_{b,t}\mu[F^r_{a,t}(x_t) - F^{\bar{r}}_{b,t}(x_t)]$$
$$= x^r_{a,t} + \alpha\mu\Delta t x^r_{a,t}\left[F^r_{a,t}(x_t) - \sum_{(\bar{r},b)\in\mathcal{N}_{(r,a)}} x^{\bar{r}}_{b,t}F^{\bar{r}}_{b,t}(x_t)\right] \quad (5)$$

where $\mathcal{I} = \left\{(\bar{r}, b) \in \mathcal{N}_{(r,a)},\ F^{\bar{r}}_{b,t}(x_t) > F^r_{a,t}(x_t)\right\}$. For large population, we can replace $\mathbb{E}x^r_{a,t+\Delta t}$ by $x^r_{a,t+\Delta t}$. Taking the limit of $\frac{x^r_{a,t+\Delta t} - x^r_{a,t}}{\Delta t}$ when Δt goes to zero, we then obtain

Discrete time: $x^r_{a,t+\Delta t} = x^r_{a,t} + \alpha\mu G_t(x_t)\Delta t$

Continuous time: $\frac{d}{dt}x^r_{a,t} = \alpha\mu G_t(x_t)$

where

$$G_t = x^r_{a,t}\left[F^r_{a,t}(x_t) - \sum_{(\bar{r},b)\in\mathcal{N}_{(r,a)}} x^{\bar{r}}_{b,t}F^{\bar{r}}_{b,t}(x_t)\right].$$

The constant $\mu\alpha$ changes the rate of adjustment to stationary point. In the classical replicator these parameters are fixed to one. The two parameters α and μ give us a framework for controlling game dynamics (changing or upgrading policy) through the choice of a gain parameter governing the replicator dynamics.

2) Decomposable dynamics for hybrid power control games : We now describe more general decomposable dynamics with migration respectively to apply in hybrid power control games.

We assume that during any time-step Δt, each individual among a fraction $\kappa^r\Delta t$ of the region r takes part in some local power control game and receives payoffs. We allow a fraction of users to migrate to a region in which their strategies have higher payoffs. The flow in the region r is specified in terms of some functions $\rho^r_{\bar{a},a}$ which determine the rates at which a player who is considering a change in strategies opts to switch to his various alternatives in the region r and some function $\eta^{(r,a)}_{(\bar{r},b)}$ which determine the rates at which a player who is considering a change in strategies opts to switch to his various alternatives from other regions into region r. The functions ρ and η are called *revision protocols*. The two revision protocols depend on the state of the population and the payoff functions. The inflow inside the region r is $k^r\sum_{\bar{a}\in\mathcal{A}^r} x^r_{\bar{a}}\rho^r_{\bar{a},a}$ and outflow inside the region r, $k^r x^r_a \sum_{\bar{a}\in\mathcal{A}^r} \rho^r_{a,\bar{a}}$. We denote by M^r_a the function representing an increase number of users inside of region r due to higher fitness. M^r_a is the difference between the *intra-inflow* and the *intra-outflow*.

$$M^r_a(x^r) := k^r\left[\sum_{\bar{a}\in\mathcal{A}^r} x^r_{\bar{a}}\rho^r_{\bar{a},a} - x^r_a\sum_{\bar{a}\in\mathcal{A}^r}\rho^r_{a,\bar{a}}\right] \quad (6)$$

The *inter-inflow* of the region r is $k^r\gamma^r\sum_{(\bar{r},b)} x^{\bar{r}}_b\eta^{(\bar{r},b)}_{(r,a)}$, and *inter-outflow* of the region r is $\gamma^r k^r x^r_a\sum_{(\bar{r},b)}\eta^{(r,a)}_{(\bar{r},b)}$. By ϕ^r_a we denote a function which describes changes of the numbers of users playing the pure strategy a in the region r due to migration from other region to the region r.

$$\phi^r_a(x^r, x^{-r}) = k^r\gamma^r\left[\sum_{(\bar{r},b)} x^{\bar{r}}_b\eta^{(\bar{r},b)}_{(r,a)} - x^r_a\sum_{(\bar{r},b)}\eta^{(r,a)}_{(\bar{r},b)}\right] \quad (7)$$

where γ^r is a parameter which represents the migration rate of region r and k^r is a growth parameter.

By combining the two equations (6), (7) we obtain the continuous time mean dynamics with migration or mean dynamics with *multicomponent strategies*.

$$\frac{d}{dt}x^r_a(t) := V^{r,a}_F(x(t)) = M^r_a(x^r(t)) + \phi^r_a(x(t)) \quad (8)$$

We denote by β the following revision protocol

$$\beta^{(\bar{r},b)}_{(r,a)} = \begin{cases} k^r\rho^r_{ba} & \text{if } \bar{r} = r \\ k^r\gamma^r\eta^{(\bar{r},b)}_{(r,a)} & \text{if } \bar{r} \neq r \end{cases}$$

Constrained Nash Stationarity(CNS) property All rest points of the mean dynamic (8) are precisely the CCEs of the game being played i.e $(M + \phi)(x) = 0$ if and only if x is CCE.

The replicator dynamics is one of the most studied dynamics in evolutionary game theory to describe evolution of the frequencies in the population but it is known that the replicator dynamics may not lead to equilibria (see [10]). To guarantee *constrained Nash stationary*(CNS) properties, the mean dynamics must to satisfy some conditions.

Positive Correlation (PC)

$$\sum_{r,a} F^r_a(x)(M^r_a(x^r) + \phi^r_a(x^r, x^{-r})) \geq 0.$$

Positive correlation guarantees that every Nash equilibrium of the game is a stationary point of the dynamics (8) as shown in Result III-C.

To see why this condition is so named, observe that by the condition $\sum_{r,a} M^r_a(x^r) + \phi^r_a(x^r, x^{-r}) = 0$,

$$\sum_{r,a} F^r_a(x)(M^r_a(x^r) + \phi^r_a(x^r, x^{-r}))$$
$$= \sum_{r,a}\left[F^r_a(x) - \frac{1}{l^r}\sum_{(\bar{r},b)} x^{\bar{r}}_b F^{\bar{r}}_b(x)\right]\left[M^r_a(x^r) + \phi^r_a(x^r, x^{-r}) - 0\right]$$
$$= \sum_r l^r Cov(M^r(x^r) + \phi^r(x), F^r(x))$$

where Cov denotes the covariance between strategy growth rates and payoffs in region r. Hence, condition (PC) holds if there is a positive correlation between growth rates and payoffs in each region.

Monotonicity condition: sign preserving (SP) Monotonicity condition on ρ and η defines the following class of dynamics. The couple of functions (ρ, η) preserves the sign if for all region r

$$\rho^r_{\bar{a},a}(x)\begin{cases} > 0 & \text{if } \bar{a}, a \in \mathcal{A}^r \text{ and } F^r_a(x) > F^r_{\bar{a}}(x) \\ = 0 & \text{otherwise} \end{cases},$$

$$\eta_{r,a}^{(\bar{r},b)}(x)\begin{cases} >0 & \text{if } a\in\mathcal{A}^r, b\in\mathcal{A}^{\bar{r}}, F_a^r(x)>F_b^{\bar{r}}(x)\\ =0 & \text{otherwise}\end{cases}$$

The sign preserving property says that the inflow rate from the strategy a to $\bar{a}$ inside the region r is positive if and only if the payoff to a exceeds the payoff to $\bar{a}$ and the inflow rate from other regions to the region r is positive for a given strategy a if and only the payoff to r exceeds the payoff to $\bar{r}$.

III. EQUILIBRIUM AND STATIONARY POINT

Sandholm showed in [10] that in absence of migration ($\gamma^r = 0,\ \forall\ r$) replicator dynamics (in general imitation dynamics), Smith dynamics, Brown-von Neumann-Nash dynamics (excess payoff dynamics) satisfy positive correlation property. Moreover Brown-von Neumann-Nash dynamics and Smith dynamics satisfy (CNS) property. Here, we extend these results to evolutionary game dynamics with migrations.

Result III-A: Suppose that ρ and η generate one of the following dynamics: replicator, Smith, BNN dynamics. Then, the resulting dynamics with multicomponent (ρ,η) in (8) is (PC).

Result III-B: Suppose that the functions ρ and η satisfy (SP). Then, The multicomponent dynamics (8) satisfies (PC). In particular, Smith dynamics is (PC).

Proof: Sign preserving property implies that $\rho_{\bar{a},a}^r[F_a^r - F_{\bar{a}}^r]\geq 0$ and $\eta_{(r,a)}^{(\bar{r},b)}[F_a^r - F_b^{\bar{r}}]\geq 0$. $\sum_{(r,a)}\dot{x}_a^r F_a^r = \sum_{r,a,\bar{a}} k^r x_{\bar{a}}^r \underbrace{\rho_{\bar{a},a}^r[F_a^r - F_{\bar{a}}^r]}_{\geq 0} + \sum_{(r,a),(\bar{r},b)} k^r\gamma^r x_b^{\bar{r}} \underbrace{\eta_{(r,a)}^{(\bar{r},b)}[F_a^r - F_b^{\bar{r}}]}_{\geq 0}$ ■

Result III-C: If V_F is positively correlated then x is CCE implies that x is a stationary point of mean dynamics (8).

Proof: x is a CCE is equivalent to $\forall z,$ such that $\sum_{(r,a)} z_a^r = 0,$ one has $\sum_{(r,a)} F_a^r(x) z_a^r \leq 0$. Now fix a region r, and define the vector z as follows

$$z_b^{\bar{r}} = \begin{cases} V_F^{\bar{r},b}(x) & \text{if } \bar{r}=r,\ b\in\mathcal{A}^r\\ 0 & \text{if } b\notin\mathcal{A}^r \text{ or } \bar{r}\neq r\end{cases}$$

Then $\sum_{r,a} z_a^r = \sum_{\substack{(r,a)\\(\bar{r},b)}} x_b^{\bar{r}}\beta_{(r,a)}^{(\bar{r},b)} - \sum_{\substack{(r,a)\\(\bar{r},b)}} x_a^r \beta_{(\bar{r},b)}^{(r,a)} = 0$. So,

$$\sum_{(r,a)} V_F^{r,a}(x)F_a^r(x) = \sum_{(r,a)} z_a^r F_a^r(x) \leq 0$$

By (PC), this inequality implies that $V_F^{r,a}(x)=0$. ■

Result III-D (Characterization of stationary points): Suppose that the functions ρ and η satisfy (SP). Let x be a stationary point of the mean dynamics (8). Then x is a CCE.

$$x_a^r > 0\ \Rightarrow F_a^r(x) = \max_{\substack{\bar{r},\\ b\in\mathcal{A}^{\bar{r}}}} F_b^{\bar{r}}(x) \quad (9)$$

Proof: We show that every stationary point of the mean dynamics is a CCE. x is a CCE if and only for all $(r,a),\ x_a^r = 0$ or there is no outflow from strategy (r,a) : $\sum_{(\bar{r},b)}\beta_{(\bar{r},b)}^{(r,a)} = 0$ Suppose that $V_F(x)=0$. If $(\bar{r},b)$ is an optimal strategy then sign preserving assumption implies that there is no outflow from $(\bar{r},b)$ i.e $x_b^{\bar{r}}\sum_{(r,a)}\beta_{(r,a)}^{(\bar{r},b)} = 0$. Since $V_F(x)=0$, one has $\sum_{(r,a)} x_a^r\beta_{(\bar{r},b)}^{(r,a)} = 0$. This condition is exactly

$$\forall (r,a),\ x_a^r = 0 \text{ or } \beta_{(\bar{r},b)}^{(r,a)} = 0,\ \forall(\bar{r},b),\ b\in\mathcal{A}^{\bar{r}}.$$

This last alternative says that

$$F_a^r(x)\geq F_b^{\bar{r}}(x),\ \forall(\bar{r},b),\ b\in\mathcal{A}^{\bar{r}}.$$

■

E. Stable Population Games

We say that F is a stable game if for all $x,y\in\mathcal{X}, \sum_{r,a}(x_a^r - y_a^r)(F_a^r(x)-F_a^r(y))\leq 0$.

Proposition III-F: In stable population games, the set of GNE is convex and coincides with the set of neutrally ESSs.

Proof: Suppose that F satisfies $x,y\in\mathcal{X},\ \sum_{r,a}(x_a^r - y_a^r)(F_a^r(x)-F_a^r(y))\leq 0$ and let x be a GNE. It is easy to see that every neutrally ESS is a GNE. Now we show that x is a neutrally ESS. So fix, an arbitrary vector y. Since F is stable and x is GNE, one has the system

$$\begin{cases} \sum_{r,a}(y_a^r - x_a^r)(F_a^r(y)-F_a^r(x))\leq 0\\ \sum_{r,a}(y_a^r - x_a^r)F_a^r(x)\leq 0\end{cases}$$

Adding the two inequalities of the last system, we obtain that $\sum_{r,a}(y_a^r - x_a^r)F_a^r(y)\leq 0$. Taking $y=(1-\epsilon)x+\epsilon\ mut$ for arbitrary $mut\neq x$, we conclude that x is a neutrally ESS. To prove the convexity, we rewrite the GNE set as an intersection of convex set $\{GNE\}=\bigcap_y A_y$ where

$$A_y = \{x\in\mathcal{X},\ \sum_{r,a}(y_a^r - x_a^r)F_a^r(y)\leq 0\}.$$

■

Proposition III-G: Suppose that the revision protocol has the form

$$\beta_{(r,a)}^{(\bar{r},b)}(x) = \begin{cases} \xi_a^r(F_a^r(x)-F_b^{\bar{r}}(x)) & \text{if } b\in\mathcal{A}^{\bar{r}}, a\in\mathcal{A}^r\\ 0 & \text{otherwise}\end{cases}$$

for some functions $\xi_a^r:\ \mathbb{R}\to\mathbb{R}_+$ then the set of CCE is globally asymptotically stable in stable games.

Proof: Let the function $B:\ \mathcal{X}\to\mathbb{R}_+$ be defined by

$$B(x) = \sum_{\substack{(r,a),\ a\in\mathcal{A}^r\\(\bar{r},b),\ b\in\mathcal{A}^{\bar{r}}}} x_a^r\int_0^{-F_a^r(x)+F_b^{\bar{r}}(x)} \xi_b^{\bar{r}}(\theta)\ d\theta.$$

The function B has the following properties:

- The set $\{x,\ B(x)=0\}$ is exactly the set of CCE.
- $\frac{\partial}{\partial x_{a'}^{r'}}B = \sum_{(\bar{r},b)}\int_0^{-F_{a'}^{r'}+F_b^{\bar{r}}}\xi_b^{\bar{r}}(\theta)\ d\theta + \sum_{(r,a)}\dot{x}_a^r\frac{\partial F_a^r}{\partial x_{a'}^{r'}}$
- $\frac{d}{dt}B(x(t)) = \sum_{(r',a')}\frac{\partial B}{\partial x_{a'}^{r'}}\dot{x}_{a'}^{r'} = \langle\dot{x},DF(x)\dot{x}\rangle + \sum_{\substack{(r',a')\\(r,a)}}\beta_{(r',a')}^{(r,a)}\sum_{(\bar{r},b)}\left[\int_0^{-F_{a'}^{r'}+F_b^{\bar{r}}}\xi_b^{\bar{r}} - \int_0^{-F_a^r+F_b^{\bar{r}}}\xi_b^{\bar{r}}\right]$

Since F is a stable population game, one has $\langle\dot{x},DF(x)\dot{x}\rangle\leq 0$. In other hand, $\beta_{(r',a')}^{(r,a)}>0\Leftrightarrow F_{a'}^{r'}>F_a^r$. Then, for all actions $(\bar{r},b)$, one has $F_b^{\bar{r}}-F_{a'}^{r'}\leq F_b^{\bar{r}}-F_a^r$. Hence, the term $\left[\int_0^{-F_{a'}^{r'}+F_b^{\bar{r}}}\xi_b^{\bar{r}}(\theta)\ d\theta - \int_0^{-F_a^r+F_b^{\bar{r}}}\xi_b^{\bar{r}}(\theta)\ d\theta\right]$ is negative. We conclude that $\frac{d}{dt}B(x)\leq 0$ and $\frac{d}{dt}B(x)=0$ if and only $x_a^r\beta_{(r',a')}^{(r,a)}=0,\ \forall(r,a),(r',a')$ i.e $V_F(x)=0$. Thus, the function satisfies Lyapunov stability criterion for the set of stationary point of V_F. ■

H. Potential Population Games

We say that F is a full potential game if it exists a C^1 function $f: \mathcal{X} \to \mathbb{R}$ such that $\frac{\partial}{\partial x_a^r} f = F_a^r(x)$.

Two strategies: if the game has only two strategies in each r, all strategy distributions lie on a line. Given any continuous payoff functions

$$F_1^r, F_2^r : \mathcal{X}^r = \{(x_1^r, x_2^r),\ x_i^r \geq 0,\ x_1^r + x_2^r = l^r\} \to \mathbb{R},$$

a potential function is given by

$$f(x) = \sum_{r=1}^{N} \int_0^{x_1^r} (F_1^r(t, l^r - t) - F_2^r(t, l^r - t))\ dt$$

Proposition III-I: global convergence holds in potential games under (PC).

Proof: Suppose that f is a potential function of the game and the dynamic $\dot{x} = V_F(x)$ is (PC). Then f is a Lyapunov function of this dynamic.

$$\frac{d}{dt} f(x) = \sum_r \sum_{a \in \mathcal{A}^r} V_F^{r,a}(x) F_a^r(x).$$

Moreover f satisfies $\frac{d}{dt} f(x) \geq 0$ with equally if and only if x is a CCE. It is known that if a dynamic admits a such Lyapunov function, all solution trajectories of the dynamic converge to an equilibrium. Combining with results III-D and (PC), we obtain the announced result. ■

J. Migration with constraints

In this subsection, we assume that when updating a strategy can change only one component of the strategies at a time. Players with the action (r, a) can use all actions in $\mathcal{N}_{(r,a)} = \cup_{\bar{r} \neq r,(}\{(\bar{r}, a),\ a \in \mathcal{A}^{\bar{r}}\} \cup \cup_{\bar{a} \in \mathcal{A}^r} \{(r, \bar{a})\}$. Taking these considerations, a strategy x is a CCE if for all (r, a) such that $x_a^r > 0$ one has

$$F_a^r(x) = \max_{(\bar{r},b) \in \mathcal{N}_{(r,a)}} F_b^{\bar{r}}(x). \quad (10)$$

and the constrained dynamic becomes

$$\begin{aligned} \dot{x}_a^r &= \textstyle\sum_{(\bar{r},b) \in \mathcal{N}_{(r,a)}} x_b^{\bar{r}} \beta_{(r,a)}^{(\bar{r},b)} - x_a^r \sum_{(\bar{r},b) \in \mathcal{N}_{(r,a)}} \beta_{(\bar{r},b)}^{(r,a)} \\ &= \textstyle\sum_{\bar{r}} x_b^{\bar{r}} \eta_{(r,a)}^{(\bar{r},a)} - x_a^r \sum_{\bar{r}} \eta_{(\bar{r},a)}^{(r,a)} \\ &\quad + \textstyle\sum_{\bar{a} \in \mathcal{A}^r} x_{\bar{a}}^r \rho_{\bar{a},a}^r - x_a^r \sum_{\bar{a} \in \mathcal{A}^r} \rho_{a,\bar{a}}^r \end{aligned}$$

K. Inverse problem: reachable regions of a power level

In this subsection, we model the power levels as first strategies and regions as secondary strategies. Let $\mathcal{P}$ a finite set of power levels, $\mathcal{A}_p = \{r_p^1, \ldots, r_p^{n^p}\}$ the pure secondary strategies set of the power level $p \in \mathcal{P}$ and, $\mathcal{S}_p = \{(r, a),\ r \in \mathcal{A}_p\}$ the pure strategies of a $p-$player. Then $\mathcal{S} = \cup_{p \in \mathcal{P}} \mathcal{S}_p$. Given an energy (power level), the player will migrate from its location to a reachable region of this power level.

A strategy x is a CCE of this inverse problem if for all (r, a) such that $x_a^r > 0$, one has $F_a^r(x) = \max_{\substack{b \in \mathcal{P},\\ \bar{r} \in \mathcal{A}_b}} F_b^{\bar{r}}(x)$, and mean dynamic becomes

$$\dot{x}_a^r = \textstyle\sum_{b \in \mathcal{P}} \sum_{\bar{r} \in \mathcal{A}_b} x_b^{\bar{r}} \beta_{(r,a)}^{(\bar{r},b)} - x_a^r \sum_{b \in \mathcal{P}} \sum_{\bar{r} \in \mathcal{A}_b} \beta_{(\bar{r},b)}^{(r,a)}$$

IV. A HYBRID EVOLUTIONARY GAME IN MULTICELL CDMA SYSTEM

We consider a large population of mobile terminals and many distributed base stations in a multicell CDMA wireless network model. The system consists of N cells which are called "regions". The number of users which transmit to a base station is a random variable. Each mobile connects to a base station which it chooses from of the set of base stations $\{1, 2, \ldots, N\}$ with an uplink power level from the set $\mathcal{A}$. The action space of a mobile is given by $\{1, 2, \ldots, N\} \times \mathcal{A}$. The level of services a mobile receives is described in terms of signal-to-interference ratio (SIR). The SIR obtained by mobile m at a base station located in region r is given by

$$SIR_m^{r,k}(a^m, a^{-m}) = \frac{L a^m h^{r,m}}{\sigma_r^2 + \sum_{0 \leq l \leq k, l \neq m} a^l h^{r,l}}$$

where a^{-m} denote the vector $(a^1, \ldots, a^{m-1}, a^{m+1}, \ldots, a^k)$ the power levels of the others mobiles, σ_r^2 is a constant which represents the variance of a noise power due to the factors other than the transmissions of other mobiles at the base station r. The term $a^m h^{r,m}$ represents the power level received at base station from mobile using the power level a^m and $L = W/B > 1$ is the spreading gain of the CDMA system, W is the chip rate and B is the data rate of the users.

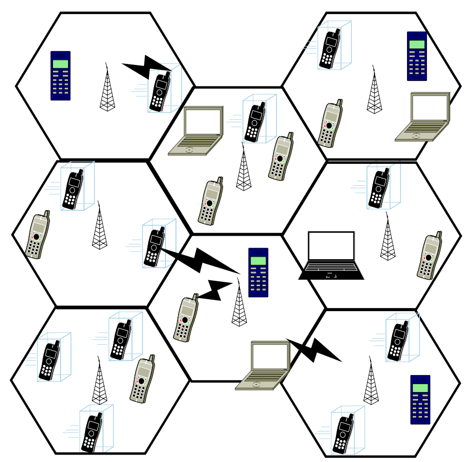

Fig. 1. The hybrid model

The payoff that mobile using the power level a^m can send to the base station in region r at a given slot is given by

$$J^{r,k}(a^m, a^{-m}) = C^r \log\left(1 + SIR_m^{r,k}(a^m, a^{-m})\right) - c^r(a^m)$$

where C^r is the channel bandwidth of r and $c^r(.)$ is the cost function of region r. We assume that c^r is an increasing cost function. The expression of $J^{r,k} + c^r$ is known as Shannon capacity. The expected fitness of mobile m at the base station r when the state of the population at region r is x^r is given by

$$F_{a^m}^r(x^r) = \sum_k P(K = k) \sum_{a^{-m}} \left(\prod_{l \neq m} x_{a^l}^r \right) F^{r,k}(a^m, a^{-m}).$$

Proposition IV-A (two power levels): Let $\mathcal{A} = \{0, P\}$ and $h^{r,m} = h^r$, $r = 1, 2$. Let $W_j^r = C^r \log(1 + \frac{h^r P}{\sigma_r^2 + j h^r P}), j = 0, \ldots, k.$, and $c^r(P)$ positive reals satisfying $c^r(P) \in (\sum_k p_k W_k^r, a_0^r)$ where $p_k = P(K = k)$. Then, the polynomial

$$Q^r(\xi) := -c^r(P) + \sum_k p_k \sum_{j=0}^{k} W_j^r \binom{k}{j} \xi^j (1-\xi)^{k-j}$$

has unique root x_*^r on the interval $(0, 1)$.

Proposition IV-B: The unique solution $(x_*^r)_{r=1,\ldots,N}$ given by the result IV-A is a the unique interior Wardrop equilibrium.

Proof: The fitness of the strategy P in the cell r is given by $F_P^r(x_P^r, 1 - x_P^r) = Q^r(x_P^r)$, the fitness of the strategy 0 is zero. Then the expected fitness of the cell r is $x_P^r Q^r(x_P^r)$. Hence, every interior Wardrop equilibrium $(x_*^r)_{r=1,\ldots,N}$ must satisfy $Q^r(x_P^r) = 0,\ \forall r$. From result IV-A, these equations have a unique solution on $(0, 1)$ under the conditions:

$$\forall r,\ Q^r(0) = -c^r(P) + W_0^r > 0,$$

$$Q^r(1) = -c^r(P) + \sum_k p_k W_k^r < 0$$

■

Proposition IV-C: The Wardrop equilibrium $(x_*^r)_{r=1,\ldots,N}$ is also a global ESS.

Proof: For $x = x_*$ and every cell r, one has $(x_P^r - mut_P^r) F_P^r(\epsilon\, mut^r + (1-\epsilon)x^r) > 0$ for small ϵ. Hence, for all r,

$$\sum_{a \in \mathcal{A}} x_a^r F_a^r(\epsilon\, mut^r + (1-\epsilon)x^r) > \sum_{a \in \mathcal{A}} mut_a^r F_a^r(\epsilon\, mut^r + (1-\epsilon)x^r)$$

■

V. Numerical investigation: Convergence to the equilibrium

Our numerical experiment studies the behavior of the replicator dynamics and smith dynamics with migration. We consider the following fixed parameters : we took $N = 2$ cells, $K \in \{0, 1, \ldots, 500\}$, $\mathcal{A} = \{0, P\}$, $h^{r,m} = h^r$, $r = 1, 2$, $\sigma_r^2 = r \times 10^{-3}$, $p_k = 2 \times 10^{-3}$, $k = 1, \ldots, 500$, $\gamma_1 = 0.5$, $\gamma_2 = 1$, $c^r(P) = r$, $P = 1$. An interior Wardrop equilibrium exists for these parameters, for which the fractions of the population using P is given by the result IV-B. The resulting trajectories of the population ratio each cell using the power level P, as a function of time, is given in Fig. 2.

The throughput of transmitters of the whole population is respectively in Fig.3. In Fig. 3, we can see that the throughput of players which use the strategy P decreases when the number of transmitters increases. This because the SINR decreases when the number of transmitters increases. The throughput increases with the power level P. When the number of transmitters is high, a player will increase its SINR, but will decrease the SINRs of the others players, and the total cost will decrease.

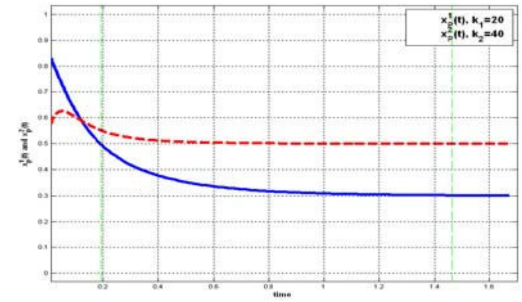

Fig. 2. RD: fraction of mobiles using the power levels P in cells 1, 2

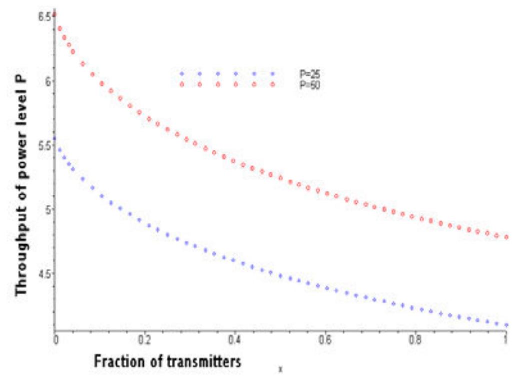

Fig. 3. Expected throughput of the transmitters.

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