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Nonlinear Control of a Hypersonic Vehicle with Structural Flexibility

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Abstract— **The design of a nonlinear robust adaptive controller for a flexible air-breathing hypersonic vehicle model is considered in this work. Due to the complexity of a first-principle model of the vehicle dynamics, for design and stability analysis a simplified model is adopted, which nonetheless retains the dominant features of the higher fidelity model, including non-minimum phase behavior, flexibility effects and strong coupling between the engine and flight dynamics. A combination of nonlinear sequential loop-closure and adaptive dynamic inversion is adopted to design a dynamic state-feedback controller that provides stable tracking of velocity and altitude reference trajectories and imposes a desired setpoint for the angle of attack. The proposed methodology addresses the issue of robustness with respect to both parametric model uncertainty, which naturally arises in adopting reduced-complexity models for control design, and dynamic perturbations due to the flexible dynamics. Simulation results on the full nonlinear model are included to show the effectiveness of the controller.**

I. Introduction

Air-breathing hypersonic vehicles represent a promising and cost-effective technology for reliable access to space. Vehicles of this type can carry more cargo/payload than equivalent rocket-powered systems since they do not need to carry oxygen tanks. In turn, the scramjet engine allows these vehicle to operate at very high Mach number, by maximizing the efficiency of the combustion process.

Notwithstanding the recent success of NASAs X-43A experimental vehicle, the design of robust guidance and control systems for hypersonic vehicles is still an open problem, due to the peculiar characteristics of the vehicle dynamics. The slender geometries required for these aircraft cause significant flexible effects; strong coupling between propulsive and aerodynamic forces results from the underslung location of the scramjet engine; in addition, because of the variability of the vehicle characteristics with flight conditions (e.g., fuel consumption and thermal effects on the structure), significant uncertainties affect the vehicle model [1]–[3].

For linearized models, several results are available in the literature, which consider control solutions of various degree of complexity [4]–[9]. As far as nonlinear control is concerned, sliding-mode control [10] and robust inversion-based design [11], [12] has been proposed for simpler vehicle models than the one considered in this paper. In particular, the specific vehicle models employed in [10]–[12] do not include elevator-to-lift coupling, coupling between thrust and pitch moment, and structural dynamics, as found in the recent model of the vehicle longitudinal dynamics developed in [2]. For this specific model, a nonlinear controller achieving local regulation has been designed in [13] by applying approximate feedback linearization techniques.

Building upon our previous work [14], [15], we consider in this paper a complete robust adaptive nonlinear control design for the model given in [2], [16]. Following [13], a simplified model has been derived for control design which retains all the dominant features of the higher fidelity model, including non-minimum phase behavior, flexibility effects and coupling between the propulsion system and the airframe. A nonlinear controller is designed to achieve robust tracking of altitude and velocity references, and regulation of the angle of attack to a desired setpoint. Since the controller does not depend on the model parameters, the design satisfactorily addresses the issue of robustness with respect to parameter model uncertainties. In addition, the proof of stability now includes the flexible dynamics, thereby showing the robustness with respect to the considered class of dynamic uncertainty. This is in sharp contrast to design based on feedback linearization, where model uncertainties may lead to poor closed-loop performance or even instability, as shown in [13].

The paper is organized as follows: in Section II the first-principle model and the simplified control-oriented model are introduced and the control objective is stated. Section III elaborates on the internal dynamics of the system, while the tracking error dynamics is in section IV. Section V presents the controller design and the stability analysis of the overall system. Finally, simulation results are discussed in Section VI, and Section VII offers a brief summary of the results.

II. Vehicle model

Throughout this work, two different models are considered: a higher-fidelity Simulation Model (SM) is used for closed-loop simulation, while a Control-design Model (CDM), which approximates the SM with reduced complexity, is used for control design and for stability analysis.

A. Simulation Model

The SM adopted in this study, taken from [2] and [16], reads as follows

$$\begin{aligned} \dot{V} &= \frac{T\cos\alpha - D}{m} - g\sin\gamma \\ \dot{h} &= V\sin\gamma \end{aligned}$$

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$$\begin{aligned}\dot{\gamma} &= \frac{L+T\sin\alpha}{mV}-\frac{g}{V}\cos\gamma\\ \dot{\alpha} &= -\frac{L+T\sin\alpha}{mV}+Q+\frac{g}{V}\cos\gamma\\ \dot{Q} &= \frac{M}{I_{yy}}\\ \ddot{\eta}_i &= -2\zeta_i\omega_i\dot{\eta}_i-\omega_i^2\eta_i+N_i\,, \qquad i=1,2,3\,. \end{aligned} \quad (1)$$

This model comprises five rigid-body state variables $x = [V,h,\gamma,\alpha,Q]^T$, six flexible states $\eta = [\eta_1,\dot{\eta}_1,\eta_2,\dot{\eta}_2,\eta_3,\dot{\eta}_3]^T$ and three control inputs $u = [\Phi,\delta_e,\delta_c]^T$ which enter (1) through the thrust, T, the pitching moment about the body y-axis, M, lift, L, and drag, D. The output to be controlled is selected as $y=[V,h]^T$. The meaning of the state variables and the input vector is given in Table I.

B. Control-design Model

For control design and stability analysis, a simplified model has been derived from the SM following the approach used in [13]. The approximations of the forces and moments employed in the CDM are given as follows

$$\begin{aligned}&T\approx \bar{q}S\left[C_{T,\Phi}(\alpha)\Phi+C_T(\alpha)+C_T^{\eta}\eta\right], \quad L\approx\bar{q}SC_L(\alpha,\delta,\eta)\\ &D\approx\bar{q}SC_D(\alpha,\delta,\eta)\,, \qquad M\approx z_T T+\bar{q}\bar{c}SC_M(\alpha,\delta,\eta)\\ &N_i\approx\bar{q}S\left[N_i^{\alpha^2}\alpha^2+N_i^{\alpha}\alpha+N_i^{\delta_e}\delta_e+N_i^{\delta_c}\delta_c+N_i^0+N_i^{\eta}\eta\right] \end{aligned}\quad (2)$$

for $i=1,2,3$, where $\delta=[\delta_c,\delta_e]^T$, and

$$\begin{aligned}C_{T,\Phi}(\alpha) &= C_T^{\Phi\alpha^3}\alpha^3+C_T^{\Phi\alpha^2}\alpha^2+C_T^{\Phi\alpha}\alpha+C_T^{\Phi}\\ C_T(\alpha) &= C_T^3\alpha^3+C_T^2\alpha^2+C_T^1\alpha+C_T^0\\ C_M(\alpha,\delta,\eta) &= C_M^{\alpha^2}\alpha^2+C_M^{\alpha}\alpha+C_M^{\delta_e}\delta_e+C_M^{\delta_c}\delta_c+C_M^0+C_M^{\eta}\eta\\ C_L(\alpha,\delta,\eta) &= C_L^{\alpha}\alpha+C_L^{\delta_e}\delta_e+C_L^{\delta_c}\delta_c+C_L^0+C_L^{\eta}\eta\\ C_D(\alpha,\delta,\eta) &= C_D^{\alpha^2}\alpha^2+C_D^{\alpha}\alpha+C_D^{\delta_e}\delta_e+C_D^{\delta_c}\delta_c+C_D^0+C_D^{\eta}\eta\\ C_j^{\eta} &= [C_j^{\eta_1}\ 0\ C_j^{\eta_2}\ 0\ C_j^{\eta_3}\ 0] \qquad j=T,M,L,D\\ N_i^{\eta} &= [N_i^{\eta_1}\ 0\ N_i^{\eta_2}\ 0\ N_i^{\eta_3}\ 0] \qquad i=1,2,3\,. \end{aligned}\quad (3)$$

It is assumed that all the coefficients of the CDM and all vehicle parameters are subject to uncertainty. The vector of all uncertain parameters is denoted by $\mathscr{P}\in\mathbb{R}^p$ and it is assumed that $\mathscr{P}\in\Xi_{\mathscr{P}}$ where $\Xi_{\mathscr{P}}$ is a compact convex set that includes the nominal value $\mathscr{P}^0$ of $\mathscr{P}$ and represents the admissible range of variation of $\mathscr{P}$. The mass of the vehicle m, which varies due to fuel consumption on a slower time scale with respect to the references to be tracked, will be considered constant during each tracking maneuver as all the other model parameters.

C. Control Objectives and Problem Formulation

The goal pursued in this study is to design a dynamic controller using feedback from the rigid-body states only, to steer the output of system (1) from a given set of initial values of velocity and altitude to desired trim conditions V^* and h^* along smooth exogenous reference trajectories $y_{\text{ref}}(t)=[V_{\text{ref}}(t),h_{\text{ref}}(t)]^T$, robustly with respect to the considered model parameter uncertainty. In addition, the control system should provide boundedness of all internal trajectories and regulation of the angle of attack to a desired trim value, α^*. The velocity and altitude references and the set-point for the angle of attack are generated to satisfy the bounds shown in Table I, which determine the considered flight envelope of the vehicle. The initial conditions of the vehicle, $x_0=[V_0,h_0,\gamma_0,\alpha_0,Q_0]^T$, are also assumed to take arbitrary values within given compact subsets of the set in Table I. From the reference trajectories, desired commands $\gamma_{\text{cmd}}(t)$, $\alpha_{\text{cmd}}(t)$ and $Q_{\text{cmd}}(t)$ will be issued by the controller to regulate the corresponding intermediate state variables. Consequently, the tracking error $\tilde{x}=[\tilde{V},\tilde{h},\tilde{\gamma},\tilde{\alpha},\tilde{Q}]^T$ is defined as $\tilde{x}:=[V-V_{\text{ref}},h-h_{\text{ref}},\gamma-\gamma_{\text{cmd}},\alpha-\alpha_{\text{cmd}},Q-Q_{\text{cmd}}]^T$. The reference and command trajectories are defined in such a way that their asymptotic values yield the desired trim condition of the rigid-body state, $x^*=[V^*,h^*,0,\alpha^*,0]^T$.

TABLE I

ADMISSIBLE RANGES FOR STATE, INPUT, AND DYNAMIC PRESSURE

Variable		Min Value	Max Value
V	Vehicle Velocity	7500 ft/s	11000 ft/s
h	Vehicle Altitude	85000 ft	135000 ft
γ	Flight Path Angle	-3 deg	3 deg
α	Angle of Attack	-5 deg	10 deg
Q	Pitch Rate	-10 deg/s	10 deg/s
Φ	Fuel-to-air Ratio	0.05	1.5
δ_c	Canard Deflection	-20 deg	20 deg
δ_e	Elevator Deflection	-20 deg	20 deg
$\bar{q}$	Dynamic Pressure	182.5 psf	2200 psf
M	Mach Number	6	12

III. INTERNAL DYNAMICS

Because of the selected control objectives, there is no direct control authority on the flexible dynamics. Since the system has vector relative degree $r=(1,2,2)$ with respect to the output (V,h,α), the system possesses a nontrivial internal dynamics, which is related to the structural dynamics. To compute the internal dynamics, we begin by substituting the expression of the generalized forces N_i in (2) into the last equation of (1), obtaining

$$\dot{\eta} = A_\eta\,\eta+\bar{q}S[A_1\,\alpha+A_2\,\alpha^2+A_3]+\bar{q}SA_4\,\delta \quad (4)$$

where

$$A_\eta = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0\\ a_{\eta_1} & b_{\eta_1} & \bar{q}SN_1^{\eta_2} & 0 & \bar{q}SN_1^{\eta_3} & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ \bar{q}SN_2^{\eta_1} & 0 & a_{\eta_2} & b_{\eta_2} & \bar{q}SN_2^{\eta_3} & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ \bar{q}SN_3^{\eta_1} & 0 & \bar{q}SN_3^{\eta_2} & 0 & a_{\eta_3} & b_{\eta_3}\end{bmatrix}$$

$$\begin{aligned}a_{\eta_i} &= -\omega_i^2+\bar{q}SN_i^{\eta_i} \quad \text{for } i=1,2,3\\ b_{\eta_i} &= -2\zeta_i\omega_i \qquad\quad \text{for } i=1,2,3\\ A_1 &= \begin{bmatrix}0 & N_1^{\alpha} & 0 & N_2^{\alpha} & 0 & N_3^{\alpha}\end{bmatrix}^T\\ A_2 &= \begin{bmatrix}0 & N_1^{\alpha^2} & 0 & N_2^{\alpha^2} & 0 & N_3^{\alpha^2}\end{bmatrix}^T\\ A_3 &= \begin{bmatrix}0 & N_1^{0} & 0 & N_2^{0} & 0 & N_3^{0}\end{bmatrix}^T\end{aligned}$$

$$A_4 = \begin{bmatrix} 0 & N_1^{\delta_c} & 0 & N_2^{\delta_c} & 0 & N_3^{\delta_c} \\ 0 & N_1^{\delta_e} & 0 & N_2^{\delta_e} & 0 & N_3^{\delta_e} \end{bmatrix}^T .$$

Using similar arguments as in [15], it is possible to show that, within the ranges given in Table I,

$$\bar{q}SC_L^{\alpha}\,\alpha + T\sin\alpha := \bar{q}SK_{\alpha_1}(x,\Phi)\,\alpha \tag{5}$$

where $K_{\alpha_1}(x,\Phi)$ satisfies $K_{\alpha_1}^{min} < K_{\alpha_1}(x,\Phi) < K_{\alpha_1}^{max}$ for some positive constants $K_{\alpha_1}^{min}$ and $K_{\alpha_1}^{max}$. As a result, using (1) and (3), the (α,Q)-dynamics can be written in the form

$$\begin{bmatrix} \dot{\alpha} \\ \dot{Q} \end{bmatrix} = \bar{q}S\,G_1(V)\,\delta + G_2\,\eta + G_3$$

where

$$G_1(V) = \begin{bmatrix} -\dfrac{C_L^{\delta_c}}{mV} & -\dfrac{C_L^{\delta_e}}{mV} \\ \dfrac{\bar{c}C_M^{\delta_c}}{I_{yy}} & \dfrac{\bar{c}C_M^{\delta_c}}{I_{yy}} \end{bmatrix}$$

$$G_2 = \bar{q}S \begin{bmatrix} -\dfrac{C_L^{\eta_1}}{mV} & 0 & -\dfrac{C_L^{\eta_2}}{mV} & 0 & -\dfrac{C_L^{\eta_3}}{mV} & 0 \\ \dfrac{\bar{c}C_M^{\eta_1}}{I_{yy}} & 0 & \dfrac{\bar{c}C_M^{\eta_2}}{I_{yy}} & 0 & \dfrac{\bar{c}C_M^{\eta_3}}{I_{yy}} & 0 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} -\dfrac{\bar{q}S}{mV}\left[K_{\alpha_1}\alpha + C_L^0\right] + Q + \dfrac{g}{V}\cos\gamma \\ \dfrac{\bar{q}S\bar{c}}{I_{yy}}\left(C_M^{\alpha^2}\alpha^2 + C_M^{\alpha}\alpha + C_M^0\right) + \dfrac{z_T}{I_{yy}}T \end{bmatrix} .$$

The following change of coordinates will be applied to (4)

$$\chi = \eta - B_X\,G_1^{-1}(V_{\text{ref}}) \begin{bmatrix} \alpha \\ Q \end{bmatrix} + \frac{m}{\cos\alpha^*} D_X\,\tilde{V} \tag{6}$$

where

$$B_X = \begin{bmatrix} 0 & B_{X_{11}} & 0 & B_{X_{21}} & 0 & B_{X_{31}} \\ 0 & B_{X_{12}} & 0 & B_{X_{22}} & 0 & B_{X_{32}} \end{bmatrix}^T$$

$$D_X = \frac{C_A z_T}{\bar{c}} \begin{bmatrix} 0 & N_1^{L\delta} & 0 & N_2^{L\delta} & 0 & N_3^{L\delta} \end{bmatrix}^T$$

$$B_{X_{i1}} = \frac{N_i^{\delta_c}[1 - C_B C_D^{\delta_e} C_L^{\delta_c}] + N_i^{\delta_e}[C_B C_D^{\delta_c} C_L^{\delta_c}]}{[1 + C_B C_D^{\delta_c} C_L^{\delta_e}][1 - C_B C_D^{\delta_e} C_L^{\delta_c}] + C_B^2 C_D^{\delta_c} C_D^{\delta_e} C_L^{\delta_c} C_L^{\delta_e}}$$

$$B_{X_{i2}} = \frac{N_i^{\delta_e}[1 + C_B C_D^{\delta_c} C_L^{\delta_e}] - N_i^{\delta_c}[C_B C_D^{\delta_e} C_L^{\delta_e}]}{[1 + C_B C_D^{\delta_c} C_L^{\delta_e}][1 - C_B C_D^{\delta_e} C_L^{\delta_c}] + C_B^2 C_D^{\delta_c} C_D^{\delta_e} C_L^{\delta_c} C_L^{\delta_e}}$$

$$N_i^{M\delta} := C_M^{\delta_e} B_{X_{i1}} - C_M^{\delta_c} B_{X_{i2}}, \qquad N_i^{L\delta} = C_L^{\delta_e} B_{X_{i1}} - C_L^{\delta_c} B_{X_{i2}}$$

$$C_A := \frac{1}{C_M^{\delta_c} C_L^{\delta_e} - C_L^{\delta_c} C_M^{\delta_e}}, \qquad C_B = \frac{C_A z_T}{\bar{c}\cos\alpha^*} .$$

Noticing that $\cos\alpha \approx \cos\alpha^*$ and $\sin\gamma \approx \gamma$ for the values of α, α^* and γ satisfying the bounds given in Table I, the reader can verify that change of coordinates (6) transforms (4) into

$$\begin{aligned} \dot{\chi} &= [A_{ST} + A_P]\,\chi + J_0 + J_1\,\alpha + J_2\,\alpha^2 + J_3\,Q \\ &\quad + J_4\,\tilde{V} + J_5\,\gamma + J_6\bar{V}\,\delta \end{aligned} \tag{7}$$

where $\bar{V} = \tilde{V}/V$. The matrices A_{ST} and A_P are respectively given by $A_{ST} = \text{diag}\{A_{ST_1}, A_{ST_2}, A_{ST_3}\}$, where

$$A_{ST_i} = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\zeta_i\omega_i \end{bmatrix} \qquad i = 1,2,3$$

and

$$A_P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ P_{11} & 0 & P_{12} & 0 & P_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ P_{21} & 0 & P_{22} & 0 & P_{23} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ P_{31} & 0 & P_{32} & 0 & P_{33} & 0 \end{bmatrix}$$

where, for $1 \le j,k \le 3$,

$$P_{jk} = \bar{q}S\left\{N_j^{\eta_k} + C_A\left[\frac{V_{\text{ref}}}{V} N_j^{M\delta} C_L^{\eta_k} - N_j^{L\delta} C_M^{\eta_k}\right] - C_B N_j^{L\delta} C_D^{\eta_k}\right\}.$$

Since the damping ratios ζ_i are positive, the matrix A_{ST} is Hurwitz. The expression of the matrices J_i , $0 \le i \le 5$, is not reported here for reasons of space limitation. Because of the presence of the term $J_6\bar{V}\delta$, the χ-dynamics do not represent the internal dynamics yet. Since $\bar{V} \ll 1$, it is possible to verify that the maximum norm of $J_6\bar{V}$ is at least 3 orders of magnitude smaller than the maximum norm of the other J_i matrices. Thus, the term $J_6\bar{V}\delta$ will be neglected, and the internal dynamics of the system are given by

$$\dot{\chi} = [A_{ST} + A_P]\,\chi + J_0 + J_1\alpha + J_2\alpha^2 + J_3 Q + J_4\tilde{V} + J_5\gamma. \tag{8}$$

A. Flexible States Stability Analysis

Consider the unforced internal dynamics

$$\dot{\chi} = [A_{ST} + A_P]\,\chi\,. \tag{9}$$

Since the matrix A_{ST} is Hurwitz, there exist a positive definite matrix P that satisfies the Lyapunov equation $PA_{ST} + A_{ST}^T P = -I_{6\times6}$. In particular, $P = \text{diag}\{P_1, P_2, P_3\}$, where

$$P_i = \begin{bmatrix} \dfrac{1 + \omega_i^2 + 4\zeta_i^2}{4\,\zeta_i\,\omega_i} & \dfrac{1}{2\,\omega_i^2} \\ \dfrac{1}{2\,\omega_i^2} & \dfrac{1 + \omega_i^2}{4\,\zeta_i\,\omega_i^3} \end{bmatrix}$$

As a consequence, the Lyapunov function candidate for system (9) is selected as $W_F(\chi) = \sigma_\chi\,\chi^T P \chi$, where σ_χ is a positive scaling factor. By construction, the Lie derivative of W_F along the trajectories of system (9) is given by $\dot{W}_F(\chi) = -\sigma_\chi\,\chi^T\left[I_{6\times6} - (PA_P^T + A_P P)\right]\chi$, therefore the origin of system (9) is asymptotically stable if $PA_P^T + A_P P < I_{6\times6}$. It has been verified numerically that the matrix $I_{6\times6} - (PA_P^T + A_P P)$ is positive definite for any feasible $\bar{q}, V$ and V_{ref} in Table I, and for all values of the parameter vector $\mathscr{P} \in \Xi_{\mathscr{P}}$. As a result, there exists a $\lambda_A > 0$ such that $\dot{W}_F(\chi) \le -\sigma_\chi\,\lambda_A \|\chi\|^2$ along trajectories of system (9).

IV. Tracking Error Dynamics

Let $\chi = \tilde{\chi} + \chi^*$ where χ^* represent the steady state value of the flexible states in the new coordinates. The structure of the matrices $C_T^\eta, C_M^\eta, C_D^\eta, C_L^\eta$ and B_4 is such that $C_i^\eta\,\eta = C_i^\eta\,\chi$ for $i = T, M, L, D$. As a result, one can directly substitute η with χ in the force and moment approximations in (2).

A. Rigid-body Error Dynamics

Substituting the expression of thrust in (2) into the first equation of system (1), the velocity error dynamics read as

$$\begin{aligned} m\dot{\tilde V} &= \bar q S C_{T,\Phi}(\alpha)\cos\alpha\,\Phi + \bar q S\left[C_T(\alpha)+C_T^{\eta}\chi\right]\cos\alpha \\ &\quad -D - mg\sin\gamma - m\dot V_{\rm ref}\,. \end{aligned} \tag{10}$$

By introducing the vector of uncertain parameters $\theta_1 \in \mathbb{R}^{14}$

$$\begin{aligned}\theta_1 = \big[&SC_T^{\Phi\alpha^3}, SC_T^{\Phi\alpha^2}, SC_T^{\Phi\alpha}, SC_T^{\Phi}, SC_T^3, SC_T^2, SC_T^1, \\ &S(C_T^0+C_T^{\eta}\chi^*), SC_D^{\alpha^2}, SC_D^{\alpha}, SC_D^{\delta_e}, SC_D^{\delta_c}, S\big(C_D^0+C_D^{\eta}\chi^*\big), m\big]^T\end{aligned}$$

equation (10) transforms into

$$m\dot{\tilde V} = \theta_1^T B_1\ \Phi - \Psi_1^T\theta_1 + \bar q S[C_T^{\eta}\cos\alpha + C_D^{\eta}]\,\tilde\chi \tag{11}$$

where the regressor $\Psi_1(x,u,y_{\rm ref})$ and the input matrix $B_1(\alpha,\bar q)$ are given respectively by

$$\begin{aligned}\Psi_1 &= \big[0_{1\times4}, -\alpha^3\cos\alpha, -\alpha^2\cos\alpha, -\alpha\cos\alpha, -\cos\alpha, \\ &\qquad \bar q\alpha^2, \bar q\alpha, \bar q\delta_e, \bar q\delta_c, \bar q, g\sin\gamma + \dot V_{\rm ref}\big]^T \\ B_1 &= \big[\bar q\alpha^3\cos\alpha, \bar q\alpha^2\cos\alpha, \bar q\alpha\cos\alpha, \bar q\cos\alpha, 0_{1\times10}\big]^T.\end{aligned}$$

The dynamics of the tracking error $\tilde h$ can be written as $\dot{\tilde h} = V\sin\gamma - \dot h_{\rm ref} \approx V_{\rm ref}\gamma - \dot h_{\rm ref} + \tilde V\sin\gamma$, where the approximation $\sin\gamma\approx\gamma$ is valid for all γ in the range given in Table I. Choosing the flight-path angle command as $\gamma_{\rm cmd} = -k_2\tilde h + \frac{1}{V_{\rm ref}}\dot h_{\rm ref}$, where $k_2>0$ is a gain parameter, yields the following expression of the dynamics of the altitude error

$$\dot{\tilde h} = -k_2 V_{\rm ref}\tilde h + V_{\rm ref}\tilde\gamma + \tilde V\sin\gamma. \tag{12}$$

Using (1)-(3), the dynamics of $\tilde\gamma$ reads as

$$\begin{aligned}\dot{\tilde\gamma} &= \frac{1}{mV}\Big[\bar q S C_L^{\alpha}\alpha + T\sin\alpha - mg\cos\gamma - mV\,\dot\gamma_{\rm cmd} \\ &\qquad + \bar q S\big[C_L^{\delta_e}\delta_e + C_L^{\delta_c}\delta_c + C_L^0 + C_L^{\eta}\chi\big]\Big].\end{aligned}$$

Similarly to (5), along the trajectories of the system compatible with the conditions given in Table I

$$\begin{aligned}\bar q S C_L^{\alpha}\,\alpha + T(\alpha,\Phi)\sin\alpha - \bar q S C_L^{\alpha}\,\alpha^* - T(\alpha^*,\Phi)\sin\alpha^* \\ := K_{\alpha_2}(x,\Phi)\,V^2\,[\alpha-\alpha^*]\end{aligned}$$

where $K_{\alpha_2}(x,\Phi)$ satisfies $K_{\alpha_2}^{min} \le K_{\alpha_2}(x,\Phi) \le K_{\alpha_2}^{max}$ for some positive constants $K_{\alpha_2}^{min}$ and $K_{\alpha_2}^{max}$. The command trajectory for α is selected as $\alpha_{\rm cmd} = \alpha^* - \tilde\gamma$, where α^* can be set to arbitrary values in the envelope of the feasible flight conditions. As a result $\alpha = \tilde\alpha + \alpha_{\rm cmd} = \alpha^* - \tilde\gamma + \tilde\alpha$, and

$$\bar q S C_L^{\alpha}\,\alpha + T\sin\alpha := K_{\alpha_2}V^2[\tilde\alpha - \tilde\gamma] + \bar q S\,c_1(\alpha^*) + \bar q S\,c_2(\alpha^*)\Phi$$

where

$$\begin{aligned}c_1(\alpha^*) &= \big[C_T^3\,\alpha^{*3} + C_T^2\,\alpha^{*2} + C_T^1\,\alpha^* + C_T^0\big]\sin\alpha^* + C_L^{\alpha}\alpha^* \\ c_2(\alpha^*) &= \big[C_T^{\Phi\alpha^3}\alpha^{*3} + C_T^{\Phi\alpha^2}\alpha^{*2} + C_T^{\Phi\alpha}\alpha^* + C_T^{\Phi}\big]\sin\alpha^*.\end{aligned}$$

Introducing the vectors of uncertain parameters θ_2 and the regressor $\Psi_2(x,u,y_{\rm ref})$ defined as

$$\begin{aligned}\theta_2 &= \left[\frac{C_L^{\delta_e}}{C_L^{\delta_c}}, \frac{C_L^0 + C_L^{\eta}\chi^* + c_1(\alpha^*)}{C_L^{\delta_c}}, \frac{c_2(\alpha^*)}{C_L^{\delta_c}}, \frac{m}{SC_L^{\delta_c}}\right]^T \\ \Psi_2 &= \left[-\delta_e, -1, -\Phi, \frac{1}{\bar q}\big[g\cos\gamma + V\,\dot\gamma_{\rm cmd}\big]\right]^T\end{aligned}$$

the $\tilde\gamma$-dynamics assumes the following form

$$m\dot{\tilde\gamma} = K_{\alpha_2}V[\tilde\alpha - \tilde\gamma] + \frac{\bar q S C_L^{\delta_c}}{V}\left[\delta_c - \Psi_2^T\theta_2 + \frac{C_L^{\eta}}{C_L^{\delta_c}}\tilde\chi\right] \tag{13}$$

Using (1) and the definition of the $\alpha_{\rm cmd}$ it follows that

$$\begin{aligned}\dot{\tilde\alpha} &= Q - \dot\gamma_{\rm cmd} \\ \dot Q &= \frac{M}{I_{yy}}.\end{aligned} \tag{14}$$

By choosing $Q_{\rm cmd} = \dot\gamma_{\rm cmd} - k_4\tilde\alpha$, where k_4 is a positive gain parameter, system (14) reads as

$$\begin{aligned}\dot{\tilde\alpha} &= -k_4\tilde\alpha + \tilde Q \\ \dot{\tilde Q} &= \frac{M}{I_{yy}} - \ddot\gamma_{\rm cmd} - k_4^2\tilde\alpha + k_4\tilde Q.\end{aligned} \tag{15}$$

Using (2)-(3) and introducing the vector of uncertain parameters θ_3 defined as

$$\begin{aligned}\theta_3 = \frac{S}{I_{yy}}\big[&\bar c C_M^{\delta_e}, \bar c C_M^{\delta_c}, z_T C_T^{\Phi\alpha^3}, z_T C_T^{\Phi\alpha^2}, z_T C_T^{\Phi\alpha}, z_T C_T^{\Phi}, z_T C_T^3, \\ &z_T C_T^2 + \bar c C_M^{\alpha^2}, z_T C_T^1 + \bar c C_M^{\alpha}, z_T C_T^0 + \bar c C_M^0 \\ &+ (z_T C_T^{\eta} + \bar c C_M^{\eta})\,\chi^*, I_{yy}/S\big]^T\end{aligned}$$

system (15) reads as

$$\begin{aligned}\dot{\tilde\alpha} &= -k_4\tilde\alpha + \tilde Q \qquad (16)\\ \dot{\tilde Q} &= \theta_3^T B_3\,\delta_e - \Psi_3^T\theta_3 + k_4\tilde Q - k_4^2\tilde\alpha + \frac{\bar q S}{I_{YY}}[z_T C_T^{\eta} + \bar c C_M^{\eta}]\tilde\chi\end{aligned}$$

where the regressor $\Psi_3(x,u,y_{\rm ref})$ and the input matrix $B_3(\bar q)$ are given by

$$\begin{aligned}\Psi_3 &= -\bar q\big[0, \delta_c, \alpha^3\Phi, \alpha^2\Phi, \alpha\Phi, \Phi, \alpha^3, \alpha^2, \alpha, 1, -\ddot\gamma_{\rm cmd}/\bar q\big]^T \\ B_3 &= \bar q\big[1, 0_{1\times10}\big]^T.\end{aligned}$$

B. Flexible States Error Dynamics

Using the definition of the error variables and command trajectories for γ, α and Q, system (8) is transformed into

$$\begin{aligned}\dot\chi &= [A_{ST} + A_P]\,\chi + J_2(\tilde\alpha - \tilde\gamma)^2 + \big[k_2^2 V_{\rm ref}J_3 - k_2 J_5\big]\tilde h \\ &\quad -\big[J_1 + 2J_2\,\alpha^* + k_2 V_{\rm ref}J_3 + J_5\big]\tilde\gamma + \big[J_1 + 2J_2\,\alpha^* - k_4 J_3\big]\tilde\alpha \\ &\quad + J_3\tilde Q + \big[J_4 - k_2\sin\gamma J_3\big]\tilde V + \bar J\end{aligned} \tag{17}$$

where $\bar J := J_0 + J_1\alpha^* + J_2\alpha^{*2} - \left(\frac{\dot V_{\rm ref}}{V_{\rm ref}^2}\dot h_{\rm ref} - \frac{1}{V_{\rm ref}}\ddot h_{\rm ref}\right)J_3 + \frac{\dot h_{\rm ref}}{V_{\rm ref}}J_5$ is a bounded non-vanishing perturbation that determines the steady-state value χ^* of $\chi(t)$. As a result the analysis of the stability of the equilibrium point $\chi = \chi^*$ of (17) is reduced to the study of the stability of the origin of

$$\begin{aligned}\dot{\tilde{\chi}} \;=\;& \left[A_{ST}+A_P\right]\tilde{\chi} + \left[k_2^2 V_{\mathrm{ref}} J_3 - k_2 J_5\right]\tilde{h} + J_2\left(\tilde{\alpha}-\tilde{\gamma}\right)^2 \\ & -\left[J_1 + 2J_2\,\alpha^* + k_2 V_{\mathrm{ref}} J_3 + J_5\right]\tilde{\gamma} + \left[J_4 - k_2 \sin\gamma J_3\right]\tilde{V} \\ & +\left[J_1 + 2J_2\,\alpha^* - k_4 J_3\right]\tilde{\alpha} + J_3\tilde{Q} \qquad (18)\end{aligned}$$

V. Controller design and stability analysis

Let $\hat{\theta}_1 \in \mathbb{R}^{14}$, $\hat{\theta}_2 \in \mathbb{R}^4$ and $\hat{\theta}_3 \in \mathbb{R}^{11}$ be vectors of estimates of the corresponding uncertain parameters, and define $\tilde{\theta}_1 := \hat{\theta}_1 - \theta_1$, $\tilde{\theta}_2 = \hat{\theta}_2 - \theta_2$ and $\tilde{\theta}_3 := \hat{\theta}_3 - \theta_3$. Let $\Theta_1 \subset \mathbb{R}^{14}$, $\Theta_2 \subset \mathbb{R}^4$ and $\Theta_3 \subset \mathbb{R}^{11}$ be compact convex sets obtained by respectively letting the entries of θ_1, θ_2 and θ_3 vary within the parameter set $\Xi_{\mathscr{P}}$, and denote by Θ the set $\Theta_1 \times \Theta_2 \times \Theta_3$. The dynamic nonlinear controller is defined by

$$\begin{aligned}\dot{\hat{\theta}}_1 &= \operatorname*{Proj}_{\theta_1\in\Theta_1}\left\{\tilde{V}\,\Gamma_1\left[B_1(\alpha,\bar{q})\Phi - \Psi_1(x,u,y_{\mathrm{ref}})\right]\right\} \\ \dot{\hat{\theta}}_2 &= \operatorname*{Proj}_{\theta_2\in\Theta_2}\left\{-\frac{\bar{q}\,\tilde{\gamma}}{V}\,\Gamma_2\,\Psi_2(x,u,y_{\mathrm{ref}})\right\} \\ \dot{\hat{\theta}}_3 &= \operatorname*{Proj}_{\theta_3\in\Theta_3}\left\{\Gamma_3\left[B_3(\bar{q})\delta_e - \Psi_3(x,u,y_{\mathrm{ref}})\right]\tilde{Q}\right\} \\ \Phi &= \frac{1}{\hat{\theta}_1^T B_1(\alpha,\bar{q})}\left[\Psi_1(x,u,y_{\mathrm{ref}})^T\,\hat{\theta}_1 - k_1\tilde{V}\right] \\ \delta_c &= \Psi_2(x,u,y_{\mathrm{ref}})^T\,\hat{\theta}_2 - k_3\tilde{\gamma} \\ \delta_e &= \frac{1}{\hat{\theta}_3^T B_3(\bar{q})}\left[\Psi_3(x,u,y_{\mathrm{ref}})^T\,\hat{\theta}_3 - k_5\tilde{Q}\right]\end{aligned}$$

where k_1, k_2 and k_3 are gain parameters, $\Gamma_1 \in \mathbb{R}^{14\times 14}$, $\Gamma_2 \in \mathbb{R}^{4\times 4}$ and $\Gamma_3 \in \mathbb{R}^{11\times 11}$ are symmetric positive definite matrices, and $\mathrm{Proj}_{\theta_k\in\Theta_k}$ is a smooth parameter projection [17] for $k = 1,2,3$. The parameter projections ensure non-singularity of the control laws over the flight conditions. Consider the Lyapunov function candidate

$$\begin{aligned}W(\tilde{x},\tilde{\chi},\tilde{\theta}) =\;& \frac{m\sigma_V}{2}\,\tilde{V}^2 + \frac{\sigma_h}{2}\,\tilde{h}^2 + \frac{m}{2}\,\tilde{\gamma}^2 + \frac{\sigma_\alpha}{2}\,\tilde{\alpha}^2 + \frac{\sigma_Q}{2}\,\tilde{Q}^2 \\ & + \dot{W}_F(\tilde{\chi}) + \frac{\sigma_V}{2}\,\tilde{\theta}_1^T\Gamma_1^{-1}\tilde{\theta}_1 + \frac{SC_L^{\delta_c}}{2}\,\tilde{\theta}_2^T\Gamma_2^{-1}\tilde{\theta}_2 + \frac{\sigma_Q}{2}\,\tilde{\theta}_3^T\Gamma_3^{-1}\tilde{\theta}_3\end{aligned}$$

where $\sigma_V, \sigma_h, \sigma_\alpha, \sigma_Q$ are positive scaling factors. The role of the scaling factor is to allow flexibility in the design by shaping the level sets $\Omega_c = \{W(\tilde{x},\tilde{\chi},\tilde{\theta}) \le c\}$ of the Lyapunov function W. In particular, given a fixed compact set $\mathscr{K}_x$ of initial conditions for $\tilde{x}(t)$ compatible with Table I, and a compact set $\mathscr{K}_\chi$ of initial conditions for $\tilde{\chi}(t)$, a constant $c > 0$ and a choice of the scaling factors $\sigma_\chi, \sigma_V, \sigma_h, \sigma_\alpha, \sigma_Q$ should be determined such that $(\tilde{x},\tilde{\chi}) \in \mathscr{K}_x \times \mathscr{K}_\chi \implies (\tilde{x},\tilde{\chi}) \in \Omega_c$ and for any $\tilde{x} \in \Omega_c$ the corresponding value x remains within the bounds of Table I.

Proposition 5.1: Let compact sets $\mathscr{K}_x$, $\mathscr{K}_x$ of feasible initial conditions for $\tilde{x}(t)$ and $\tilde{\chi}(t)$ be given. Fix $c > 0$ and values $\sigma_\chi, \sigma_V, \sigma_h, \sigma_\alpha, \sigma_Q$ as in the discussion above. Then, there exist $k_1^* > 0$, $k_2^* > 0$, $k_3^* > 0$, $k_5^* > 0$ and $\underline{k}_4^*, \overline{k}_4^* > 0$, such that for any $k_1 > k_1^*$, $k_2 < k_2^*$, $k_3 > k_3^*$, $k_5 > k_5^*$ and $\underline{k}_4^* < k_4 < \overline{k}_4^*$, the trajectories of the closed loop system originating within the set $\mathscr{K}_x \times \mathscr{K}_\chi \times \Theta$ are bounded, and satisfy $\lim_{t\to\infty}(\tilde{x}(t),\tilde{\chi}(t)) = (0,0)$. □

The existence of a finite interval for the stabilizing values of k_4 (i.e., a conditional stability) is a consequence of the peaking phenomenon exhibited by the internal dynamics, due to the appearance of both α and Q. As a result, the system does not have an infinite gain margin when either a low-gain or a high-gain feedback is applied [18]. It is worth noticing that if the peaking phenomenon had not occurred, the term R_{α_0} would have been zero and therefore Δ_3 could have been rendered positive by small values of the gain k_4.

VI. Simulations

To validate the controller derived in the previous section, simulations have been performed on the HFM model implemented in SIMULINK®. As a representative case study, the vehicle is initially trimmed at $h = 85000$ ft and $V = 7850$ ft/s; the reference trajectory $h_{\mathrm{ref}}(t)$ is generated to let the vehicle climb 25000 ft in about 250s, and $V_{\mathrm{ref}}(t)$ is generated to increase the vehicle velocity by 2650 ft/s in the same time interval. The desired trim value for the angle of attack has been selected as $\alpha^* = 2$ deg. The initial conditions of the plant parameter estimates have been randomly selected within 50% of their nominal values. The controller gains have been chosen as

$$\begin{array}{lllllllll} k_1 &=& 120 & k_4 &=& 50 & \Gamma_1 &=& 0.1\times I_{14\times 14} \\ k_2 &=& 0.0001 & k_5 &=& 80 & \Gamma_2 &=& 0.1\times I_{4\times 4} \\ k_3 &=& 10 & & & & \Gamma_3 &=& 0.1\times I_{11\times 11}\,. \end{array}$$

Fig. 1(a)-(b) show that the tracking performance in closed-loop for the velocity and altitude are excellent in spite of parameter uncertainty. It is worth noticing that the parameter uncertainty considered here (perturbation within 50% of the nominal values), leads to instability in [13]. The inputs and the flexible states remain well-behaved, as shown in Fig.1(c)-(d), while the angle of attack is regulated to the desired trim value. The parameter estimates are settled to constant values, even though for reasons of space limitations their plots are not reported here.

VII. Conclusions

In this paper we have presented a nonlinear controller design for the longitudinal motion of an air-breathing hypersonic vehicle. The controller provides robustness with respect to model uncertainty, both parametric and dynamics. In particular, the work fully addresses the presence of structural flexibilities in the proof of stability of the closed-loop system. The controller combines robust adaptive control, high-gain feedback, and low-gain feedback. Based on the analysis, it is shown that there exists a finite range of the values of the controller gains that guarantees stable tracking of velocity and output references. Simulation results conducted on a high-fidelity vehicle model validate the proposed methodology.

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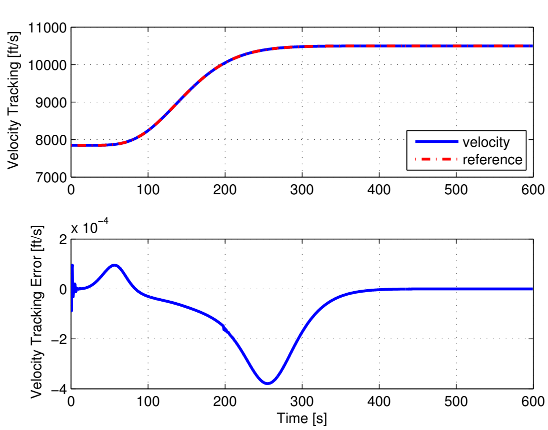
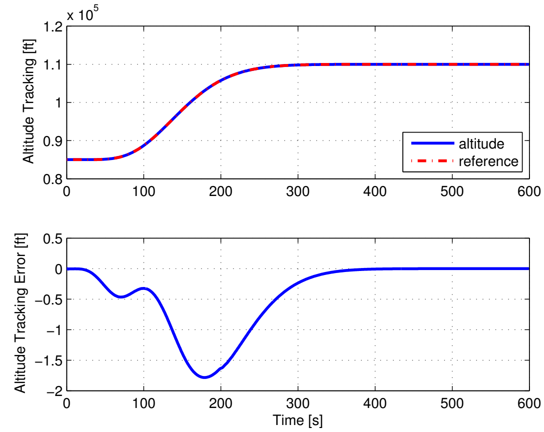
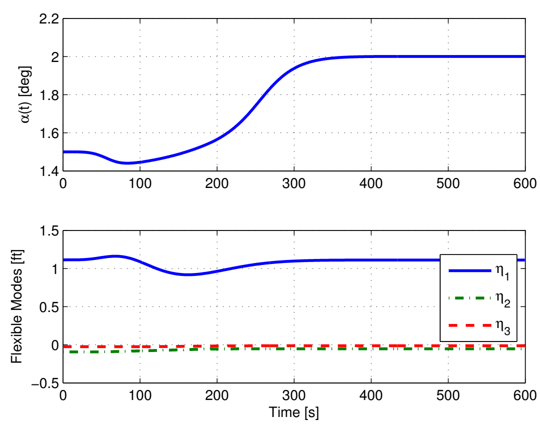


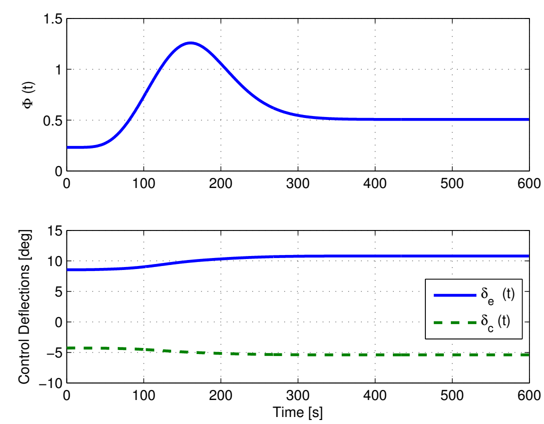
(a) Velocity Tracking and Tracking Error



(b) Altitude Tracking and Tracking Error



(c) Angle of Attack and Flexible States



(d) Control Inputs

Fig. 1. Simulation Results.

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