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Disturbance Rejection and Set-point Tracking of Sinusoidal Signals using Generalized Predictive Control

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Abstract— **This paper seeks to extend Generalized Predictive Control (GPC) to tracking of trajectories in a periodic nature. The initial focus is on sinusoidal trajectories, but the work may be extended later on to a signal with bandlimited frequencies. In addition, this paper proposes strategies for optimizing the prefilter in GPC to improve the transient performance in set-point tracking.**

I. INTRODUCTION

Far and away the large part of publications [12], [2] in model predictive control (MPC) focus on either regulation or tracking of constant set points (step changes), although there are a few exceptions [11], [1]. This focus is a serious limitation as some of the main purported advantages of MPC are its ability to handle the multivariable case and constraint handling. These advantages are still required in systems where the set points may be quite complex and certainly are not piecewise constant; essentially constant for times well beyond the settling time. Of course one conundrum is that MPC is a time based approach and thus is not ideally suited to meet criteria that may be frequency domain based. This includes consideration of robustness where methods for adding this into the MPC framework tend to work well only in the constraint free case [8], [6], [16], or are computationally very demanding [7], [9], [10] and often highly conservative.

This paper seeks to make an initial step in the field of tracking non-standard trajectories. Although the reader may think that seminal works e.g. [3] suggested that MPC automatically took account of future set point trajectories within the optimisation, it has been shown by subsequent works [5], [12] that the default choice of pre-filter making use of this information is often poor. The fundamental reason for this is that the class of trajectories over which the performance is optimised must include some which are close to the ideal behaviour [14], [13], but this was not fully appreciated by the community until the late 1990s. On the other hand, it is well known through the internal model control principle that in order to reject a periodic disturbance or following a periodic reference signal with zero steady-state error, the generator for the disturbance or the reference is included in the stable closed-loop control system [4]. Recent work in a continuous-time predictive control system has shown that indeed the internal model control is required to achieve zero steady-state error [15]. This paper investigates the tracking of periodic signals in discrete-time using the framework of Generalized Predictive Control [3], where the special issues about the prefilter design are examined.

A further motivation for this work is a specific case study, that of a cutting tool. Initial work is focusing on scenarios where the cutting tool must track sinusoids, with no offset and subject to constraints. More discussion of this case study and applications will be covered in future work, whereas this paper focuses on development of the requisite theory.

In summary then, this paper makes two contributions. First, after some background in section II, in section III it shows how MPC can be set up to track sinusoids, with no asymptotic offset and secondly, section IV shows how the prefilter may be optimised in a systematic, but simple, fashion. The paper is completed with numerical examples and conclusions.

II. BACKGROUND ON CONVENTIONAL MPC

This section gives the MPC mathematical background necessary to discuss the main issues in this paper. In order to give a better context, first a conventional algorithm is introduced so that the reader can see how the proposed algorithm differs.

A. *Properties of modelling, prediction and optimisation*

Assume that the model takes the form

$$A(z)y = B(z)u + \frac{C(z)}{D(z)}\zeta \qquad (1)$$

where $D(z)$ contains the dynamics of the disturbances/set points which we desire to reject; assume herein that $C(z) = 1$. For conventional MPC [3] assume that $D(z) = 1 - z^{-1}$ and this models disturbances with a non-zero steady-state.

In order to get offset free tracking (asymptotically) it is necessary for both the predictions and the performance index to be unbiased in the presence of the expected set points and disturbances. Using the proposed model for prediction achieves this if modified to the following incremental form:

$$A(z)D(z)y = B(z)D(z)u + \zeta \qquad (2)$$

as ζ is a zero mean random variable. It is usual to combine the $D(z)$ and u terms to define an 'incremental' input as $\Delta u = D(z)u$ or $\Delta u_k = u_k - u_{k-1}$. In this case it can be observed that:

$$\lim_{k\to\infty} \Delta u_k = 0 \quad \Rightarrow \quad \lim_{k\to\infty} u_k = u_{k-1} \qquad (3)$$

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Thus the asymptotic input contains the requisite dynamics (a steady value) to reject disturbances and track piecewise constant setpoints, when the incremental term converges to zero. Also, $\Delta u = 0$ implies from (2) that $\Delta y = 0$ which allows for y to have a non zero steady-state. Let the performance index be similar to a standard GPC form:

$$J = \sum_{k=1}^{n_y} (r_k - y_k)^2 + \sum_{k=0}^{n_u-1} (\Delta u_k)^2 \tag{4}$$

Again, this can be seen to be unbiased asymptotically in that a zero tracking error implies $r = y$ and can also be achieved with $Du = 0,\ \Delta y = 0$; thus a zero cost and zero control increments are consistent with the desired objective.

So in summary, conventional MPC algorithms such as GPC achieve offset free tracking of constant set points and disturbance rejection of constant disturbances by including two requirements in the algorithm set up.

1) Predictions must be unbiased in the steady-state, so that if the system is already at steady-state the prediction model predicts it remains at that steady-state. Using model (2) achieves this.
2) The performance index J must be such that, if the system is at the correct steady-state, then minimising J gives an optimum control trajectory that causes the system to remain at that steady-state. This is achieved by using offset terms, e.g. $(r_k - y_k)^2,\ \Delta u_k^2,\ (u_k - u_{ss})^2$, where u_{ss} is an unbiased expected steady value of the input.

B. Predictions and control law

Many details are omitted here as standard in the literature [12]. In summary the predictions take the form:

$$\underset{\rightarrow}{y}_k = H\underset{\rightarrow}{\Delta u}_{k-1} + P\underset{\leftarrow}{\Delta u}_{k-1} + Q\underset{\leftarrow}{y}_k \tag{5}$$

where the matrices H, P, Q depend on the model parameters and horizons and the arrow notation (right for future, left for past) is defined as follows (dimensions are always appropriate to the usage):

$$\underset{\rightarrow}{x}_k = \begin{bmatrix} x_{k+1} \\ x_{k+2} \\ \vdots \end{bmatrix}; \quad \underset{\leftarrow}{x}_k = \begin{bmatrix} x_k \\ x_{k-1} \\ \vdots \end{bmatrix} \tag{6}$$

Substitution into the performance index J gives:

$$J = \|\underset{\rightarrow}{r}_k - \underset{\rightarrow}{y}_k\|_2^2 + \lambda\|\underset{\rightarrow}{Du}_{k-1}\|_2^2 \tag{7}$$

Minimising with respect to the future control increments (assuming $\Delta u_{k+n_u+i} = 0,\ i \geq 0$) gives:

$$\underset{\rightarrow}{\Delta u}_{k-1} = (H^T H + \lambda I)^{-1} H^T [\underset{\rightarrow}{r}_k - P\underset{\leftarrow}{\Delta u}_{k-1} - Q\underset{\leftarrow}{y}_k] \tag{8}$$

The control law is given from only the first component of $\underset{\rightarrow}{\Delta u}_{k-1}$, that is Δu_k and can easily be represented in transfer function form as:

$$D_k(z)\Delta u = P_r(z)r - N_k(z)y; \quad u = \frac{1}{D(z)}\Delta u \tag{9}$$

where $P_r(z)$ is anti-causal.

Remark 2.1: The closed-loop poles are given from

$$P_c(z) = A(z)D(z)D_k(z) + b(z)N_k(z). \tag{10}$$

and the reader is reminded that often, for a GPC type law, the P_r is not well designed and is often better replaced by its gain.

III. Model predictive control for tracking sinusoids

This section focuses on what is different when the disturbances and/or set point are of sinusoidal form. For the context of this paper these could be described as e.g. $r_k = R\sin(wk + \theta)$ or in transfer function form:

$$r(z) = \frac{\alpha(z)}{D(z)}; \quad D(z) = 1 - 2z^{-1}\cos w + z^{-2} \tag{11}$$

where $\alpha(z) = \alpha_0 + \alpha_1 z^{-1}$ defines the gain and phase and w is the frequency (rad/sample).

The previous section summarised the two key points for offset free tracking as both the predictions and the performance index must be be unbiased in the presence of the expected set points and disturbances. This was achieved by basing predictions on an appropriate incremental model and using appropriate terms in the performance index.

Consider the incremental model:

$$A(z)y = B(z)u + \frac{C(z)}{D(z)}\zeta; \quad D(z) = 1 - 2z^{-1}\cos w + z^{-2} \tag{12}$$

which implicitly allows for disturbances with a specific sinusoidal component. Change this into incremental form:

$$A(z)D(z)y = B(z)D(z)u + \zeta; \tag{13}$$

where ζ is a zero mean random variable and make the usual combination of the $D(z)$ and u terms to define an 'incremental' input as $\Delta u = D(z)u$. In this case it can be observed that:

$$\lim_{k\to\infty} \Delta u_k = 0 \quad \Rightarrow \quad \lim_{k\to\infty} u_k = E\sin(wk + \phi) \tag{14}$$

for some, as yet unknown, constants E, ϕ. Thus the asymptotic input contains the requisite sinusoidal dynamics when the incremental term converges to zero. A similar statement can be made for the output. Thus the use of model (13) for prediction incorporates the required sinusoidal dynamics while also having $\lim_{k\to\infty} \Delta u_k = 0$; it allows for unbiased prediction!

Let the performance index be similar to a standard GPC form:

$$J = \sum_{k=1}^{n_y} (r_k - y_k)^2 + \sum_{k=0}^{n_u-1} (\Delta u_k)^2 \tag{15}$$

Again, this can be seen to be unbiased asymptotically in that a zero tracking error implies $r = y$ and can also be achieved with $\Delta u = 0$; thus a zero cost and zero control increments are consistent with the desired objective.

Remark 3.1: So far the only difference between the proposed algorithm and standard GPC is the use of a different $D(z)$, that is we use $D(z) = 1 - 2z^{-1}\cos w + z^{-2}$ as opposed to the more common $D(z) = 1 - z^{-1}$ used for step type targets. The algebra for the predictions and control law take the same form as section II-B except that the matrices H, P, Q will be different as the prediction model is now (13).

Remark 3.2: The reader should note that the design will only give offset free tracking if frequency is known exactly and hence, if the frequency were to change, new controller parameters would be needed. This should not be surprising as the conventional algorithm only gave offset free tracking for the zero frequency case. If there are multiple frequencies, one could include all of these in $D(z)$, but this may introduce sensitivity issues and is not considered here. Sinusoidal type of disturbance with a known frequency can also be rejected without steady-state errors.

IV. Predictability of future set point: redesign of feed forward stage 1

One purported advantage of predictive control strategies is that, where available, information about future set point changes can be incorporated systematically. This information enters through the feedforward compensator $P_r(z)$ which is anti-causal of order n_y. Hence typical terms in the control law are:

$$P_r(z)r \equiv [P_1, P_2, ..., P_{n_y}] \underrightarrow{r}_k \tag{16}$$

In the case of sinusoidal set points (and disturbances) one major difference is necessary; as mentioned in the previous remark it is allowable, if not essential, to assume that the frequency is known. Without this assumption it would not be possible to enter the requisite controller poles into $D(z)$ to give offset free tracking. However, given this, it is also possible to predict exactly the future set point trajectory from any two values; moreover given this is a provided signal there are no issues to do with sensitivity/noise. A neat way of doing this which greatly simplifies the implementation of $P_r(z)$ is given next.

Algorithm 4.1: Reducing $P_r(z)$ to its simplest equivalent form:

1) Assume that $r(k) = R\sin(wk + \theta)$ where R, θ are not provided explicitly, but instead r_k is provided.
2) By definition, $[1 - 2z^{-1}\cos w + z^{-2}]r_k = 0 = r_k - [2\cos w]r_{k-1} + r_{k-2}$.
3) One can determine all future set points from the two most recent values using an iteration:

$$\begin{array}{rcl} r_{k+2} &=& [2\cos w]r_{k+1} - r_k \\ r_{k+3} &=& [2\cos w]r_{k+2} - r_{k+1} \\ \vdots && \vdots \end{array} \tag{17}$$

4) Simple algebra can present this in compact form as:

$$\underrightarrow{r}_{k+1} = M \underbrace{\begin{bmatrix} r_{k+1} \\ r_k \end{bmatrix}}_{\underleftarrow{r}_{k+1}} \tag{18}$$

for suitable M.

5) It is possible to reconstruct the full $\underrightarrow{r}_k$ vector from:

$$\underrightarrow{r}_k = \underbrace{\begin{bmatrix} 1 \quad 0 \\ M \end{bmatrix}}_{\hat{M}} \underleftarrow{r}_{k+1} \tag{19}$$

6) The operation of the original n_y-order anti-causal prefilter is reduced to a 1st order filter using the upcoming and past values of the set point.

$$[P_1, P_2, ..., P_{n_y}] \underrightarrow{r}_k = \underbrace{[P_1, P_2, ..., P_{n_y}]\hat{M}}_{\hat{P}_r} \underleftarrow{r}_{k+1} \tag{20}$$

In summary, the coefficients of the modified $P_r(z)$ are related to the coefficients of the original P_r as

$$\hat{P}_r = P_r\hat{M} = [\hat{P}_0, \hat{P}_1] \tag{21}$$

and the feedforward term in the predictive control law is exactly equivalent to:

$$\hat{P}_r \underleftarrow{r}_{k+1} \tag{22}$$

Again, the reader is reminded that this analysis assumes the frequency is known.

Remark 4.1: It is interesting to note that, in the case of sinusoidal set points, there is no advantage from anti-causal terms in $P_r(z)$ because these can all be subsumed into an equivalent dependence on r_{k+1} and r_k.

V. Improving transients: redesign of feed forward stage 2

This section considers how we can make better use of the flexibility in the feedforward compensator $P_r(z)$. As mentioned earlier, the claim that conventional GPC handles future input systematically is more limited than immediately obvious ([5]) and the default strategy is suboptimal due to the use of finite horizons. However, earlier work also demonstrated very clearly that transient performance during set point changes could be improved much further still by adopting a more systematic design for P_r. This paper develops a simple means of doing this for sinusoidal tracking which can be coded on elementary processors and without recourse to optimal control theory where the full trajectory would need to be modelled and embedded.

In essence this section exploits the freedom in the design of the prefilter $P_r(z)$ and shows how this freedom can be used in a very simple two stage design; thus easy to manage and implement. First a brief subsection derives the conditions on $P_r(z)$ for offset free tracking and then the next subsection shows how the freedom is exploited.

A. Conditions for asymptotic tracking

It is well known that in the case of tracking step wise set points and disturbances, the required conditions on the controller are that:

1) An integrator is placed in the forward loop (i.e. $1/D(z), \quad D(z) = 1 - z^{-1}$).

2) The steady-state gains of $P_r(z)$, $N_k(z)$ must match (i.e. $P_r(1) = N_k(1)$). Alternatively one could state that

$$N_k(z) - P_r(z) = [1 - z^{-1}]\gamma(z) \tag{23}$$

with $\gamma(z)$ any polynomial.

In the case of requiring no offset to sinusoidal signals, very similar conditions apply:

1) The two poles modelling the sinusoid are placed in the forward loop (i.e. $1/D(z)$, $D(z) = 1 - 2\cos wz^{-1} + z^{-2}$). [This happens automatically with control law (9).
2) The gains of $P_r(z)$, $N_k(z)$ must match at the specified frequency or, more simply stated:

$$N_k(z) - P_r(z) = [1 - 2z^{-1}\cos w + z^{-2}]\gamma(z) = D(z)\gamma(z) \tag{24}$$

with $\gamma(z)$ any polynomial.

Thus, $P_r(z)$ can be modified as the user pleases, without affecting the asymptotic tracking, as long as condition (24) holds. However, these modifications will clearly have an impact on transient behaviour. In fact this flexibility is akin to a Youla parameterisation [8].

Lemma 5.1: The flexibility in the prefilter can be summarised by the equation

$$P_r(z) \rightarrow \; P_r(z) + D(z)Q(z) \tag{25}$$

for $Q(z)$ any stable (causal[1]) function.

Proof: Assume that $P_r(z)$ satisfies condition (24) with $\gamma(z) = \gamma_0(z)$. Next substitute in the modified $P_r(z)$ into (24) to give:

$$\begin{aligned} N_k(z) - P_r(z) - D(z)Q(z) &= D(z)\gamma_0(z) - D(z)Q(z) \\ &= D(z)\underbrace{[\gamma_0(z) - Q(z)]}_{\gamma(z)} \end{aligned} \tag{26}$$

Clearly the condition is still satisfied. □

Hence, the user can modify $Q(z)$ to meet any objectives they wish. The next subsection explores how this could be used to reduce transient errors.

B. Optimising the free parameter in the prefilter

A simple objective would be to choose $Q(z)$ to minimise the transient errors when tracking a sinusoid which may occur at start up. For the simplicity of presentation this paper we assume a pure sinusoid signal of the form:

$$r(k) = \sin wk,\; k > 0; \quad r(k) = 0,\; k < 0 \tag{27}$$

It will be obvious from the algebra that one could also deal with signals that have a smoother transition of gradients should that be desirable. However, the 'optimal' design for transient behaviour is specific to the set point signal provided so the user would be advised to update the prefilter dynamically each time a new trajectory is required. As the reader will see, the algebra is trivial so this extra computation would not be an obstacle.

[1] In fact this restriction is not necessary, but pragmatic.

Lemma 5.2: The nominal error dynamics are given as follows:

$$e(z) = \frac{\beta_0(z)}{P_c(z)}; \quad e(z) = r(z) - y(z) \tag{28}$$

where $\beta_0(z) = [b(z)\hat{P}_r(z) - P_c(z)]/D(z)$ for the $\hat{P}_r(z)$ determined in section 3 and $r(z) = 1/D(z)$.

Proof: The closed-loop output response is defined from

$$y(z) = \frac{b(z)\hat{P}_r(z)}{P_c(z)} r(z) \tag{29}$$

However, because we have asymptotic tracking, we also know that the output breaks down to the set point signal plus some other transients, where those transients therefore form the error $e(z)$:

$$y(z) = \frac{b(z)\hat{P}_r(z)}{P_c(z)} r(z) = r(z) + \underbrace{\frac{\beta_0(z)}{P_c(z)}}_{e(z)} \tag{30}$$

for some β to be determined. For $r(z) = 1/D(z)$, a simple rearrangement of the output formulae (30) gives $b(z)\hat{P}_r(z) = P_c(z) + D(z)\beta_0(z)$ or

$$\beta_0(z) = \frac{b(z)\hat{P}_r(z) - P_c(z)}{D(z)} \tag{31}$$

There must be an exact cancellation of $D(z)$ so $\beta_0(z)$ is polynomial. □

Lemma 5.3: The flexibility in the error dynamics in relation of parameter $Q(z)$ is given as follows:

$$e(z) = \frac{\beta_0(z) + b(z)Q(z)}{P_c(z)}; \quad e(z) = r(z) - y(z) \tag{32}$$

Proof: The nominal solution β_0 was derived from

$$b(z)\hat{P}_r(z) = P_c(z) + D(z)\beta_0(z) \tag{33}$$

Substituting in $\hat{P}_r(z) \;\rightarrow\; \hat{P}_r(z) + D(z)Q(z)$ this equation is modified to

$$\begin{aligned} & b(z)[\hat{P}_r(z) + D(z)Q(z)] \\ & = P_c(z) + D(z)\beta_0(z) + b(z)D(z)Q(z) \\ & = P_c(z) + D(z)\underbrace{[\beta_0(z) + b(z)Q(z)]}_{\beta(z)} \end{aligned} \tag{34}$$

□

Corollary 5.1: The input sequence will also be affine in the parameters of Q. However, as the input is sinusoidal this sequence would only be of interest during transients and it is not immediately obvious how one would judge 'optimum' and thus include this into the optimisation of Q_i.

Theorem 5.1: The errors in transients can be minimised by minimising the signal $e(z)$. This signal comprises a fixed term $\beta_0(z)/P_c(z)$ and a free affine term $b(z)Q(z)/P_c(z)$. As the dependence on the free parameters $Q(z) = [Q_0 + Q_1 z^{-1} + Q_2 z^{-2}, ..., Q_n z^{-n}]$ is affine, the 2-norm error can be minimised using a straightforward least squares optimisation over the parameters Q_i.

Proof: Obvious. □

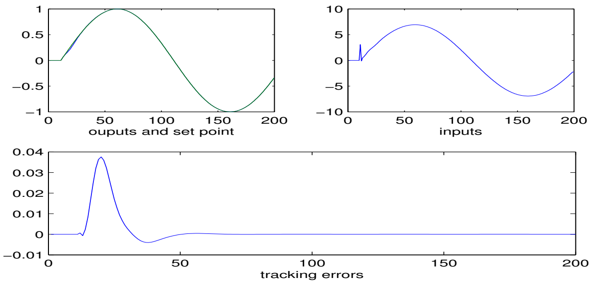

Fig. 1. Responses with a frequency of $w = 0.0314$ (rad/sample).

Remark 5.1: One could also optimise the parameters of $Q(z)$ with a view to satisfying input and output constraints for pre-specified transients, however that is not pursued in this paper.

VI. Examples

This section will first illustrate the efficacy of MPC in tracking sinusoidal targets and rejecting sinusoidal disturbances. Secondly, it will demonstrate the potential benefits of introducing the Q parameter to improve transient behaviour.

A. Model and set points

Consider the model given as

$$A(z) = 1 - 1.8z^{-1} + 0.81z^{-2};\ B(z) = 0.01z^{-1} + 0.003z^{-2} \tag{35}$$

Assume a sample time of $T = 1$ and define: (i) set point 1 with $w = 2\pi/200$ and (ii) set point 2 with $w = 2\pi/20$.

B. Asymptotic tracking and disturbance rejection

This section focuses on the efficacy of the basic algorithm using the default prefilter $\hat{P}_r$ and demonstrates that asymptotic offset tracking is achieved, in the presence of unknown disturbances.

The default prefilter $\hat{P}_r(z)$ is used with the sinusoidal set points specified above and the corresponding closed-loop responses are displayed in figures 1 and 2. It is perhaps unsurprising that the transient errors are larger with the faster frequency, however the key point in both cases is that the control clearly performs well.

Figure 3 demonstrates the case of disturbance rejection, where the disturbance is a cosine curve with a frequency $w = 2\pi/200$; again this is rejected.

C. Reducing transient errors

This section demonstrates the potential benefits of modifying the default prefilter as discussed in section V. In this case we try $Q(z)$ with various numbers of terms $n_Q = 0, 2, 4, 7$ and overlay the response plots on the same figures: figures 4,5 for slow and fast frequencies respectively. It is clear that including $Q(z)$ has greatly reduced transient errors, but there is little benefit above four terms for this case. Table I shows the 2-norms of the transient errors and backs up this message.

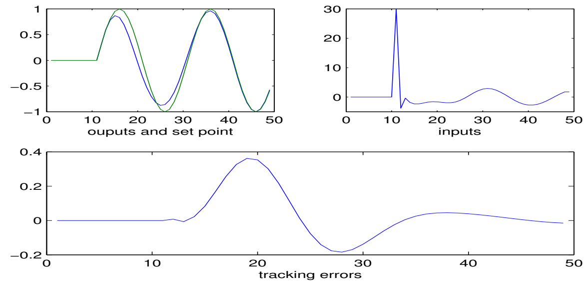

Fig. 2. Responses with a frequency of $w = 0.314$ (rad/sample).

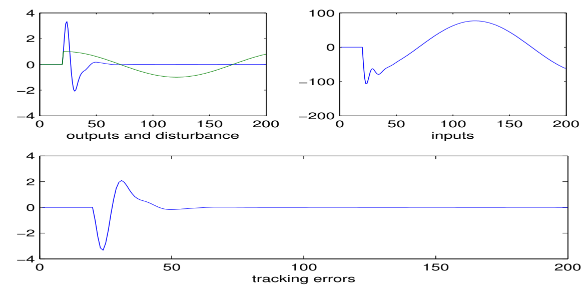

Fig. 3. Disturbance responses with a frequency of $w = 0.0314$ (rad/sample).

TABLE I

2-NORMS OF TRANSIENT ERRORS

	$n_Q = 0$	$n_Q = 2$	$n_Q = 4$	$n_Q = 7$
w=0.0314	0.0273	0.0037	0.0026	0.0026
w=0.314	1.95	0.3033	0.2117	0.2115

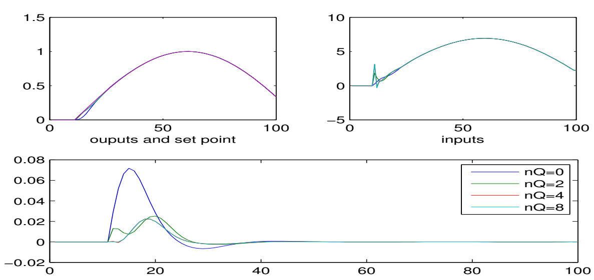

Fig. 4. Responses with a frequency of $w = 0.0314$ (rad/sample).

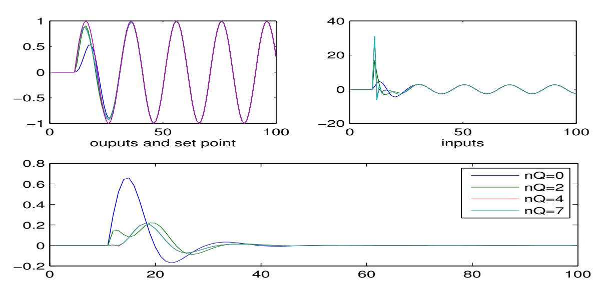

Fig. 5. Responses with a frequency of $w = 0.314$ (rad/sample).

VII. Conclusions

This paper has developed an MPC algorithm for tracking sinusoidal set points, a scenario that has been largely ignored in the predictive control community. The algorithm works well and also rejects disturbances with the same sinusoidal component. Further contributions are to show how the pre-filter can be reduced to a minimal order, without any loss of performance and also to provide simple, and therefore potentially adaptive, mechanisms for augmenting the prefilter dynamics systematically to improve transient behaviour.

This paper is an initial work and in future studies we intend to look more carefully at issues such as constraint handling, nuances with multivariable systems and also some case studies such as the cutting tool.

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