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# Multi-objective Flight Controller Design for an Airplane with Multiple Operating Points

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*Abstract*—**A multi-objective flight controller design method for an airplane with multiple operating points (MOP) is proposed via Lyapunov theory, which takes account of handling quality requirement and robust stability simultaneously. A handling quality criterion for all flights that correspond to multiple operating points is derived by linear matrix inequality (LMI) approach. Based on parameter-dependent Lyapunov functions combined with a descriptor system approach, a robust stability condition for the flight control system is obtained. At last, a flight controller is designed by solving a set of LMIs and the adjusting range of the parameter that influences the sensitivity of output responses and feasibility of LMIs is obtained by introducing convex optimization algorithms. Simulation results illustrate the effectiveness of the proposed method.**

## I. INTRODUCTION

ONE of the necessary performances of a flight control system is handling the operating quality, which means the easiness of carrying out maneuvers. Hence, there have been several criteria established for handling the operating quality that quantify the desirable aircraft performance from the pilot's point of view.

In [1], a handling quality criterion is developed for a high AOA maneuvering. The $C^*$ criterion has been applied to design flight control systems of airplanes for civil aviation [2]. The specification ADS-33C, which is dependent on the task to be performed, essentially quantifies acceptable parameters defining aircraft dynamics. Besides the above-mentioned criteria, control anticipation parameter (CAP), which characterizes the easiness of incorporation into the design, is also a handling quality metric that is used widely. Consequently, various control techniques have been exploited to design a flight controller that satisfies existing handling quality specifications.

In [3], a model reference adaptive control scheme for an aircraft is developed by using modified time response parameters as a handling quality criterion. In [4], a flight controller for a helicopter is designed via the implicit model following approach, where the $H_\infty$ norm between the actual system outputs and outputs of the desired handling quality model is minimized. In [5], a design method of translational rate commanded controller is proposed for a helicopter in hover using sliding mode, where hyper-plane is designed by means of a multivariable linear quadratic optimization. In [6], with the ADS-33D as the design criteria, the problem of fuzzy switching control for UH-60A helicopter is investigated via $H_\infty$ control approach. In [7], a controller architecture, which combines an online adaptive neural network with model inversion control, is presented for a tilt rotor aircraft. This controller can provide a pilot with consistent handling quality during conversion from fixed wing flight to hover. In [8], combined with dynamic inversion, stochastic robust nonlinear control is proposed for an aircraft, which gives good handling qualities without the use of gain scheduling. In [9], based on the $H_\infty$control, a multivariable design method is proposed for a helicopter. In [10], by $H_\infty$ model matching approach, a single degree of freedom controller for a UAV is designed. In [11], an adaptive back-stepping flight control law design to achieve a satisfactory CAP for all flight conditions is presented.

Robust stability is another important performance property of flight control systems. Some results on robust control design for an aircraft dynamic system have been reported in [12, 13] recently. However, with the intention of ensuring both handling quality and robust stability, a multi-objective controller design for an airplane with multiple operating points has not been considered as yet. On the other hand, the LMI approach has significant advantages of simplifying the design procedure and achieving a multi-objective design. Thus, if both the handling quality requirements and the robust stability conditions are formulated in terms of LMIs, a multi-objective controller design is facilitated.

An LMI framework is proposed in this work for an airplane with multiple operating points aimed at designing a multi-objective flight controller that guarantees both handling quality requirement and robust stability. The main feature of the proposed method is that by a single controller the control objective is satisfied for all operating points instead of computing respective controllers for different operating points. This enables to avoid the controller switch from one operating point to another. A handling quality criterion for all flights across the envelope containing multiple operating points is formulated in the form of linear matrix inequality (LMI). In the sense of Lyapunov theory, a robust stability condition for the flight control system is derived by a

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descriptor system approach. Consequently, the existence of such controller depends on the solvability of a set of LMIs. Moreover, the adjusting range of a parameter needed in the design can be easily obtained using the convex optimization algorithm, which has an influence on the sensitivity of output responses and solving feasibility of LMIs above. Finally, the necessity and feasibility of the obtained controller are verified on the grounds of computer simulation results.

## II. The Airplane Model

It is well known from the literature, an airplane dynamic system typically has multiple flight conditions that correspond to the convex combination of given operating points. The values of system matrix parameters are constant for each operating point and vary from one operating point to another. The longitudinal dynamics in the flight envelope containing three operating points as apexes are described [14] by

$$\begin{aligned} \dot{x}(t) &= A(\lambda)x(t) + B(\lambda)u(t) \\ y(t) &= C(\lambda)x(t) \end{aligned}, \tag{1}$$

where $x(t) \in R^2$ is the state vector containing angle of attack (AOA) $\alpha$ and pitch rate $q$, $u(t) \in R^2$ is the control input vector containing elevator angle $\delta_e$ and power lever deflection $\delta_t$, and $y(t) \in R$ is pitch rate $q$. The system matrices $A(\lambda) \in R^{2\times 2}$, $B(\lambda) \in R^{2\times 2}$ and $C(\lambda) \in R^{1\times 2}$ belong to a polytopic uncertainties domain $S$ and can be written as a convex combination of the polytope vertex $(A_i, B_i, C_i)$, that is,

$$(A(\lambda), B(\lambda), C(\lambda)) = \sum_{i=1}^{3} \lambda_i (A_i, B_i, C_i) \in S, \tag{2}$$

where uncertain vector $\lambda = [\lambda_1, \lambda_2, \lambda_3]^{\mathrm{T}} \in R^3$ is a fixed but unknown parameters satisfying

$$\lambda \in \Xi \triangleq \{\lambda \in R^3 : \sum_{i=1}^{3} \lambda_i = 1, \lambda_i \geq 0\}. \tag{3}$$

## III. The Multi-Objective Control Law

The design problem considered in this section is to find a two-block controller which consists of a feed-forward controller $u_K(t) = Kr$ and a static state feedback controller $u_F(t) = Fx(t)$ such that the closed-loop system satisfies

(i) good time response to pilot step stick input $r$, i.e. $\gamma_1 < \mathrm{CAP} < \gamma_2$ for given scalars $\gamma_1$ and $\gamma_2$.

(ii) robust stability in the presence of parameters in (3).

***Remark 1***. In the closed-loop system, the trajectory evolves in two phases. When the maneuver is initiated (i.e. $r \neq 0$ is imposed on the system), requirement (i) is satisfied by $u_K(t)$ and $u_F(t)$. This is referred to as the *first phase*. Once the maneuver is achieved (i.e. $r = 0$), requirement (ii) is secured by $u_F(t)$ only, which is called the *second phase*. It is worth mentioning that $F$ needs not to be changed from the first phase to the second since it is obtained by our multi-objective design method presented below.

Consider airplane system (1) with $u(t) = u_K(t) + u_F(t)$ and let $C_1 = C_2 = C_3 = C$. Then the closed-loop system is given by

$$\begin{aligned} \dot{x}(t) &= A_{cl}(\lambda)x(t) + B(\lambda)Kr \\ y(t) &= Cx(t) \end{aligned}, \tag{4}$$

where $A_{cl}(\lambda) = A(\lambda) + B(\lambda)F$, $K \in R^{2\times 1}$, $F \in R^{2\times 2}$, $r \in R$ is step stick input from a pilot. Using the closed-loop system, design requirements (i) and (ii) can be formulated in terms of LMIs.

### A. Handling Quality Criterion

CAP is defined upon the assumption that a pilot initiates a maneuver predicting the steady state normal acceleration with initial pitch acceleration, that is,

$$\mathrm{CAP} = \frac{\ddot{\theta}(\delta_{ec}|_{t=0})}{\Delta n(\delta_{ec}|_{t=\infty})}, \tag{5}$$

where $\delta_{ec}$ denotes elevator deflection command, $\ddot{\theta}(\delta_{ec}|_{t=0})$ represents initial pitch acceleration and $\Delta n(\delta_{ec}|_{t=\infty})$ is steady state normal acceleration change.

***Remark 2***. In order to illustrate the necessity of the constraint on CAP, (5) is rewritten as

$$\mathrm{CAP} = \frac{\ddot{\theta}(\delta_{ec}|_{t=0})}{F_z} \cdot \frac{F_z}{\Delta n(\delta_{ec}|_{t=\infty})} = M_{\mathrm{FS}} \cdot F_z^{n_y}, \tag{6}$$

where $F_z$ denotes stick force, stick sensitivity $M_{\mathrm{FS}}$ represents initial pitch acceleration generated by unit stick force and unit acceleration stick force $F_z^{n_y}$ is stick force that generates unit steady state acceleration. It follows from (6) that CAP has a close relation with $M_{\mathrm{FS}}$ and $F_z^{n_y}$ which reflect handling sense of a pilot directly. Hence, the value of CAP influences handling quality of an airplane. If the value of CAP is too small, $F_z^{n_y}$ has to be increased to keep proper $M_{\mathrm{FS}}$ or $M_{\mathrm{FS}}$ is enlarged to maintain satisfactory $F_z^{n_y}$. It is also difficult for a big CAP to get satisfactory tradeoff between $F_z^{n_y}$ and $M_{\mathrm{FS}}$. In a word, the value of CAP must be restricted to an appropriate range in order to obtain satisfactory handling quality.

The following theorem rewrites the CAP equivalently using system matrices of the closed-loop system (4).

***Theorem 1.*** For the closed-loop system (4) and pilot step stick input $r$, CAP becomes

$$\mathrm{CAP} = -\frac{CB(\lambda)K}{CA_{cl}^{-1}(\lambda)B(\lambda)K} \cdot \frac{g}{U_0} \tag{7}$$

under the condition of zero initial states, where $U_0$ and $g$ denotes equilibrium velocity along the $X$ axis and acceleration of gravity, respectively.

***Proof:*** Under the condition of zero initial states, the transfer matrix from $r$ to $\alpha$ is $G_\alpha$ and that from $r$ to $q$ is $G_q$.

$$\begin{aligned} G_\alpha &= C_\alpha (sI - A_{cl}(\lambda))^{-1} B(\lambda)K \\ G_q &= C\,(sI - A_{cl}(\lambda))^{-1} B(\lambda)K \end{aligned}, \tag{8}$$

where $C_\alpha$ is a matrix extracting $\alpha$ from $x$. According to (8), the step response of (4) is given as follows

$$\dot{\alpha}(\infty) = \lim_{s\to 0}(s \times s \times G_\alpha / s) = 0 \tag{9}$$

$$q(\infty)=\lim_{s\to 0}(s\times G_q/s)=-CA_{cl}^{-1}(\lambda)B(\lambda)K \quad (10)$$

$$\begin{aligned}\dot q(0) &= \lim_{s\to\infty}(s\times s\times G_q/s)\\ &= \lim_{s\to\infty}[Cs(sI-A_{cl}(\lambda))^{-1}B(\lambda)K]\\ &= \lim_{s\to\infty}[Cs\frac{\text{adj}(sI-A_{cl}(\lambda))}{\det(sI-A_{cl}(\lambda))}B(\lambda)K].\end{aligned}$$

Since, $\det(sI-A_{cl}(\lambda))$ is a 2-th order monic polynomial of $s$ and $\text{adj}(sI-A_{cl}(\lambda))$ is a transfer matrix whose diagonal elements are 1-th order monic polynomials and other elements are lower than 1-th order polynomial,

$$\dot q(0)=CB(\lambda)K \quad (11)$$

is derived. From $\Delta n=-(\dot\alpha(\infty)-q(\infty))g/U_0$, (5) and (9)~(11), Eq. (7) is satisfied. This completes the proof.

***Remark 3***. It follows from the above-mentioned dimension of $A(\lambda)$, $B(\lambda)$, $C$, $F$ and $K$ that both numerator $CB(\lambda)K$ and denominator $CA_{cl}^{-1}(\lambda)B(\lambda)K$ in (7) are scalars, which indicates that (7) in Theorem 1 is valid.

Using this equivalent expression (7) of CAP, the next theorem formulates the CAP requirement as solvable LMIs.

***Theorem 2:*** For given scalars $0<\gamma_1<\gamma_2$ and $\varepsilon>0$, if there exist symmetric matrix $Q$, $W$ and matrices $K$ and $R$ such that

$$\begin{bmatrix}\rho & C(\delta A_i+I)W+\delta CB_iR\\ * & Q\end{bmatrix}>0, \quad (12)$$

$$\begin{bmatrix}-W & B_iK\\ * & -I\end{bmatrix}<0, \quad (13)$$

$$A_iW+WA_i^{\mathrm T}+B_iR+(B_iR)^{\mathrm T}+W+Q<0, \quad (14)$$

$$W>0, \quad (15)$$

$$\varepsilon^2+CB_iK+(CB_iK)^{\mathrm T}+I<0,\ i=1,2,3, \quad (16)$$

where $\bar\gamma_1=\gamma_1U_0/g$, $\bar\gamma_2=\gamma_2U_0/g$, $\delta=(\bar\gamma_1+\bar\gamma_2)/(2\bar\gamma_1\bar\gamma_2)$, $\rho=\varepsilon^2(\bar\gamma_2-\bar\gamma_1)^2/(4\bar\gamma_1^2\bar\gamma_2^2)$, then CAP and control law satisfy

$$\gamma_1<\text{CAP}<\gamma_2. \quad (17)$$

$$u(t)=Kr(t)+RW^{-1}x(t). \quad (18)$$

***Proof:*** According to (15), we have

$$\begin{aligned}&WA_{cl}^{\mathrm T}(\lambda)W^{-1}A_{cl}(\lambda)W+A_{cl}(\lambda)W+WA_{cl}^{\mathrm T}(\lambda)+W=\\ &(WA_{cl}^{\mathrm T}(\lambda)+W)W^{-1}(A_{cl}(\lambda)W+W)>0.\end{aligned} \quad (19)$$

From (12), (14), $F=RW^{-1}$ and $A_{cli}=A_i+B_iF$, it follows that

$$\begin{bmatrix}\rho & C(\delta A_{cli}+I)W\\ * & Q\end{bmatrix}>0, \quad (20)$$

$$A_{cli}W+WA_{cli}^{\mathrm T}+W+Q<0. \quad (21)$$

By (20), (21), (13) and (16), we get

$$\begin{bmatrix}\rho & C(\delta A_{cl}(\lambda)+I)W\\ * & Q\end{bmatrix}=\sum_{i=1}^{3}\lambda_i\begin{bmatrix}\rho & C(\delta A_{cli}+I)W\\ * & Q\end{bmatrix}>0 \quad (22)$$

$$\begin{aligned}&A_{cl}(\lambda)W+WA_{cl}^{\mathrm T}(\lambda)+W+Q=\\ &\sum_{i=1}^{3}\lambda_i(A_{cli}W+WA_{cli}^{\mathrm T}+W+Q)<0,\end{aligned} \quad (23)$$

$$\begin{bmatrix}-W & B(\lambda)K\\ * & -I\end{bmatrix}=\sum_{i=1}^{3}\lambda_i\begin{bmatrix}-W & B_iK\\ * & -I\end{bmatrix}<0, \quad (24)$$

$$\begin{aligned}&\varepsilon^2+CB(\lambda)K+(CB(\lambda)K)^{\mathrm T}+I=\\ &\sum_{i=1}^{3}\lambda_i(\varepsilon^2+CB_iK+(CB_iK)^{\mathrm T}+I)<0.\end{aligned} \quad (25)$$

From (19) and (23), we have

$$WA_{cl}^{\mathrm T}(\lambda)W^{-1}A_{cl}(\lambda)W>-(A_{cl}(\lambda)W+WA_{cl}^{\mathrm T}(\lambda)+W)>Q. \quad (26)$$

From $-\rho<0$ and by Schur complement with (22), we get

$$Q>W^{\mathrm T}(\delta A_{cl}(\lambda)+I)^{\mathrm T}C^{\mathrm T}\rho^{-1}C(\delta A_{cl}(\lambda)+I)W$$

From this inequality and (26),

$$\begin{bmatrix}\rho & C(\delta A_{cl}(\lambda)+I)W\\ * & WA_{cl}^{\mathrm T}(\lambda)W^{-1}A_{cl}(\lambda)W\end{bmatrix}>0. \quad (27)$$

is derived by Schur complement. By left and right multiplying both sides of (27) by $\text{diag}\{I,A_{cl}^{-\mathrm T}(\lambda)W^{-1}\}$ and $\text{diag}\{I,W^{-1}A_{cl}^{-1}(\lambda)\}$, respectively, we obtain

$$\begin{bmatrix}\rho & C(\delta I+A_{cl}^{-1}(\lambda))\\ * & W^{-1}\end{bmatrix}>0. \quad (28)$$

By Schur complement, (24) and (28) are equivalent to

$$W>B(\lambda)K(B(\lambda)K)^{\mathrm T}, \quad (29)$$

$$\rho-C(\delta I+A_{cl}^{-1}(\lambda))W(\delta I+A_{cl}^{-\mathrm T}(\lambda))C^{\mathrm T}>0. \quad (30)$$

From (29) and (30), we obtain

$$|C(\delta I+A_{cl}^{-1}(\lambda))B(\lambda)K|<\sqrt{\rho}. \quad (31)$$

According to (25) and $(CB(\lambda)K+I)(CB(\lambda)K+I)^{\mathrm T}>0$,

$$|CB(\lambda)K|>\varepsilon \quad (32)$$

is satisfied. From (31) and (32), we have the following inequality

$$\left|\frac{C(\delta I+A_{cl}^{-1}(\lambda))B(\lambda)K}{CB(\lambda)K}\right|<\frac{\sqrt{\rho}}{\varepsilon}. \quad (33)$$

According to $\rho=\varepsilon^2(\bar\gamma_2-\bar\gamma_1)^2/(4\bar\gamma_1^2\bar\gamma_2^2)$, (33) becomes

$$\left|\delta+\frac{CA_{cl}^{-1}(\lambda)B(\lambda)K}{CB(\lambda)K}\right|<\frac{\bar\gamma_2-\bar\gamma_1}{2\bar\gamma_1\bar\gamma_2}. \quad (34)$$

From $\delta=(\bar\gamma_1+\bar\gamma_2)/(2\bar\gamma_1\bar\gamma_2)$, (34) is equivalent to

$$-\frac{1}{\bar\gamma_1}<\frac{CA_{cl}^{-1}(\lambda)B(\lambda)K}{CB(\lambda)K}<-\frac{1}{\bar\gamma_2}. \quad (35)$$

From $\bar\gamma_1=(\gamma_1U_0)/g>0$, $\bar\gamma_2=(\gamma_2U_0)/g>0$ and (35),

$$\frac{CA_{cl}^{-1}(\lambda)B(\lambda)K}{CB(\lambda)K}<0 \quad (36)$$

is derived. And (36) is equivalent to two cases below.

case I: $CA_{cl}^{-1}(\lambda)B(\lambda)K>0, CB(\lambda)K<0$,

case II: $CA_{cl}^{-1}(\lambda)B(\lambda)K<0, CB(\lambda)K>0$.

In both case I and case II, (35) is equivalent to

$$-\bar\gamma_2<\frac{CB(\lambda)K}{CA_{cl}^{-1}(\lambda)B(\lambda)K}<-\bar\gamma_1. \quad (37)$$

According to $\bar\gamma_1=\gamma_1\dfrac{U_0}{g}$, $\bar\gamma_2=\gamma_2\dfrac{U_0}{g}$ and (37), we have

$$\gamma_1<-\frac{CB(\lambda)K}{CA_{cl}^{-1}(\lambda)B(\lambda)K}\cdot\frac{g}{U_0}<\gamma_2. \quad (38)$$

From (7) and (38), we know that (17) is satisfied.

***Remark 4***. In Theorem 2, according to (11) and (32), we have $|\dot q(0)|>\varepsilon$. So, if the free scalar $\varepsilon$ is increased, the ratio of initial pitch rate becomes larger, which improves the sensitivity of output response $q$ .

### B. Robust Control Synthesis

Since (ii) is secured only by $u_F(t)$, we get the closed-loop system with $u(t) = Fx(t)$ by a descriptor system approach [15]

$$\dot{x}(t) = \eta(t),$$
$$\eta(t) = (A(\lambda) + B(\lambda)F)x(t). \quad (39)$$

***Theorem 3.*** The system (39) is robustly stable if there exist a symmetric matrix $W$, a matrix $R$ and symmetric positive definite matrices $X_i (i = 1,2,3)$ such that

$$\begin{bmatrix} A_iW + WA_i^{\mathrm{T}} + B_iR + R^{\mathrm{T}}B_i^{\mathrm{T}} & WA_i^{\mathrm{T}} + R^{\mathrm{T}}B_i^{\mathrm{T}} + X_i - W \\ * & -2W \end{bmatrix} < 0, \quad (40)$$

The state-feedback gain is then given by $F = RW^{-1}$.

***Proof:*** Defining $P_2 = W^{-1}$, $P_{1i} = W^{-1}X_iW^{-1} > 0$, $R = FW$, multiplying (40) by $\mathrm{diag}\{W^{-1}, W^{-1}\}$ and $\mathrm{diag}\{W^{-1}, W^{-1}\}$ on the left and the right, respectively, we obtain

$$\begin{bmatrix} A_{cli}^{\mathrm{T}}P_2 + P_2A_{cli} & A_{cli}^{\mathrm{T}}P_2 + P_{1i} - P_2 \\ * & -2P_2 \end{bmatrix} < 0. \quad (41)$$

According to (41), we have

$$\begin{bmatrix} A_{cl}^{\mathrm{T}}(\lambda)P_2 + P_2A_{cl}(\lambda) & A_{cl}^{\mathrm{T}}(\lambda)P_2 + P_1(\lambda) - P_2 \\ * & -2P_2 \end{bmatrix} =$$

$$\sum_{i=1}^{3}\lambda_i \begin{bmatrix} A_{cli}^{\mathrm{T}}P_2 + P_2A_{cli} & A_{cli}^{\mathrm{T}}P_2 + P_{1i} - P_2 \\ * & -2P_2 \end{bmatrix} < 0. \quad (42)$$

Choose a parameter-dependent Lyapunov function

$$V(x,\lambda) = x^{\mathrm{T}}(t)P_1(\lambda)x(t), \quad (43)$$

where $P_1(\lambda) = \sum_{i=1}^{3}\lambda_iP_{1i} > 0$, $P_{1i} = P_{1i}^{\mathrm{T}} > 0$. Differentiating $V(x,\lambda)$ with respect to $t$ and using (39), we have

$$\begin{aligned} \dot{V}(x,\lambda) &= 2x^{\mathrm{T}}(t)P_1(\lambda)\dot{x}(t) \\ &= 2\begin{bmatrix} x^{\mathrm{T}}(t) & \eta^{\mathrm{T}}(t) \end{bmatrix} \begin{bmatrix} P_1(\lambda) & P_2 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ 0 \end{bmatrix} \\ &= 2\zeta^{\mathrm{T}}(t) \begin{bmatrix} P_1(\lambda) & P_2 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} \eta(t) \\ -\eta(t) + (A(\lambda) + B(\lambda)F)x(t) \end{bmatrix} \\ &= \zeta^{\mathrm{T}}(t) \begin{bmatrix} A_{cl}^{\mathrm{T}}(\lambda)P_2 + P_2A_{cl}(\lambda) & A_{cl}^{\mathrm{T}}(\lambda)P_2 + P_1(\lambda) - P_2 \\ * & -2P_2 \end{bmatrix} \zeta(t) \quad (44) \end{aligned}$$

where $\zeta(t) = \begin{bmatrix} x^{\mathrm{T}}(t) & \eta^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$. Using (42) and (44), we have

$$\dot{V}(x,\lambda) < 0, \forall \zeta(t) \neq 0, \quad (45)$$

which means that system (39) is robustly stable.

### C. Multi-objective Controller Design

The multi-objective controller satisfying (i) and (ii) can be obtained by solving LMIs (12)-(16) and (40). Hence, we can calculate $K$ and $F$ according to the design program below.

i) Scalars $\bar{\gamma}_1$ and $\bar{\gamma}_2$ are computed with $\gamma_1$ and $\gamma_2$.

ii) The scalar $\varepsilon$ is determined.

iii) $\delta$ and $\rho$ are figured from parameters obtained above.

iv) $K$, $F$ are gotten by solving LMIs (12)-(16) and (40).

***Remark 5.*** Since $F$ satisfies conditions of Theorem 2 and Theorem 3 simultaneously, it needs not to be changed from the first phase to the second.

***Remark 6.*** If the solution of LMIs is feasible, there exists a multi-objective flight controller for system (4). Moreover, the controller gain depends on the selection of $\varepsilon$ value. So in order to secure feasibility of LMIs and sensitivity of output responses, it is crucial to choose $\varepsilon$ in a reasonable range.

Next, by constructing and resolving convex optimization problems, the adjusting range of $\varepsilon$ can be obtained. By replacing $\varepsilon^2$ in Theorem 2 with a decision variable $\bar{\varepsilon} > 0$, the proofs of theorems below are the same as for Theorem 2.

***Theorem 4.*** Consider system (4) and given scalars $\gamma_1 < \gamma_2$. The following convex optimization problem:

$$\min_{R,Q,W,K,X_i} \bar{\varepsilon} \quad (46)$$

subject to (13)-(15), (40),

$$\begin{bmatrix} \bar{\varepsilon}(\bar{\gamma}_2 - \bar{\gamma}_1)^2/(4\bar{\gamma}_1^2\bar{\gamma}_2^2) & C(\delta A_i + I)W + \delta CB_iR \\ * & Q \end{bmatrix} > 0, \quad (47)$$

$$\bar{\varepsilon} + CB_iK + (CB_iK)^{\mathrm{T}} + I < 0, i = 1,2,3, \quad (48)$$

$$\bar{\varepsilon} > 0 \quad (49)$$

has a solution $\bar{\varepsilon}, R, Q, W, K, X_i$. Then, $\varepsilon$ satisfies $\varepsilon \geq \sqrt{\bar{\varepsilon}}$.

***Theorem 5.*** Consider system (4) and given scalars $\gamma_1 < \gamma_2$. The following convex optimization problem:

$$\min_{R,Q,W,K,X_i} -\bar{\varepsilon} \quad (50)$$

subject to (13)-(15), (40) and (47)-(49)

has a solution $\bar{\varepsilon}, R, Q, W, K, X_i$. Then, $\varepsilon$ satisfies $\varepsilon \leq \sqrt{|-\bar{\varepsilon}|}$.

## IV. Illustrative Example And Simulation Results

This section presents simulation results and result analysis from the multi-objective controller described in the previous section. The flying qualities analyses based on MIL-F-8785C are presented first. Then, the robustness to uncertain system vector is analyzed in the simulation. For various operating points, values of system parameters [14] are given in Table I.

TABLE I
SYSTEM MATRIX PARAMETERS

| Operating point | 1 | 2 | 3 |
|---|---|---|---|
| $a_{11}$ | -0.886 | -1.328 | -1.726 |
| $a_{12}$ | 0.987 | 0.983 | 1.278 |
| $a_{21}$ | -2.039 | -6.44 | -8.37 |
| $a_{22}$ | -0.878 | -1.493 | -1.941 |
| $b_{11}$ | -0.186 | -0.301 | -0.392 |
| $b_{12}$ | -0.036 | -0.059 | -0.076 |
| $b_{21}$ | -22.06 | -44.43 | -57.76 |
| $b_{22}$ | -3.803 | -7.815 | -10.16 |

First, we design a simple robust controller with no consideration of CAP. Consequently, by solving LMIs (40) the corresponding controller gain is

$$F = \begin{bmatrix} 2.4388 & 1.2667 \\ -14.7025 & -7.2310 \end{bmatrix}.$$

Second, we design a multi-objective controller taking account of CAP. According to the handling qualities requirements guideline of MIL-F-8785C [15], CAP satisfies requirement of category A and level 1, i.e. 0.28<CAP<3.6 . By applying Theorem 4 and Theorem 5, the adjusting range of $\varepsilon$ is $0.3441 \le \varepsilon \le 2.9063$ . Letting $\varepsilon = 0.4$ and according to the above-mentioned design procedure, the controller gain is

$$K = \begin{bmatrix} 0.8962 & -5.0187 \end{bmatrix}^{\mathrm{T}}, \; F = \begin{bmatrix} 2.4847 & 22.5169 \\ -14.9503 & -126.4054 \end{bmatrix}$$

### A. Handling Quality Analysis

In order to illustrate the necessity of constraints on CAP, on different operating points, CAP is computed for the open-loop system, the closed-loop system with a simple robust controller, and the closed-loop system with a multi-objective controller. The obtained results are shown in table II. For simplicity, three systems above are named system 1, system 2 and system 3, respectively. For system 1 and system 2, CAP on each operating point satisfies

$$\mathrm{CAP} = -(CB_{\delta_e} / C\bar{A}^{-1} B_{\delta_e}) \cdot (g/U_0), \qquad (51)$$

where $\bar{A}$ in system 1 is $A_i$, $\bar{A}$ in system 2 is $A_i + B_i F$ and $B_{\delta_e}$ is vector in $B_i$ according to elevator input.

TABLE II
CAP FOR EACH OPERATING POINT (OP)

| | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | system 1 | system 2 | system 3 |
|---|---|---|---|---|---|---|
| OP1 | 1 | 0 | 0 | 0.1031 | 0.0396 | 0.5431 |
| OP2 | 0 | 1 | 0 | 0.2078 | 0.0376 | 0.4513 |
| OP3 | 0 | 0 | 1 | 0.2702 | 0.0489 | 0.5867 |

From Table II, it follows that CAP values of system 1 and system 2 are not within level 1 boundary. However, CAP values of system 3 are just in the desirable range, which satisfies requirement of category A and level 1. So, it is easy to perceive the contrast between the results with and without the consideration of CAP.

### B. Robust Stability Analysis

For robust stability, response on different operating points is shown in Figure 1, where $r$ is 0.3rad input for 0.3s.

From Figure 1, it is obvious that attack of angle $\alpha$ and pitch rate $q$ converge to equilibrium by the designed controller on given operating points. This result has shown that the proposed multi-objective controller ensures robust stability in spite of fixed but unknown parameters in the model.

If the pilot feels that the airplane responses to stick input are sluggish, according to Remark 4 the responses can be improved by designing a new controller with a larger $\varepsilon$ . Figures 2-4 show responses for different $\varepsilon$ on given operating points. As seen, the sensitivity of output response $q$ improves with acceptable overshoot and the flight state $\alpha$ can be still stabilized in the meanwhile. However, as the $\varepsilon$ value increases further, sensitivity of $q$ enhances slightly with a remarkable overshoot. Hence, in the design for the controller, $\varepsilon$ should be determined to secure an appropriate trade-off between the sensitivity and overshoot of output response $q$ .

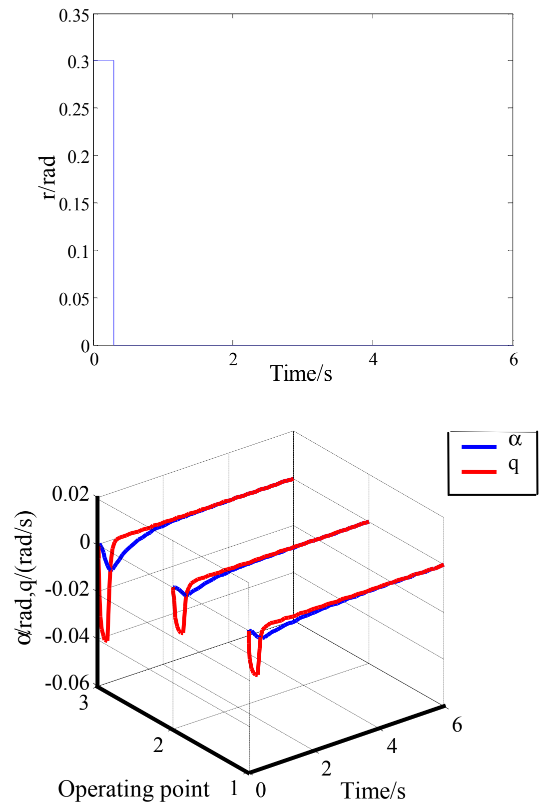


Fig. 1. Response of the airplane on different operating points.

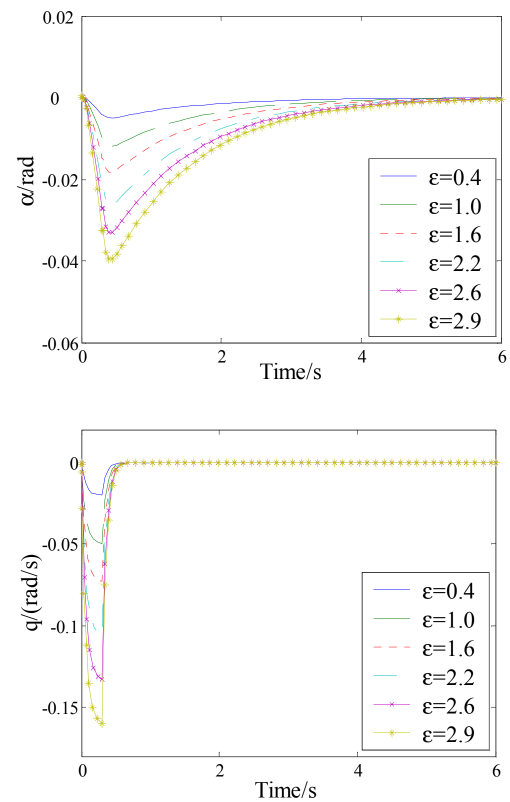


Fig. 2. Response for different $\varepsilon$ on operating point 1.

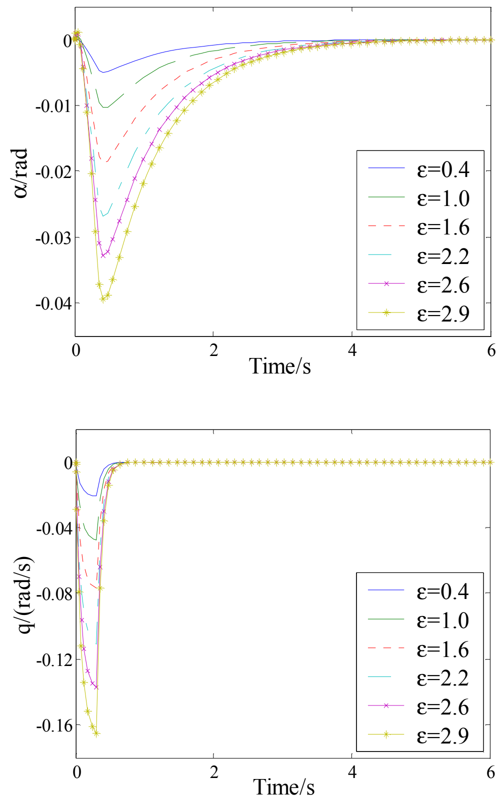


Fig. 3. Response for different $\varepsilon$ on operating point 2.

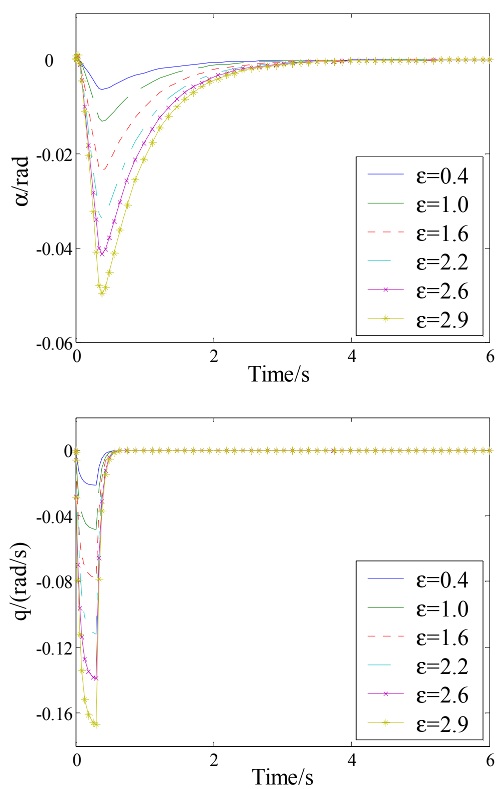


Fig. 4. Response for different $\varepsilon$ on operating point 3.

## V. CONCLUSION

A multi-objective control strategy using LMIs to design a robust flight controller for an airplane with multiple operating points has been presented. In order to guarantee handling quality requirement while considering the robust stability in the presence of fixed but unknown system parameters, a multi-objective controller is introduced, which can be solved via a convex problem with common solutions. The handling quality of airplane dynamics in the flight envelope containing multiple operating points is determined by CAP in LMI form. Furthermore, the design parameter range, in which the value guarantees the sensitivity of output responses and feasibility of LMIs, is given by solving a convex optimization problem. The simulation study demonstrates that the proposed controller design is able to satisfy the typical design requirements.

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