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Discrete-time nonlinear observer and output feedback design for a combustion engine test bench

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Abstract— **Combustion engine control design depends strongly on the availability and the quality of the measurement of quantities involving in the controller construction. In general, not all quantities are available through direct measurement, and therefore an observer is often necessary to realize the controller. In this paper, a discrete-time partial state observer design for a combustion engine test bench is proposed. The observer is used to estimate the torque and the torsion angle of the engine, based on the measurement of the engine and the dynamometer speeds. The convergence of the observer is shown, and separation principle is also proved. The observer is used for constructing an output feedback controller for set point tracking of the test bench. Some numerical simulations are performed, showing the performance of the observer and also comparing the performance of the output feedback controller with the state feedback controller. A discussion, comparing a so called "static observer" and a "dynamic observer" which are possible to construct using the same approach, is also presented.**

Keywords: Combustion engine test bench control; Controller redesign; Discrete-time systems; Extended Hammerstein systems; Setpoint tracking; Static and dynamic observers.

I. INTRODUCTION

When designing a controller for a dynamical system, it is commonly assumed that all states are available from measurement, and a state feedback controller is feasible for design. However, this assumption is often unrealistic in practice. In many applications not all states can be measured, hence control using output feedback or dynamic feedback becomes necessary. While design tools mainly aim at designing a state feedback controller, designing an observer is a useful solution to provide the estimates of the unmeasured states to be used for constructing an output feedback controller. In other cases even when the states may be available from measurement, observer is still playing an important role in reducing the number of sensors applied to the plant, and hence reducing the data acquisition complexity and the cost when the required sensors are complicated and expensive.

In this work, we study an output feedback control design problem for a combustion engine testbench. As combustion engines are widely used in automotive as well as industrial applications, the topic has attracted a lot of researchers to study the control problems of the engine as well as the engine test bench (see for instance [1]–[3] and references therein).

The issue of the partially available state measurement is addressed. To handle the problem of unmeasured signals, a discrete-time observer design is proposed in this paper. The observer is applied in a sampled-data setting, where the proposed discrete-time partial state nonlinear observer is used to estimate the unmeasured states or the continuous-time model of the testbench. Although it is possible to construct the discrete-time observer by emulation, namely discretizing the continuous-time observer by sampling, this approach often results in performance degradation. Therefore, direct discrete-time design is favorable in many situation (see for instance [4]–[6]). This paper can be seen as the discrete-time counterpart of [7], where a continuous-time observer design for the same system was proposed.

In this paper, we prove the convergence of the observer by showing the convergence of the observation error. We also prove that separation principle holds in our construction. This is very important as we will use the observer to build an output feedback controller for setpoint tracking of the speed and the torque of the test bench. While the proof of the separation principle is not presented in [7], the proof presented in this paper serves to complete the results of [7].

The application of the main results to the output feedback control design of the combustion engine testbench is presented and some simulation results to test the performance of the proposed observer and output feedback design are also provided. A comparison between the so called "static observer" and "dynamic observer" constructed using a similar approach is also provided to complete the discussion.

II. NOTATION AND DEFINITIONS

The set of real and natural numbers (including 0) are denoted respectively by $\mathbb{R}$ and $\mathbb{N}$. A function $\gamma : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{K}$ if it is continuous, strictly increasing and zero at zero. It is of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and unbounded. Functions of class $\mathcal{K}_\infty$ are invertible. A function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{KL}$ if $\beta(\cdot, t)$ is of class $\mathcal{K}$ for each $t \geq 0$ and $\beta(s, \cdot)$ is decreasing to zero for each $s > 0$ [8]. Note that we often drop the arguments of a function whenever they are clear from the context.

Consider a general input affine nonlinear system

$$\dot{x} = f(x) + g(x)u \ , \qquad y = h(x) \ , \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input and $y \in \mathbb{R}^p$ is the output. The functions f, g and h are smooth and f is zero at zero. The family of discrete-time model of the system can be written as

$$x_{k+1} = F_T(x_k, u_k) \ , \qquad y_k = h(x_k) \ , \tag{2}$$

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with the parameter $T > 0$ is the sampling period. If the control input is a state feedback controller $u_k = u_k(x_k)$, we write the closed loop system of (1) as

$$x_{k+1} = \mathcal{F}_T(x_k) \tag{3}$$

We use the following definitions throughout the paper.

Definition 2.1: (Semiglobal practical asymptotic (SPA) stability) A discrete-time system (3) is SPA stable in a Lyapunov sense if there exists a continuously differentiable function $V_T : \mathbb{R}^n \to \mathbb{R}$ such that there exist class K_∞ functions $\underline{\alpha}$, $\overline{\alpha}$, α such that for any compact and invariant set $\mathcal{O}_x \subseteq \mathbb{R}^n$ and a sufficiently small real number $\nu > 0$ there exists $T^* > 0$ such that for all $T \in (0, T^*)$ and for all $x \in \mathcal{O}_x$ the following holds.

$$\underline{\alpha}(|x|) \le V_T(x) \le \overline{\alpha}(|x|) \tag{4}$$

$$V_T(F_T(x)) - V_T(x) \le -T\alpha(|x|) + T\nu \ , \tag{5}$$

and V_T is called a SPA stability Lyapunov function. ■

Definition 2.2: (SPA stabilizability) A discrete-time system (2) is SPA stabilizable by means of state feedback if there exists a state feedback control law $u_k = u_k(x_k)$ s.t. the closed-loop system (2) with the control u_k is SPA stable. ■

Consider now another discrete-time system

$$z_{k+1}{=}\Gamma_T(z_k, y_k, u_k); \quad \hat{x}_k{=}\gamma(z_k, y_k, u_k); \quad z_k{\in}\mathbb{R}^l. \tag{6}$$

Definition 2.3: (SPA stable observer) The system (6) is a (SPA) stable observer for (2) if for any compact and invariant sets $\mathcal{O}_x \subseteq \mathbb{R}^n$, $\mathcal{O}_z \subseteq \mathbb{R}^l$, $\mathcal{O}_u \subseteq \mathbb{R}^m$ and a sufficiently small number $\nu > 0$, there exists $T^* > 0$ s.t. the following hold.

1) For all $x_0 \in \mathcal{O}_x$, $u_k \in \mathcal{O}_u$ and $T \in (0, T^*]$, there exists $z_0 \in \mathcal{O}_z$ such that $\|\hat{x}_k - x_k\| \le T\nu$, $\forall k \ge 1$.
2) For all $x_0 \in \mathcal{O}_x$, $u_k \in \mathcal{O}_u$, $z_0 \in \mathcal{O}_z$ and all $T \in (0, T^*]$, $\lim_{k\to\infty} \|\hat{x}_k - x_k\| \to T\nu$.

If $\hat{x}_k = z_k$, the system (6) is called an identity observer, and if the convergence of $\hat{x}$ to x is exponential, then (6) is called an exponential observer. ■

Remark 2.1: The property in Definition 2.3 holds globally if $\mathcal{O}_x = \mathbb{R}^n$, $\mathcal{O}_z = \mathbb{R}^l$, $\mathcal{O}_u = \mathbb{R}^m$ and $\nu = 0$. Note that the parameter ν which takes nonnegative value, defines the practical property. On the other hands, it holds locally if $\mathcal{O}_x \subset \mathbb{R}^n$, $\mathcal{O}_z \subset \mathbb{R}^l$, $\mathcal{O}_u \subset \mathbb{R}^m$. It is called quasilocal [9] if given $\mathcal{O}_x \subset \mathbb{R}^n$, $\mathcal{O}_u \subset \mathbb{R}^m$ for the state and control respectively, for every initial condition $x_0 \in \mathcal{O}_x$ there exists an open subset $\mathcal{O}_z(x_0) \subset \mathbb{R}^l$ such that conditions 1) and 2) of Definition 2.3 holds $z_0 \in \mathcal{O}_z(x_0) \subset \mathbb{R}^l$ for all $k \ge 0$. ■

The following property is a direct implication of the existence of a SPA observer for a system.

Definition 2.4: (SPA observability) A discrete-time system (2) is SPA observable if there exists a SPA observer for the system. ■

III. MAIN RESULT

A. Engine test bench model

A simple diagram of the combustion engine test bench is illustrated in Figure 1. The main parts of such a dynamical engine test bench are the dynamometer, the connection shaft and the combustion engine itself. One of the control design objectives for a dynamical engine test bench control is to stabilize the engine torque and the engine speed.

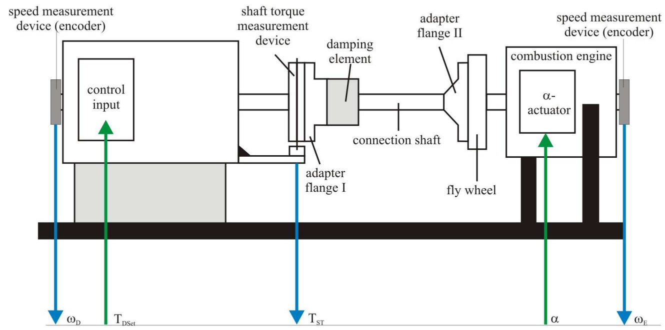

Fig. 1. The combustion engine test bench system

Considering the torque of the combustion engine and the air gap torque of the dynamometer as the inputs to the mechanical part of the test bench, the dynamical model of the engine can be represented by a two mass oscillator

$$\dot{\psi}_\Delta = \omega_E - \omega_D \tag{7}$$

$$\dot{\omega}_E = \frac{1}{\theta_E}(T_E - c\psi_\Delta - d(\omega_E - \omega_D)) \tag{8}$$

$$\dot{\omega}_D = \frac{1}{\theta_D}(c\psi_\Delta + d(\omega_E - \omega_D) - T_{DSet}) \ , \tag{9}$$

where ψ_Δ is the torsion angle, ω_E and ω_D are respectively the engine and the dynamometer angular velocity, T_E is the engine's torque, T_{DSet} is the air gap torque of the dynamometer, θ_E and θ_D are the inertia of the engine and the dynamometer, respectively. The parameters c and d are the damping constant and stiffness of the shaft, respectively. The dynamical model of the combustion engine test bench is approximated by the class of the extended Hammerstein systems (see [10] for more details)

$$\dot{T}_E = -(c_0 + c_1\omega_E + c_2\omega_E^2)T_E + m(\omega_E, T_E, \alpha) \ . \tag{10}$$

with c_0, c_1 and c_2 are some positive coefficients and $m(\omega_E, T_E, \alpha)$ is a continuous nonlinear function. From the continuity of m, without lose of generality, we assume that it is locally Lipschitz with respect to T_E.

B. Discrete-time observer design

From Subsection III-A we have obtained the dynamic model (7)-(10) of the testbench. In practice, from the four state variables of the system, only the engine angular velocity ω_E and the dynamometer angular velocity ω_D are available through measurement. Hence, we can write the output equation for the system as

$$y_1 = \omega_E \qquad y_2 = \omega_D \ . \tag{11}$$

As the control problem of the testbench usually involves the torque control, in order to design a feedback controller the knowledge of the torque T_E is necessary. Hence, an observer is required to estimate the unmeasured states T_E and ψ_Δ.

In [7] a continuous-time observer design has been proposed. However, this observer needs to be implemented digitally in the real application. Although as have been shown in [7] that the observer works very well in continuous-time simulation, sampling inevitably degrades the performance. It is known from numerous experiences that direct discrete-time design yields in improved performance [6], [11]. Therefore, in this paper we design a discrete-time observer based on the discrete-time model of the test bench. Suppose the family of the exact model of the test bench (7)-(10) is

$$\begin{aligned} x^e_{1,k+1} &= F^e_{1T}(x^e_{1k}, x^e_{2k}, u_k)\\ x^e_{2,k+1} &= F^e_{2T}(x^e_{1k}, x^e_{2k}, u_k) \end{aligned} \tag{12}$$

where x_1 is the measured state and x_2 is the unmeasured state. The output is then $y^e_k = x^e_{1k}$. The exact model (12) is obtained by integrating the initial value problem of (7)-(10) over the sampling interval $[kT, (k+1)T]$ with initial condition $x(0) = x_\circ := [x_{1\circ},\ x_{2\circ}]$ and a constant input u_k. However, since the explicit solution of the nonlinear initial value problem is in general impossible to compute, the exact discrete-time model (12) is not available. Therefore an approximate discrete-time model is used. We use the Euler approximate model of the dynamic of the unmeasured states

$$\begin{aligned} T_{E,k+1} &= T_{Ek} + T\big[m(\omega_E, T_E, \alpha) - (c_0 + c_1\omega_E + c_2\omega_E^2)T_E\big]\\ \psi_{\Delta,k+1} &= \psi_{\Delta k} + T\,(\omega_E - \omega_D)\ , \end{aligned} \tag{13}$$

as the base of the observer structure. The following theorem provide the observer construction. To simplify notation, we denote the state vector $x = [T_E, \psi_\Delta, \omega_E, \omega_D]^\top$, and we drop the argument k whenever it is clear from the context.

Theorem 3.1: Given the exact discrete-time model (12) of a continuous-time model of an engine testbench (7)-(10) with the measured output (11). For any compact and invariant set $D \subset R^4$, there exist $T^* > 0$, so that the following reduced order observer

$$\begin{aligned} \hat{T}_{E,k+1} &= \hat{T}_{Ek} + T\Big[m(\omega_E, \hat{T}_E, \alpha) - (c_0 + c_1\omega_E + c_2\omega_E^2)\hat{T}_E + L_1 e_1\Big]\\ \hat{\psi}_{\Delta,k+1} &= \hat{\psi}_{\Delta k} + T\,(\omega_E - \omega_D + L_2 e_2)\ , \end{aligned} \tag{14}$$

with $L_1 > 0$, $L_2 > 0$ sufficiently large and

$$\begin{aligned} e_1 &= \theta_E \mathring{\omega}_E + \theta_D \mathring{\omega}_D + T_{DSet} - \hat{T}_E\\ e_2 &= \frac{1}{c}(\theta_D \mathring{\omega}_D - d(\omega_E - \omega_D) + T_{DSet} - c\hat{\psi}_\Delta)\\ \mathring{\omega}_E &:= \frac{\omega_{Ek} - \omega_{E(k-1)}}{T}, \quad \mathring{\omega}_D := \frac{\omega_{Dk} - \omega_{D(k-1)}}{T}, \end{aligned} \tag{15}$$

is a SPA stable observer for the exact discrete-time model (12) for all $x \in D$ and $T \in (0, T^*]$. ■

Remark 3.1: Note that in practice the signals $\dot{\omega}_E$ and $\dot{\omega}_D$ are not measured. Although theoretically it is possible to use a differentiator to obtain these signals from the output ω_E and ω_D, it is not practical as a differentiator needs two input signals. The common practice is by approximating the derivatives as follows

$$\dot{\omega}_E \approx \frac{\omega_E(t) - \omega_E(t-T)}{T}, \quad \dot{\omega}_D \approx \frac{\omega_D(t) - \omega_D(t-T)}{T},$$

with $T > 0$ sufficiently small. This approach is also applied in this paper. ■

Proof of Theorem 3.1: Given the exact discrete-time model system (12) of the system (7)-(10), with measured output (11) and the observer (14). To prove Theorem 3.1, we first show that the observer (14) is a SPA stable observer for the Euler approximate discrete-time model (13). We define the estimation errors as $e_1 := T_E - \hat{T}_E$ and $e_2 := \psi_\Delta - \hat{\psi}_\Delta$. First, we will show that the error terms satisfy (15). It is straight forward that from (9) we can obtain

$$\psi_\Delta = \frac{1}{c}\Big(\theta_D\dot{\omega}_D - d(\omega_E - \omega_D) + T_{DSet}\Big)\ . \tag{16}$$

Moreover, from (8) and (16) we have

$$\begin{aligned} T_E &= \theta_E\dot{\omega}_E + c\psi_\Delta + d(\omega_E - \omega_D)\\ &= \theta_E\dot{\omega}_E + \theta_D\dot{\omega}_D + T_{DSet}\ . \end{aligned} \tag{17}$$

Therefore using (17) we obtain

$$e_1 = T_E - \hat{T}_E = \theta_E\dot{\omega}_E + \theta_D\dot{\omega}_D + T_{DSet} - \hat{T}_E\ , \tag{18}$$

and using (16), we obtain

$$e_2 = \psi_\Delta - \hat{\psi}_\Delta = \frac{1}{c}\Big[\theta_D\dot{\omega}_D - d(\omega_E - \omega_D) + T_{DSet} - c\hat{\psi}_\Delta\Big]\ . \tag{19}$$

Following Remark 3.1, we replace $\dot{\omega}_E$ and $\dot{\omega}_D$ with $\mathring{\omega}_E$ and $\mathring{\omega}_D$ respectively to arrive at (15). Now, considering the Euler model of the testbench we can write the error dynamics

$$\begin{aligned} e_{1,k+1} &= T_{E,k+1} - \hat{T}_{E,k+1}\\ &= e_1 + T\left[-(c_0 + c_1\omega_E + c_2\omega_E^2)e_1\right.\\ &\qquad \left. + m(\omega_E, T_E, \alpha) - m(\omega_E, \hat{T}_E, \alpha) - L_1 e_1\right], \end{aligned} \tag{20}$$

and

$$e_{2,k+1} = \psi_{\Delta,k+1} - \hat{\psi}_{\Delta,k+1} = e_2 - TL_2e_2 \tag{21}$$

To show the asymptotic stability of the error system, we use choose $V_T = \frac{1}{2}e^\top e$ as the Lyapunov function. The Lyapunov difference ΔV_T is

$$\Delta V_T = \frac{1}{2}\big[e_{1,k+1}^\top e_{1,k+1} + e_{2,k+1}^\top e_{2,k+1} - e_{1,k}^\top e_{1,k} - e_{2,k}^\top e_{2,k}\big]\ .$$

From the local Lipschitzity of m with respect to T_E, there exist $L_m > 0$ such that for all $x \in D$

$$m(\omega_E, T_E, \alpha) - m(\omega_E, \hat{T}_E, \alpha) \le L_m(T_E - \hat{T}_E) = L_m e_1\ .$$

Following very similar steps as in [7], after some standard calculation (see for instance [12]), for all $x \in D$ and $T \in (0, T^*]$ we can obtain

$$\begin{aligned} \Delta V_T &< T\left[-(c_0 + c_1\omega_E + c_2\omega_E^2)e_1^2\right.\\ &\qquad \left. + L_m e_1^2 - L_1 e_1^2 - L_2 e_2^2\right] + T\nu\\ &< T\left[-\Big(C(\omega_E) + L_1 - L_m\Big)e_1^2 - L_2 e_2^2\right] + T\nu\\ &< T\left[-Le_1^2 - L_2 e_2^2\right] + T\nu\ , \end{aligned} \tag{22}$$

with $\nu > 0$ sufficiently small. The existence of $L > 0$ is guaranteed by choosing L_1 large enough so that $C(\omega_E) + L_1 > L_m$ for all ω_E. Therefore the Lyapunov difference is negative definite with a small offset $T\nu$. Hence, the observer (14) is a SPA stable observer for the Euler model (13).

Moreover, since the Euler model (13) is one step consistent with the exact model (12) we have

$$|F^e_{2T} - F^{Euler}_{2T}| < T\rho(T)\ . \tag{23}$$

It is then obvious that the observer (14) is also a SPA stable observer for the exact model (12) with an offset $\tilde{\nu} > \nu$. ■

C. Separation Principle

Separation principle needs to hold if an observer is used for designing an output feedback controller. For the separation principle to hold, asymptotic stabilizability and uniform observability of the system w.r.t the observer is required.

In nonlinear sampled-data stabilization problems, we are interested in stabilizing a nonlinear continuous-time system using a discrete-time controller. However, under certain conditions the stability analysis for sampled-data systems can be derived from the stability analysis of its closed-loop exact discrete-time model, or further from its closed-loop approximate discrete-time model [13], [14].

In this section, while we are interested in the stabilization and separation principle for the continuous-time system (7)-(10) and observer (14), we study the properties for the exact discrete-time model (12) and observer (14) and use the results from [13], [14] to derive conclusions for the sampled-data system (7)-(10) and (14).

Given a state feedback control u_k for the system (7)-(10). To guarantee that the estimated state $\hat{T}_E$ and $\hat{\psi}_\Delta$ can be used to replace the unmeasured state T_E and ψ_Δ in a feedback control construction, the separation principle must hold. The separation principle required to solve the stabilization problem is stated in the following result.

Proposition 3.2: (Separation Principle) Consider the exact discrete-time model of the engine test bench (12). Suppose there exists a controller $u_k = u_k(T_E, \psi_\Delta, \omega_E, \omega_D) := u_k(x)$ that SPA stabilizes the system. Assume that u_k is continuous, and zero at zero. The SPA stabilization for the system using an output feedback $u_k = \hat{u}_k(\hat{T}_E, \hat{\psi}_\Delta, \omega_E, \omega_D) := u_k(\hat{x}_k)$ from the observer (14) is solvable if the closed-loop system is uniformly observable. ■

Proof of Proposition 3.2: The origin is a SPA stable equilibrium point of the system (7)-(10) with the controller $u_k = u_k(x)$. Hence using the Lyapunov converse theorem we can claim that there exists a continuous and differentiable function $V_T : R^n \to R$ and positive constants Δ_1 and ν such that for all $|x| \leq \Delta_1$ there exists $T^* > 0$ such that for all $T \in (0, T^*)$ we have

$$\underline{\alpha}(|x|) \leq V_T(x) \leq \overline{\alpha}(|x|) \tag{24}$$

$$V_T(F_T(x, u_k(x))) - V_T(x) \leq -T\alpha(|x|) + T\nu \tag{25}$$

where $\underline{\alpha}, \overline{\alpha}, \alpha$ are class $\mathcal{K}$ functions.

Applying $\hat{u}_k = u_k(\hat{x}_k)$ to the system (12) yields the closed-loop exact discrete-time model

$$\begin{aligned} x_{k+1} &= F_T^e(x_k, u_k(\hat{x}_k) \\ \hat{x}_{2(k+1)} &= \hat{x}_{2k} + T\dot{x}_2 + TLe \ , \end{aligned} \tag{26}$$

with L some positive constant. Since the system is SPA observable, Definition 2.3 follows. Moreover, we can write the Lyapunov difference for the closed-loop system as

$$\begin{aligned} \Delta V_T &= V_T(F_T^e(x, \hat{u}_k)) - V_T(x) \\ &\leq -T\alpha(|x|) + T\nu + V_T(F_T^e(x, \hat{u}_k)) - V_T(F_T^e(x, u_k)). \end{aligned}$$

Although $F_T^e(\cdot)$ is not known explicitly, since the Euler approximate model $F_T^a(\cdot)$ that we use is one step consistent [15], we have that

$$F_T^e(x, u_k) - F_T^a(x, u_k) \leq T\rho(T) \ . \tag{27}$$

From the continuity of V_T, we can then write

$$\begin{aligned} \Delta V_T &\leq -T\alpha(|x|) + T\nu + L_v(T\rho(T)) \\ &\quad + V_T(F_T^a(x, \hat{u}_k)) - V_T(F_T^a(x, u_k)) \\ &\leq -T\alpha(|x|) + T\nu + L_v(T\rho(T)) \\ &\quad + L_v(F_T^a(x, \hat{u}_k) - F_T^a(x, u_k)) \\ &\leq -T\alpha(|x|) + T\nu + L_v(T\rho(T)) \\ &\quad + TL_v(f(x, \hat{u}_k) - f(x, u_k)) \end{aligned} \tag{28}$$

Once more, due to the continuity of $f(\cdot)$, we have that

$$f(x, \hat{u}_k) - f(x, u_k) \leq L_f(\hat{u}_k - u_k) \ , \tag{29}$$

and without lose of generality we assume the continuity of u_k so that we have

$$\hat{u}_k - u_k = u_k(\hat{x}) - u_k(x) \leq L_u \, |e| \ . \tag{30}$$

Hence we can write

$$\Delta V_T \leq -T\alpha(|x|) + T\nu + L_v(T\rho(T)) + TL_v L_f L_u \, |e| \, . \tag{31}$$

As it has been proved in Theorem 3.1 that the observer is SPA stable, which means the first and second conditions of Definition 2.3 hold. Moreover, we can pick a sufficiently small $\tilde{\nu} > \nu$ such that there exists $0 < T_1^* < T^*$ such that for all $T \in (0, T_1^*)$ we have

$$T\nu + L_v(T\rho(T)) + TL_v L_f L_u(e) \leq T\tilde{\nu} \ , \tag{32}$$

and hence $\Delta V_T \leq -T\alpha(|x|) + T\tilde{\nu}$, that completes the proof of Proposition 3.2. ■

Remark 3.2: We emphasize that the separation principle stated in Proposition 3.2 is for the discrete-time observer (14) and the exact discrete-time model of the testbench (12). Indeed, in practice we are interested in the stability of the sampled-data system that consists of the continuous-time system (7)-(10) and the observer (14). To conclude the SPA stability of the sampled-data system, we apply [13, Theorem 2]. The theorem states that uniform global asymptotic stability of the exact discrete-time model together with the uniform global boundedness of the control signal with respect to the time sampling $T > 0$ implies uniform global asymptotic stability for the sampled-data system. This result can be applied directly for the case of SPA stability considered in Proposition 3.2, provided that the controllers $u_k = u_k(T_E, \psi_\Delta, \omega_E, \omega_D)$ and $u_k = \hat{u}_k(\hat{T}_E, \hat{\psi}_\Delta, \omega_E, \omega_D)$ are uniformly globally bounded for all $T > 0$. ■

D. Comparison between "static" and dynamic observer

It is obvious from (17) and (16) that the states T_E and ψ_Δ can be constructed by manipulating the measured states ω_E and ω_D. Rewriting (17) and (16) respectively as

$$\begin{aligned} T_E &= \theta_E \dot{\omega}_E + \theta_D \dot{\omega}_D + T_{DSet} \\ \psi_\Delta &= \frac{1}{c}\Big(\theta_D \dot{\omega}_D - d(\omega_E - \omega_D) + T_{DSet}\Big) \ , \end{aligned} \tag{33}$$

one may argue that (17) and (16) can also function as an observer to estimate the states T_E and ψ_Δ. Indeed, this is true. A similar construction has been used in [12], in which case the observer was shown to be a semiglobal observer to the system being estimated. However, although (33) might be used as an observer, it carries some limitations. First, the initial condition of the state estimates cannot be set independently from the initial condition of the two measured states ω_E and ω_D. Second, we cannot set the speed of convergence of the observer (33) as the convergence terms do not exist in the formula. In this case, if the measured states do not converge to a certain value, the estimation convergence of the observer (33) is also not guaranteed.

Comparing the observer (14) and (33), we can say that (33) is a static observer and (14) is a dynamic observer for the system (10)-(9) with output (11). In Section IV-B we will show a comparison between the observers (14) and (33) in one simulation setting.

IV. Set point tracking using output feedback

A. Output feedback controller design

As separation principle is valid for the state feedback controller and the observer, we can use the state estimate to substitute the original state to construct an output feedback controller for the engine. In [16] we have designed a controller that guarantee asymptotic stability for a setpoint tracking problem of the engine. The controller is designed via a model transformation approach as briefly describe in the followings.

We define the state normalization as follows

$$\begin{aligned} x_1 &= \frac{T_E - T_{E0}}{\Delta T_E} , \qquad x_2 = \frac{\psi_\Delta - \psi_{\Delta 0}}{\max(\psi_\Delta)} , \\ x_3 &= \frac{\omega_E - \omega_{E0}}{\Delta \omega_E} , \qquad x_4 = \frac{\omega_D - \omega_{D0}}{\Delta \omega_D} , \end{aligned} \tag{34}$$

with T_{E0}, $\psi_{\Delta 0}$, ω_{E0} and ω_{D0} defines the operating point and ΔT_E, $\max(\psi_\Delta)$, $\Delta\omega_E$ and $\Delta\omega_D$ the maximum expected distance from the equilibrium point. With this scaling and taking $c \max(\psi_\Delta) = \Delta T_E$, the system (7)-(10) can now be represented as an extended Hammerstein system as follows

$$\begin{aligned} \dot{x}_1 &= -(\tilde{c}_0 + \tilde{c}_1 x_3 + \tilde{c}_2 x_3^2)x_1 - \gamma_1 x_3 - \gamma_2 x_3^2 + u_1 \\ \dot{x}_2 &= b(x_3 - x_4) \\ \dot{x}_3 &= \frac{1}{\theta_E}\left(\frac{c}{b}x_1 - \frac{c}{b}x_2 - d(x_3 - x_4)\right) \\ \dot{x}_4 &= \frac{1}{\theta_D}\left(\frac{c}{b}x_2 + d(x_3 - x_4)\right) + u_2 , \end{aligned} \tag{35}$$

with the inputs

$$u_1 = \frac{m(x_1, x_3, \alpha) - m(0, 0, \alpha_0)}{\Delta T_E} , \; u_2 = -\frac{T_{DSet} - T_{D0}}{\theta_D \Delta\omega_D} ,$$

and $\tilde{c}_0$, $\tilde{c}_1$, $\tilde{c}_2$, b, γ_1, γ_2 are positive constants.

In [16] a continuous-time controller has been constructed to satisfy some robust optimal design criteria. The control Lyapunov function used for designing the controller is

$$V(x_1, x_2, x_3, x_4) = k_1 x_1^2 + k_2 x_2^2 + k_3 x_3^2 + k_4 x_4^2 + k_5 x_2 x_4, \tag{36}$$

with $k_i \in \mathbb{R}^+$, $i = 1 \cdots 4$ and $k_5 \in \mathbb{R} - \{0\}$. The positive definiteness of $V(\cdot)$ is guaranteed for some k_5 with $|k_5|$ sufficiently small. The controller takes form

$$u_k = -[R(x)g(x)]^\top \left[\frac{\partial V(x)}{\partial x}\right]^\top = -\begin{bmatrix} 2r_1 k_1 x_1 \\ r_2(2k_4 x_4 + k_5 x_2) \end{bmatrix} \tag{37}$$

with a positive matrix $R = \mathrm{diag}[r_1, 0, 0, r_2]$. The controller has been proved to SPA stabilize the system.

Note that the controller (37) is designed to SPA stabilize the normalized model (35) of the engine. As our main objective is to apply the controller to the engine test bench, we need to transform back the normalized model of the test bench and test the stability of tracking of the original system. From the state transformation (34), we have the relations

$$\begin{aligned} m(\omega_E, T_E, \alpha) &= u_1 \Delta T_E + T_{E0}(c_0 + c_1 \omega_{E0} + c_2 \omega_{E0}^2) \\ T_{DSet} &= -u_2 \theta_D \Delta\omega_D + T_{D0} , \end{aligned} \tag{38}$$

where we have chosen $\psi_{\Delta 0} = \frac{T_{E0}}{c}$, $T_{D0} = T_{E0}$ and $\omega_{E0} = \omega_{D0}$. The setpoint tracking aims to follow the changing of operating points (T_{E0}, ω_{E0}) of the engine.

Replacing the unmeasured states with their estimate value, and applying the transformation (34), the output feedback controller takes the form

$$\begin{aligned} m(\omega_E, \hat{T}_E, \alpha) &= -2r_1 k_1(\hat{T}_E - T_{E0}) \\ &\quad + T_{E0}(c_0 + c_1\omega_{E0} + c_2\omega_{E0}^2) \\ T_{DSet} &= 2k_4 r_2 \theta_D(\omega_D - \omega_{D0}) \\ &\quad + k_5 r_2 \theta_D \Delta\omega_D \frac{c\hat{\psi}_\Delta - T_{E0}}{\Delta T_E} + T_{D0} . \end{aligned} \tag{39}$$

B. Simulation results

In this subsection, by simulation we first show the convergence of the observer in estimating the states T_E and ψ_Δ. Further, we will apply the output feedback controller (39) to control the engine test bench (10), (7)-(9). The performance of the output feedback controller (39) is compared to the state feedback controller (37) for a setpoint tracking assignment.

In the simulation we have used the engine parameters $\theta_E = 0.32$ kgm^2, $\theta_D = 0.28$ kgm^2, $d = 3.5505$ Nms/rad and $c = 1.7441 \times 10^3$ Nm/rad, which are based on a dynamic test bench with a production BMW M47D diesel engine. The controller parameters are $k_1 = 1.56860$, $k_2 = 0.00174$, $k_3 = 0.88000$, $k_4 = 1.05000$ and $k_5 = -0.01450$, and the coefficients of the approximate dynamic model after scaling are $\tilde{c}_0 = 6.3466$, $\tilde{c}_1 = 3.2096$, $\tilde{c}_2 = 2.7744$, $b = 1.8264 \times 10^3$, $\gamma_1 = 4.8143$ and $\gamma_2 = 4.1616$. We apply the controller for a setpoint tracking when changing the operating point (T_E, ω_E) of the engine each following a square wave reference signal. Moreover, we have chosen $R = \mathrm{diag}[1, 0, 0, 2]$. The initial condition of the engine $(50,\ 50/c,\ 3000,\ 3000)$, the initial condition of the observer $(100,\ 100/c)$, choosing $L_1 = 1$, $L_2 = 1$ and $T = 0.01$ sec.

In Figure 2, it is shown that the observer can estimate the unmeasured states T_E and ψ_Δ very well as the discrete-time observer converges very quickly to the engine test bench system, even when the initial condition of the observer is very different from the initial condition of the test bench.

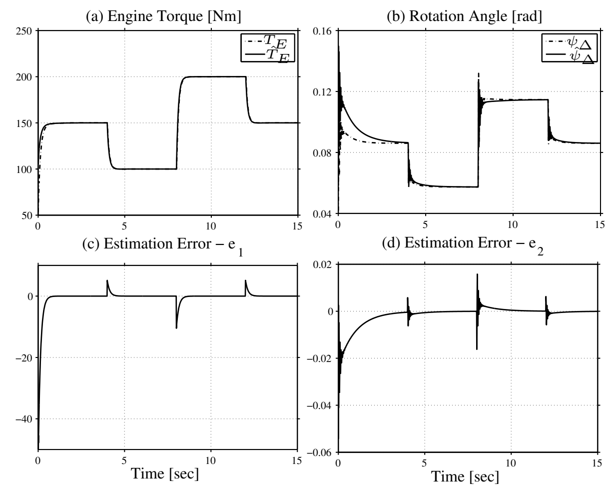

Fig. 2. Convergence test of the observer.

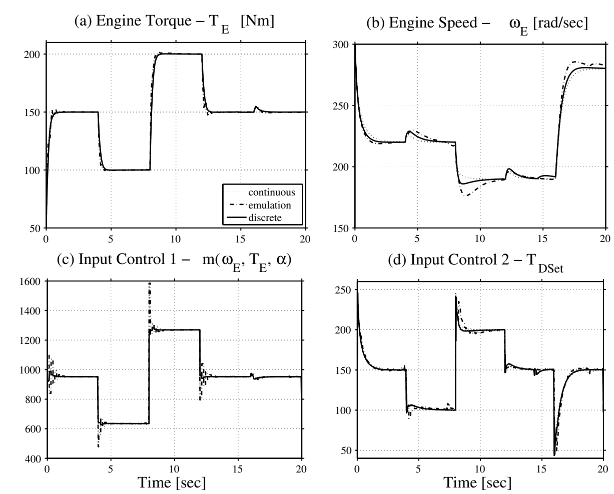

Fig. 3. Tracking using output feedback controller.

The response of the system, comparing the discrete-time observer with the emulation observer and the continuous-time observer in the output feedback configuration is shown in Figure 3. In the simulation we have used the parameters $T = 0.01$ sec. It apprears that with the discrete-time observer the closed-loop response of the system is closer to the continuous-time system rather than with the emulation observer. A clear comparison can be seen from the engine speed response in Figure 3(b). On the other hand, the output feedback control effort with the discrete-time observer is lighter than the effort with the emulation observer, which can be seen clearly in Figure 3(c).

To complete the comparison analysis, we also compare the discrete-time observer (14) with the discrete-time static observer (33). Figure 4 show the performance of the two observers in an output feedback tracking. We have used sampling time $T = 0.1$ sec in the simulation.

V. Summary

We have presented a nonlinear discrete-time partial state observer design for a combustion engine test bench system and have tested the performance of an output feedback controller constructed based on the observer. We have also

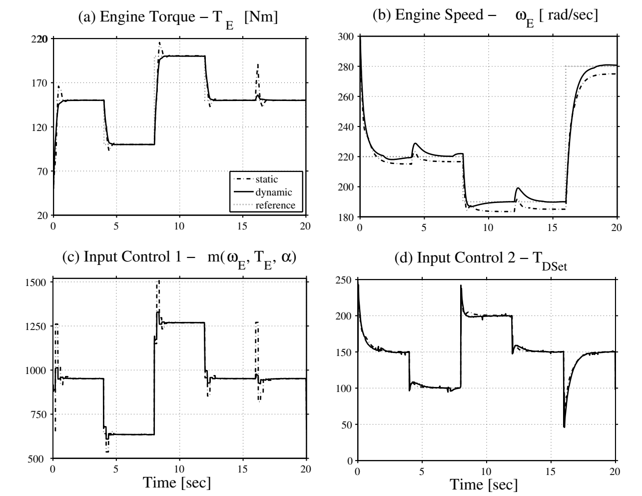

Fig. 4. Tracking using output feedback with dynamic and static observers.

introduced the so called static and dynamic observer, and present a comparison of these two type of observers showing that dynamic observers give more flexibility than static observers. As this study is done only based on simulation, the next step will be to implement the observer and output feedback controller design to a real engine test bench for solving the setpoint tracking problem.

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