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Coverage Control for Mobile Networks with Limited-Range Anisotropic Sensors

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***Abstract*— In this paper the coverage control for mobile sensor networks is studied. The novelty is to consider an anisotropic sensor model where the performance of the sensor depends not only on the distance but also on the orientation from the sensor to the target. Moreover we consider sensors with limited-range sensing defined by a probabilistic model and we assume that each robot is equipped with omni-directional communication capability. A gradient-based distributed algorithm is designed to maximize the joint detection probabilities of the events in the region of interest by the sensors. Simulations illustrate the results.**

I. INTRODUCTION

Stimulated by the technological advances and the development of relatively inexpensive communication, computation, and sensing devices, the interest in the research area of coordinated networked control has majorly increased over the past years. One example is the deployment of autonomous vehicles to perform challenging tasks such as search and recovery operations, manipulation in hazardous environments, surveillance and also environmental monitoring for pollution detection and estimation. Deploying multiple agents to perform tasks is advantageous compared to the single agent case: It provides robustness to agent failure and allows to handle more complex tasks.

In this paper, we consider a mobile sensing network of vehicles equipped with sensors to sample the environment. The goal is to drive the sensors/agents to the position such that a given region is optimally covered by the sensors. Some relevant works on the coverage control problem are [1]–[12]. In [1] the agents move to the optimal configuration which minimizes an objective function. The approach is based on Voronoi tessellation and Lloyd algorithm. Briefly speaking, the agents partition the given region into subregions given by Voronoi partitions and move towards the centroid of its subregion and increase its sensing radius until all the area is covered. The same problem is considered in [2] with a more realistic model by introducing "limited-range interactions" of the sensors, i.e the sensing range is restricted to a bounded region. Power-aware coverage algorithms for mobile networks are proposed in [3] in order to balance the energy expenditure accross the network and make nodes with high power compensate for nodes with low power. The advantage of the Voronoi approach is that the control law is distributed by its nature.

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Alternative approaches are introduced in [4], [5], [7]. In [4] the authors consider a probabilistic network model and a density function to represent the frequency of random events taking place over the mission space. The authors develop an optimization problem that aims at maximizing coverage using sensors with limited ranges, while minimizing communication cost. A potential-field-based approach to deployment problem in an unknown environment is presented in [5]. An algorithm also based on artificial potential-field is proposed in [6] that maximizes the area coverage of a network while satisfying the constraint that every node has at least K neighbors. Dimarogonas, et.al [7] proposed an inverse agreement control strategy that forces the agents to disperse in the workspace. Here each agent follows a flow, whose inverse would lead the team to an agreement. Coverage control problem based on receding horizon control is considered in [8].

Moreover, dynamic coverage is considered in [9]. Here, the agents move such that every point in a given area is sensed with a pre-specified coverage level C^*. The same problem is introduced in [10] by considering information decay i.e. each point in the area is decaying w.r.t. time so that the robots must revisit them periodically. Dynamic coverage under some practical assumptions such as bounded sensing and actuation capacities of the vehicles are addressed in [11], [12].

However, in the works mentioned above, only a uniform (isotropic) sensor model is considered. In this paper, in contrast to the above papers, we consider the coverage problem with an anisotropic sensor model where the performance of the sensor depends not only on the distance but also on the orientation to the target. This model is more realistic since most of the sensors such as cameras, directional microphones, radars etc are anisotropic. In [13] we consider coverage control with anisotropic sensor based on Voronoi tessellation combined with an adapted Lloyd algorithm and a gradient descent, similar to the approach in [1]. The consideration of a general anisotropic sensor model results in an anisotropic Voronoi tesselation which is difficult to analyze. In [13], the distributed optimal control law for the coverage problem is derived assuming a fixed and equal sensor orientation and we assume a specific class of anisotropic sensors with elliptic sensing performance level sets instead of circles as for the isotropic case.

In this paper, a more realistic model of anisotropic sensor is considered. As surveyed above, most of coverage control schemes are based on the gradient flow approach and this paper also takes this approach. To the best of our knowledge, only an exception is [8], where receding horizon control strategy is adopted. In addition, instead of using the Voronoi approach where each sensor is assumed to have its own sensing region, we apply the joint detection probability approach motivated by [4]. The advantage of this approach is that it does away with the hard partitions of the Lloyd algorithm and can be extended to the three dimensional case in a straightforward manner as will be shown later. Moreover, the sensor is assumed to have a limited range which is defined by a probabilistic model. This model depends on the distance and orientation from the sensor to the target in the region of interest. A deployment algorithm is applied to each mobile sensor in order to maximize the joint detection probabilities of the events in the region of interest. In [14], the problem of coverage control with limited anisotropic sensors is considered, which is an extension of [2]. However, the sensing performance of the sensor depends only on the distance from the sensor to the target to be sensed, which is different with the definition of the anisotropic sensor in this paper.

This paper is organized as follows: The problem formulation for the limited-range anisotropic sensor model is presented in section 2. The distributed coverage control laws are derived in section 3. The extension of the approach to the three dimensional case is discussed in section 4. We verify the effectiveness of the control strategy through numerical simulations in section 5. Finally we present concluding remarks and address some future works in section 6.

II. Problem Formulation

A. Region of Interest and Sensor Model

Let Q be a polyhedron in $\mathscr{R}^2$ including its interior. $\phi(q): Q \to \mathscr{R}_+$ is a density function which represents the probability that some event takes place in Q. Regions with a large value of ϕ are regions of higher chances of finding a target. $\phi(q)$ satisfies $\phi(q) \geq 0$ for all $q \in Q$ and $\int_Q \phi(q) < \infty$. We consider a robotic network where each robot is equipped with limited-range omnidirectional communication and anisotropic sensing capabilities. In this paper, we interchangeably refer to the elements of the network as sensors, agents, vehicles, or robots. Furthermore it is assumed that all the sensors are identical i.e. all sensors have the identical capabilities for sensing, communication, computation, and mobility. Let $\mathrm{s} = (s_1, ..., s_N)$ be the location of the N identical robots/sensors moving in the region Q. Let $\theta = (\theta_1, ..., \theta_N)$ be the orientation/attitude of N sensors. The kinematic model of the agents are given by

$$s_i(k+1) = s_i(k) + u_i(k), \tag{1}$$
$$\theta_i(k+1) = \theta_i(k) + v_i(k). \tag{2}$$

where k is the iteration index, $u_i(k)$ and $v_i(k)$ are the control input for the position and the orientation of sensor i respectively. When an event occurs at point q, it emits a signal and this signal is observed by sensor i at location s_i. The received signal strength (performance of the sensor) is assumed to be decayed not only with the distance from the sensor but also with the orientation of the target to the sensor. We define this type of sensor as an anisotropic sensor model. The degradation of the sensor performance is represented by a monotonically decreasing differentiable function $p_i(q)$, which expresses the probability that sensor i detects the event occuring at q or indicates how poor the sensing performance is. Lower value of $p_i(q)$ means that point q is sensed poorly by sensor i and vice versa. Formally, the anisotropic sensor model (see Fig. 1) is given as follows:

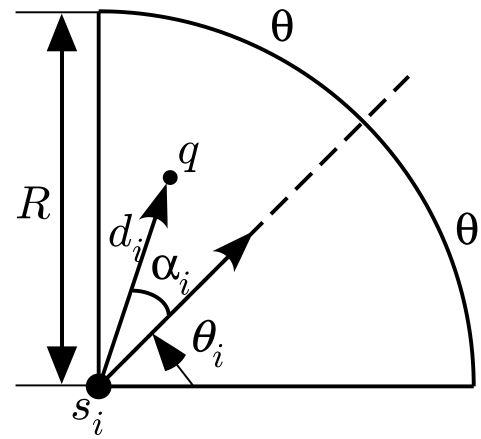

Fig. 1. Limited-range anisotropic sensor model. The sensing performance of sensor i depends on the distance d_i and the orientation α_i from sensor i to the target q

Sensor Model 1: Each sensor has a limited sensory domain Q_i with the maximum sensing range R and the maximum sensing direction Θ. The sensing ability of each sensor declines along the radial distance and the radial angle from the sensor to the point to be sensed. Mathematically, the sensory domain of each sensor is given by

$$Q_i = \{q \in Q : d_i \leq R \bigwedge |\alpha_i| \leq \Theta\}, \tag{3}$$

where

$$\begin{aligned} d_i &= \|q - s_i\|, \\ \alpha_i &= cos^{-1}\left(\frac{(q-s_i)(\cos\theta_i, \sin\theta_i)}{\|q-s_i\|}\right), \\ \Theta &\in \left(0, \frac{\pi}{2}\right]. \end{aligned}$$

Moreover we make the following assumption on the sensing performance of the above sensor model.

Assumption 1:

$$p_i(q) = 0, \frac{\partial p_i(q)}{\partial d_i(q)} = 0, \frac{\partial p_i(q)}{\partial \alpha_i(q)} = 0 \text{ if } q \notin Q_i. \tag{4}$$

The assumption tells us that the sensor i can only sense the point inside its region of sensing Q_i. One example of the sensor model that later will be used in the simulation of this paper is

$$p_i(q) = \begin{cases} \frac{(d_i-R)^2(\alpha_i-\Theta)^2}{R^2\Theta^2} & \text{if } q \in Q_i \\ 0 & \text{otherwise} \end{cases}$$

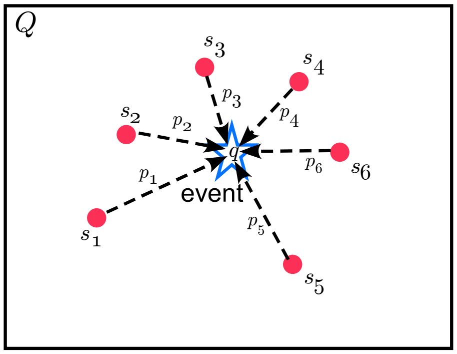

Fig. 2. Joint detection probability. Each point in Q is sensed by all sensors

This sensor model is similar to the one which combines camera and ultrasonic sensor used in YAMABICO robot (see [15] for the details).

B. Optimal Coverage Formulation

The optimal coverage is achieved by deploying mobile sensors into the region of interest so that the probability that events are detected is maximized. In this paper, sensors are assumed to make observations independently. Given the region of interest and sensor model, when an event takes place at q and it is observed by the sensors, the joint probability that this event is detected by the sensors can be written as (see Fig. 2)

$$P(q,\mathrm{s},\theta) = 1 - \prod_{i=1}^{N}[1-p_i(q)]. \tag{5}$$

Then the optimal coverage problem can be formulated as an optimization problem of maximizing the objective function defined by

$$F(\mathrm{s},\theta) = \int_Q \phi(q)P(q,\mathrm{s},\theta)dq, \tag{6}$$

which is the expected event detection probability by the sensors over Q. In the optimization problem, the controllable variables are the locations and the orientations of mobile sensors contained in s and θ respectively. The goal of this paper is to design the control laws u_i and v_i such that the region of interest Q is optimally covered i.e. (6) is maximized.

III. Distributed Coverage Control

In this paper we apply the gradient flows approach [16] with the aggregate objective function (6) in order to design the control laws $u_i(k)$ and $v_i(k)$ for achieving optimal coverage i.e. to drive the agents such that (6) is maximized. For this purpose let us take the partial derivatives with respect to s_i and θ_i respetively which lead to

$$\frac{\partial F}{\partial s_i} = \int_Q \phi(q)\frac{\partial P(q,\mathrm{s},\theta)}{\partial s_i}dq, \tag{7}$$

$$\frac{\partial F}{\partial \theta_i} = \int_Q \phi(q)\frac{\partial P(q,\mathrm{s},\theta)}{\partial \theta_i}dq. \tag{8}$$

In the view of (5), without applying assumption 1, the partial derivative (7) and (8) can be written as

$$\begin{aligned}\frac{\partial F}{\partial s_i} &= \int_Q \phi(q) \prod_{k=1,k\neq i}^{N}[1-p_k(q)] \\ &\quad \left(\frac{\partial p_i(q)}{\partial d_i}\frac{\partial d_i}{\partial s_i} + \frac{\partial p_i(q)}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial s_i}\right)dq, \end{aligned} \tag{9}$$

$$\begin{aligned}\frac{\partial F}{\partial \theta_i} &= \int_Q \phi(q) \prod_{k=1,k\neq i}^{N}[1-p_k(q)] \\ &\quad \left(\frac{\partial p_i(q)}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial \theta_i}\right)dq. \end{aligned} \tag{10}$$

The gradient above provides direction information for a mobile sensor to decide its next movement. Thus the control laws $u_i(k)$ and $v_i(k)$ are given by

$$u_i(k) = \beta_k \frac{\partial F}{\partial s_i(k)}, \tag{11}$$

$$v_i(k) = \gamma_k \frac{\partial F}{\partial \theta_i(k)}. \tag{12}$$

Here the step size β_k, γ_k are selected in order to guarantee the convergence of the motion trajectories by using the standard method (see e.g. [17]).

Note that the control laws (11) and (12) can not be computed locally by the robots since they require global information of the environment such as the value of $\phi(q)$ over the whole region of interest Q and the position of all other robots as observed from (9), (10). By applying assumption 1, (9) and (10) can be written as

$$\begin{aligned}\frac{\partial F}{\partial s_i} &= \int_{Q_i} \phi(q) \prod_{k\in N_i}[1-p_k(q)] \\ &\quad \left(\frac{\partial p_i(q)}{\partial d_i}\frac{\partial d_i}{\partial s_i} + \frac{\partial p_i(q)}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial s_i}\right)dq, \end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\partial F}{\partial \theta_i} &= \int_{Q_i} \phi(q) \prod_{k\in N_i}[1-p_k(q)] \\ &\quad \left(\frac{\partial p_i(q)}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial \theta_i}\right)dq, \end{aligned} \tag{14}$$

where N_i denotes the neighbors of sensor i and is defined as follows:

Definition 1: The neighbor set of sensor i (N_i) is defined by:

$$N_i = \{r : \|s_i - s_r\| < 2R, r = 1,\ldots,N, r\neq i\}. \tag{15}$$

Thus we have the following theorem:

Theorem 3.1: Under the assumption 1, each agent can compute u_i and v_i locally by using the information $(s_j,\theta_j), j\in N_i$.

Proof: Sensors which do not satisfy the condition of N_i (i.e. $\|s_i - s_r\| \geq 2R$) will not contribute to the integral in (13) and (14) since the detection probability p_r for a point $q \in Q_i$ is equal to zero which is clear from assumption 1. From (13) and (14), it can be seen that by assuming limited-range of the sensors, each robot can compute the derivative by requiring only the local information i.e. each robot only

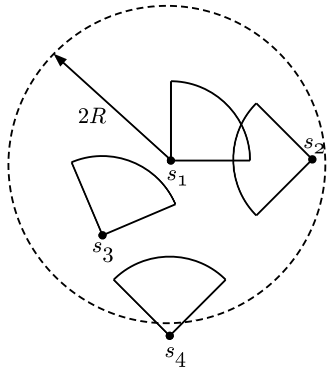

Fig. 3. Neighbor Set of sensor 1. Only the information of the sensors which are in the range $2R$ from sensor 1 are required to compute the control laws

needs to have the information of other robots and density function which are in the range of $2R$ from its position to compute the next step. ■

Note that not all agents r that satisfy (15) contribute to (13) and (14) (i.e. the condition (15) is redundant) since the condition on the limitation of the sensing direction Θ is neglected. For example, as shown in Fig. 3, the sensor located in s_3 does not contribute to the integral in (13) and (14) of sensor 1 even though the condition to be the neighbor of sensor 1 is satisfied.

Next, we mention some remarks regarding the distributed control law (13) and (14).

Remark 1: For sensors with different maximum sensing ranges and directions R_i and Θ_i respectively, the definition of the neighbor set of robot i (N_i) in (15) can be generalized into

$$N_i = \{r : \|s_i - s_r\| < 2R_{max}, r = 1, ..., N, r \neq i\},$$

where

$$R_{max} = \max_i R_i, i \in \{1, ..., N\}$$

is the maximum sensing range of all sensors.

Remark 2: The derivative in (13) and (14) can be fully written as follows:

$$\begin{aligned}\frac{\partial F}{\partial s_i} &= \int_{Q_i} \phi(q) \prod_{k \in N_i} [1 - p_k(q)] \\ &\quad \left(\frac{\partial p_i(q)}{\partial d_i} \frac{s_i - q}{\|s_i - q\|} + \frac{\partial p_i(q)}{\partial \alpha_i} \frac{a + b}{\sqrt{m^2 - l^2}} \right) dq, \\ \frac{\partial F}{\partial \theta_i} &= \int_{Q_i} \phi(q) \prod_{k \in N_i} [1 - p_k(q)] \\ &\quad \left(\frac{\partial p_i(q)}{\partial \alpha_i} \frac{-z}{\sqrt{m^2 - l^2}} \right) dq,\end{aligned}$$

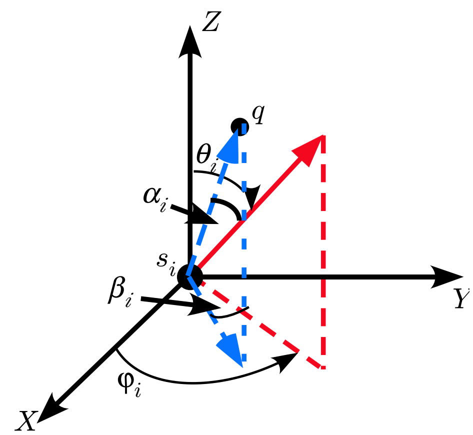

Fig. 4. Limited-range anisotropic sensor model in three dimensional space

where

$$\begin{aligned} a &= (\cos\theta_i, \sin\theta_i), \\ b &= \frac{(q - s_i)(\cos\theta_i, \sin\theta_i)(s_i - q)}{y^2}, \\ l &= (q - s_i)(\cos\theta_i, \sin\theta_i), \\ m &= \|q - s_i\|, \\ z &= (q - s_i)(-\sin\theta_i, \cos\theta_i). \end{aligned}$$

Remark 3: In order to make it computable on-line, we discretize it and transform the global coordinate into the local coordinate system of each sensor in the same way as [4].

Remark 4: One remaining issue for the distributed control law is the dependency on the global density function in (13) and (14). To deal with this problem, the same argument can also be applied as in [4]. At the beginning of the deployment, all sensors have the same local map i.e. the same copy of the estimated event density function. During the deployment, each sensor updates its local map based on the collected data and the information received from its neighbors. Thus the control law can be computed by only using the local information of the sensors.

IV. Three Dimensional Case

The limited-range anisotropic sensor model in this paper can be generalized into three dimensional case in a straightforward manner by working with spherical coordinates as shown in Fig. 4. Here the orientation of the sensor is defined by the angles θ_i, φ_i and each of them has some maximum value of Θ, Ψ respectively similar to the two dimensional model. One example of the three dimensional conic sensor model is given by

$$p_i(q) = \begin{cases} \frac{(d_i - R)^2(\alpha_i - \Theta)^2(\beta_i - \Psi)^2}{R^2\Theta^2\Psi^2} & \text{if } q \in \bar{Q}_i \\ 0 & \text{otherwise,} \end{cases}$$

where

$$\bar{Q}_i = \{q \in Q : d_i \leq R \bigwedge |\alpha_i| \leq \Theta \ \bigwedge |\beta_i| \leq \psi\}$$

is the sensory domain. The objective function is defined by

$$F(s,\theta,\varphi) = \int_Q \phi(q)P(q,s,\theta,\varphi)dq. \quad (16)$$

The kinematic model of the agents are given by

$$\begin{aligned} s_i(k+1) &= s_i(k)+u_i(k), & (17)\\ \theta_i(k+1) &= \theta_i(k)+v_i(k), & (18)\\ \varphi_i(k+1) &= \varphi_i(k)+w_i(k), & (19)\end{aligned}$$

and the neighbor of the sensor is defined in a similar manner as in (15). Moreover, assumption 1 is extended as follow

Assumption 2:

$$p_i(q)=0, \frac{\partial p_i(q)}{\partial d_i(q)}=0, \frac{\partial p_i(q)}{\partial \alpha_i(q)}=0, \frac{\partial p_i(q)}{\partial \beta_i(q)}=0 \text{ if } q\notin \bar{Q}_i.$$

The control laws can be computed as follows:

$$u_i(k) = \beta_k \frac{\partial F}{\partial s_i(k)}, \quad (20)$$

$$v_i(k) = \gamma_k \frac{\partial F}{\partial \theta_i(k)}, \quad (21)$$

$$w_i(k) = \eta_k \frac{\partial F}{\partial \varphi_i(k)}, \quad (22)$$

where the step size $\beta_k, \gamma_k, \eta_k$ are selected in order to guarantee the convergence of the motion trajectories. By applying assumption 2, the following corollary holds.

Corollary 1: For all agent i, the present control laws (20), (21) and (22) can be computed by using their local information $(s_j,\theta_j,\varphi_j), j\in N_i$ if $D>2R$.

The derivatives in (20), (21) and (22) can be computed as follows.

$$\begin{aligned}\frac{\partial F}{\partial s_i} &= \int_{Q_i}\phi(q)\prod_{k\in N_i}[1-p_k(q)]\\ &\quad \left(\frac{\partial p_i(q)}{\partial d_i}\frac{\partial d_i}{\partial s_i}+\frac{\partial p_i(q)}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial s_i}+\frac{\partial p_i(q)}{\partial \beta_i}\frac{\partial \beta_i}{\partial s_i}\right)dq,\\ \frac{\partial F}{\partial \theta_i} &= \int_{Q_i}\phi(q)\prod_{k\in N_i}[1-p_k(q)]\\ &\quad \left(\frac{\partial p_i(q)}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial \theta_i}\right)dq,\\ \frac{\partial F}{\partial \varphi_i} &= \int_{Q_i}\phi(q)\prod_{k\in N_i}[1-p_k(q)]\\ &\quad \left(\frac{\partial p_i(q)}{\partial \beta_i}\frac{\partial \beta_i}{\partial \varphi_i}\right)dq.\end{aligned}$$

V. SIMULATION

Here a simulation is presented to verify our algorithm. Assume that there are four identical mobile sensors that will cover an area Q of 40 x 40 (meter). The initial position and orientation of each agent are shown in Fig. 5. The density function $\phi(q)$ is given by :

$$\phi(q) = 3-0.1\|q-x_0\|,$$

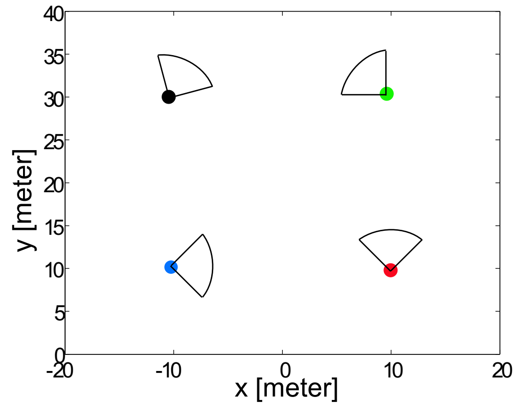

Fig. 5. Region of interest Q and the initial configuration of four mobile sensors

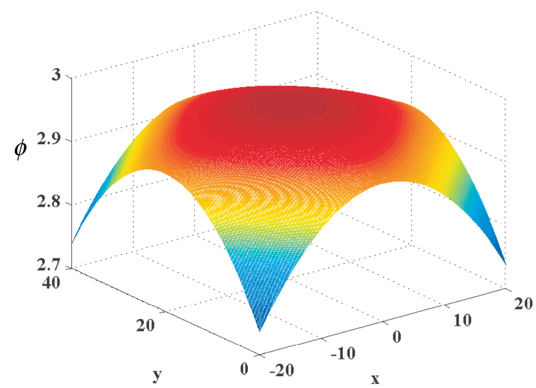

Fig. 6. Density function of the region of interest Q. The point in the middle of Q has the highest value.

where $x_0 = [0,20]$ (see Fig. 6). Sensing model of each agent is defined by :

$$p_i(q) = \begin{cases} \frac{(d_i-R)^2(\alpha_i-\Theta)^2}{R^2\Theta^2} & \text{if } d_i\le R \bigwedge |\alpha_i|\le\Theta \\ 0 & \text{otherwise}\end{cases}$$

where the maximum sensing range $R=5$ (meter) and the maximum sensing direction $\Theta=\frac{\pi}{4}$. The results of applying the control laws (11), (12) are shown in Fig. 7 and Fig. 8 which depict the trajectories and the evolution of the orientations of the mobile sensors respectively. The mobile sensors move until they reach the optimal configuration i.e. the objective function is maximized as shown in Fig. 9. It can be seen that each agent tried to sense the area with the highest density function.

VI. CONCLUSION AND FUTURE WORKS

In this paper the coverage control with anisotropic sensor model is presented. The anisotropic sensors are assumed to have limited-range sensing and each agent is equipped with omni-directional communication capability. Moreover, the sensor is defined by a probabilistic model. Distributed control algorithms are developed using gradient-based approach which maximize the joint detection probabilities of

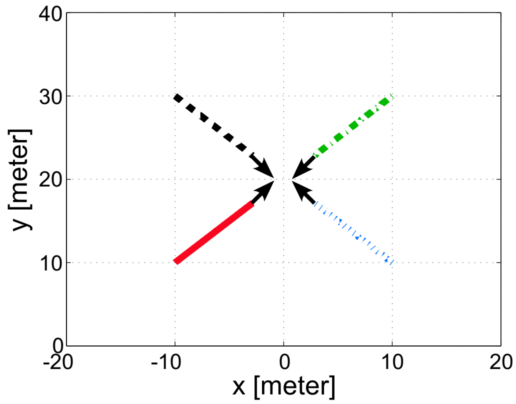

Fig. 7. Trajectories of the mobile sensors. The mobile sensors are moving to the center of Q

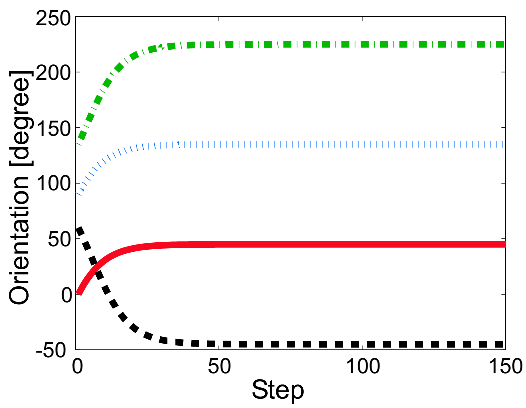

Fig. 8. Evolution of the orientations of the mobile sensors. The mobile sensors are rotating to the center of Q

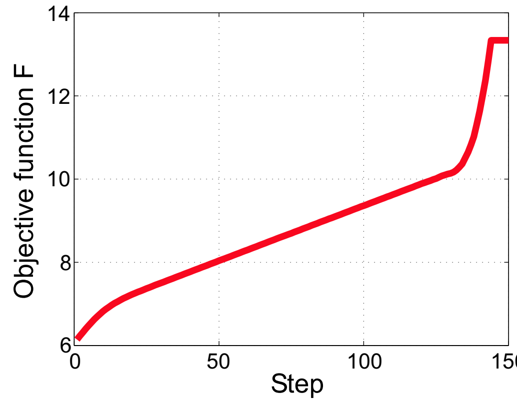

Fig. 9. Objective function. The agents move such that the objective function is maximized

the events. The efficiency of the proposed algorithm is confirmed by simulation. In the future, we will incorporate the energy consumption and communication cost of the sensors into our coverage control problem.

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