

# On Stability in the Presence of Analog Erasure Channels

V. Gupta and N. C. Martins

**Abstract**—Consider a discrete-time networked control scheme, in which the controller has direct access to noisy measurements of the plant's output, but the controller and the actuator are connected via a link that features Bernoulli erasure events. We determine necessary and sufficient conditions for the stabilizability of an unstable linear and time-invariant plant. We show that these conditions are identical for two types of actuators:

- (Type I) Processing at the actuator has access to the plant's model;
- (Type II) Processing at the actuator uses a *universal* algorithm that does not depend on the model of the plant.

We also identify cases where availability of acknowledgements over the controller-actuator channel is not required for stability. We also consider decentralized networked control structures, where two or more independent controller-link-actuator assemblies have access to different measurements of the plant's output.

## I. INTRODUCTION

Networked control systems have now become an active area of research (see, e.g., [1], [3], [18] and the references therein). The performance of such systems is adversely affected by the reliability limitations of the underlying communication network. Hence, networked control systems must be designed so as to combat detrimental effects such as quantization error, random delays, data loss and data corruption to name a few. These adverse effects may also lead to stability loss.

In this paper, we focus on the problem of estimation and control across communication links that exhibit data loss. Preliminary work in this area has largely concentrated on the case with one sensor and one controller. In particular, stability [23], [27] and performance [17], [23], [15] have been characterized for a single sensor-controller pair. Approaches to compensate for the data loss, so as to counteract the degradation in performance, have also been proposed, see, e.g., [13], [21], [17] among others. Also relevant are the results by Azimi-Sadjadi [2], Schenato et al. [22] and Imer et al. [14] who looked at controller structures to minimize quadratic costs for systems in which both sensor-controller and controller-actuator channels exhibit erasure. The related problem of optimal estimation across an erasure link was considered by Sinopoli et al. in [24] for the case of one sensor and erasures occurring in an i.i.d. fashion, while Gupta et al. [8] considered multiple sensors and more general erasure models. Recent research has also addressed the case where multiple sensors and erasure channels are present [12], [4], although we would like to argue that the

simplifying assumptions made in these (notably a single controller with access to information from all the sensors and generating inputs for all the actuators) render them essentially in the domain of centralized control.

Yet other lines of investigation have taken a more general approach to the control of networked control systems, by exploring the possibility of pre-processing information prior to transmission, and transmission of extra data to improve the performance of a networked control system. This view was taken, e.g., in [10], [9], [12], [5]. This pre-processing strategy can also be seen in the recent results on receding horizon networked control, in which a few future control inputs are transmitted at every time step by the controller and buffered at the actuator to be used in case subsequent control updates are dropped by the network and do not arrive at the actuator(s) [6], [7], [16], [19], [20].

In this paper, we consider the effect of such information processing on the controller-actuator channel. Clearly, any processing that is done by the controller must depend on the complexity and information assumed to be available at the actuator. From the spectrum of assumptions that can be made about the actuator, we choose two cases: We consider a *smart* actuator (also referred to as a type I actuator) that has access to the system model and also has computational capabilities. We also investigate the use of a *logical* actuator (also referred to as a type II actuator) that has access to computational capabilities but not the system model. We analyze stabilizability conditions in the presence of these two forms of actuators. We show that the necessary and sufficient conditions for stabilizability are identical for type I and type II actuators. Since the type II actuator does not require the system model, this result shows that a *universal* actuator is optimal from the point of view of stability.

In addition, we also concentrate on the effect of lack of acknowledgements to the controller about any data transmitted over the controller-actuator channel. It has been shown [22], [14] that the absence of acknowledgements breaks the separation principle for our problem. Limited work has been done on the design problem for this case. As an example, [25] restricted the controller design to be linear and showed via some simulations that the loss in performance may not be huge in certain cases. Some cases when stabilizability does not depend on availability of acknowledgements were also considered in [11]. The root of the difficulty can be traced to the fact that if no acknowledgements are available to the controller, it does not know the control input applied by the actuator and hence the control input begins to have a dual effect. We show that both for actuators of type I or type II, absence of acknowledgements do not have any impact on the stability

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conditions. Since the dual effect of control is also important in distributed control, we also show how some of our results can extend to distributed control problems. We would like to point out, however, that except for the smart actuator case, the general distributed control problem is still open.

The paper is organized as follows. We formulate the problem for both the centralized and the decentralized cases in the next section. We then consider the centralized case. We present a necessary condition for stabilizability with type I actuator. We then show that the condition is sufficient for stabilizability with actuator of type II. Together, these results imply that the stabilizability conditions for type I and type II actuators are identical. We then repeat the process for distributed controllers.

## II. PROBLEM FORMULATION

We consider two distinct problem set-ups. Consider first the problem set-up shown in Figure 1 with the following associated assumptions. In the sequel, we refer to this set-up the centralized control problem.

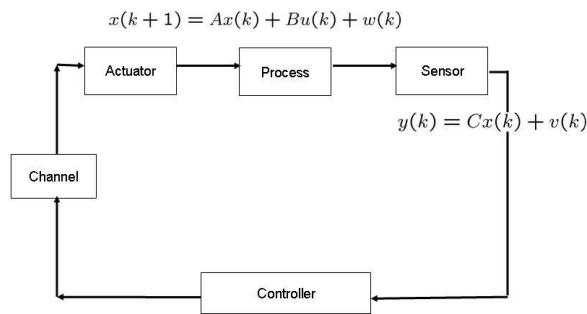


Fig. 1. Basic framework for the centralized control problem.

### A. Centralized Control Problem

**Process and Sensor:** Consider a process that evolves as

$$x(k+1) = Ax(k) + Bu(k) + w(k), \quad x(0)$$

where  $x(k) \in \mathbf{R}^n$  is the process state,  $u(k) \in \mathbf{R}^m$  is the control input and  $w(k)$  is the process noise modeled as Gaussian, white, zero mean with covariance  $Q > 0$ . The initial condition  $x(0)$  is drawn from a Gaussian distribution with zero mean and covariance  $P_0$ . The process is observed using a sensor that generates measurements of the form

$$y(k) = Cx(k) + v(k),$$

where  $v(k)$  is the measurement noise modeled Gaussian, white, zero mean with covariance  $R > 0$ . The noises  $\{w(k)\}$  and  $\{v(k)\}$  and the initial condition  $x(0)$  are assumed mutually independent. We assume that the pair  $(A, B)$  is controllable and the pair  $(A, C)$  is observable.

**Controller and Channel:** The control input is generated by a controller that has access to the measurement set  $\{y(0), y(1), \dots, y(k)\}$  at time  $k$ . Thus, for simplicity, we do not assume the presence of a communication channel between the sensor and the controller. Most of the results,

however, extend to the case when the channel is present. The control input is transmitted over an analog erasure channel to the actuator. The channel, at every time step  $k$ , accepts as input a vector  $t(k) \in \mathbf{R}^{n_1}$  of finite dimension. The output vector  $r(k)$  is given by

$$r(k) = \begin{cases} t(k) & \text{with probability } 1-p \\ \phi & \text{otherwise,} \end{cases}$$

where we have used the symbol  $\phi$  to indicate that no information is received at the output. Note that while we assume that the erasure events occur according to a Bernoulli process, more complicated models such as when the erasure events occur according to a Markov chain are possible and can be analyzed using similar techniques as presented in the paper. Also note that we have not assumed that the controller has explicit knowledge about the erasure events. Thus, a mechanism such as acknowledgements has not been implemented.

**Actuator and Stability Definition:** The actuator at time  $k$  has access to the vector set  $\{r(0), r(1), \dots, r(k)\}$ . It calculates and applies the control input  $u(k)$  to the process. Depending on the information and computational capabilities at the actuator, we consider two types of actuator as defined below:

- 1) *Type I or a Smart actuator:* The actuator has access to the system model, and has some computational capability. This is a reasonable assumption for any device that communicates using wireless. In any case, the stability conditions obtained using a smart actuator provide necessary conditions for stability for any other actuator capabilities.
- 2) *Type II or a Logical actuator:* The actuator has access to limited computational capability. However, it does not have access to the system model.

Note that this list is not exhaustive and other variations on the actuator capabilities can be thought of. As an instance, when the actuator has access to a buffer, the optimal design problem was solved in [7].

The aim of the system is to stabilize the process. We will be interested in mean squared stability. Thus, the system is stable if and only if

$$\limsup_{k \rightarrow \infty} E[x(k)x^T(k)] \leq c,$$

where  $c$  is a constant. The expectation is taken over all the sources of randomness in the system. If there exists a controller such that with a finite value of  $n_1$ , the closed-loop system becomes stable, the system is said to be stabilizable.

### B. Distributed Control Problem

The distributed control framework that we consider is shown in Figure 2. The process now evolves as

$$x(k) = Ax(k) + \sum_{i=1}^N B_i u_i(k) + w(k),$$

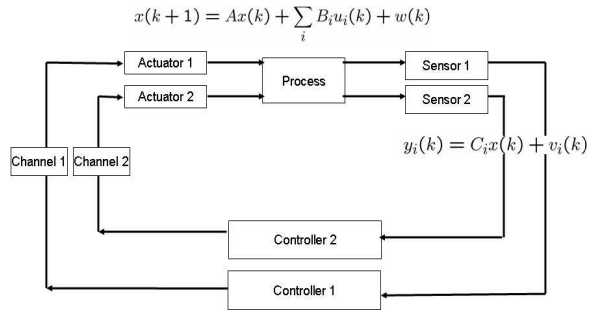


Fig. 2. Basic framework for the distributed control problem.

where  $x(k) \in \mathbf{R}^n$  is once again the process state and  $w(k)$  is white Gaussian noise modeled zero mean with covariance  $Q$ . The vector  $u_i(k) \in \mathbf{R}^{m_i}$  refers to the control applied by the  $i$ -th controller. The  $i$ -th controller has access to measurements from a sensor of the form

$$y_i(k) = C_i x(k) + v_i(k),$$

where  $v_i(k)$  is the measurement noise modeled white Gaussian with zero mean and covariance  $R_i$ . The control input  $u_i(k)$  is transmitted to the actuator through an analog erasure channel with erasure probability  $p_i$  at every time step. We assume that all the sources of randomness in the system including the noises, data erasure events and the initial condition are mutually independent. Furthermore, if  $C = [C_1^T \ \cdots \ C_N^T]^T$ , and  $B = [B_1 \ \cdots \ B_N]$ , then we assume that the pairs  $(A, C)$  and  $(A, B)$  are observable and controllable, respectively. Thus, if all sensors shared their measurements, the process will be observable and if no communication channels were present, the process will be controllable.

We are once again interested in the mean squared stability of the system for various types of actuator as defined above. The actuator now has access to the messages from all the channels. Compared to the centralized control problem, this problem is more complicated because of the presence of multiple sensor-controller pairs that can not only observe and control different modes, but also may be able to obtain better performance through signaling. To isolate the two effects, we will sometimes make one of the following simplifying assumptions.

**Assumption B:** All actuators have access to all the signals received from the different controllers.

**Assumption C:** All controllers have access to all the measurements received from the various sensors.

### III. CENTRALIZED CONTROLLER

We present the results for the single centralized controller in this section. Note that an actuator of type I can emulate an actuator of type II. Thus, the conditions for stability for the smart actuator are necessary for stability for a logical actuator. Similarly, the stability conditions obtained for an actuator of type II will be sufficient for stability with an actuator of type I. With this fact in mind, we begin with

the necessary conditions for stabilizability for actuator of type I.

#### A. Necessary condition for stabilizability

Before we state the necessity result, we state the following result from [9] about the optimal information processing for LQG control across an analog erasure channel.

*Theorem 3.1 (From [9]):* Consider the centralized problem set-up shown in Figure 1, except with the channel between the sensor and the controller rather than the controller and the actuator. For any quantity that is calculated using a causal algorithm by the sensor and transmitted over the channel, and for any design of the controller, the process is stabilizable only if

$$p |\rho(A)|^2 < 1, \quad (1)$$

where  $\rho(A)$  denotes the spectral radius of the matrix  $A$ . We will denote this problem as  $\mathcal{P}_2$  to compare it with the centralized problem set-up discussed in this paper, that we denote by  $\mathcal{P}_1$ . It was also shown in [9] that the result continues to be true even if the sensor has access to the previous control inputs while calculating the quantity it transmits at every time step.

Using this result, we obtain the following necessary condition for stabilizability. We assume that an actuator of type I is being used.

*Theorem 3.2:* Consider the centralized control problem stated above when the actuator is a smart actuator. Then, the process is stabilizable in the mean-squared sense defined above only if

$$p |\rho(A)|^2 < 1, \quad (2)$$

where  $\rho(A)$  denotes the spectral radius of the matrix  $A$ . Moreover, this condition remains necessary for stabilizability even if the control has access to the received vector  $r(k)$  at time  $k$ . Thus, the necessity of the stability condition does not depend on the existence of acknowledgements for the controller-actuator channel.

*Proof:* The proof is based on mapping the problem to one considered in [9]. Suppose that equation (2) does not hold, but the process is stabilizable when the controller transmits the quantity obtained by using algorithm  $\mathcal{C}$  and the actuator applies the control input obtained by using the algorithm  $\mathcal{A}$ . Now consider the problem set-up considered in Theorem 3.1. Let the sensor implement algorithm  $\mathcal{C}$  and the controller implement algorithm  $\mathcal{A}$ . Note that this is possible since the information available at the sensor (resp. controller) in problem  $\mathcal{P}_2$  is identical to the information available to the controller (resp. actuator) in problem  $\mathcal{P}_1$ . In this situation, identical control inputs are applied to the process in problems  $\mathcal{P}_1$  and  $\mathcal{P}_2$ . Thus, our supposition implies that the process in problem  $\mathcal{P}_2$  is stabilizable. But, this contradicts the result in Theorem 3.1. Thus, our supposition must be incorrect and the process cannot be stabilized when equation (2) does not hold.

That the necessity does not depend on availability of the acknowledgements can be proved similarly by noting that

Theorem 3.1 continues to hold even if the sensor has access to the control inputs in problem  $\mathcal{P}_2$ . ■

### B. Sufficient condition for stabilizability

Interestingly, we can prove that the condition in equation (2) is almost sufficient for stabilizability. We demonstrate this by constructing an algorithm that leads to bounded covariance of the state, in the presence of an actuator of type II, even if acknowledgements are not available to the controller. We have the following theorem.

*Theorem 3.3:* Consider the centralized control problem stated above when the actuator is a logical actuator. Then a sufficient condition that the system is stabilizable is that (2) holds.

*Proof:* We provide a specific algorithm to be followed by the controller and the actuator. To begin with, we note that since the process is observable, if no control input were applied to the process, the state can be determined with a bounded error covariance given  $n$  consecutive measurements. Similarly, since the process is controllable,  $n$  control inputs can be calculated such that when they are applied consecutively to the process, the covariance of the resulting state value lies in a bounded set centered at origin. Note that were measurement and process noises not be present, the above two statements could be strengthened to say that the covariance value would be zero. Denote the vector formed at time  $k$  by stacking such control inputs to be applied from time  $k$  to  $k + n - 1$  by  $\mathcal{U}(k)$ .

The algorithm proceeds in batches of time. Each batch consists of  $n + T$  time steps, where  $T$  is a parameter assuming positive integral values.

- 1) For the first  $n$  time steps, no control input is applied to the process. Using the measurements obtained over these time steps, the state value is estimated.
- 2) For the next  $T$  time steps, the controller calculates and transmits the vector  $\mathcal{U}(k)$  for  $k = n - 1$  to  $n + T - 1$ . The controller can calculate this vector as long as it can estimate the state with a bounded error covariance. This is true at time  $n$  because of the observation taken from step 0 to  $n - 1$ . At all subsequent time steps, the controller assumes that no control input was applied at any previous time step to estimate the state value and then calculate  $\mathcal{U}(k)$ .
- 3) If the transmission is not successful, no control input is applied. As soon as the actuator receives a vector for the first time, it extracts and applies the control inputs over the next  $n$  time steps. At all remaining time steps, it applies no control input.

At time  $n + T + 1$ , the entire process is repeated.

Using this algorithm, the state covariance at time  $n + T$  can be calculated by conditioning on the event that transmission was successful for the first time at step  $n, n + 1, \dots, n + T - 1$ , and the event that transmission was not successful at any time step. For the event that transmission was not successful at any time step, the state covariance at time  $n + T$  can be upper bounded by  $\| \| A^n \| \|^2 \| A \|^{2T} P_0$ ,

where  $\| M \|$  is the spectral norm of matrix  $M$ . For any other event, due to the controllability assumption, the error covariance is upper bounded by a positive definite matrix independent of  $P_0$  at the time when a packet is received by the actuator. If  $T$  is finite, the error covariance at time  $n + T$  is also bounded. Thus, for all events for which transmission was successful at least once, the error covariance can be jointly upper bounded by a positive value  $\Sigma_1$  that is independent of  $P_0$  and is bounded if  $T$  is finite. Thus, we can bound

$$P(n + T) \leq p^T \| A^n \|^2 \| A^T \|^2 P_0 + \Sigma_1.$$

Similarly, the state covariance at time  $2(n + T)$  can be bounded as

$$\begin{aligned} P(2(n + T)) &\leq p^T \| A^n \|^2 \| A^T \|^2 P(n + T) + \Sigma_1 \\ &\leq p^{2T} \| A^n \|^4 \| A^T \|^4 P_0 \\ &\quad + (p^T \| A^n \|^2 \| A^T \|^2 + 1) \Sigma_1. \end{aligned}$$

Continuing this process, we see that the state covariance remains bounded if

$$p^T \| A^n \|^2 \| A^T \|^2 < 1.$$

Since  $\lim_{T \rightarrow \infty} \| A^T \|^{\frac{1}{T}} = \rho(A)$ , the stabilizability condition is  $p | \rho(A) |^2 < 1 + \epsilon$ , where  $\epsilon$  can be made as small as we wish by choosing a large enough (but finite) value of  $T$ . Thus, the sufficient condition for stability can be cast as close to (2) as we wish. ■

### C. Comments

The above two theorems together imply that the condition (2) is necessary and sufficient for stability with actuators of type I and II. This interesting result means that the knowledge of the plant at the actuator does not help for expanding the stability region. This also has interesting implications, e.g., for situations in which the controller might be estimating the plant parameters as they change. This result implies that, for stability purposes, the controller does not need to send the plant matrices to the actuator.

Also note that for actuators of type I and II, acknowledgements for the controller-actuator channel will not gain anything in terms of stability.

For actuators of type I, we can extend the results of [9] to solve for the optimal controller and actuator design for minimizing a quadratic cost (and not merely the stability criterion). We refer the reader to [9] for details of the algorithm. Roughly speaking, the controller serves as an information relaying device and transmits an estimate of the state to the actuator. The actuator calculates the control input.

## IV. DISTRIBUTED CONTROL PROBLEMS

Note that if either the assumption B or assumption C holds and, moreover, if each individual controller has access to data transmitted by every other controller, the problem reduces to the cases considered by [12], [4]. While

interesting first steps, the problem formulation in those works is not truly distributed. In this work, we make further inroads into the problem.

For the presentation of the results below, we assume  $N = 2$ . The case for general  $N$  is similar. We begin with the case when assumption B holds.

#### A. Distributed control with assumption B

If assumption B holds, the following stability results hold irrespective of assumption C.

1) *Actuator of type I:* We have the following result for actuator of type I.

*Theorem 4.1:* Consider the distributed control problem stated above with assumption B being true with  $N = 2$  and a smart actuator. Let  $A_i$  denote the unobservable part of  $A$  when the pair  $(A, C_i)$  is put in the observer canonical form. Then a necessary and sufficient condition for the process to be stabilizable is that the probability of erasures  $p_1$  and  $p_2$  satisfy the following set of inequalities

$$\begin{aligned} p_1 |\rho(A_2)|^2 &< 1 \\ p_2 |\rho(A_1)|^2 &< 1 \\ p_1 p_2 |\rho(A)|^2 &< 1, \end{aligned}$$

where  $\rho(X)$  denotes the spectral radius of matrix  $X$ .

Note that the stabilizability conditions do not involve the control matrices  $B_i$ 's. We can also consider the effect of allowing the controllers to communicate with each other.

*Theorem 4.2:* Consider the distributed control problem stated above with  $N = 2$  and a smart actuator. Additionally, suppose that the controllers can transmit data to each other over an analog erasure channel with erasure probability  $p_{12}$ . Then a necessary and sufficient condition for the process to be stabilizable is that the probabilities  $p_1$ ,  $p_2$  and  $p_{12}$  satisfy the following set of inequalities

$$\begin{aligned} \max(p_2, p_{12}) p_1 |\rho(A_2)|^2 &< 1 \\ \max(p_1, p_{12}) p_2 |\rho(A_1)|^2 &< 1 \\ p_1 p_2 |\rho(A)|^2 &< 1. \end{aligned}$$

The above two theorems quantify the stability region loss because of decentralization in this case. The increase in stability region by allowing the controllers to communicate with each other is similar to that observed for digital noiseless channels in [26].

Both the above two theorems can be proven in a similar fashion as the centralized case by using the results from [12]. We omit the proof here.

2) *Actuator of type II:* Similar to the centralized controller case, the stability region for a logical actuator is identical to the region for a smart actuator. We can state the following result.

*Theorem 4.3:* Consider the distributed control problem stated above with assumption B being true with  $N = 2$  and actuator of type II. Let  $A_i$  denote the unobservable part of  $A$  when the pair  $(A, C_i)$  is put in the observer canonical form. Then a necessary and sufficient condition for the process to

be stabilizable is that the probability of erasures  $p_1$  and  $p_2$  satisfy the following set of inequalities

$$\begin{aligned} p_1 |\rho(A_2)|^2 &< 1 \\ p_2 |\rho(A_1)|^2 &< 1 \\ p_1 p_2 |\rho(A)|^2 &< 1, \end{aligned}$$

where  $\rho(X)$  denotes the spectral radius of matrix  $X$ .

The proof follows that of the centralized control case. The necessity of the conditions is due to the conditions being necessary for actuator of type I. For the sufficiency, each controller transmits control inputs that will transfer all those modes to zero that it can observe. Note that some of these modes may not be controllable from the particular controller. However, because of assumption B, the control inputs can still be applied to the plant using all those actuators that correspond to controllers that can control the modes.

#### B. Distributed control with assumption C

The stability conditions alter when assumption B does not hold. We present the results for the case when assumption B is not true, but assumption C is.

1) *Actuator of type I:* With assumption C being true, each controller has access to the same measurements. However, it can only control certain modes. Since the assumption B does not hold, there is no benefit in transferring information about the modes that it cannot control to the corresponding actuator. We have the following result.

*Theorem 4.4:* Consider the distributed control problem stated above with assumption C being true,  $N = 2$  and an actuator of type I. Let  $\bar{A}_i$  denote the uncontrollable part of  $A$  when the pair  $(A, B_i)$  is put in the controller canonical form. The process is stabilizable if the following inequalities hold

$$\begin{aligned} p_1 |\rho(\bar{A}_2)|^2 &< 1 \\ p_2 |\rho(\bar{A}_1)|^2 &< 1 \\ p_1 p_2 |\rho(A)|^2 &< 1. \end{aligned}$$

The proof of theorem is very similar to the case of assumption B being true. It relies on a decomposition of the system into modes that can be stabilized only by controller 1, only by controller 2 and by either of the controllers. For modes that can be stabilized by controller 1, the stabilization problem is then identical to the centralized control case and the result readily follows.

2) *Actuator of type II:* Once again, a universal algorithm by an actuator of type II can obtain the same stability region as a smart actuator.

*Theorem 4.5:* Consider the distributed control problem stated above with assumption C being true,  $N = 2$  and a logical actuator. Let  $\bar{A}_i$  denote the uncontrollable part of  $A$  when the pair  $(A, B_i)$  is put in the controller canonical form. The process is stabilizable if the following inequalities

hold

$$\begin{aligned} p_1 | \rho(\bar{A}_2) |^2 &< 1 \\ p_2 | \rho(\bar{A}_1) |^2 &< 1 \\ p_1 p_2 | \rho(A) |^2 &< 1. \end{aligned}$$

Once again, sufficiency can be proven using an algorithm similar to the one constructed for the centralized case, with the difference that for the distributed case, each controller can only transmit control values to drive the modes it can control to the origin. Considering the modes controlled only by controller 1, only by controller 2 and by both the controllers, gives the three inequalities respectively. Necessity follows from the condition for the smart actuator.

The case when neither assumption B nor assumption C holds is more complicated since signalling is allowed. We do not yet have a crisp characterization for the stabilizability conditions for that case.

## V. CONCLUSIONS AND FUTURE DIRECTIONS

This paper deals with some cases of centralized and distributed control problems where the control is being done across analog erasure channels. We were interested primarily in mean square stability in such cases. We considered two types of actuators based on the complexity and information available to them. For smart actuators, we provided stability conditions for both the cases when acknowledgements were available and when they were not. For logical actuators, we showed that knowledge of plant at the actuator does not alter the stability region. We identified some cases where the stability conditions were identical even if acknowledgements were not available. We also presented some results for the distributed control case. Although the distributed control results were presented for 2 controllers, they can be easily extended for more controllers being present.

This work is simply a first step in this interesting arena. For distributed control problems, we are currently working on stability results when neither assumption B nor assumption C holds. It will also be interesting to derive the controller design from a LQG performance perspective for the actuator of type II.

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