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Feedback Error Learning with Insufficient Excitation

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Abstract— **Control-theoretical studies on Feedback Error Learning (FEL) have been active recently. The authors generalized this scheme to multi-input multi-output (MIMO) systems with application to writing one-stroke characters by a two-link manipulator. After mentioning those related works, this paper studies further issues on MIMO-FEL, with focus on insufficient excitation and plant parameter estimation by means of frequency response.**

I. INTRODUCTION

Feedback Error Learning (FEL) originates from the pioneering work by Kawato *et al.* [1] on brain motor control. Our neural system is too slow in transmission to control motion only by feedback, hence it also uses feedforward with learning, whose mechanism was modeled as FEL.

This scheme soon attracted much attention in control engineering as well. Miyamura and Kimura [2] have established a control theoretical validity of the FEL method in the frame of adaptive control for single-input single-output (SISO) systems, proving its stability based on strict positive realness, whereas Muramatsu and Watanabe [3] have relaxed this condition by using the error signal between the reference and the output signal as well as the feedback input. Then Alali *et al.* [5] developed a MIMO-FEL scheme, which was further applied to a two-link manipulator, thereby showing that MIMO-FEL is effective for learning how to write one-stroke characters with the manipulator.

A striking feature in FEL is its excellent tracking performance without precise knowledge on the plant. This is so even without sufficient excitation of signals, which is clear both in simulation and experiment [5], [6]. So far, however, stability has been shown only in the case of sufficient excitation for the MIMO case.

The objectives of this paper are two-fold: First, we prove that the tracking error in [5] converges exponentially to zero even without full PE condition; i.e., even if signals are insufficiently rich[1]. This is important since in practice, PE condition is undesirable or even impossible to satisfy while good tracking performance is required.

Secondly, by making full use of this performance, we estimate plant parameter while in closed-loop operation. Our scenario is as follows: We first apply a sinusoidal signal with a single frequency component, a typical example of insufficient excitation, as a reference signal. We make the feedforward controller learn to track this reference. After convergence the trained controller reflects the inverse response of the plant *only* at this frequency. We repeat this process for various frequencies. Rapid convergence of FEL allows us to do it efficiently. The plant parameter is finally computed by solving a linear equation from these data.

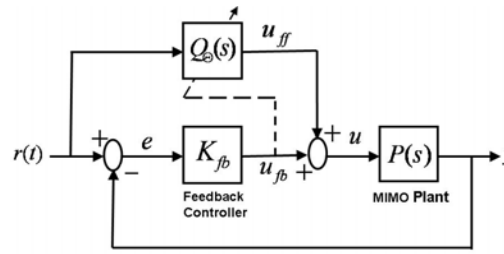

Fig. 1. Feedback Error Learning Architecture for MIMO Biproper Systems

It is widely known that closed-loop identification is difficult since the input/output signals have correlation. Under sensor noise, our method does not solve the problem perfectly, yet gives a new insight from an adaptive viewpoint. We cancel the feedback signal by feedfoward control, thus leading to virtual open-loop operation. We do not even require knowledge of feedback controller for estimation.

II. REVIEW OF MIMO-FEL

Consider the following state-space plant model

$$\dot{x}(t) = Ax(t) + Bu(t),\ y(t) = Cx(t) + Du(t) \qquad (1)$$

where $x \in \mathbf{R}^n$, $u \in \mathbf{R}^m$, $y \in \mathbf{R}^m$, (A, B, C) controllable and observable. In this paper, we confine ourselves to the biproper case [2] , *i.e.*, det $D \neq 0$. Assume that (1) is minimum phase. Then, there exists a proper stable system $Q(s)$ such that $P(s)Q(s) = I$, where $P(s) = D + C(sI - A)^{-1}B$.

The feedback error learning architecture of the plant (1) is shown in Fig. 1. Here we assume that we have already given the feedback controller $K_{\rm fb}$ to stabilize the plant, where det $K_{\rm fb} \neq 0$. However, this is not expected to give a good tracking performance because we do not know the model precisely. Thus, we need to adapt a feedforward controller $Q_\Theta(s)$ to generate $u_{\rm ff}$ in order to improve the performance; *i.e.*, to minimize the error signal $e(t)$ in Fig. 1.

In fact, we can attain $e = 0$ in steady state by some $Q_\Theta(s)$. The "optimal" feedforward controller in this sense is

$$u_0(t) = P(s)^{-1}r(t) = Q(s)r(t). \qquad (2)$$

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[1]But we need certain approximation which is carried over since [2].

[2]It is known that biproper systems can be made positive real by some feedback [2], which we will use in a stability proof. If a MIMO system is not biproper, we put a prefilter to compensate the delay [5], [6].

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In reality, however, we cannot construct $Q(s)$ in advance since (A, B, C, D) in (1) is assumed to be unknown. We consequently need adaptation to realize (2). We first take a pair (A_f, B_f) such that $B_f \in \mathbf{R}^{\nu m \times m}$ and

$$(sI - A_f)^{-1} B_f = \frac{1}{d(s)} \begin{bmatrix} S(s) & & O \\ & \ddots & \\ O & & S(s) \end{bmatrix}, \ S = \begin{bmatrix} 1 \\ \vdots \\ s^{\nu-1} \end{bmatrix} \tag{3}$$

for a Hurwitz polynomial $d(s)$ and an integer ν (see [5] for concrete forms of A_f and B_f). Consider

$$\eta = \begin{bmatrix} (sI - A_f)^{-1} B_f r \\ (sI - A_f)^{-1} B_f u_0 \\ r \end{bmatrix} \in \mathbf{R}^{\ell}, \quad \ell := 2\nu m + m. \tag{4}$$

If ν is large enough, then there exist $F_0, G_0 \in \mathbf{R}^{m \times \nu m}$, and $H_0 \in \mathbf{R}^{m \times m}$ such that

$$Q(s) = \{I - G_0 (sI - A_f)^{-1} B_f\}^{-1} \{H_0 + F_0 (sI - A_f)^{-1} B_f\}.$$

Namely, we attain

$$u_0(t) = \Theta_0 \eta(t), \quad \Theta_0 = [F_0 \ G_0 \ H_0] \in \mathbf{R}^{m \times \ell}. \tag{5}$$

However, F_0 , G_0 and H_0 are unknown matrices to be estimated (which will be called "optimal" later), so that we replace them with tunable matrices:

$$\hat{u}_0(t) = \Theta(t)\eta(t), \ \ \Theta(t) = [F(t) \ \ G(t) \ \ H(t)] \tag{6}$$

where u_0 in (4) is replaced by $\hat{u}_0$ [3] and $\Theta(t)$ is tuned by the MIMO-version learning law [5]

$$\frac{d\Theta}{dt} = \alpha u_{\text{fb}}(t) \eta^T(t) \tag{7}$$

with a small positive constant α that gives adaptation speed. We apply $\hat{u}_0$ as u_{ff} in Fig. 1.

MIMO-FEL has turned out to be highly effective for tracking reference signals. Various simulation results show that tracking error converges to zero very fast [5]. This scheme has also been applied to a two-link manipulator for learning to write one-stroke characters [6]. For the experimental hardware and written characters, see Figs. 2 and 3 respectively. It is clear that in such cases reference signals lack sufficient excitation (see §4.2 in conference version of [6] for a detail), which motivates our development below.

III. Exponential Tracking Error Convergence

Persistent excitation (PE) is a sufficient condition to ensure parameter convergence as well as tracking error convergence for FEL [2], [3], [5]. In this paper, we relax this condition for the latter convergence [7].

We start by considering the dynamics of the parameter estimation error defined by

$$\Psi(t) := \Theta_0 - \Theta(t), \tag{8}$$

[3]Conference vesion of [5] used u for u_0 in (4), which is not desirable, as pointed out by reference [12]. On the other hand, journal version of [5] and both versions of [6] used $\hat{u}_0$ and applied it as u_{ff} correctly.

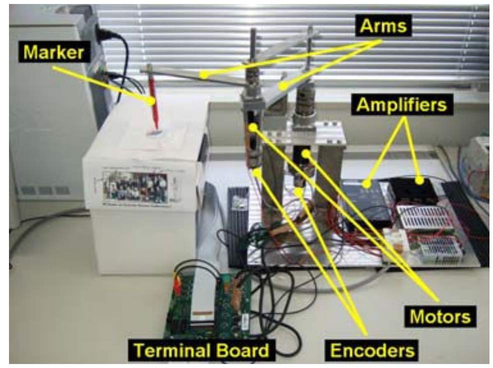

Fig. 2. Two-link Manipulator

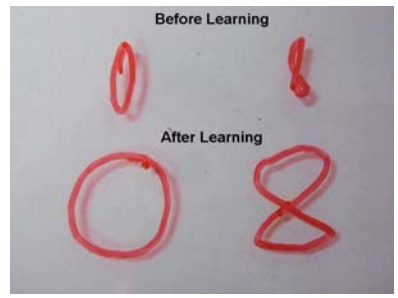

Fig. 3. Written Characters 0 and 8 by the Manipulator

as in [2], [3]. By (7) we have

$$\dot{\Psi}(t) = -\alpha u_{\text{fb}}(t) \eta^T(t). \tag{9}$$

In our scheme (see Fig. 1), we have

$$u_{\text{fb}} = u - u_{\text{ff}} = P^{-1} y - \hat{u}_o. \tag{10}$$

Furthermore,

$$P^{-1} y = Qy \cong Qr = u_o, \tag{11}$$

in the neighborhood of the optimal parameter (*i.e.*, when $\Psi \cong 0$). By (8), (10), and (11) we obtain

$$u_{\text{fb}}(t) = u_0(t) - \hat{u}_0(t) = \Psi(t)\eta(t) \tag{12}$$

Substituting (12) in (9) we have

$$\dot{\Psi}(t) = -\alpha \Psi(t) \eta(t) \eta^T(t). \tag{13}$$

Note that (13) is a matrix differential equation with variable coefficients. Now let us define the Lyapunov function $V = \frac{1}{2}\text{tr}\{\Psi^T \Psi\}$. Its derivative is computed as

$$\dot{V}(t) = \text{tr}[\Psi^T(t) \dot{\Psi}(t)] = -\alpha \text{tr}[\Psi^T(t) \Psi(t) \eta(t) \eta^T(t)] \leq 0. \tag{14}$$

This proves that Ψ remains bounded. If η is bounded, then from (12) the error

$$e(t) = K_{\text{fb}}^{-1} u_{\text{fb}}(t) = K_{\text{fb}}^{-1} \Psi(t) \eta(t), \tag{15}$$

remains bounded. Furthermore, if $\dot{\eta}$ is bounded, then $\ddot{V}$ is also bounded, $\dot{V}$ is uniformly continuous, then we can apply Barbalat lemma [4] (see Appendix A) to ensure that $\lim_{t\to\infty} e(t) = 0$. But this does not guarantee the exponential convergence of $e(t)$ to zero. An additional condition is usually imposed to this end, as follows [2], [3], [5].

In general, we say that a vector signal $\xi(t) = [\xi_1(t)\ \xi_2(t)\ \cdots \xi_p(t)]^T$ satisfies the PE condition, if there exists $\delta > 0$ such that

$$\Xi(t_0, \delta) = \int_{t_0}^{t_0+\delta} \xi(t)\xi^T(t)dt > 0, \tag{16}$$

for arbitrary initial time t_0. Now we first show that if $\eta(t)$ in (4) is PE, then $e(t)$ converges to zero exponentially; *i.e.*, there exist constants $\phi \geq 0$, $\sigma > 0$ such that

$$\|e(t)\| \leq \phi e^{-\sigma t}. \tag{17}$$

After that, we will relax this sufficient condition.

For $\Psi = (\psi_{ij})$ in (8), let $\mathbf{vec}(\Psi)$ denote the vector formed by stacking its columns into one long vector:

$$\mathbf{vec}(\Psi) = (\psi_{11} \cdots \psi_{m1}\ \psi_{12} \cdots \cdots \psi_{m\ell})^T. \tag{18}$$

Then for any matrices X, Y and Z with appropriate dimensions, it is known that [9]

$$\mathbf{vec}(XYZ) = (Z^T \otimes X)\mathbf{vec}(Y), \tag{19}$$

where $\otimes$ is Kronecker product. Hence (13) is written as

$$\begin{aligned}
&\mathbf{vec}(\dot{\Psi}(t)) = (\eta(t)\eta^T(t) \otimes (-\alpha I))\mathbf{vec}(\Psi(t)) \\
&= -\alpha \begin{bmatrix} \eta_1(t)I \\ \vdots \\ \eta_\ell(t)I \end{bmatrix} [\eta_1(t)I\ \cdots\ \eta_\ell(t)I]\, \mathbf{vec}(\Psi(t))
\end{aligned} \tag{20}$$

This means that (13) is the system of linear differential equations. Now we have the following lemma.

Lemma 1:
If $\eta(t)$ satisfies the PE condition, then there exists $\delta > 0$ such that

$$\int_{t_0}^{t_0+\delta} \eta(t)\eta^T(t) \otimes I dt > 0 \tag{21}$$

for arbitrary t_0.

Proof: see the Appendix B. $\diamond$

Corollary:
If $\eta(t)$ is PE, then (20) is globally exponentially stable, which implies that $e(t)$ also converges to zero exponentially.

Proof: The first statement holds by applying Exponential Stability Theorem [10], see Appendix C, to the vector differential equation (20), with Lemma 1. The second holds by (15). $\diamond$

Now we proceed to the case where η is not fully excited, the first main objective of the paper. We start by defining the correlation matrix:

$$M = \int_{t_0}^{\infty} \eta(t)\eta^T(t)dt \geq 0. \tag{22}$$

The constant matrix M fails to be positive definite but is positive semidefinite anyway. Hence there exists an eigenvalue decomposition:

$$M = R \begin{bmatrix} \Lambda & 0 \\ 0 & 0 \end{bmatrix} R^T, \quad \Lambda = \mathrm{diag}(\lambda_1, \cdots, \lambda_p) \tag{23}$$

where R is an $\ell \times \ell$ orthogonal matrix and $\lambda_1 \geq \cdots \geq \lambda_p > 0$. Now let us show that the error system (12) and (13) can be written equivalently as a reduced system.

Theorem 1:
Using the MIMO-FEL adaptive law (7), the smallest nonzero eigenvalue λ_p always exists unless $\eta \equiv 0$, $e \equiv 0$. Further, the tracking error e converges to zero exponentially.

Proof: If $M = 0$ then, from (22) $\eta \equiv 0$, and from (15) the error $e \equiv 0$. Hence $M \neq 0$ and $\lambda_p > 0$ exists. Defining

$$\rho(t) = R^T\eta(t) \in \mathbf{R}^\ell, \quad \Omega(t) = \Psi(t)R \in \mathbf{R}^{m\times\ell}, \tag{24}$$

$$\rho(t) = \begin{bmatrix} \rho_1(t) \\ \rho_2(t) \end{bmatrix}, \quad \Omega(t) = \begin{bmatrix} \Omega_1(t) & \Omega_2(t) \end{bmatrix} \tag{25}$$

in block sizes compatible with (23), the error (12) can be written as follows:

$$u_{\mathrm{fb}}(t) = \Psi R R^T \eta = \Omega\rho = \Omega_1\rho_1 + \Omega_2\rho_2. \tag{26}$$

From (23), the correlation matrix of ρ is computed as

$$\int_{t_0}^{\infty} \rho(t)\rho^T(t)dt = \begin{bmatrix} \Lambda & 0 \\ 0 & 0 \end{bmatrix}, \tag{27}$$

which implies from (25) that

$$\int_{t_0}^{\infty} \rho_1(t)\rho_1^T(t)dt = \Lambda > 0, \quad \rho_2 \equiv 0. \tag{28}$$

Substituting (28) in (26) gives

$$u_{\mathrm{fb}}(t) = \Omega_1(t)\rho_1(t). \tag{29}$$

This implies that we have reduced ρ to ρ_1, which has a smaller dimension and is persistently excited by (28).

A similar reduction can be shown for Ω as follows. By post-multiplying both sides of (13) by R we obtain

$$\dot{\Psi}(t)R = -\alpha\Psi(t)\eta(t)\eta^T(t)R. \tag{30}$$

Using (24) and (12), we have

$$\dot{\Omega}(t) = -\alpha u_{\mathrm{fb}}(t)\rho^T(t) = -\alpha\Omega_1(t)\rho_1(t)\rho^T(t) \tag{31}$$

by (29), which can be partitioned using (25) and (28) as

$$\dot{\Omega}_1(t) = -\alpha\Omega_1(t)\rho_1(t)\rho_1^T(t), \quad \dot{\Omega}_2(t) = 0. \tag{32}$$

It follows that the reduced parameter error Ω_1 in (32) converges exponentially based on Corollary to Lemma 1 . As $\Omega_1 \to 0$, $u_{\mathrm{fb}}(t) \to 0$ by (29), which ensures the exponential convergence of e by (15). $\diamond$

The proof concludes that there is no need for convergence of the full parameter matrix $\Theta(t)$ for the purpose of the exponential convergence of the tracking error. Rather, partial excitation of η, *i.e.,* that of ρ_1, is enough for the exponential convergence of $e(t)$.

IV. Parameter Estimation by Frequency Response

A main advantage of FEL as mentioned above is the exponential convergence of the tracking error by means of learning feedforward controller. It is then natural to expect that the acquired $Q_\Theta(s)$ has some knowledge on the plant model. If, in particular, we apply a sinusoidal reference, then $Q_\Theta(s)$ works as the inverse of the plant at this specific frequency.

In this section we show that, by testing various frequencies, we can identify the plant parameter from such knowledge while in closed-loop operation. To be specific, we consider a left factorization

$$P(s) = \mathcal{D}^{-1}(s)\mathcal{N}(s), \tag{33}$$

where $\mathcal{D}(s)$ and $\mathcal{N}(s)$ are polynomial matrices defined by

$$\begin{array}{rcl}\mathcal{D}(s) & = & s^\mu I + s^{\mu-1}D_1 + \ldots + D_\mu, \\ \mathcal{N}(s) & = & s^\mu N_0 + s^{\mu-1}N_1 + \ldots + N_\mu.\end{array} \tag{34}$$

N_0 is nonsingular because (1) is biproper. Our objective here is to estimate the coefficient matrices in (34) based on MIMO-FEL. Now we apply the reference signals of the form

$$r_i{}^k(t) = \gamma_i{}^k \sin(\omega_i t), \quad k = 1, \cdots, m, \tag{35}$$

with linearly independent vectors $\gamma_i{}^1, \cdots, \gamma_i{}^m$ for $i = 1, \cdots, \mu$. Then, we make the feedforward controller learn to track such references. After convergence the trained controller satisfies

$$P(j\omega_i)Q_{\Theta_i{}^k}(j\omega_i)\gamma_i{}^k = \gamma_i{}^k. \tag{36}$$

Hence from (33) we obtain

$$\mathcal{N}(j\omega_i)[\xi_i{}^1 \cdots \xi_i{}^m] = \mathcal{D}(j\omega_i)[\gamma_i{}^1 \cdots \gamma_i{}^m]. \tag{37}$$

where $\xi_i{}^k := Q_{\Theta_i{}^k}(j\omega_i)\gamma_i{}^k$. (37) is a linear equation with respect to the coefficient matrices of (34). By testing (35) for various frequencies and solving (37) for $i = 1, \cdots, \mu$, we obtain those coefficient matrices.

In order to explain more specifically, take the second order bi-prober SISO case for example. Then, we have four parameters to be estimated. We apply two different frequencies in order to estimate the parameters. By taking $\gamma_i{}^1 = 1$, we obtain $\xi_i{}^1 = Q_{\Theta_i{}^1}(j\omega_i) = \alpha_i + j\beta_i$. Then (37) becomes

$$(-\omega_i^2 + j\omega_i n_1 + n_2)(\alpha_i + j\beta_i) = -\omega_i^2 + j\omega_i d_1 + d_2, \tag{38}$$

where the scalar coefficients are written in lowercase. Comparing its real and imaginary parts, we have

$$\left.\begin{array}{l}-\omega_i^2\alpha_i + \alpha_i n_2 - \omega_i\beta_i n_1 = -\omega_i^2 + d_2 \\ -\omega_i^2\beta_i + \beta_i n_2 + \omega_i\alpha_i n_1 = \omega_i d_1\end{array}\right\} i = 1,\ 2. \tag{39}$$

The plant parameter is then computed by solving the following linear equation:

$$\begin{bmatrix}\omega_1 & 0 & -\alpha_1\omega_1 & -\beta_1 \\ \omega_2 & 0 & -\alpha_2\omega_2 & -\beta_2 \\ 0 & 1 & \beta_1\omega_1 & -\alpha_1 \\ 0 & 1 & \beta_2\omega_2 & -\alpha_2\end{bmatrix}\begin{bmatrix}d_1 \\ d_2 \\ n_1 \\ n_2\end{bmatrix} = \begin{bmatrix}-\beta_1\omega_1^2 \\ -\beta_2\omega_2^2 \\ \omega_1^2 - \alpha_1\omega_1^2 \\ \omega_2^2 - \alpha_2\omega_2^2\end{bmatrix}.$$

In general, the algorithm is summarized as follows:

1) Put $i = 1$.
2) Take a frequency ω_i. Put $k = 1$.
3) Apply sinusoidal reference input $r_i{}^k(t)$ in (35). Use the learning law (7) to tune the linear filter parameter in (6). After convergence, compute $Q_{\Theta_i{}^k}(j\omega_i)$.
4) Put $k := k + 1$ and go to Step 3 until $k = m$.
5) Obtain the equation (37) for i. Put $i := i + 1$ and go back to Step 2 until $i = \mu$.
6) Based on the obtained data, solve the linear system of equations (37).

Note that the proposed algorithm cancels the feedback effect by adjusting feedforward control, thereby obtaining i/o relations of the plant itself. The rapid convergence of FEL allows us to achieve this process efficiently. An advantage of the proposed method is that it does not require knowledge of the feedback controller.

V. Examples

To illustrate the proposed method, we perform numerical simulation. Here we confine ourselves in SISO case for simplicity. Consider the second order plant:

$$P(s) = \frac{s^2 + 3s + 2}{s^2 + 7s + 12}$$

We apply the reference input $r_i(t) = \sin(\omega_i t)$, for $\omega_1 = 1$ [rad/s] and $\omega_2 = 2$ [rad/s]. We choose a suitable feedback controller gain $K_{\mathrm{fb}} = 5$ which maintains the closed-loop stability.

A. Proposed Method

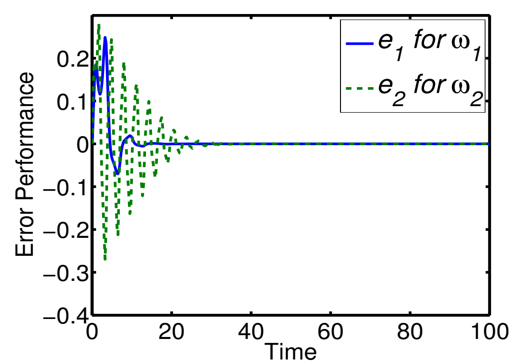

Fig. 4. Tracking Errors

Based on the order of the plant, we set

$$A_f = \begin{bmatrix} 0 & 1 \\ -5 & -5 \end{bmatrix}, B_f = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Then, we tune $\Theta(t)$ using the learning law (7) so that we can improve the tracking performance. Fig. 4 shows that the error signals $e_i(t)$ for the two frequencies converge to zero very fast. In view of this, we take the value of $\Theta_i(t)$ at the time when $e_i(t)$ vanishes; $e_1(t)$ and $e_2(t)$ vanish around $t_1 = 20$ and $t_2 = 40$, respectively. The learned feedforward controllers for the two frequencies are as follows:

$$Q_{\Theta_1}(s) = \frac{3.421s^2 + 16.77s + 18.28}{s^2 + 5.428s + 1.232},$$

$$Q_{\Theta_2}(s) = \frac{1.66s^2 + 7.978s + 10.6}{s^2 + 2.666s + 0.3708}.$$

Note that the resulting $Q_{\Theta_i}(s)$ does *not* represent the inverse of the plant, *i.e.*, $P(s)Q_{\Theta_i}(s) \neq 1$, because $\Theta(t)$ converges to a constant matrix not necessarily optimal. This is because $r_i(t)$ lacks full excitation or the PE condition, nonetheless the tracking is successful, due to the first result of this paper. In fact, the correlation matrix is computed as

$$M = \begin{bmatrix} 1.2161 & 0.012 & 3.8782 & 3.0599 & 4.9343 \\ 0.012 & 1.2022 & -2.9742 & 3.8524 & 6.0719 \\ 3.8782 & -2.9742 & 20.1335 & 0.1532 & 0.6543 \\ 3.0599 & 3.8524 & 0.1532 & 20.1224 & 31.5214 \\ 4.9343 & 6.0719 & 0.6543 & 31.5214 & 50.2183 \end{bmatrix},$$

which gives rank $M = 4$.

Now, let us estimate the plant parameter. The Bode plot in Fig. 5 shows that the estimated gain and phase at the particular frequencies coincide with the true values; $P(j\omega_i) = Q_{\Theta_i}^{-1}(j\omega_i)$. From these data, the linear equation in Section IV has been solved with respect to the parameter. The result is as follows:

$$P_{FEL}(s) = \frac{s^2 + 2.8387s + 1.8611}{s^2 + 6.8567s + 11.1334}.$$

If we take more time than t_1 and t_2, or if we test more frequencies, then the result of estimation becomes better.

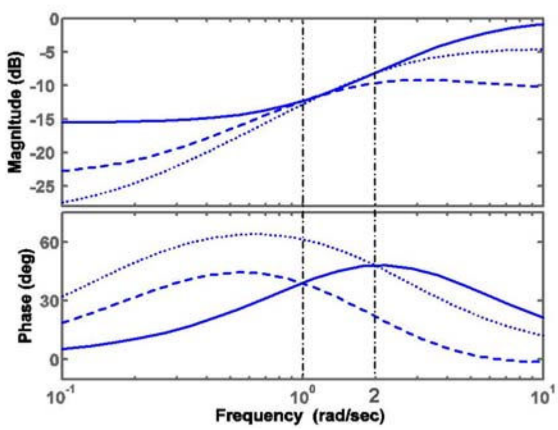

Fig. 5. FEL Analysis Results: solid line: $P(s)$, dashed line: $Q_{\Theta_1}^{-1}(s)$, dotted line: $Q_{\Theta_2}^{-1}(s)$

B. Conventional Method

We adopt an approach based on one-degree of freedom control scheme (1DOF); *i.e.*, without the feedforward controller in Fig. 1. The frequency response data from the reference input and the output of the plant are used to estimate the plant parameter. In this simulation, the spectrum analyzer in MATLAB is used to measure the frequency response data. The data is collected at the same time as Section V-A for fair comparison between both methods. We also use the same feedback controller, reference input and frequencies. From the frequency response data, we solve the linear equation as in the proposed algorithm to estimate the closed-loop transfer function. Then, by using the feedback controller knowledge, the plant is estimated as follows:

$$P_{con.}(s) = \frac{s^2 + 3.405s + 2.5697}{s^2 + 7.15s + 14.51}.$$

The result is biased from the actual plant parameter. Of course, the estimation is much improved if we take more time.

C. Results Comparison

It can be seen clearly that the estimation based on the FEL is better than the conventional approach as shown in Figs. 6 and 7.

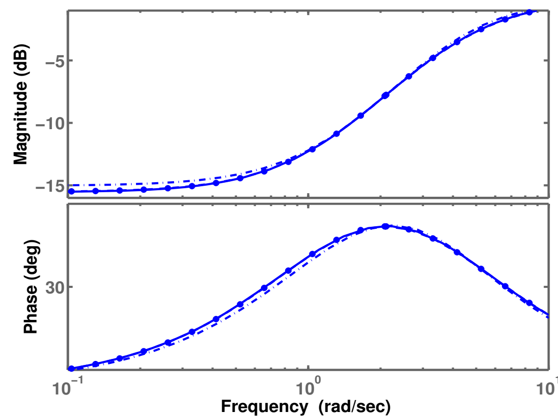

Fig. 6. True and estimated plants: solid line: $P(s)$, dashed line with *: $P_{FEL}(s)$, dash-dotted line: $P_{con.}(s)$

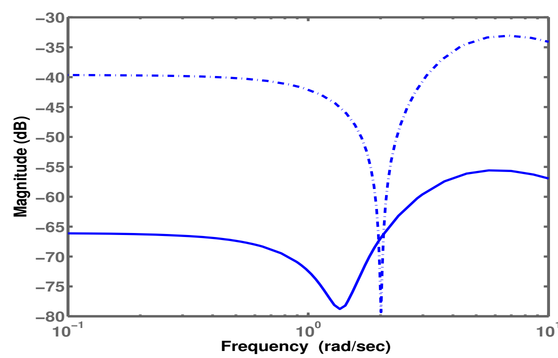

Fig. 7. Estimation error: solid line: $P(s) - P_{FEL}(s)$, dash-dotted line: $P(s) - P_{con.}(s)$

The former is fairly close to the actual one. We conclude that the proposed algorithm gives good approximation of the plant model faster than the conventional method. Another merit of FEL estimation over the conventional one is that it does not require the feedback controller knowledge.

D. Further Verification by Experiment

It is also possible to verify the proposed method by experiment with the two-link manipulator. Instead of writing characters, sinusoidal signals are applied as reference in X and Y axes, so that the manipulator learns to draw various ellipsoids. Fig. 8 shows one such result. The authors are currently analyzing the data, but have not finished identification of the manipulator at the moment of final version submission for some reasons.

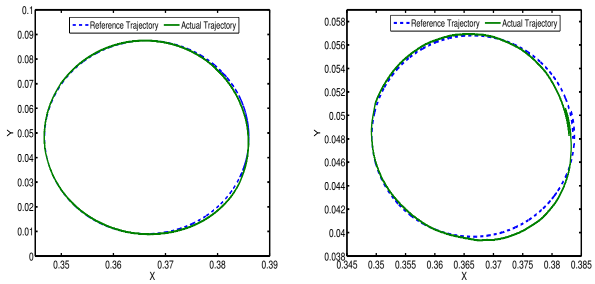

Fig. 8. Ellipsoids drawn by manipulator

VI. Conclusion

The main objective of this work is to prove the exponential convergence of the tracking error without full excitation in FEL. The merit of this result is that in many applications good tracking performance is required, while it is not desirable or even impossible to satisfy PE condition. Furthermore, the parameter estimation using FEL shows better result than conventional approach after the error convergence without using the knowledge of the feedback controller as required by most of the conventional methods.

It is worth to compare the proposed method with the recent work by Kaneko *et al.* [11], where two degree of freedom structure is used as in our approach for closed-loop identification based on fictitious reference iterative tuning (FRIT). FRIT requires only one-shot experimental data to identify the plant parameter. However, the parameter tuning is done off-line using the collected data while the proposed method works on-operation.

A drawback in this paper is that we have proved merely local stability, as is also the case in [2], since we have used approximation in (11). Positive realness also seems a conservative condition, because extensive simulation results suggest stability in a more general case. It is future work to generalize in this direction, together with establishing an on-line estimation method.

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References

[1] M. Kawato, K. Furukawa and R. Suzuki, "A hierarchical neural-network model for control and learning of voluntary movement," *Biol. Cybern.*, vol. 57, pp. 169-185, 1987.

[2] A. Miyamura and H. Kimura, "Stability of feedback error learning scheme," *Systems & Control Letters*, vol. 45, pp. 303-316, 2002.

[3] E. Muramatsu and K. Watanabe, "Feedback error learning control without recourse to positive realness," *IEEE Trans. on Automatic Control*, vol. 49 (10), pp. 1762-1767, 2004.

[4] K. Astrom and B. Wittenmark, "Adaptive Control," Addison Wesley, Reading, MA, U.S.A, 1995.

[5] B. Alali, K. Hirata and K. Sugimoto, "Generalization of Feedback Error Learning (FEL) to MIMO Systems," *Trans. of the Society of Instrument and Control Engineers (SICE)*, vol. 43 (4), pp. 293-302, 2007, also appeared in American Control Conference, pp. 5451-5456, Minneapolis, Minnesota USA, 14-16 Jun., 2006.

[6] B. Alali, K. Hirata and K. Sugimoto, "Feedback Error Learning for Writing One Stroke Characters by Two-Link Manipulator," *Transactions of the Institute of Systems, Control and Information Engineers (ISCIE)*, Vol. 20, No. 12, pp. 475-481, 2007, also appeared in 17th IFAC World Congress, pp. 4084-4089, Seoul, Korea, 6-11 Jul., 2008.

[7] S. Bayard, J. Spanos and Z. Rahman, "A Result on exponential tracking error convergence and persistent excitation," *IEEE Trans. on Automatic Control*, vol. 43 (10), pp. 1334-1338, 1998.

[8] G. Tao, "Adaptive Control: Design and Analysis," John Wiley & Sons, Hoboken, NJ, 2003.

[9] K. Zhou, J. Doyle and K. Glover, "Robust and Optimal Control," Prentice Hall, NJ, U.S.A, 1996.

[10] S. Sastry, "Adaptive Control: Stability, Convergence, and Robustness," Prentice Hall, NJ, U.S.A, 1989.

[11] O. Kaneko, S. Souma and T. Fujii, "A fictitious reference iterative tuning (FRIT) in the two-degree of freedom control scheme and its application to a facile closed loop system identification," *Trans. of the Society of Instrument and Control Engineers (SICE)*, vol. 42 (1), pp. 17-25, 2006.

[12] M. Nilsson and B. Egardt, "A clarifying analysis of feedback error learning in an LTI framework," to appear in *Int. J. Adaptive Control and Signal Processing*, now available on line.

Appendix

A. Barbalat Lemma (page 205 in [4]):

If a function $g : [0, \infty) \to \mathbf{R}$ is uniformly continuous and $\int_0^\infty g(s)ds$ has a finite value, then $\lim_{t\to\infty} g(t) = 0$.

B. Proof of Lemma 1:

In order to prove that $\int_{t_0}^{t_0+\delta} \eta(t)\eta^T(t) \otimes I dt =$

$$\begin{bmatrix} \int_{t_0}^{t_0+\delta} \eta_1^2 dtI & \int_{t_0}^{t_0+\delta} \eta_1\eta_2 dtI & \cdots \\ \int_{t_0}^{t_0+\delta} \eta_2\eta_1 dtI & \int_{t_0}^{t_0+\delta} \eta_2^2 dtI & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} > 0,$$

it is enough to show that if $A = (a_{ij}) > 0$ then

$$A \otimes I = \begin{bmatrix} a_{11}I & a_{12}I & \cdots \\ a_{21}I & a_{22}I & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} > 0.$$

There exists an orthogonal matrix V such that

$$V^TAV = \mathrm{diag}(\alpha_1, \alpha_2, \cdots) \quad \text{with} \quad \alpha_i > 0.$$

Hence we obtain $(V \otimes I)^T A \otimes I(V \otimes I)$

$$= \begin{bmatrix} \alpha_1 & & 0 \\ & \alpha_2 & \\ 0 & & \ddots \end{bmatrix} \otimes I = \begin{bmatrix} \alpha_1 I & & 0 \\ & \alpha_2 I & \\ 0 & & \ddots \end{bmatrix} > 0.$$

C. Exponential Stability Theorem (page 73 in [10]):

Let $w(t) \in \mathbf{R}^n$ be piecewise continuous: If $w(t)$ is PE, then the solution of $\dot{\phi}(t) = -gw(t)w^T(t)\phi(t)$ for $g > 0$ is globally exponentially stable.