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Passivity-based Output Synchronization and Flocking Algorithm in SE(3)

Yuji Igarashi, Takeshi Hatanaka, Masayuki Fujita and Mark W. Spong

***Abstract*— This paper addresses passivity-based output synchronization and a collision avoidance problem of rigid bodies in the Special Euclidean group $SE(3)$ under the assumption that the rigid bodies exchange information over weighted digraphs. We first develop a passivity-based distributed velocity input law to achieve output synchronization. Using the notion of algebraic connectivity, we then establish a connection between the speed of convergence and the structure of the interconnection graph in $SE(3)$. We also prove output synchronization in the presence of temporary communication failures. We next develop a flocking algorithm by modifying the present velocity input so that it embodies cohesion, separation and alignment rules introduced by Reynolds. Finally, we demonstrate the effectiveness of the present input through numerical simulations.**

I. INTRODUCTION

Cooperative control and motion coordination are received a lot of attention recently [1], [2] with numerous practical applications such as sensor networks, robot networks, coordinated control of satellites and formation control of aircraft [3–5]. In addition, motion coordination is also motivated by scientific interest in cooperative behavior in nature such as flocking of birds and schooling of fishes [6]. More recently, passivity and passivity-based control have proven useful for the problem of motion coordination of multi-agent systems [7–9]. In [7] passivity-based control laws are presented for output synchronization of networks of passive nonlinear systems. Output synchronization is proved by employing the sum of storage functions of each agent as a Lyapunov function candidate.

Attitude synchronization is gaining increasing interest [8–11]. The reference [8–10] consider multiple rigid bodies with attitude dynamics represented by Euler-Lagrange equations, while in [11] with kinematics. The control laws which attain almost global convergence are proposed in [10]. The passivity-based control laws also are presented in [8], [9], [11]. The proposed control law in [8], [11] relies only on relative attitude information with respect to neightbors. In [11] we also establish a connection between the graph structure and the speed of convergence and prove the convergence in the case of the communication failures. In [12], we generalize the results in [11] in the following respects. We propose a control input achieving output synchronization in $SE(3)$,

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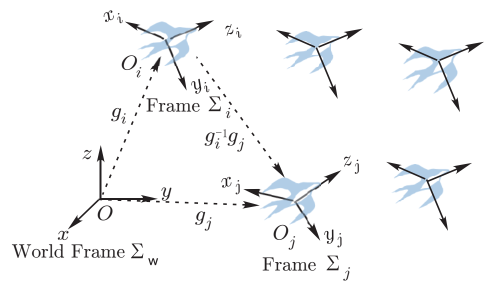

Fig. 1. Rigid Body Motion in $SE(3)$

which requires the convergence of not only orientations but also positions. We show the convergence in strongly connected digraphs in contrast to balanced digraphs in [11]. However analyses of connectivity and switching topology remain open problems in the case of output synchronization.

On the other hand, flocking has been also studied by many researchers e.g. [13–19]. In [18], Reynolds introduced three rules to achieve flocking, which are cohesion, separation(collision avoidance) and alignment rules. In [19], Vicsek *et al*. claimed that a discrete model of n agents in the plane with the same speed but with different headings achieves flocking and its theoretical proof was presented by Jadbabaie *et al*. [13]. In [14], [15], [17], the flocking problem was studied in continuous model of n agents. However, the references [15], [17] mainly focused on an alignment rule and did not take into account other rules (separation and cohesion) explicitly. On the other hand Olfati-Saber [16] proposed the input with all of the three rules.

In this paper, we address the output synchronization with collision avoidance in $SE(3)$ based on some techniques developed in [7], [11], [12], [20], [21]. We consider a group of rigid bodies in $SE(3)$ whose interconnection is represented by weighted digraphs. First, we present output synchronization base of the fact that the kinematics of a rigid body in $SE(3)$ has a passivity-like property. Then, we propose a velocity control scheme only requires error of outputs with respect to neighbors defined by the digraph. We show that output synchronization is achieved using the proposed velocity input under milder assumptions than in [11], [12]. We also introduce the notion of *algebraic connectivity* in order to clarify a relationship between the speed of convergence and the graph structure in $SE(3)$. The speed of convergence is a useful metric for the design of the information graph as well as for the analysis of the performance of cooperative control for a given network. We also address the practically important case of temporary communication failures using brief connectivity.

We next address the flocking problem in $SE(3)$, which requires aforementioned three rules. From this point of view, the above law achieving synchronization does not include the separation rule. Thus the velocity input is modified to incorporate this rule based on the avoidance control approach [20]. We show the modified velocity input embodies all three rules introduced by Reynolds.

This paper is organized as follows. Section II formulates rigid-body motion in $SE(3)$ and graph structure considered in this paper. We show that the rigid-body motion in $SE(3)$ has a passivity-like property and introduce the output synchronization problem. In Section III, a body velocity control law is proposed based passivity-like property and achievement of output synchronization is proven. Moreover we analyzes the connectivity and possibility of communication losses. In Section IV we develop the flocking algorithm in $SE(3)$. We demonstrate our results through numerical simulations in Section V and present conclusions in Section VI.

II. PROBLEM STATEMENT

A. System Description

Throughout this paper, we consider the motion of a group of n rigid bodies in 3-dimensional space (see Fig. 1). Let $\Sigma_{\rm w}$ be an inertial coordinate frame and $\Sigma_i, i \in \{1, \cdots, n\}$ a body-fixed coordinate frame whose origin is located at the center of mass of body i. Assume that all the coordinate frames are right-handed and Cartesian. We denote by $p_i \in \mathcal{R}^3$ the position of the rigid body $i \in \{1, \cdots, n\}$ in a fixed inertial coordinate frame $\Sigma_{\rm w}$. We will use $e^{\hat{\xi}_i\theta_i} \in \mathcal{R}^{3\times 3}$ to represent the rotation matrix of a body-fixed frame Σ_i relative to an inertial coordinate frame $\Sigma_{\rm w}$. Here, $\xi_i \in \mathcal{R}^3$, $\xi_i^T\xi_i = 1$ and $\theta_i \in \mathcal{R}$ specify the direction of rotation and the angle of rotation, respectively. The notation '$\wedge$' (wedge) is the skew-symmetric operator such that $\hat{a}b = a\times b$ for the vector cross-product $\times$ and any vector $a, b \in \mathcal{R}^3$. The notation '$\vee$' (vee) denotes the inverse operator to '$\wedge$'. The transformation $e^{\hat{\xi}_i\theta_i}$ is orthogonal with unit determinant i.e. an element of the Special Orthogonal group $SO(3)$. A configuration consists of the pair $(p_i, e^{\hat{\xi}_i\theta_i})$ and hence the configuration space of the rigid-body motion is the Special Euclidean group $SE(3)$, which is the product space of $\mathcal{R}^3$ with $SO(3)$. We use the 4×4 matrix

$$g_i = \begin{bmatrix} e^{\hat{\xi}_i\theta_i} & p_i \\ 0 & 1 \end{bmatrix},\ i \in \{1, \cdots, n\}$$

as the homogeneous representation of $(p_i, e^{\hat{\xi}_i\theta_i}) \in SE(3)$.

Let us now introduce the velocity of each rigid body to represent the rigid-body motion of the frame Σ_i relative to $\Sigma_{\rm w}$. Define the body velocity $V_i^b := (v_i, \omega_i)$ and

$$\hat{V}_i^b = \begin{bmatrix} \hat{\omega}_i & v_i \\ 0 & 0 \end{bmatrix},\ i \in \{1, \cdots, n\},$$

where $v_i \in \mathcal{R}^3$ and $\omega_i \in \mathcal{R}^3$ are the linear and angular velocities of body i relative to Σ_i respectively. Then, each rigid-body motion is represented by the kinematic model

$$\begin{aligned} \dot{g}_i &= g_i\hat{V}_i^b, \\ y_i &= \begin{bmatrix} e^{\hat{\xi}_i\theta_i} & p_i + d_i \\ 0 & 1 \end{bmatrix},\ i \in \{1, \cdots, n\}. \end{aligned} \quad (1)$$

where $y_i \in \mathcal{R}^3$ is a controlled output and $d_i \in \mathcal{R}^3$ is a bias of body i. If $d_i = 0$ then $y_i = g_i$, which is the same as the controlled output in [12]. We need the bias d_i in order to address the collision avoidance problem discussed in Section IV. For more details on the rigid-body motion in $SE(3)$, refer to [22], [23].

The interconnection of a network of rigid bodies is represented by a weighted digraph $G_O = (\mathcal{V}, \mathcal{E}_\mathcal{O}, \mathcal{W})$, where $\mathcal{V} := \{1, \cdots, n\}$ is the node set, $\mathcal{E}_\mathcal{O} \subset \mathcal{V}\times\mathcal{V}$ is the edge set containing pairs of nodes that represent communication and $\mathcal{W}$ is the weight set. The neighbors of body i are defined as $\mathcal{N}_{Oi} := \{j \in \mathcal{V} \mid (j, i) \in \mathcal{E}_\mathcal{O}\}$ [2]. $\mathcal{N}_{Oi}$ means that agent i receives information from agent j if $j \in \mathcal{N}_{Oi}$. We moreover define the weighted graph Laplacian matrix of the digraph G_O

$$L_{\rm w} := [L_{{\rm w}ij}] = \begin{cases} \sum_{k\in\mathcal{N}_{Oi}} w_{ik} & \text{if } j = i, \\ -w_{ij} & \text{if } j \in \mathcal{N}_{Oi}, \\ 0 & \text{if } j \notin \mathcal{N}_{Oi}, \end{cases}$$

which plays an important role in this paper.

B. Passivity-like Property in $SE(3)$

We show that the kinematic model (1) possesses a passivity-like property. This property is used to develop output feedback law for output synchronization which will be defined in the subsequent subsection.

We first define the total energy of translation and rotation

$$\begin{aligned} \psi(y_i) &:= \|(I_4 - y_i)J\|_F^2, \qquad J := \begin{bmatrix} \frac{1}{2}I_3 & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \\ &= \frac{1}{2}\|q_i\|^2 + \phi(e^{\hat{\xi}_i\theta_i}),\ \phi(e^{\hat{\xi}_i\theta_i}) := \frac{1}{2}{\rm tr}(I_3 - e^{\hat{\xi}_i\theta_i}) \end{aligned}$$

where $q_i := p_i + d_i$, I_n is the $n\times n$ identity matrix, $\|\cdot\|_F$ represents the Frobenius matrix norm ($\|A\|_F = {\rm tr}(A^TA)^{1/2}$) and $\|\cdot\|$ the Euclidean vector norm. By the definition, $\psi(y_i) = 0$ if and only if $y_i = I_4$.

Lemma 1: The time derivative of $\psi(y_i)$ along the trajectories of (1) satisfies

$$\dot{\psi}(y_i) = (V_i^b)^T\Pi_i,\ V_i^b = \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}, \Pi_i := \begin{bmatrix} e^{-\hat{\xi}_i\theta_i}q_i \\ \mathbf{sk}(e^{\hat{\xi}_i\theta_i})^\vee \end{bmatrix},$$

where $\mathbf{sk}(e^{\hat{\xi}_i\theta_i})$ is a skew-symmetric part of the matrix $e^{\hat{\xi}_i\theta_i}$, i.e. $\mathbf{sk}(e^{\hat{\xi}_i\theta_i}) := \frac{1}{2}(e^{\hat{\xi}_i\theta_i} - e^{-\hat{\xi}_i\theta_i})$.

Proof: Immediate from ([24, pp. 42 Lemma 1]). ■

If we now consider the velocity V_i^b as an input and the vector form of the rigid-body motion Π_i as an output, Lemma 1 says that the rigid-body motion in $SE(3)$ (1) is passive from the input V_i^b to the output Π_i (This property is called a *passivity-like* property throughout this paper) in the sense defined in [25].

C. Output Synchronization in $SE(3)$

In the next section, we will investigate the output synchronization defined as follows.

Definition 1 (Output Synchronization): A group of n rigid bodies is said to achieve output synchronization, if

$$\lim_{t\to\infty} \psi(y_i^{-1}y_j) = 0\ \forall i, j \in \{1, \cdots, n\}. \quad (2)$$

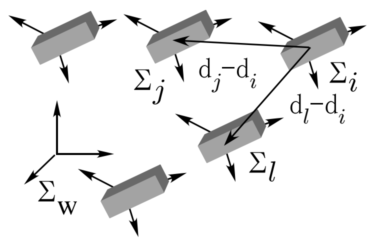

Fig. 2. Output Synchronization in $SE(3)$

By the definition of the function ψ, equation (2) implies the outputs of all the rigid bodies converge to a common value. From the definition of the output, it means that relative positions between rigid bodies converge to desired ones $d_i - d_j\ \forall i, j$, while orientations converge to a common value (Fig. 2). The convergence of only the orientations is called attitude synchronization and investigated in [8–11].

III. Convergence and Connectivity Analysis

A. Velocity Control Law and Convergence

The goal of this section is to design a body velocity input V_i^b so that the group of rigid bodies achieves output synchronization.

We propose the body velocity input

$$V_i^b = -K_i \sum_{j\in\mathcal{N}_{Oi}} \left(w_{ij} \begin{bmatrix} e^{-\hat{\xi}_i\theta_i} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} q_i - q_j \\ \mathbf{sk}(e^{-\hat{\xi}_j\theta_j} e^{\hat{\xi}_i\theta_i})^\vee \end{bmatrix} \right) + \begin{bmatrix} e^{-\hat{\xi}_i\theta_i} & 0 \\ 0 & e^{-\hat{\xi}_i\theta_i} \end{bmatrix} \begin{bmatrix} v_{\rm d} \\ e^{\hat{\xi}_{\rm d}\theta_{\rm d}}\omega_{\rm d} \end{bmatrix},\ i \in \{1, \cdots, n\} \tag{3}$$

where $K_i = \begin{bmatrix} k_{pi}I_3 & 0 \\ 0 & k_{ei}I_3 \end{bmatrix}$, $k_{pi} > 0$, $k_{ei} > 0$ and the vectors $v_{\rm d}$ and $\omega_{\rm d} := \left(e^{-\hat{\xi}_{\rm d}\theta_{\rm d}} \frac{d}{dt} e^{\hat{\xi}_{\rm d}\theta_{\rm d}} \right)^\vee$ represent desired linear and angular velocities, respectively. The input (3) consists of two parts. The first term assures the output synchronization. The second term specifies a common desired velocity after the output synchronizes. Thus, the functions $v_{\rm d}$ and $\omega_{\rm d}$ should be common among all the rigid bodies.

Theorem 1: Consider the n rigid bodies represented by (1) and suppose that $v_{\rm d}$ and $\omega_{\rm d}$ represent desired group trajectories. Then, under the assumption that there exists $e^{\hat{\xi}_\alpha\theta_\alpha}$ such that $e^{\hat{\bar{\xi}}_i\bar{\theta}_i} := e^{-\hat{\xi}_{\rm d}\theta_{\rm d}} e^{-\hat{\xi}_\alpha\theta_\alpha} e^{\hat{\xi}_i\theta_i}\ \forall i$ are positive definite [1] and the interconnection digraph G_O is fixed and strongly connected, the velocity input (3) achieves output synchronization in the sense of (2).

Proof: It is easy to prove this theorem using the following potential function and similar calculation in [12]

$$U_O := \sum_{i=1}^n \gamma_i \left(\frac{1}{2k_{pi}} \|\bar{q}_i\|^2 + \frac{1}{k_{ei}} \phi(e^{\hat{\bar{\xi}}_i\bar{\theta}_i}) \right),$$

where $\bar{q}_i := q_i - \int_0^t v_{\rm d} d\tau$ and γ_i is an element of vectors satisfying

$$\gamma^T L_{\rm w} = 0\ \gamma^T = [\gamma_1, \cdots, \gamma_n],\ \gamma_i > 0\ \forall i. \tag{4}$$

[1]Throughout this paper, we refer to a real matrix Q which is not necessarily symmetric, as a positive definite matrix if and only if $x^TQx > 0$ for any nonzero vector x.

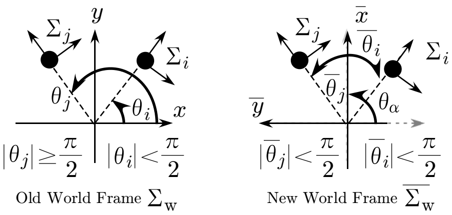

Fig. 3. Coordinate transform of the world frame

From the assumption of strongly connected, there exists vectors satisfying (4) [12, Lemma 2]. ■

In the proof of Theorem 1, the potential function $U_{\rm O}$ is defined as a weighted sum of the energy functions $\psi(y_i)$ based on the passivity-like property (Lemma 1).

If $e^{\hat{\xi}_{\rm d}\theta_{\rm d}} = I_3$, i.e. $e^{\hat{\bar{\xi}}_i\bar{\theta}_i} = e^{-\hat{\xi}_\alpha\theta_\alpha} e^{\hat{\xi}_i\theta_i}\ \forall i$, then existence of $e^{\hat{\xi}_\alpha\theta_\alpha}$ satisfying the condition of Theorem 1 means that there is a coordinate transformation of the world frame such that all rigid bodies orientation matrices become positive definite (Fig. 3). Clearly, this condition is milder than the assumption in [11], [12] that all rigid bodies' orientation matrices are positive definite.

In the following, we make a physical interpretation of the proposed velocity input (3). The first term can be rewritten as

$$-K_i \begin{bmatrix} e^{-\hat{\xi}_i\theta_i} & 0 \\ 0 & I \end{bmatrix} \Big(\sum_{j\in\mathcal{N}_{Oi}} w_{ij} \Big) \begin{bmatrix} q_i - \langle q_i \rangle \\ \mathbf{sk}(\langle e^{-\hat{\xi}_i\theta_i} \rangle e^{\hat{\xi}_i\theta_i})^\vee \end{bmatrix}\ \forall i,$$

where $\langle q_i \rangle$ and $\langle e^{\hat{\xi}_j\theta_j} \rangle$ are weighted average position and orientation of neighbors of rigid body i, i.e., $\langle q_i \rangle := (\sum_{j\in\mathcal{N}_{Oi}} w_{ij}q_j)/(\sum_{j\in\mathcal{N}_{Oi}} w_{ij})$ and $\langle e^{-\hat{\xi}_i\theta_i} \rangle := (\sum_{j\in\mathcal{N}_{Oi}} w_{ij} e^{-\hat{\xi}_j\theta_j})/(\sum_{j\in\mathcal{N}_{Oi}} w_{ij})$, respectively. Hence each rigid body moves and rotates toward the weighted center of outputs with respect to the neightbors. This means the first term of the body velocity input (3) embodies the cohesion rule introduced by Reynolds. Moreover, the second term incorporates the alignment rule. In fact, if output synchronization is achieved, then the body velocity input (3) of each rigid body becomes

$$V_i^b = \begin{bmatrix} e^{-\hat{\xi}_i\theta_i} & 0 \\ 0 & e^{-\hat{\xi}_i\theta_i} \end{bmatrix} \begin{bmatrix} v_{\rm d} \\ e^{\hat{\xi}_{\rm d}\theta_{\rm d}}\omega_{\rm d} \end{bmatrix}\ \forall i. \tag{5}$$

Thus all rigid bodies have the same linear and angular velocity because output synchronization implies $e^{-\hat{\xi}_i\theta_i} = e^{-\hat{\xi}_j\theta_j}\ \forall i, j$. This property is called flocking in [14], [16], [17]. However it is somewhat different from the original definition of flocking by Reynolds [18] as pointed out by Olfati-Saber [16]. From the viewpoint of flocking by Reynolds, the velocity input (3) does not embody the separation rule which is introduced section IV.

Remark 1: The result can be extend to the case of time delays in communication in the same way as [12].

B. Algebraic Connectivity and Switching Topology

In [11] we analyze a relation between the graph structure G_O and the speed of convergence of the network to its final configuration and possibility of communication losses in attitude synchronization case. However these have not been investigated in output synchronization case yet. The purpose of this section is to study these issues in output synchronization case based on the same ideas in [11]. We first introduce an index $U_{\exp}$ evaluating the speed of convergence. In order to measure the speed, $U_{\exp}$ should satisfy the following two conditions:

1) output synchronization is achieved if and only if $\lim_{t\to\infty} U_{\exp} = 0$ holds.
2) $U_{\exp}(0)$ is independent of the graph.

Functions of the output errors between rigid bodies also satisfy the condition 1), and such a function can be easily constructed by using the graph Laplacian $L_{\rm w}$. However, the use of $L_{\rm w}$ is prohibited due to the condition 2). For the above the reason, we define the function

$$U_{\exp}(t) := Q(t)^T(M\otimes I_3)Q(t) + \operatorname{tr}\left((e^{\hat{\xi}\theta(t)})^T(M\otimes I_3)e^{\hat{\xi}\theta(t)}\right) \geq 0$$

to evaluate the speed of convergence, where M denotes the graph Laplacian of the nonweighted complete graph (i.e. $w_{ij} = 1\ \forall i,j$), namely

$$M := nI_n - \mathbf{1}\mathbf{1}^T,$$

where $\mathbf{1} := [1,\cdots,1]^T$, $Q := [q_1^T,\cdots,q_n^T]^T$ and $e^{\hat{\xi}\theta(t)} := [e^{-\hat{\xi}_1\theta_1(t)},\cdots,e^{-\hat{\xi}_n\theta_n(t)}]^T$. This function evaluates the relative outputs for all rigid bodies regardless of the actual connectivity, and satisfies both of the above conditions.

Before stating the main result of this section, we introduce the following notation. The notation $\operatorname{diag}(\gamma_1,\cdots,\gamma_n)$ represents the diagonal matrix with diagonal elements $\gamma_1,\cdots,\gamma_n$ and L_γ denotes $L_\gamma := \operatorname{diag}(\gamma_1,\cdots,\gamma_n)L_{\rm w}$, where $\gamma^T L_{\rm w} = 0,\ \gamma := [\gamma_1,\cdots,\gamma_n]^T$. Moreover $L_{\gamma{\rm sym}} := \frac{1}{2}(L_\gamma + L_\gamma^T)$ denotes the symmetric part of a matrix L_γ. Let $\lambda_{\min}(L)$ and $\lambda_{\min 2}(L)$ be the smallest eigenvalue and the second smallest eigenvalue, respectively, of any real symmetric square matrix L. We can now state the following,

Theorem 2: Consider the n rigid bodies represented by (1) *and the input* (3). *Then, under the assumption that the relative orientations, $e^{-\hat{\xi}_i\theta_i}e^{\hat{\xi}_j\theta_j}\ \forall i,j$, are positive definite and the interconnection digraph G_O is strongly connected, there exist positive real numbers a and b such that*

$$U_{\exp}(t) \leq aU_{\exp}(0)e^{-\lambda_{\min 2}(L_{\gamma{\rm sym}})bt}. \tag{6}$$

Proof: Omitted due to page limitations. ■

Because of the independence of $U_{\exp}(0)$ on the graph structure, Theorem 2 implies that the larger the value of $\lambda_{\min 2}(L_{\gamma{\rm sym}})$ for a given digraph, the faster the right hand side of (6) converges to 0. In general, $\lambda_{\min 2}(L_{\gamma{\rm sym}})$ is called *algebraic connectivity* of a graph, and it is well-known in consensus [2] that it is a measure of the speed of convergence.

Theorem 2 also gives another important insight into convergence analysis. In the inequality (6), the error of outputs between rigid bodies exponentially converges to 0, though Theorem 1 only shows asymptotic convergence. Notice that Theorem 2 assumes the positive definiteness of the *relative* orientations, while Theorem 1 that of the *individual* orientations. If $e^{\hat{\xi}_{\rm d}\theta_{\rm d}} = I_3$, the latter assumption includes the former. In fact, we can confirm that if $e^{-\hat{\xi}_i\theta_i}e^{\hat{\xi}_j\theta_j} > 0\ \forall i,j$, then $e^{\hat{\bar{\xi}}_i\bar{\theta}_i} = e^{-\hat{\xi}_\alpha\theta_\alpha}e^{\hat{\xi}_i\theta_i}\ \forall i$ become positive definite by choosing $e^{\hat{\xi}_\alpha\theta_\alpha} = e^{\hat{\xi}_1\theta_1}$.

We next investigate the situation where the information graph changes over time. To study the effect of switching topology we utilize the concept of brief instability developed in [26]. This concept will be instrumental in capturing the fraction of the time that the graph may remain disconnected. Let $\mathcal{G} \subseteq G_O$ be a certain set of possible state independent graphs with n nodes and let $s(t) : [0,\infty] \to \mathcal{G}$ be the piecewise constant switching signal with consecutive switching times separated by a dwell time, $\tau_D > 0$. Namely, any two consecutive switching times t_l and $t_{l+1},\ l \geq \{0,1,2,\cdots\}$ satisfy $t_{l+1} - t_l \geq \tau_D,\ l \geq 0$. The signal $s(t)$ belongs to either the following subsets of $\mathcal{G}$,

1) $\mathcal{G}_{\rm c} \subseteq \mathcal{G}$: a subset of strongly connected digraphs,
2) $\mathcal{G}_{\rm dc} \subseteq \mathcal{G}$: a subset of not strongly connected digraphs.

It is obvious from the definitions that $\mathcal{G} = \mathcal{G}_{\rm c} \cup \mathcal{G}_{\rm dc}$ holds true. Let us now introduce the connectivity loss time $T(\tau,t)$, which is the length of the time when the digraph belongs to $\mathcal{G}_{\rm dc}$ over any time interval $[\tau,t]$. The function $T(\tau,t)$ is clearly given by

$$T(\tau,t) = \int_\tau^t \mathcal{X}(s(r))dr,\ \ \mathcal{X}(s(t)) := \begin{cases} 0 & s(t) \in \mathcal{G}_{\rm c} \\ 1 & s(t) \in \mathcal{G}_{\rm dc} \end{cases}.$$

Brief connectivity losses [26] means

$$T(\tau,t) \leq \alpha(t-\tau) + T_0\ \ \forall t \geq \tau \geq 0 \tag{7}$$

holds for some $T_0 \geq 0$ and $0 \leq \alpha < 1$.

Theorem 3: Consider the n rigid bodies represented by (1) *and the input* (3). *Assume that the relative orientations, $e^{-\hat{\xi}_i\theta_i}e^{\hat{\xi}_j\theta_j}\ \forall i,j$, are positive definite. Then, if the inequalities* (7) *holds, there exists a lower bound of τ_D such that the velocity input* (3) *achieves output synchronization in the sense of* (2).

Proof: Omitted due to page limitations. ■

Roughly speaking, the inequality (7) means that the fraction of the connectivity loss time is small, and the existence of a lower bound of τ_D assures that the digraph does not switch frequently.

IV. Flocking Algorithm in SE(3)

In Section III-A, we show that flocking in the sense of velocity matching results from the input (3). This flocking is only taken account of an alignment rule. However, Olfati-Saber [16] insists that distributed algorithms for creation of flocking should be embodied three rules introduced by Reynolds, which are cohesion, separation and alignment rules. The cohesion and alignment rules are already incorporated in the proposed input (3), however the separation rule, or collision avoidance is not. In this section we thus incorporate the third rule into the input (3) based on the ideas in [20] and [21], which guarantees collision avoidance. Accordingly we reformulate the problem in Section III

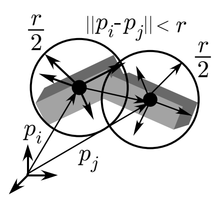

Fig. 4. Collision

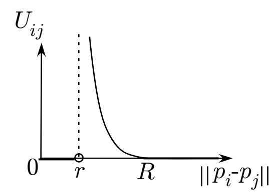

Fig. 5. Potential function for collision avoidance

A. Problem Reformulation

In this section, we investigate not only output synchronization but also collision avoidance. The collision in this paper is defined as follows.

Definition 2: (*Collision*) Rigid body i and j are said to collide (Fig. 4) , if and only if

$$\|p_i - p_j\| \leq r,\ r > 0.$$

Next we define the avoidance region [20], [21] as

$$\Omega := \bigcup_{j>i} \Omega_{ij},\ \Omega_{ij} := \{P : P \in \mathcal{R}^{3n},\ \|p_i - p_j\| \leq r\},$$

and sensing region as

$$D := \bigcup_{j>i} D_{ij},\ D_{ij} := \{P : P \in \mathcal{R}^{3n}, r < \|p_i - p_j\| \leq R\}$$

where $P := [p_1^T, \cdots, p_n^T]^T$. By definition, $P(t) \notin \Omega$ means that collision does not occur between rigid bodies at time t. Additionally, we define a sensing network as a position dependent graph $G_C = (\mathcal{V}, \mathcal{E}_\mathcal{C})$ where $\mathcal{E}_\mathcal{C} := \{(j,i) \in \mathcal{V} \times \mathcal{V} \mid r < \|p_i - p_j\| \leq R\}$. Its neightbors of rigid body i is defined as $\mathcal{N}_{Ci} := \{j \in \mathcal{V} \mid (j,i) \in \mathcal{E}_C\}$. Throughout this section, we assume each rigid body can get the information about rigid body j, if $j \in \mathcal{N}_{Oi} \cup \mathcal{N}_{Ci}$ and $\|d_i - d_j\| > r\ \forall i, j$. Note that if $d_i = 0\ \forall i$, then the condition $\|d_i - d_j\| > r\ \forall i, j$ is violated.

B. Flocking Algorithm

In the following we propose the body velocity input to guarantee collision avoidance. For this purpose, we use the following functions [20].

$$U_{ij}(p_i, p_j) = \left(\min \left\{ 0, \frac{\|p_i - p_j\|^2 - R^2}{\|p_i - p_j\|^2 - r^2} \right\} \right)^2,\ i \neq j,$$

where $0 < r < R$ (Fig 5). Under $P(0) \notin \Omega$, U_{ij} grows infinitely as rigid body i approaches rigid body j. The partial derivative of $U_{ij}(p_i, p_j)$ with respect to p_i is given in [20].

$$\begin{aligned} \frac{\partial U_{ij}}{\partial p_i} &= \begin{cases} 0 & \text{if } R \leq \|p_i - p_j\| \\ s_{ij} & \text{if } r < \|p_i - p_j\| < R \\ \text{not definitd} & \text{if } \|p_i - p_j\| = r \\ 0 & \text{if } \|p_i - p_j\| < r \end{cases}, \\ s_{ij} &= 4 \frac{(R^2 - r^2)(\|p_i - p_j\|^2 - R^2)}{(\|p_i - p_j\|^2 - r^2)^3}(p_i - p_j). \end{aligned}$$

By employing this function, we modify the body velocity input (3) as

$$V_i^b = \begin{bmatrix} e^{-\hat{\xi}_i \theta_i} & 0 \\ 0 & e^{-\hat{\xi}_i \theta_i} \end{bmatrix} \begin{bmatrix} v_\mathrm{d} \\ e^{\hat{\xi}_\mathrm{d} \theta_\mathrm{d}} \omega_\mathrm{d} \end{bmatrix} - K_{pi} \begin{bmatrix} e^{-\hat{\xi}_i \theta_i} & 0 \\ 0 & I \end{bmatrix} \left(\sum_{j \in \mathcal{N}_{Oi}} w_{ij} \begin{bmatrix} q_i - q_j \\ \mathbf{sk}(e^{-\hat{\xi}_j \theta_j} e^{\hat{\xi}_i \theta_i})^\vee \end{bmatrix} + \sum_{j \in N_{Ci}} \begin{bmatrix} \frac{\partial U_{ij}}{\partial p_i} \\ 0 \end{bmatrix} \right),\ i \in \{1, \cdots, n\} \qquad (8)$$

where $K_{pi} > 0 \in R^{3 \times 3}$.

The first and second terms of the modified body velocity input (8) are the same as the input (3). If $j \in \mathcal{N}_{Ci}$, then $s_{ij} < 0$, that is, the term $\frac{\partial U_{ij}}{\partial p_i}$ works so that rigid body i moves away from rigid body j. Thus, the third term represents a separation rule. Therefore, the input (8) embodies all three rules introduced by Reynolds as the same way as the reference [16].

Theorem 4: Consider the n rigid bodies represented by (1). *Assume that the $e^{\hat{\bar{\xi}}_i \bar{\theta}_i}$ are positive definite, the interconnection graph G_O is fixed, undirected and connected , $w_{ij} = w_{ji}$ and $P(0) \notin \Omega$. Then the velocity input* (8) *guarantees collision avoidance and achieves attitude synchronization.*

Proof: Omitted due to page limitations. ■

In this theorem, we prove collision avoidance and attitude synchronization. However positions of all rigid bodies do not always converge to the desired value. These finally converge to the positions satisfying the following condition

$$\sum_{j \in \mathcal{N}_{Oi}} w_{ij}(q_i - q_j) + \sum_{j \in \mathcal{N}_{Ci}} \frac{\partial U_{ij}}{\partial p_i} = 0\ \forall i. \qquad (9)$$

If attitude synchronization is achieved and (9) is satisfied, then the velocity input of each rigid body becomes the equation (5). Therefore, the linear and angular velocities of all rigid bodies asymptotically synchronize.

Theorem 4 assumes the graph G_O is undirected while Theorem 1 strongly connected. A similar assumption appears in [14], which studies collision avoidance.

Remark 2: The result in this section can be extended to guarantee obstacle avoidance in the same way as [16].

V. Simulation

In this section, we numerically demonstrate that the present input (8) achieves collision avoidance and attitude synchronization. In our simulation we assume that each rigid body is a rectangular solid whose breadth, width, height and circumradius are 1.2[m], 0.8[m], 0.6[m] and 1.5[m] respectively (Fig. 6). Therefore, we select the radius of collision region as $r = 3.0$[m]. The group consists of five rigid bodies with the kinematics described by (1) and a graph structure is depicted in Fig. 7. This graph is undirected and connected. The input (8) with $v_\mathrm{d} = [1\ 0\ 0]^T, \omega_\mathrm{d} = [0\ 0\ 0]^T$, $K_{pi} = 0.001 I_3$ and $k_{ei} = 0.001\ \forall i$ is applied to each rigid body under the following conditions $p_1(0) = \begin{bmatrix} 4 & 3 & 1 \end{bmatrix}^T$, $\xi_1\theta_1(0) = \begin{bmatrix} 0.21 & 0.50 & 0.77 \end{bmatrix}^T$, $p_2(0) = \begin{bmatrix} 2 & 3 & -5 \end{bmatrix}^T$, $\xi_2\theta_2(0) = \begin{bmatrix} 0.60 & 0.04 & 0.83 \end{bmatrix}^T$, $p_3(0) = \begin{bmatrix} 3 & -1 & 2 \end{bmatrix}^T$, $\xi_3\theta_3(0) = \begin{bmatrix} 0.21 & -0.77 & -0.50 \end{bmatrix}^T$, $p_4(0) = \begin{bmatrix} -1 & 2 & -3 \end{bmatrix}^T$, $\xi_4\theta_4(0) = \begin{bmatrix} -0.32 & 0.48 & -0.32 \end{bmatrix}^T$, $p_5(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$, $\xi_5\theta_5(0) = \begin{bmatrix} 0.51 & -0.51 & -0.14 \end{bmatrix}^T$ and output biases $d_1 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$, $d_2 = \begin{bmatrix} 7 & 5 & 0 \end{bmatrix}^T$, $d_3 = \begin{bmatrix} 7 & -5 & 0 \end{bmatrix}^T$, $d_4 = \begin{bmatrix} 14 & 10 & 0 \end{bmatrix}^T$, $d_5 = \begin{bmatrix} 14 & -10 & 0 \end{bmatrix}^T$. We remark that the orientation matrices are positive definite at

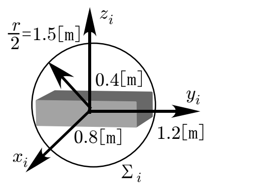

Fig. 6. Rigid body in simulation

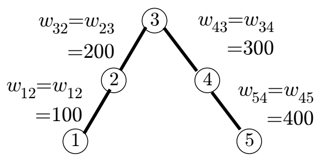

Fig. 7. Graph structure in simulation

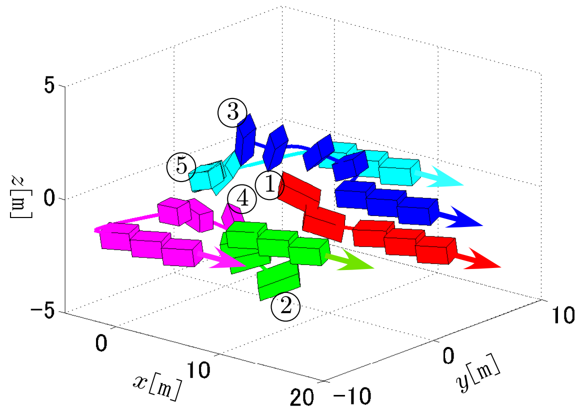

Fig. 8. Trajectory and orientation of each rigid body

the initial time. Fig. 8 shows the trajectories of the rigid bodies. In Fig. 8, the encircled number is associated with the corresponding one in Fig. 7. We see from Fig. 8 that the rigid bodies smoothly adjust their orientation without collisions. From this figure we can confirm that attitude synchronization and collision avoidance are achieved by the body velocity input (8). In this example, positions of all rigid bodies also converge to the desired relative one. Note that if collision avoidance is not token account, rigid bodies 1 and 4 collide with rigid bodies 2 and 5, respectively.

VI. Conclusions

In this paper, we have investigated output synchronization in $SE(3)$ based on a passivity-like property of the kinematics of rigid bodies. We first have developed a passivity-based control law attaining output synchronization. The passivity-like property has been also employed in connectivity analysis and we established a connection between the speed of convergence and the graph structure. We have also shown the fact that the passivity-based control input still attains output synchronization as the case with a topology switching. Moreover, we have extended the above results to incorporate three flocking rules introduced by Reynolds. The simulation results have demonstrated the validity of our results.

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