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Passification of Linear Systems with Respect to Given Output

Alexander Fradkov
Institute for Problems of Mechanical Engineering,
Russian Academy of Sciences,
61 Bolshoy ave. V.O., St. Petersburg, 199178, RUSSIA
Fax : +7 (812) 321-4771, E-mail : fradkov@mail.ru

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ABSTRACT

The concepts related to passifiability (feedback passivity) of systems with the given output are introduced. They are intended to study passifiability of systems with respect to output y by output feedback based on output y_1 for the case when y_1 differs from y (extended passifiability). Necessary and sufficient conditions for extended passifiability of linear systems by linear output feedback are established. Links of the obtained results to the high gain stabilization are shown and an extension to passifiability with different inputs is given. The proofs are based on a version of the celebrated Yakubovich-Kalman-Popov Lemma (KYP-lemma) and Meerov's results on high-gain stabilization [1]

I. INTRODUCTION

During the last decade there was a growing interest in passification (sometimes called passivation) control design methods. A special attention was attracted to feasibility conditions for passification and to *passifiable* or *feedback passive* systems — ones that can be made passive by means of state or output feedback [8], [17], [24], [27], [30], [31], [32]. Such passifiability conditions are of interest even for linear systems: they are used not only for linear feedback design, but also for passivity-based design of cascade nonlinear systems [30], [31]. Since passivity of a linear system is equivalent to positive realness of its transfer function, the linear passifiable (strictly passifiable) systems were also called "almost positive real" ("almost strictly positive real") [22], [33]. The conditions for passifiability by output feedback are of utmost importance for applications.

Necessary and sufficient conditions for passifiability of linear systems by linear output feedback (or existence of a linear feedback rendering system strictly positive real) were proposed still in the 1970s, see [10] for SIMO systems and [11] for MIMO systems. Later they were rediscovered and applied for SISO systems [34], [39] and for MIMO systems [1], [19], [20]. State feedback case was considered in [23], [30]. In a number of works the problem of positive real synthesis for systems with feedthrough (relative degree zero case) was considered, see [35] and references therein. The obtained results have applications in robust control [1], [19], [34], [39], adaptive control [9], [12], [17], [20], [22], [36], stabilization of partially linear cascaded systems [23], [30], [31]. Note that the adaptive output feedback control algorithms proposed in [20] (and recalled in [24]) coincide with those of [11]. Surveys of this area can be found in [2], [5], [13].

In this paper the passifiability of systems with respect to output y by output feedback based on output y_1 for the case when y_1 differs from y is studied. Necessary and sufficient conditions for passifiability of linear systems by linear output feedback are given. Loosely speaking, it is established that a system is strictly passifiable if and only if the ratio of the transfer functions for y_1 and for y is strictly positive real. To the best of the author's knowledge all existing passification and passifiability results are developed for the case when y_1 coincides with y. An extension to passifiability with different inputs is given.

The formulation of the problem and some auxiliary concepts are introduced in Section 2. The main results are presented in Section 3, while some extensions and applications are discussed in Section 4.

II. PASSIVITY AND PASSIFICATION

Consider an affine in control system

$$\dot{x} = f(x) + g(x)u, \qquad y = h(x), \tag{1}$$

where $x = x(t) \in \mathbb{R}^n$, $u = u(t) \in \mathbb{R}^m$, $y = y(t) \in \mathbb{R}^l$ are state, input and output vectors, respectively, f, h are smooth vector-valued functions of x and g is smooth matrix-valued function of x. In some applications (e.g. in control of quantum-mechanical systems) the case with complex-valued variables and parameters ($x = x(t) \in \mathbb{C}^n$, $u = u(t) \in \mathbb{C}^m$, $y = y(t) \in \mathbb{C}^l$) is also important. Such a case will be called *complex case* while the case of real-valued variables and parameters will be referred to as *real case*.

We denote conjugate matrix (transposed matrix A with complex conjugate elements) as A^*, Euclidean norm of

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a vector or matrix as $|x|$, degree of a polynomial φ as $\deg\varphi$. Notation $\mathrm{col}(x_1,\dots,x_N)$ means that x is a column vector composed of all components of $x_1,\dots,x_N$. Most definitions and facts of this section can also be found in [11], [13].

Definition 1. Let G be a prespecified $m\times l$-matrix. System (1) is called *G-passive* if there exists a nonnegative scalar function $V(x)$ (*storage function*) such that

$$V(x) \le V(x_0) + \int_0^t u(t)^* G y(t)\,dt \tag{2}$$

for any solution of the system (1) satisfying $x(0) = x_0,\ x(t) = x$. In (2) and below the asterisk denotes transposition of the matrix and complex conjugation of its elements which is just transposition in real case.

Definition 2. System (1) is called *strictly G-passive*, if there exist a nonnegative scalar function $V(x)$ and a scalar function $\mu(x)$, where $\mu(x) > 0$ for $x \ne 0$, such that

$$V(x) \le V(x_0) + \int_0^t \left[u(t)^* G y(t) - \mu(x(t))\right]\,dt \tag{3}$$

for any solution of system (1) satisfying $x(0) = x_0, x(t) = x$.

Obviously, if $l = m$ and $G = I_m$ is identity matrix, then G-passivity coincides with conventional passivity property. In general case introduction of G allows designer to balance inputs and outputs and to increase flexibility of controller design.

In this paper we will be dealing with strict version of G-passivity property for linear systems

$$\dot{x} = Ax + Bu, \qquad y = Cx, \tag{4}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^l$, A, B, C are matrices of appropriate size. For linear systems the storage function $V(x)$ is quadratic form $V(x) = 0.5x^*Hx$ (or Hermitian form in complex case), while function $\mu(x)$ is just Euclidean norm of the vector: $\mu(x) = \mu|x|^2,\ \mu > 0$.

In turn, passivity is closely related to *hyperstability*, introduced by V.M.Popov [28].

If the storage function $V(x)$ is smooth, the integral dissipation inequalities (2), (3) are equivalent to their differential forms. For a nonlinear system (1) integral inequality (3) is equivalent to fulfillment of the differential dissipation inequality

$$\frac{\partial V}{\partial x}(f(x) + g(x)u) \le u^* G y - \mu(x). \tag{5}$$

For linear system (4) and for quadratic storage function $V(x) = 0.5x^*Hx$ integral inequality (3) is equivalent to

$$x^* H(Ax + Bu) \le u^* G y - \mu|x|^2 \tag{6}$$

for some $\mu > 0$ and all $x \in \mathbb{R}^n, u \in \mathbb{R}^m$.

In its turn, dissipation inequality (5) is equivalent to the relations

$$\frac{\partial V}{\partial x} f(x) \le -\mu|x|^2, \quad \frac{\partial V}{\partial x} g(x) = (Gh(x))^*, \tag{7}$$

while inequality (6) is equivalent to the following matrix relations

$$HA + A^*H < 0, \qquad HB = (GC)^*. \tag{8}$$

The solvability conditions for (8) and related versions of the dissipation inequalities for linear systems are given by the seminal Yakubovich–Kalman–Popov or Kalman–Yakubovich-Popov (KYP) lemma (for this special case also called positive real lemma). In what follows we need the "semi-singular" version of the KYP lemma established by V.A.Yakubovich [38] in 1966. Introduce the following notations[2]:

$$G = HA + A^*H + R,\ g = -Ha - b,\ Q(H) = \begin{bmatrix} -G & g \\ g^* & \varrho \end{bmatrix},$$

$$\pi(s) = \varrho + 2\mathrm{Re}\, b^*(sI_n - A)^{-1}a + a^*(s^*I_n - A^*)^{-1}R(sI_n - A)^{-1}a,$$

where $H = H^*$ is $n\times n$-matrix, $R = R^*$ is $n\times n$-matrix, $\varrho = \varrho^*$ is $m\times m$-matrix, a, b are $n\times m$-matrices. Let $m = m_1 + m_2$, where m_1, m_2 are integer numbers and let matrices ϱ, π, a be split into the corresponding blocks as follows:

$$\varrho = \begin{bmatrix} \varrho_{11} & \varrho_{12} \\ \varrho_{21} & \varrho_{22} \end{bmatrix},\ \pi = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix},\ a = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

and $\varrho_{12} = \varrho_{21}^* = 0,\ \varrho_{22} = 0$.

Theorem 1 (Yakubovich, 1966). Let A be a Hurwitz matrix, $\varrho_{11} \ge 0$ and $\mathrm{rank}\, a_2 = m_2$. Necessary and sufficient conditions for existence of matrix $H = H^*$ such that $Q(H) \ge 0$ and $\mathrm{rank}\, Q = n + m_1$ are

(1) $\pi(j\omega) > 0$ for all real ω; $(j = \sqrt{-1})$

(2) $\lim\limits_{\omega\to\infty} \omega^2(\pi_{22}(j\omega) - \pi_{21}(j\omega)\,\pi_{11}^{-1}(j\omega)\,\pi_{12}(j\omega)) > 0$.

Note that for $m_2 = 0$ Theorem 1 turns into "nonsingular" KYP lemma, while for $m_1 = 0,\ C = 0$ it states that solvability of matrix inequalities $HA + A^*H < 0, Ha = -b$ is equivalent to SPR property of the matrix function $b(sI_m - A)^{-1}a$.

Let $y_1 = h_1(x) \in \mathbb{R}^{l_1}$ be another output of the system. Introduce the following *extended passification problems.*

Problem A1. Find m-vector function $\mu(y_1)$ and $m\times m$-matrix function $\nu(y_1)$ such that system (1) with the output feedback

$$u = \mu(y_1) + \nu(y_1)v, \tag{9}$$

where $v \in \mathbb{R}^m$ is new input, is strictly G-passive with respect to output y.

Problem B1. Find m-vector function $\mu(y_1)$ such that system (1) with output feedback (9) is strictly G-passive

[2]Notation $\mathrm{Re}\,K$ stands for Hermitian part of the matrix: $\mathrm{Re}\,K = (K + K^*)/2$.

with respect to output y with fixed $m\times m$-matrix function $\nu(y_1)$.

For linear systems a linear passifying feedback is considered instead of (9).

The extended passification problems are formulated as follows.

Problem AL1. Find $m \times l$-matrix K and $m \times m$-matrix L such that system (4) with the output feedback

$$u = -Ky_1 + Lv, \tag{10}$$

where $y_1 = C_1 x \in \mathbb{R}^{l_1}$ is the second output, $v \in \mathbb{R}^m$ is a new input, $\det L \neq 0$, is strictly G-passive with respect to the output y.

Problem BL1. Find $l \times m$-matrix K such that system (4) with the output feedback (10) is strictly G-passive with fixed matrix L with respect to the output y.

For complex case all the variables and functions in (9), (10) are complex valued. G-passification gives rise to *G-passifiability* problems: checking solvability of the Problems A1,B1, AL1, BL1.

In this paper the passification and passifiability problems AL1, BL1 for linear systems are studied. Obviously, if a linear system is asymptotically stabilizable by an output feedback (10), then it is strictly passifiable with respect to the output $y = B^T Px$, where $P = P^T > 0$ is the matrix of the Lyapunov function establishing its stability. However, if the output y is specified in the problem statement, the problem is not so trivial. Note that for $C = C_1$ the problems AL1, BL1 coincide with problems AL, BL, formulated and solved in [13].

III. MAIN RESULTS

In order to formulate the solutions to the above problems, introduce the following notations:

$\delta(s) = \det(sI_n - A)$, $W(s) = C(sI_n - A)^{-1}B$,$\delta(s,K) = \det[sI_n - A(K)]$, $W(s,K) = C[sI_n - A(K)]^{-1}B$, where K is $m \times l$-matrix, $A(K) = A - BKC_1$. Obviously, $\delta(s,K)$ and $W(s,K)$ are characteristic polynomial and transfer matrix, respectively, of the system (4) closed with the feedback

$$u = -Ky_1 + v. \tag{11}$$

Similarly, notations $W_1(s) = C_1(sI_n - A)^{-1}B, W_1(s,K) = C_1[sI_n - A(K)]^{-1}B$, are introduced. Let $\delta(s) = \lambda^n + \delta_{n-1}\lambda^{n-1} + \dots$.

It is easy to show that the following identities are valid:

$$\delta(s,K) = \delta(s)\det[I_m + KW_1(s)], \tag{12}$$

$$W(s,K) = W(s)[I_m + KW_1(s)]^{-1}. \tag{13}$$

Similar identities hold if W is replaced with W_1.

Let G be an $m \times l$-matrix. Define a polynomial $\varphi(s) = \delta(s)\det GW(s)$ and a matrix $\Gamma = \lim_{s\to\infty} sGW(s)$. The meaning of the parallel notations $\varphi_1(s), \Gamma_1$ is obvious. It is easy to show, see [11], [13] that $\varphi(s), \varphi_1(s)$ are polynomials of degree not exceeding $n - m$, invariant with respect to feedback transformation (11). Since $\Gamma = GCB, \Gamma_1 = GC_1B$, the $m \times m$ matrices Γ, Γ_1 are also invariant with respect to the feedback transformation (11).

Definition 3. The system (4) is called *G-minimum phase* if the polynomial $\varphi(s)$ is Hurwitz (its zeros belong to the open left half-plane). It is called *strictly G-minimum phase* if it is minimum phase and $\det\Gamma \neq 0$, and *hyper minimum phase* if it is minimum phase and $\Gamma = \Gamma^* > 0$.

Since the above terms are correctly defined using the transfer matrix of the system (4), it is no abuse to use same terms as related to the transfer function itself.

It is worth noticing that for square systems the introduced terms are related to the frequently used notion of minimum phaseness defined via *transmission zeros* of the system (4). Namely, for $m = l$ and $G = I_m$ the roots of the polynomial $\varphi(s)$ coincide with transmission zeros since $\varphi(s) = \det\Sigma$, where Σ is the Rosenbrock matrix:

$$\Sigma = \begin{bmatrix} sI_n - A & -B \\ C & 0 \end{bmatrix}.$$

For the non-square case the notions are different, see example in [13].

In this paper only the case $l = l_1 = m = 1$, i.e. SISO system will be considered. The results for general case will be presented elsewhere. For SISO systems G-passivity coincides with conventional passivity. Define $\psi(s) = \varphi_1(s)/\varphi(s)$. The function $\psi(s)$ can be interpreted as a relative gain $\psi(s) = W_1(s)/W(s)$. Obviously, problems AL1 and BL1 coincide in this case. Without loss of generality let $L = 1$. To simplify formulation introduce the following property characterizing "closeness" or "friendliness" of the outputs y and y_1 regardless their signs.

Definition 4. We say that system (4), (10) possesses *F-property* if either $\mathrm{Re}\ \psi(j\omega) \geq 0\ \forall\omega$ and $\mathrm{Re}\ \delta(j\omega)/\varphi(j\omega) > 0$ for $\mathrm{Re}\ \psi(j\omega) = 0$ or $\mathrm{Re}\ \psi(j\omega) \leq 0\ \forall\omega$ and $\mathrm{Re}\ \delta(j\omega)/\varphi(j\omega) < 0$ for $\mathrm{Re}\ \psi(j\omega) = 0$.

F-property holds, e.g. if outputs y and y_1 coincide. Checking F-property reduces to verifying a polynomial frequency-domain inequality and a finite number of rational inequalities. Alternatively, it can be done using linear matrix inequalities (LMI) approach. The following auxiliary result provides one necessary and two sufficient solvability conditions for the extended passification problem.

Theorem 2. *1. If the system (4) is strictly passifiable by output feedback (10), then the polynomial $\varphi(s)$ is Hurwitz of degree $n-1$, i.e. either function $W(s)$ or function $-W(s)$ is hyper-minimum-phase.*

2. If the function $W(s)$ is hyper-minimum-phase, $\deg(\varphi_1(s)) = n-1$ and system possesses F-property, then the system (4) is strictly passifiable by output feedback (10).

3. If the function $W(s)$ is hyper-minimum-phase, $\deg(\varphi_1(s)) = n-2$, system possesses F-property and $\delta_{n-1} \neq 0$ then the system (4) is strictly passifiable by output feedback (10).

As was mentioned in Section II, passifiability is equivalent to solvability of the following algebraic problem.

Given complex-valued matrices A, B, C, C_1, G of the dimensions $n\times n$, $n\times m$, $l\times n$, $m\times l$, respectively ($m \le n$, $l \le n$). Find existence conditions for a Hermitian $n \times n$ matrix $H = H^* > 0$ and a complex valued $m \times l$ matrix K such that

$$HA(K) + A(K)^* H < 0, \tag{14}$$

$$HB = (GC_1)^* \tag{15}$$

where

$$A(K) = A - BKC. \tag{16}$$

The case when all matrices A, B, C, G are real valued is called the *real case*. A straightforward consequence of Theorem 2 is the following statement.

Corollary. *The first condition of Theorem 2 is necessary and two other conditions are sufficient for the existence of the matrices $H = H^* > 0$, K satisfying relations (14), (15), (16) and being real valued in the real case.*

Obviously, relations (14), (15), (16) for fixed K coincide with the linear matrix inequalities (LMI) (6) arising in a version of Yakubovich-Kalman-Popov lemma. Therefore the Corollary deals with the existence of a feedback rendering the system satisfy conditions of Yakubovich-Kalman-Popov lemma. In other words, the Corollary can be called the *Feedback Yakubovich-Kalman-Popov lemma*. Its very special case for $C_1 = C$ was posed and solved in [11]. Note also that the inequalities (14) are *bilinear* matrix inequalities and the problem of their solvability is in general NP-hard. However for the above special case the solvability conditions for (14), (15), (16) are simple and constructive.

To provide necessary and sufficient conditions for extended passifiability a kind of uniformity is needed to be introduced into definitions.

Definition 5. The system (4) is called δ*-uniformly strictly passifiable by output feedback* (10), if for any $\delta(s)$ there exists gain K such that the system (4) with output feedback (10) is strictly passive.

Theorem 3. *The system (4) is δ-uniformly strictly passifiable by output feedback (10) if and only if the function $W(s)$ is hyper-minimum-phase and the frequency-domain inequality holds: $\psi(j\omega) \ne 0 \quad \forall\omega$, where $\psi(s) = \varphi_1(s)/\varphi(s)$.*

IV. Proof of main results

The proofs are based upon two auxiliary lemmas. The first one is just another version of the Yakubovich-Kalman-Popov lemma which deals with positive definite solutions of matrix relations (8) and can be easily derived from Theorem 1.

Lemma 1. *Let A_0, B, C_0 be matrices of dimensions $n\times n$, $n \times n$, $n \times m$, $m \times n$ respectively, and* $\operatorname{rank} B = m$. *Let*

$$\Pi(s) = 2\,\mathrm{Re}\, C_0(sI_n - A_0)^{-1}B. \tag{17}$$

For the existence of $n \times n$ matrix $H = H^* > 0$ such that

$$HA_0 + A_0 H < 0, \quad HB = C_0^*, \tag{18}$$

which is real valued in the real case, the following conditions are necessary and sufficient:

i) $\det(sI_n - A_0)$ *is a Hurwitz polynomial;*
ii) $\Pi(j\omega) > 0$ *for all* $\omega \in \mathbb{R}^1$,
iii) $\lim_{\omega\to\infty} \omega^2\Pi(j\omega) > 0$.

The second lemma is the core result of Meerov's high gain stabilization theory [25], [26].

Lemma 2. The polynomial $\delta(s, K) = \delta(s) + K\varphi_1(s)$, where $\delta(s), \varphi_1(s)$ are polynomials, is Hurwitz for all $K > K_0$ for some K_0 if and only if $\varphi_1(s)$ is Hurwitz with positive coefficients and either (A) $\deg\varphi_1(s) = n - 1$ or (B) $\deg\varphi_1(s) = n - 2$ and $\delta_{n-1} > 0$.

Proof of Theorem 2. In view of Lemma 1 and discussion in Section II feasibility of passification problem is equivalent to solvability of conditions *i), ii), iii)* of Lemma 1. Necessary condition (Statement 1) follows from representation of the closed loop transfer function

$$W(s, K) = \varphi(s)[\delta(s) - K\varphi_1(s)]^{-1}. \tag{19}$$

and the following property of strictly positive real (SPR) functions [29]: both numerator and denominator of an SPR function are Hurwitz polynomials and their degrees differ not greater than by one.

To prove Statement 2 assume that its conditions are fulfilled and choose K such that matrix $A_0 = A - BKC_1$, satisfies conditions i), ii), iii) of Lemma 1.

Condition i) follows immediately from Lemma 2. To prove Condition ii) note that inequality $\mathrm{Re}W(s, K) > 0$ is equivalent to inequality $\mathrm{Re}[W(s, K)^{-1}] > 0$. Evaluation of $\mathrm{Re}[W(j\omega, K)^{-1}]$ yields

$$\mathrm{Re}[W(j\omega, K)^{-1}] = \mathrm{Re}\frac{\delta(j\omega)}{\varphi(j\omega)} + K\mathrm{Re}\frac{\varphi_1(j\omega)}{\varphi(j\omega)}. \tag{20}$$

Represent $\delta(s)/\varphi(s)$ in the form $\delta(s)/\varphi(s) = s + \delta_{n-1}/\varphi_{n-1} + \bar{\delta}(s)/\varphi(s)$, where $\bar{\delta}(s)$ is a polynomial of degree less than $n - 1$. Substitute $s = j\omega$ into (20) and take real part:

$$\mathrm{Re}[W(j\omega, K)^{-1}] = 0 + \frac{\delta_{n-1}}{\varphi_{n-1}} + \mathrm{Re}\frac{\bar{\delta}(j\omega)}{\varphi(j\omega)} + K\mathrm{Re}\psi(j\omega). \tag{21}$$

Let $\mathrm{Re}\psi(j\omega_l) = 0$, $l = 1, \ldots, N$. Since $\lim_{|\omega|\to\infty} \mathrm{Re}\psi(j\omega) > 0$, there exists $\varepsilon > 0$ such that $\inf_\omega \mathrm{Re}\psi(j\omega) > 0$ for $\omega \in \Omega_\varepsilon$, where $\Omega_\varepsilon = \{\omega : |\omega - \omega_l| > \varepsilon\}$. In view of boundedness of $\mathrm{Re}(\delta(j\omega)/\varphi(j\omega)$ the required inequality is valid for sufficiently large negative K. On the other hand, for $\omega \bar{\in} \Omega_\varepsilon$ for sufficiently small ε the required inequality holds too.

Let us verify the limit relation iii). For large ω we have:

$$\omega^2\mathrm{Re}\ W(j\omega, K) =$$

$$\omega^2\mathrm{Re}\,\frac{\varphi_{n-1}(j\omega)^{n-1} + \mathcal{O}(\omega^{n-2})}{(j\omega)^n + \delta_{n-1}(j\omega)^{n-1} + K\varphi_{1,n-1}(j\omega)^{n-1} + \mathcal{O}(\omega^{n-2})} =$$

$$\omega^2 \mathrm{Re} \frac{\varphi_{n-1} + \mathcal{O}(1/\omega)}{j\omega + \delta_{n-1} + K\varphi_{1,n-1} + \mathcal{O}(1/\omega)} =$$

$$\omega^2 \Big[\frac{\varphi_{n-1}}{j\omega + \delta_{n-1} + K\varphi_{1,n-1}} + \frac{\varphi_{n-1}}{-j\omega + \delta_{n-1} + K\varphi_{1,n-1}} \Big] +$$

$$\mathcal{O}(1/\omega) = \omega^2 \frac{\varphi_{n-1}(\delta_{n-1} + K\varphi_{1,n-1})}{\omega^2 + (\delta_{n-1} + K\varphi_{1,n-1})^2} + \mathcal{O}(1/\omega).$$

Therefore $\lim_{|\omega|\to\infty} \omega^2 \mathrm{Re}\ W(j\omega) = \varphi_{n-1}(\delta_{n-1} + K\varphi_{1,n-1})$ and the required inequality is valid for sufficiently large negative K. Statement 2 is proved.

Statement 3 is proved along similar lines. Instead of $\lim_{|\omega|\to\infty} \mathrm{Re}\psi(j\omega) > 0$ the relation $\lim_{|\omega|\to\infty} \omega^2 \mathrm{Re}\psi(j\omega) > 0$ is used in this case. ■

Proof of Theorem 3. Necessity. The HMP property of $W(s)$ follows from SPR property [29]. Let $\mathrm{Re}\psi(j\omega_0) = 0$ for some real ω_0. Choosing $\delta(s)$ such that $\mathrm{Re}\frac{\delta(j\omega_0)}{\varphi(j\omega_0)} < 0$ we get a contradiction since in this case no appropriate K exists as seen from (20).

Sufficiency. Without loss of generality assume that $\mathrm{Re}\psi(j\omega) > 0$ for all real ω, i.e. $\psi(s)$ is SPR. Then either $\deg(\varphi) = n-1$, or $\deg(\varphi) = n-2$, see [29]. However, in the case $\deg(\varphi) = n-2$ condition iii) of Lemma 1 is violated for any K for sufficiently large positive δ_{n-1}, i.e. system is not passifiable. Again, without loss of generality assume that $\mathrm{Re}\psi(j\omega) > 0$ for all real ω. The arguments in the proof of the previous theorem show that $\inf_\omega \mathrm{Re}\psi(j\omega) > 0$. Since all terms in (21) depending on $\delta(s)$ are bounded from above over ω axis for any polynomial $\delta(s)$, the required K does exist. ■

V. High gain passifiability

It is seen from the proofs of the above theorems that they are based on high gain arguments, i.e. if the required gain exists, an appropriate high gain does exist too. The situation is similar to the case $C = C_1$, see [10], [11]. It motivates the formulation of the following problem: find necessary and sufficient conditions for feasibility of high gain passification. The answer is provided by the following theorem.

Theorem 4. *The system (4) is strictly passifiable by output feedback (10) with all sufficiently large K if and only if the transfer function $W(s)$ is hyper-minimum-phase (polynomial $\varphi(s)$ is Hurwitz of degree $n-1$) and either* (A) $\deg(\varphi_1(s)) = n-1$ *and system possesses F-property, or* (B) $\deg(\varphi_1(s)) = n-2$, *system possesses F-property and* $\delta_{n-1} \neq 0$.

Proof. Sufficiency follows from Theorem 2. To prove necessity, assume without loss of generality that the system is passifiable as $K \to +\infty$ and pick up a sufficiently large positive K such that $W(s,K)$ is SPR. Then $\varphi(s)$ is Hurwitz of degree $n-1$ and its coefficients are positive, i.e. $W(s)$ is hyper-minimum-phase. As in the proof of Theorem 2 note that inequality $\mathrm{Re}W(j\omega, K) > 0$ is equivalent to inequality $\mathrm{Re}[W(j\omega, K)^{-1}] > 0$. Since the left-hand part in (20) is positive, F-property follows. SPR property implies also that that either $\deg(\varphi_1(s)) = n-1$ or $\deg(\varphi_1(s)) = n-2$. In the case $\deg(\varphi_1(s)) = n-2$ condition $\delta_{n-1} > 0$ follows from representation (21). ■

VI. Passifiability with respect to given input

Since passivity is widely applied for control design, the proposed results apply to a number of problems. A dual set of results is related to passivity with respect to different inputs. Consider a system

$$\dot{x} = Ax + Bu + B_1 v, \quad y = Cx, \tag{22}$$

and the problem of existence of a feedback

$$u = -Ky \tag{23}$$

rendering the closed loop system (22), (23) passive from input v to output y. Such a problem for the state feedback case ($y = x$) was studied in [3], [4], [6] (in [6] also for full order dynamic output feedback) by LMI approach.

It is easy to see that passivity from input v to output y is equivalent to existence of a matrix $H = H^* > 0$ and a feedback gain K such that

$$HA_K + A_K^* H < 0,\ HB_1 = C^*,\ A_K = A - B_1 KC. \tag{24}$$

After the change of variables $H \to P^{-1}$, $A_K \to A_K^*$, $B_1 \to C^*$, $C \to B_1^*$ the relations (24) transform into (14), (15), (16) and Theorems 2-4 apply. Since the change $A_K \to A_K^*$ does not change conditions of Theorems 2-4, the answers to the new set of problems are given by Theorems 2-4 where function $\psi(s)$ is replaced by $\bar{\psi}(s) = \bar{\varphi}(s)/\varphi(s)$, where $\bar{\varphi}(s) = \delta(s)C(sI-A)^{-1}B_1$.

VII. Conclusions

The presented passification feasibility results can be useful for design purposes in various situations. For example, they justify design of SPR system based on providing some minimum phase property and applying high gain output feedback, extending existing applications in adaptive control designs, see [10], [11], [37], [21], [33], [22], [24] to the case of "indirect" passification and "nonmatched" nonlinearities.

One can also mention passification based synchronization for nonlinear Lurie systems

$$\dot{x} = Ax + Bf_0(y) + B_1 u, \quad y = Cx \tag{25}$$

and adaptive synchronization of systems

$$\dot{x} = Ax + f_0(y) + B\sum_{i=1}^{N} \theta_i \varphi_i(y) + B_1 u, \quad y = Cx, \tag{26}$$

where $\theta_i, i = 1, \dots, N$ are unknown parameters, including the case of imposed information constraints. For the case $B_1 = B$ such problems were studied in [15], [16], [18].

The results of this paper significantly extend the class of admissible feedbacks and, therefore, a field of possible applications for passification approach.

It would be interesting to extend the obtained results to infinite-dimensional linear systems in spirit of [7] as well as to affine nonlinear systems. A MIMO version of the above discussed results is to be presented in [14].

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