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Passivity Preserving Model Order Reduction For the SMIB

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Abstract— **We apply (linear) positive real balancing to the model of a single machine connected to an infinite bus. For that we compute the available storage and the required supply using Taylor approximation and define axis positive real singular value functions. Furthermore, we apply linear positive real balancing to the nonlinear model and analyze the results.**

I. INTRODUCTION

One of the main issues in the power systems stability/dissipativity analysis and control is the complexity of the models, see e.g. [6]. A power system consists of interconnected machines. The model of the synchronous machine consists of eight states, seven fluxes and the angular momentum and its corresponding angle, which satisfy eight nonlinear equations, see e.g [9]. The machine is usually connected through a transmission line, to a network which can be represented as an infinite bus. Such power system is called the single machine infinite bus system, as in [1], [7]. This configuration is used in transient stability studies, see [4]. However, this nonlinear model is still complicated to use as it is and model reduction is called for, [4]. In the power systems community there are two ways to simplify the SMIB, based on the physical interpretation of the behaviours of the elements that are parts of the systems, see e.g.[1], [7]. Thus, two types of models are obtained, the Flux Decay Model [4], [7] and another one based on integral manifold concepts and singular perturbation analysis [4] and references therein. However these models lack passivity. We propose another way of reducing the SMIB, based on positive real balancing for nonlinear passive systems. We compute the positive real singular value functions of the system and split the states into less and more dissipative ones. Then we project onto the space of the more/less dissipative singular values. In this paper, we calculate a nonlinear approximation of the available storage and the required supply, and then continue with the linear positive real balancing method. The method is used for determining a linear projection on a reduced order subspace, resulting in a passive reduced order system. In Section 2 we present the model of the SMIB and choose the outputs such that the system is strictly passive. In Section 3 we give a brief overview of the reduction techniques used in the power systems community to obtain the so called, Flux-Decay Model. In Section 4 we give a way to compute the passivity energy functions and define the axis positive real singular values. We also find the linear projections and obtain a reduced order model based on these linear projections. In Section 5 we give a numerical example. In the end we state some conclusions and open problems.

II. MODEL OF THE MACHINE CONNECTED TO THE INFINITE BUS

We present the model of a single machine connected to an infinite bus, e.g. [7], [1]:

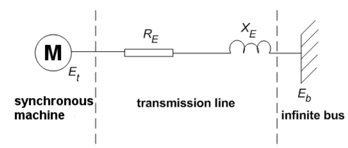


Fig. 1. A single machine connected to an infinite bus through a transmission line

The machine under consideration has one field winding, three stator windings, two q-axis amortisseur circuits and one d-axis amortisseur circuit, all magnetically coupled consisting of electrical and mechanical equations. The electrical equations are as follows

- Stator voltage equations:
$$\begin{aligned}\dot{\Psi}_d &= \omega\Psi_q + R_a i_d + e_d \\ \dot{\Psi}_q &= -\omega\Psi_d + R_a i_q + e_q,\end{aligned} \tag{1}$$
where $\Psi_d,\ e_d,\ i_d$ represent the stator flux, voltage and current in the d - axis; $\Psi_q,\ e_q,\ i_q$ are the stator flux, voltage, current in the q - axis and R_a is the stator resistance.
- Rotor field voltage equation:
$$\dot{\Psi}_{fd} = -R_{fd} i_{fd} + e_{fd}, \tag{2}$$
where $\Psi_{fd},\ i_{fd},\ e_{fd}$ represent the rotor flux due to the field circuit, field current and voltage and R_{fd} is the field circuit resistance.
- Amortisseurs rotor voltage equations
$$\dot{\Psi}_r = -R_r i_r,\ r \in \{1d, 1q, 2q\}, \tag{3}$$
with $\Psi_i,\ i_i,\ i \in \{1d, 1q, 2d\}$ the rotor fluxes due to the amortisseurs and amortisseur currents and $R_i, i \in \{1d, 1q, 2d\}$ as the amortisseurs resistances.

The mechanical (swing) equations are:

$$\begin{array}{l} j\dot{\omega} = -K_d\omega + T_m - (\Psi_d i_q - \Psi_q i_d) \\ \dot{\delta} = \omega. \end{array} \tag{4}$$

ω represents the angular velocity, δ is the rotor angle, T_m is the mechanical input power, K_D the damping constant and j the inertia of the rotor.

The network equations are obtained by applying Kirchhoff's voltage law in the figure 1 and using Park's transformation (see [1]), we have:

$$\begin{array}{l} e_d = R_E i_d - X_E\omega i_q + E_b \sin\delta + X_E \dfrac{di_d}{dt} \\ e_q = R_E i_q + X_E\omega i_d + E_b \cos\delta + X_E \dfrac{di_q}{dt}. \end{array} \tag{5}$$

R_E, X_E are transmission line resistance and reactance and E_b is the infinite bus voltage.

Substituting the network equations (5) in the stator voltage equations and denoting by $\Psi_{ds} = \Psi_d + X_E i_d$ and $\Psi_{qs} = \Psi_q + X_E i_q$, we can write the full model as:

$$\begin{array}{l} \dot{\Psi}_{ds} = \omega\Psi_q + (R_a + R_e)i_d - \omega X_e i_q + E_b \sin\delta \\ \dot{\Psi}_{qs} = -\omega\Psi_d + (R_a + R_e)i_q + \omega X_e i_d + E_b \cos\delta \\ \dot{\Psi}_{fd} = -R_{fd} i_{fd} + e_{fd} \\ \dot{\Psi}_r = -R_r i_r, \;\; r \in \{1d, 1q, 2q\} \\ j\dot{\omega} = -K_d\omega + T_m - (\Psi_d i_q - \Psi_q i_d) \\ \dot{\delta} = \omega. \end{array} \tag{6}$$

The relation between fluxes and current is given as:

$$\Psi = Li,$$

where

$$L = \begin{bmatrix} L_{ds} & 0 & L_{ad} & L_{ad} & 0 & 0 \\ 0 & L_{qs} & 0 & 0 & L_{aq} & L_{aq} \\ L_{ad} & 0 & L_{ffd} & L_{f1d} & 0 & 0 \\ L_{ad} & 0 & L_{f1d} & L_{11d} & 0 & 0 \\ 0 & L_{aq} & 0 & 0 & L_{11q} & L_{aq} \\ 0 & L_{aq} & 0 & 0 & L_{aq} & L_{22q} \end{bmatrix} > 0,$$

$$\begin{array}{l} \Psi = [\Psi_{ds} \;\; \Psi_{qs} \Psi_{fd} \;\; \Psi_{1d} \;\; \Psi_{1q} \;\; \Psi_{2q}]^T \;\; , \\ i = [-i_d \;\; -i_q \;\; i_{fd} \;\; i_{1d} \;\; i_{1q} \;\; i_{2q}]^T , \end{array} \tag{7}$$

with $L_{ds} = L_{ad} + L_l + X_e$ and $L_{qs} = L_{aq} + L_l + X_e$. Below we give the definitions of the inductances:

- L_l leakage inductance
- $L_{ai}, \; i \in \{d, q\}$ mutual inductance in the d, q axis
- L_{iiq} self inductance of rotor circuits in q axis, $i \in \{1, 2\}$
- L_{kkd} self inductance of rotor circuits in d axis, $k \in \{2, f\}$

The total energy of the system is:

$$H(i,\omega) = H(\Psi, p) = \frac{1}{2}\Psi^T i + \frac{1}{2} j\omega^2 = \frac{1}{2}\Psi^T L^{-1}\Psi + \frac{1}{2}\frac{p^2}{j}$$
$$= \frac{1}{2}[\Psi^T \;\; p] \begin{bmatrix} i \\ \omega \end{bmatrix} = \frac{1}{2}[\Psi^T \;\; p] D^{-1} \begin{bmatrix} \Psi \\ p \end{bmatrix} \geq 0, \;\; H(0) = 0, \tag{8}$$

where $D = diag\{L, j\}$ and $p = j\omega$ is the angular momentum.

Then without the dynamics of δ, system (6) can be rewritten as:

$$\begin{bmatrix} \dot{\Psi} \\ \dot{p} \end{bmatrix} = (J(\Psi, p) - R)\frac{\partial^T H(\Psi, p)}{\partial(\Psi, p)} + M(\delta)u \tag{9}$$

with

$$J(\Psi, p) = \begin{bmatrix} 0 & \omega X_E & 0 & 0 & 0 & 0 & \Psi_q \\ -\omega X_E & 0 & 0 & 0 & 0 & 0 & -\Psi_d \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\Psi_q & \Psi_d & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

skew symmetric, $\dot{\delta} = \omega = \frac{1}{j}p$,

$R = diag\{R_a + R_E, R_a + R_E, R_{fd}, R_{1d}, R_{1q}, R_{2q}, K_d\} > 0,$

$$M^T(\delta) = \begin{bmatrix} \sin\delta & \cos\delta & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ 0 & 0 & 0 & ... & 1 \end{bmatrix}, \;\; u = \begin{bmatrix} E_b \\ e_{fd} \\ T_m \end{bmatrix},$$

$\dfrac{\partial H(\Psi, p)}{\partial(\Psi, p)} = [\Psi^T \;\; p]D^{-1}$. We treat δ separately, as a parameter, since it appears only in the input vectorfield and it is in the non-minimal subspace. Making this assumption, the system with the states Ψ, p is minimal and positive real balanced truncation can be applied.

Having the total energy defined as in (8) and using equations described in (6) we have:

$$\begin{array}{l} \dot{H}(\Psi, p) = \dot{H}(i, \omega) \\ = \underbrace{-(R_a + R_E)(i_d^2 + i_q^2) - R_{fd} i_{fd}^2 - i_r^T R_r i_r - K_d\omega^2}_{\text{dissipated power}} \\ \underbrace{-E_b(i_d \sin\delta + i_q \cos\delta) + e_{fd} i_{fd} + \overline{\omega} T_m}_{\text{power supplied at terminals}}. \end{array}$$

Choosing

$$y = M^T(\delta)D^{-1}\begin{bmatrix} \Psi \\ p \end{bmatrix} + u = M^T(\delta)\frac{\partial H(\Psi, p)}{\partial(\Psi, p)} + u \tag{10}$$

then

$$\dot{H}(\Psi, p) - y^T u = -\frac{\partial H(\Psi, p)}{\partial(\Psi, p)} R \frac{\partial^T H(\Psi, p)}{\partial(\Psi, p)} - u^T u < 0, \tag{11}$$

which means that the system formed by (9), (10) with the parameter δ, is strictly passive as in e.g. [9], [11]. The feedthrough term is taken for computational reasons. It renders the problem of finding the available storage and required supply functions corresponding to the system into a nonsingular optimal control problem with the solution given by a Hamilton-Jacobi equation as it will be seen in Section IV.

Remark 1. Linearizing the model (9) around an equilibrium point $x^* = [\Psi^* \; p^*]$, we get a system with the following realization: $(A, M(\delta), M(\delta)^T D^{-1}, I_3)$, which is minimal and asymptotically stable.

III. EXISTING MODEL ORDER REDUCTION SCHEMES FOR THE SMIB

Here we review how conventional reduction is performed on (6) within the power systems community in order to obtain a model suitable for simulation and/or control as in [4], [5], [7] and references therein.

First, the dynamics of the dq fluxes, in (6), are considered small compared to the $\omega\Psi_{d,q}$ terms, so $\dot{\Psi}_d = 0,\ \dot{\Psi}_q = 0$ are taken. An assumption is also made upon the transmission line which is considered to run in steady state, rendering $\dot{\Psi}_{ds} = 0$ and $\dot{\Psi}_{qs} = 0$. Then, the dq equations become:

$$\begin{aligned}\Psi_q &= -(R_a + R_e)i_d + X_E i_q - E_b\sin(\delta)\\ \Psi_d &= (R_a + R_e)i_q + X_E i_d + E_b\cos(\delta),\end{aligned} \tag{12}$$

where $\omega[pu] = 1$ is assumed since the changes of the angular velocity are considered small and to have less significant effect on the voltages. In the power systems community this is a typical assumption used in the models for transient stability analysis and it is explained in [7] that in this way the effect of neglecting $\dot{\Psi}_d$ and $\dot{\Psi}_q$ is counterbalanced.

The second assumption is that the effect of the damper windings on the transient under study are negligible, so set $i_{1d} = i_{1q} = i_{2q} = 0$ which leads to:

$$\begin{aligned}\Psi_q &= -L_q i_q\\ \Psi_d &= -L'_d i_d + \frac{L_{ad}}{L_{ffd}}\Psi_{fd}\end{aligned} \tag{13}$$

Substituting (12) and (13) in the electrical equations of the system it is obtained that:

$$\begin{aligned}i_d = \frac{-1}{\Delta}&\left[L_{ffd}(R_a + R_E)E_b\sin\delta - L_{ad}(X_E + L_q)\Psi_{fd}\right.\\ &\left.+ L_{ffd}(X_E + L_q)E_b\cos\delta\right]\end{aligned} \tag{14}$$

with $L_q = L_{aq} + L_l,\ L_d = L_{ad} + L_l,\ L'_d = L_d - \frac{L_{ad}^2}{L_{ffd}}$, $\Delta = L_{ffd}\left[(X_E + L_q)(X_E - L'_d) + (R_a + R_E)^2\right]$. Also it can be written $i_{fd} = \frac{1}{L_{ffd}}(\Psi_{fd} + L_{ad}i_d)$. A third, typical, assumption is: $R_a + R_E = 0$. Substituting (14) in (13) the model can be written as:

$$\begin{aligned}\dot{\delta} &= \omega\\ j\dot{\omega} &= T_m - T_e - K_D\omega\\ T'_{d0}\dot{E}'_q &= \frac{(L_d - L'_d)}{(L'_d + X_E)}E_b\cos\delta - \frac{(L_d + X_E)}{(L'_d + X_E)}E'_q + E_{fd},\end{aligned} \tag{15}$$

where $T_e = \frac{1}{L'_d + X_E}E_bE'_q\sin\delta - \frac{(L_q - L'_d)E_b^2}{(X_E + L'_d)(L_q + X_E)}\sin\delta\cos\delta]$ and $E'_q = \frac{L_{ad}}{L_{ffd}}\Psi_{fd}$, $E_{fd} = \frac{L_{ad}}{R_{fd}}e_{fd}$ and $T'_{d0} = \frac{L_{ffd}}{R_{fd}}$. The equations described in (15) are known as the Flux-Decay Model. This model can be rewritten in a compact form [4]:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -b_1\sin x_1 + b_2\sin x_1\cos x_1 - Dx_2 + P\\ \dot{x}_3 &= b_3\cos x_1 - b_4x_3 + E + u,\end{aligned} \tag{16}$$

with $x_1 = \delta$, $x_2 = \omega$, $x_3 = E'_q$, u is an additional signal representing the control input and

$$b_1 = \frac{E_b}{j(L'_d + X_E)},\ b_2 = \frac{\omega_0E_b^2(L_q - L'_d)}{(L'_d + X_E)(L_q + X_E)}$$
$$b_3 = \frac{(L_d - L'_q)E_b}{(T'_{d0})(L'_d + X_E)},\ b_4 = \frac{L_d + X_E}{(T'_{d0})(L'_d + X_E)},\ P = \frac{T_m}{j},$$
$$E = \frac{E_{fd}}{T'_{d0}},\ D = \frac{K_D}{j}.$$

Remark 2. In [4] an alternative way to obtain the Flux Decay Model (15) is presented. The following assumptions are made: only one damping circuit in the q axis, i.e. the $1d$ and $2q$ equations from (6) are considered negligible; the damping circuit is expressed in terms of its proportional voltage E'_{ds} and there is no damping in the swing equation. Then the model is first reduced to 5 states and is given as in [4, Section 2.5, (2.19)-(2.24)]. Then assuming that $R_a + R_E = 0$, then $\Psi_{ds} = E_b\cos\delta,\ \Psi_{qs} = E_b\sin\delta$ defines an integral manifold for the (2.19)-(2.24) system. Substituting, the remaining differential equations yield the so called Two-Axis model, of dimension four. Then doing a singular perturbation analysis on one of the states of the Two-Axis model, yields the Flux Decay Model.

IV. ENERGY FUNCTIONS FOR THE SYSTEM

In this section we calculate a nonlinear approximation of a pair of energy functions necessary for the positive real balancing procedure, namely, the available storage and the required supply, of the system (9), see e.g. [8] for more details. They are the total internal energy plus or minus the dissipated energy of the system. However it is difficult to quantify the dissipated energy, so there is need for a computation scheme. For the sequel, we make the following notations: $x = [\Psi^T,\ p]^T$, with Ψ defined in Section II, (7) and $p = j\omega$. We write $f(x) = (J(x) - R)\frac{\partial^T H(x)}{\partial x}$, where $J(x),\ R$ and $H(x)$ are defined in Section II, (9) and choose the output from relation (10).

Definition 3. The available storage of a passive system with is given by:

$$S_a(x_0, u^Ty) = -\inf_{u,x(0)=x_0}\int_0^T u^Ty\,dt \tag{17}$$

The required supply of the system is:

$$S_r(x_0, u^Ty) = \inf_{u,x(0)=x_0}\int_{-T}^0 u^Ty\,dt \tag{18}$$

□

Remark 4. If the passive system has dissipation and the total internal energy H, then $S_a = H -$ dissipated energy and $S_r = H +$ dissipated energy. □

We will consider system (9) with δ as a parameter and we assume the equilibrium point $x = 0, u = 0$. Since the system is strictly passive, we have from (11) that for $u = 0$: $H(x) > 0,\ H(0) = 0$ and $\dot{H}(x) < 0$, and so by Lyapunov's second method, 0 is an asymptotically stable equilibrium point of the system. Then, the model (9) with the above notations is liable for applying the results in [8].

Proposition 5. *The strictly passive system (9) has the available storage $S_a(x)$ and the required supply $S_r(x)$ as the smooth stabilizing and antistabilizing, respectively, solutions of the Hamilton-Jacobi equation:*

$$\begin{aligned} &\frac{\partial S}{\partial x} f(x) + \frac{1}{2}\left(\frac{\partial S}{\partial x} M(\delta) - x^T D^{-1} M(\delta)\right) \cdot \\ &\quad \cdot \left(M^T(\delta)\frac{\partial S^T}{\partial x} - M(\delta) D^{-1} x\right) = 0 \end{aligned} \tag{19}$$

□

We compute the solutions of this PDE using an the approximation method of Lukes, based on Taylor expansion around the equilibrium point. We split the storage functions into a sum of a quadratic term plus higher order terms, making the following notations with respect to system (9):

$$F_1(x) = Ax + F^{(2)}(x),$$
$$S_i(x) = \frac{1}{2} x^T K x + S_i^{(h)}(x),\ i \in \{a, r\}$$

where h represent higher order terms. Substituting, equation (19) splits in two parts: an algebraic Riccati one for the quadratic part of S and a polynomial algebraic system for the higher order terms. For the quadratic part of the required supply the corresponding Riccati equation is:

$$AK_r + K_r A^T + (K_r - D^{-1}) M(\delta) M^T(\delta)(K_r - D^{-1}) = 0 \tag{20}$$

For the quadratic part of the available storage, we solve the dual algebraic Riccati equation:

$$A^T K_a + K_a A + (K_a D^{-1} - I) M(\delta) M^T(\delta)(D^{-1} K_r - I) = 0, \tag{21}$$

where the variables are as in Remark 1.

The higher order terms satisfy the following equation for all $h > 3$ is:

$$\begin{aligned} \mathcal{S}_i(x) F_1^{(h)}(x) = &-\frac{1}{2}\mathcal{S}_i^{(h-1)}(x)(\mathcal{M}_1)\mathcal{S}_i^{(h-1)T}(x) \\ &- \mathcal{S}_i^{(h-1)}(x)\mathcal{A}x, \end{aligned} \tag{22}$$

where $i \in \{r, a\}$, $\mathcal{A} = A + M(\delta)M(\delta)^T(K - D^{-1})$ and $\mathcal{S}_i(x) = \frac{\partial S_i}{\partial x}$. We can write $\mathcal{S}_i(x) = x^T K_i + \mathcal{S}_i^{(2)} + \mathcal{S}_i^{(3)+\dots} = x^T K_i + \mathcal{S}_i^{(h-1)}, i \in \{r, a\}$. The higher order terms of the gradient $\mathcal{S}_i(x)$ have the upper index $h-1$, since the elements of the vector are polynomials in x of order $h-1$. Then (22) is a system of polynomial equalities which is solved by finding the solution of a linear system of coefficients. For instance for $h = 3$, we can write $S_r^{(3)}(x) = \sum_{i=1, j=i, k=j}^{8} x_i x_j x_k$, which means that all the terms in $\mathcal{S}_r^{(2)}(x)$ have degree 2. The equation to be solved for $h = 3$ is:

$$x^T K_r F^{(2)}(x) = -\mathcal{S}_r^{(2)}(x)\mathcal{A}x, \tag{23}$$

yielding a linear system of equations with $c_{i,j,k}$ as unknowns.

Following the reasoning in [3] for general nonlinear systems, we define, for the strictly passive system (9) the axis positive real singular values as:

$$\rho_i(s) = \sqrt{\frac{S_a(\xi_i(s))}{S_r(\xi_i(s))}},\ x = \xi_i(s) \tag{24}$$

such that

$$\sup_s \rho_1(s) = \sup_x \sqrt{\frac{S_a(x)}{S_r(x)}}.$$

This represents the maximum energy available at the ports with respect with to the effort energy put in through the same ports. We consider the system in S_r normal S_a diagonal form if there exists $\xi_i(s)$, $i = 1, \dots, n$ such that:

$$S_r(\xi_i(s)) = \frac{s^2}{2},\ S_a(\xi_i(s)) = \frac{s^2 \rho_i(s)}{2}. \tag{25}$$

These axis singular values and energy functions in this form are used for model reduction. Computing $\Phi(z) = x$, s.t. $\Phi(0, \dots, z_i, \dots, 0) = \xi_i(z_i)$, then, if $\rho_i(s) \gg \rho_{i+1}(s)$ we have that $\rho_i^2(z_i) > \rho_{i+1}^2(z_{i+1})$, meaning that the state components $z^1 = [z_1,\ \dots,\ z_i]$ are less dissipative than the states $z^2 = [z_{i+1},\ \dots,\ z_n]$. Based on this ordering, we can retain for the reduced model either z^1 or z^2 setting $z^2 = 0$ or $z^1 = 0$ (projecting on z^1, or z^2), respectively, according to the use and accuracy of it.

V. LINEAR TRANSFORMATIONS AND TRUNCATION

In this section we continue with the linear part of the available storage and required supply approximated in the previous section, that is we use K_r solution of (20) and K_a, the solution of (21), respectively. We compute the linear positive real singular values of the strictly passive system (9): $1 \geq \pi_1 > \pi_2 > \dots > \pi_n > 0$.

Let q be such that $\pi_q \gg \pi_{q+1}$. Then the reduced order state is the projection of the original state on the space spanned by the eigenvectors corresponding to the largest singular values, i.e the less dissipative, or to the smallest singular values,i.e the least dissipative. The result is a strictly passive reduced order system. We proceed as in [2]. Since K_r and K_a are positive definite, let $K_r = UU^T$, $K_a = LL^T$. Let $\Sigma_n = diag(\pi_1, \pi_2, \dots \pi_n)$ and the singular value decomposition $U^T L = Z\Sigma_n Y^T$. Let $W = LY_q\Sigma_q^{\frac{1}{2}}$, $V = UZ_q\Sigma_q^{-\frac{1}{2}}$, where $Z_q = Z(:, 1:q)$ and $Y_q = Y(:, 1:q)$. The product $VW = \Pi$ defines a projector and $W^T V = I_q$. Then $x = Vz$, where z has dimension q and is the reduced order model state and $z = W^T x$. Substituting in (9) we obtain: $V\dot{z} = f(Vz) + Mu,\ y = M^T D^{-1} V z$. Premultiplying with W^T we obtain a reduced passive nonlinear order model given by:

$$\begin{aligned} \dot{z} &= W^T f(Vz) + W^T M u \\ \widetilde{y} &= M^T(\delta) D^{-1} V z \end{aligned} \tag{26}$$

VI. CASE STUDY

Consider a system as in figure 1, described by the equations (9) with δ considered an external parameter. We have the following parameters for the machine, i.e. matrix $D = diag\{L, j\}$, $\Psi = Li$ and the dissipation matrix R, taken from e.g. [7]:

$$D = \begin{bmatrix} 0.22 & 0 & 0.01 & 0.01 & 0 & 0 & 0 \\ 0 & 0.219 & 0 & 0 & 0.009 & 0.009 & 0 \\ 0.01 & 0 & 1.825 & 1.660 & 0 & 0 & 0 \\ 0.01 & 0 & 1.660 & 1.8313 & 0 & 0 & 0 \\ 0 & 0.009 & 0 & 0 & 0 & 0.009 & 0 \\ 0 & 0.009 & 0 & 0 & 0.009 & 0.134 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 \end{bmatrix},$$

$$R = diag\{0.031, 0.031, 0.0006, 0.0284, 0.00619, 0.023638, 10\}$$

We consider 0, as an equilibrium point which is asymptotically stable as presented in Section IV. Linearizing around this equilibrium point we obtain a minimal asymptotically stable linear realization with δ as a parameter. Solving (20), (21) and (23) and we get:

$$\begin{aligned} S_r(x) &= \frac{1}{2}x^T K_r x + S_r^{(3)}(x) = 0.105x_1^2 + 0.0041x_7^2 + \\ &0.188x_6^2 + 0.289x_2^2 - 0.095x_1x_2x_7 - 0.306x_1x_3 + \\ &0.066x_1x_4 - 0.104x_2x_5 - 0.41x_2x_6 - 0.31x_3x_4 + \\ &0.0348x_5x_6 + 0.0276x_5^2 + 0.102x_4^2 + 0.0655x_1x_6x_7 + \\ &0.001x_4x_5x_7 + 0.00183x_3x_5x_7 - 0.0172x_2x_4x_7 + \\ &0.00767x_1x_5x_7 + 0.00852x_2x_3x_7 + 0.00801x_4x_6x_7 + \\ &0.0102x_3x_6x_7 \end{aligned} \tag{27}$$

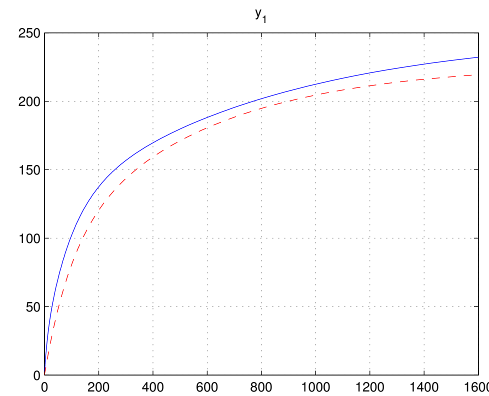
and

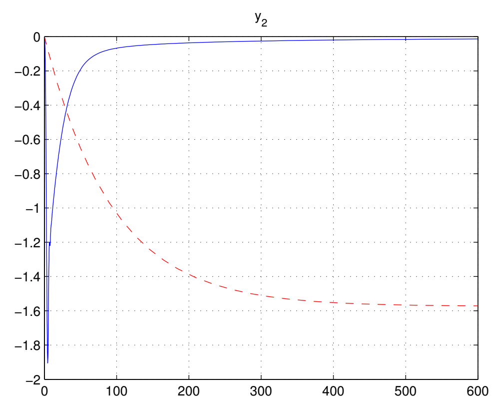
$$\begin{aligned} S_a(x) &= \frac{1}{2}x^T K_a x + S_a^{(3)}(x) = 2.1x_1^2 + 61x_7^2 + 0.67x_6^2 + \\ &0.441x_3^2 + 0.466x_2^2 + 1.26x_4^2 + 1.38x_5^2 + 0.182x_1x_3 - \\ &2.56x_1x_4 - 0.624x_2x_5 + 0.486x_2x_6 - 0.492x_3x_4 - 1.2x_5x_6 \\ &- 0.239x_1x_2x_7 - 2.53x_1x_6x_7 + 0.00213x_4x_5x_7 + \\ &0.00249x_3x_5x_7 - 0.438x_1x_5x_7 - 0.0312x_4x_6x_7 - \\ &0.0349x_3x_6x_7. \end{aligned} \tag{28}$$

We continue with the quadratic part of the solutions and we compute the linear positive real singular values of the system: $\pi_1 = 0.9715$, $\pi_2 = 0.9638$, $\pi_3 = 0.8204$, $\pi_4 = 0.5386$, $\pi_5 = 0.2413$, $\pi_6 = 0.1946$, $\pi_7 = 0.0693$. It is noticed that starting with π_4 the decay rate of the singular values is higher, thus we can choose $\pi_3 >> \pi_4$. So π_1, π_2, π_3 correspond to the less dissipative components (currents). We are going to project onto the space of these singular values. The following projection matrices are built:

$$W = \begin{bmatrix} -0.2419 & 0.2899 & -1.4972 \\ 0.0667 & -0.5219 & -0.4720 \\ -0.4359 & 0.1949 & 1.6490 \\ -0.2943 & 0.2477 & -0.2783 \\ -0.0629 & -0.3454 & 0.2 \\ -0.509 & -0.4190 & 0.3421 \\ 0 & 0 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} -0.9157 & 0.1062 & -0.3254 \\ -0.5326 & -0.6791 & -0.1646 \\ -0.9652 & 0.1568 & 0.2445 \\ -0.8922 & 0.1209 & -0.0713 \\ -0.5502 & -0.8135 & 0.0689 \\ -0.4932 & -0.6523 & -0.0052 \\ 0 & 0 & 0 \end{bmatrix}$$

Computing $x = Vz$ we notice that $x_7 = p = 0$, meaning that the angular momentum does not have any influence in the reduced order model , since it is the most dissipative. The reduced order model has 3 states in this case and with the output $\widetilde{y}_3 = 0$ that does not approximate $y_3 = \frac{1}{j}p$, at all. Simulating this model for $u = [E_b \; e_{fd} \; T_m]^T = [1 \; 0 \; 1]^T$ we obtain:





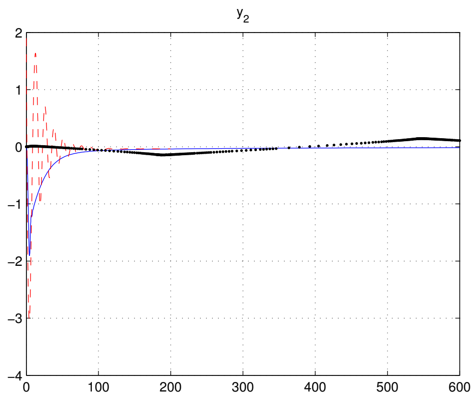
The dashed line represents the reduced order model output $\widetilde{y}$ that approximates y, represented by the continuous line. It is noted that due to very little dissipation present and the truncation of the mechanical swing equations, the second output $\widetilde{y}_2$ has a long transient component. The large unwanted error is also noticed. However, $\widetilde{y}_1$ is close to y_1 which depends on the least dissipative states. Alternatively, we explore the smaller singular values corresponding to the more dissipative states, that is $\pi_6 = 0.1946 < \pi_5$ and $\pi_7 = 0.0693 < \pi_6$ and build the corresponding projections:

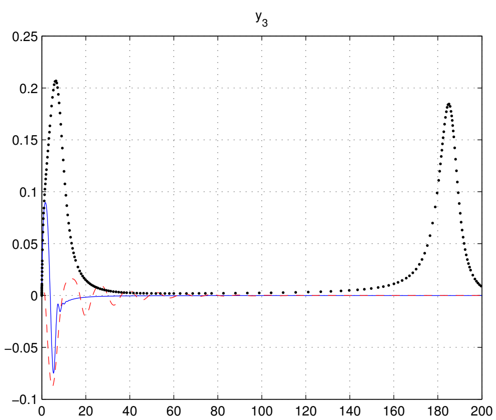
$$W = \begin{bmatrix} 0 & -1.02 \\ 0 & 1.23 \\ 0 & -0.2577 \\ 0 & 1.3115 \\ 0 & 0.2688 \\ 0 & -1.6017 \\ 0.4082 & 0 \end{bmatrix}, V = \begin{bmatrix} 0 & -0.1712 \\ 0 & 0.1476 \\ 0 & -0.0364 \\ 0 & 0.1942 \\ 0 & 0.0328 \\ 0 & -0.2314 \\ 2.4495 & 0 \end{bmatrix}$$

Substituting, we get the following passive reduced order model with the parameter δ:

$$\begin{aligned}&\dot{z}_1 = -0.00706z_2^2 - 10z_1 + 0.408T_m\\&\dot{z}_2 = -0.0175z_2 + 0.31z_1z_2 - \sin(\delta)E_b + 1.23\cos(\delta)E_b-\\&0.258e_{fd}\\&\widetilde{y}_1 = -0.171\sin(\delta)z_2 + 0.149\cos(\delta)z_2 + E_b\\&\widetilde{y}_2 = -0.0357z_2 + e_{fd}\\&\widetilde{y}_3 = 2.45z_1 + T_m\end{aligned} \tag{29}$$

For the simulation the input vector is taken $u = [E_b\ e_{fd}\ T_m]^T = [1\ 0\ 1]^T$ and we let the parameter δ vary according to the equation $\dot{\delta} = x_7 = 14.7z_1$.





The continuous line represents the output y of the full model, the dashed line is the output $\widetilde{y}$ of reduced model obtained in (29) and the dotted line represents (15). It is noticed that $\widetilde{y}_3$ of the reduced model captures the behaviour of $y_3 = \frac{1}{j}p$, however there are certain error and oscillations in the transients as can be seen in the figure above. The linear term (damping) is dominant in the first equation of (29) and this is reflected in the output $\widetilde{y}_3$. The transient in $\widetilde{y}_2$ has large oscillations which are not desired, but it approximates quite well the output y_2, the field current. $\widetilde{y}_1$ is not a satisfactory approximation of y_1. Basically, the reduced order model obtained by truncating the less dissipative states retains the mechanical part as the most important dynamics of the system, the idea being similar to the one used in the models of the power systems community, different from the model obtained projecting on the less dissipative subspace, which consists only of electrical equations and where the mechanics has no influence on the field behaviour. Still, (15) performs better than (29) in the electrical equation. We conjecture that it will be the same case when the nonlinear singular value functions will be computed and the transformations will be nonlinear. This is future work, since the computations are quite involved.

VII. CONCLUSIONS AND FUTURE WORK

In this paper we present another way to reduce the SMIB. It is based on the positive real balancing, which is a passivity preserving model order reduction technique. The reduced model is the projection of the original model on either the less or the more dissipative positive real singular values. The latter yields a third order model containing the swing equation and an electrical equation too and the reduced order model is passive. However, the transformations used here are linear. This represents a starting point for the future work where a procedure to completely compute the nonlinear positive real axis singular value functions will be constructed and consequently the nonlinear projections will be performed, in order to obtain a fully nonlinear model order reduction procedure for the SMIB.

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REFERENCES

[1] P. M. Anderson, A. A. Fouad, *Power Systems Control and Stability*, IEEE Press, New York, 1994.

[2] A. C. Antoulas, *Approximation of Large-Scale Dynamical Systems*, SIAM (2005).

[3] K. Fujimoto, J. M. A. Scherpen, Nonlinear Input-Normal Realizations Based on the Differential Eigenstructure of Hankel Operators, *IEEE Trans. On Aut. Control*, vol. 50, 2005, pp. 2-18.

[4] M. Galaz, *Nonlinear Control of Power Systems*, PhD thesis, LSS, Supelec France, 2003.

[5] M. Galaz, R. Ortega, A. Bazanella, Transient stabilization of power systems via total energy shaping: A comparative simulation study with the classical scheme, *Proc. of CLCA*, December 2002.

[6] A. Giusto, R. Ortega, A. Stankovic, On Transient Stabilization of Power Systems: A Power-Shaping Solution for Structure-Preserving Models, *Proc. of 45th IEEE CDC*, 2006, pp. 4027-4031.

[7] P. Kundur, *Power Systems Stability and Control*, McGraw-Hill, 1994.

[8] T. C. Ionescu, J. M. A. Scherpen *Positive Real Balancing for Nonlinear Systems*, SCEE Proceedings, 2006.

[9] R. Ortega, A. Stankovic, P. Stefanov, A passivation approach to power systems stabilization, *IEEE Symp. on Nonlinear Control Sysyems*, 1998.

[10] E. Pai, *Energy Function Analysis for Power Systems Stabilization*, Kluwer Academic Publishers, Boston, 1989.

[11] A. J. van der Schaft, *L_2-gain and Passivity Techniques in Nonlinear Control*, Springer-Verlag, 2000.