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Power Leader Fault Detection in Nonlinear Leader-Follower Networks

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Abstract—**This paper presents a model for fault detection of a power leader in nonlinear leader-follower networks. The fault detection method is developed for the network model proposed by Wang and Slotine [1]. Every follower is coupled with a nonlinear, neural net based observer for fault detection. Neural net tuning algorithms are derived and fault identifiers are developed using the Lyapunov approach. We consider fault detection of the power leader, and how such fault propagates through the network. We estimate the power leader fault detectability time based on the followers' observers. The paper studies properties of the fault dynamics *i.e.*, the dynamics of a fault evolution process through a network of interconnected dynamic elements. The approach for leader-follower fault detection can also be used with any other type of nonlinear systems observer. Simulation results are presented to illustrate the effectiveness of the proposed technique.**

I. INTRODUCTION

Early fault detection and identification in complex dynamical networked systems is essential to prevent a whole network system failure as well as to efficiently isolate a fault source, conduct recovery and repair of faulty components. It is a difficult task to locate the malfunctioning system components in a network because the fault effects propagate through the system, affecting other healthy modules. This paper describes the fault detection method for a dynamic coupled network where the fault occurs in a power leader.

A. *Related Work*

First, the mathematical model of a coupled power leader-follower network is presented. In [1], Wang and Slotine present a theoretical study for the various dynamic models of a coupled network according to different leader roles. The paper discusses three different types of coupled networks containing different roles of leaders: a power leader, a knowledge leader, and a pacific coexistence. Recent work for this kind of study can be also found in [2], [3], [4], [5]. Each provides an understanding for distributed networks in the natural world and emulate them in artificial systems. In this paper, a coupled network containing a power leader and *n* power followers is selected to study a fault detection and propagation properties. Details of a coupled dynamic networks with power leader are briefly reviewed in Section II.

Second, it is important to know how to detect a fault in an interconnected network of dynamic elements. Significant research effort has been concentrated on the development of fault detection methods using intelligent control techniques such as neural network (NNs) and fuzzy logic [6], [8], [9], [10]. The advantage of NNs is the nonlinear approximation and adaptive learning properties [11], [12] thereby providing the capability to learn abnormalities and failures from actual monitoring data. Other traditional approaches such as model-based fault detection are restrictive because of the requirement of an accurate system model. NNs and fuzzy logic are artificial intelligence tools able to overcome this sort of the restrictive requirements. Additionally, if a NN detects the faults, it may be able to classify the faults without a detailed system model [10], [13], [14]. Statistical analysis based on the Lyapunov function for fault tolerant control system is described in [15]. The fault tolerant control system design is presented in [16].

Recent work in adaptive control was also enabled an interesting application in actuator failure detection [6], [17]. Papers [18], [19] address fault detection, isolation, and compensation in partially known nonlinear systems. In this paper, it is assumed that the power leader and each follower are unknown non-linear coupled dynamical elements interconnected into a network. The neural networks (NN) tuning algorithms presented in [6] are applied for the fault detection of the coupled network using NN observer attached to each follower connected either directly or indirectly to the power leader.

Third, an algorithm to locate fault source in the network is required to preserve time-consuming network inspection upon fault occurrence. Fault identification can be a daunting task especially when components of the system are mutually coupled. In [22], [23], the nonlinear fault isolation problems are investigated. In [22], a fault isolation method is described to determine the particular type of known fault set. In [23], the algorithm to determine the particular faulty sensor from a set of sensors under consideration is presented and the proposed fault diagnosis architecture consists of a fault detection estimator and a bank of isolation estimators, each corresponding to a particular output sensor. In this paper, without information about the fault type and a bank of isolation estimators, another feasible fault location strategy using the fault detection time is developed and mathematically justified in coupled networks.

B. *Overview*

This paper addresses how an unknown fault of a power leader is propagated through a coupled network. The fault detection method using NN observer attached to each follower is presented. The fault detectability condition of each follower is also described. Finally, the source of the fault

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is identified by using the sequence of fault detection time for each follower. Simulation results are presented to illustrate the effectiveness of the proposed strategy.

II. Problem Formulation

Consider the dynamic model of a coupled network [1] containing a power leader and *n* power followers shown in Figure 1.

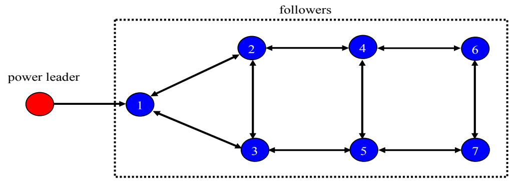

Fig. 1. A coupled leader-follower network model [1].

The mathematical model of a coupled network dynamics is given by

$$\begin{aligned} &\dot{x}_0 = f_0(x_0,t) \\ &\dot{x}_i = f_i(x_i,t) + \sum_{j=1}^{n} k_{ji}(x_j - x_i) + \gamma_0 k_{i0}(x_0 - x_i) \end{aligned}, \tag{1}$$

where $x_0 \in \Re^m$ is a state of the power leader which is independent from $x_i \in \Re^m$, the state of i-th followers, where i =1, 2, ..., n. The functions $f_i(x_i,t)$ represent a nonlinear part of the uncoupled dynamics. Coupling coefficients k_{ji} are positive coupling gains for each follower and are bidirectional with $k_{ji} = k_{ij}$. The coupling gain k_{i0} is the positive constant for the power leader to the followers. The constant γ_0 represents the connectivity to the power leader, which is equal to either 0 or 1. The followers directly connected to the power leader have $\gamma_0 = 1$, otherwise $\gamma_0 = 0$.

Assumption 1. The functions $f_i(x_i,t)$, *i*=0, 1, 2, ..., *n* are continuously differentiable.

III. Fault Detection of a Coupled Network

In order to monitor or detect the condition of the whole network, we propose a fault detection observer topology where each follower has its own observer to detect the fault for itself, neighboring followers, and the power leader. The observer in this paper is designed using NNs. However, the approach can be used with any other type of observer designed for nonlinear systems.

A. Two-layer Neural Network

A two-layer NN is used as on-line neural approximation model where only the output layer is tunable. Such NNs are linearly parameterized and can be represented by

$$y_{NN} = W^T \sigma(V^T u_{NN}), \tag{2}$$

where the input layer weights are collected into matrix *V*, the output layer weights into matrix *W*, $u_{NN} \in \Re^p$ denotes the NN input, $y_{NN} \in \Re^m$ denotes the NN output, and $\sigma(\cdot) \in \Re^L$ is the NN activation function. Many well-known results indicate that any sufficiently smooth function can be approximated arbitrarily close on a compact set using a NN with appropriate weights [7]. An adaptive estimate of the ideal NN weights *W* will be denoted by $\hat{W}$.

B. Approximation of Follower Dynamics and its Error Dynamics

The follower dynamics can be approximated as

$$\dot{x}_i = f_i(x_i,t) + \sum_{j=1}^{n} k_{ji}(x_j - x_i) + \gamma_0 k_{i0}(x_0 - x_i) = W_i^T \sigma(V_i^T u_i) + \varepsilon_i, \tag{3}$$

where ε_i is a NN approximation error. The input to the neural net $u_i = [x_i, x_j, x_0]$ depends on the network structure and signals fed into the *i*-th follower. If the follower is not connected to the power leader, the input to the NN is given by $u_i = [x_i, x_j]$.

The state observer for each follower is then given by

$$\dot{\hat{x}}_i = \hat{W}_i^T \sigma(V_i^T u_i) + A_i(x_i - \hat{x}_i), \tag{4}$$

where matrix A_i is a diagonal gain matrix with positive real numbers [6]. Let the error between the state of the actual system (1) and the observer be defined as $e_i = x_i - \hat{x}_i$. The error dynamics can then be written as

$$\dot{e}_i = -A_i e_i + \tilde{W}_i^T \sigma(V_i^T u_i) + \varepsilon_i, \tag{5}$$

where the neural network weights error is $\tilde{W}_i = W_i - \hat{W}_i$. Figure 2 shows the network with two followers and their observers (one directly connected to the power leader).

C. NN Observer Tuning Law and a Bound

The next theorem provides NN tuning laws and a bound on the state observer error using *e*-modification type of adaptation [20]. Note that the NN tuning equations for the nonlinear system identifier are similar to Lewis' NN robotic control tuning algorithms [21].

Theorem 1. (Tuning of Follower Observers). Given the nonlinear system (1) and the NN observer (4), let the estimated NN weights be provided by the NN tuning algorithm

$$\dot{\hat{W}}_i = C_i \sigma(V_i^T u_i) e_i^T - h_i C_i \|e_i\| \hat{W}_i, \tag{6}$$

with any constant matrix $C_i = C_i^T > 0$, and a design parameter h_i. Then the state observer error e_i and the NN weight estimation errors $\tilde{W}_i$ are uniformly and ultimately bounded.

Proof: Select the candidate Lyapunov function for the *i*-th follower as

$$L_i = \frac{1}{2} e_i^T e_i + \frac{1}{2} tr\left[\tilde{W}_i^T C_i^{-1} \tilde{W}_i^T\right], \tag{7}$$

where the gain C_i is a positive definite constant matrix. Lyapunov function derivative with respect to time is given by

$$\dot{L}_i = e_i^T \dot{e}_i + tr\left[\tilde{W}_i^T C_i^{-1} \dot{\tilde{W}}_i^T\right]. \tag{8}$$

Substituting error dynamics (5) into (8) and applying tuning law (6) yields

$$\dot{L}_i \le h_i \|e_i\| \|\tilde{W}_i\| (W_i - \|\tilde{W}_i\|) + e_i^T \varepsilon_i - A_{i\min} \|e_i\|^2, \tag{9}$$

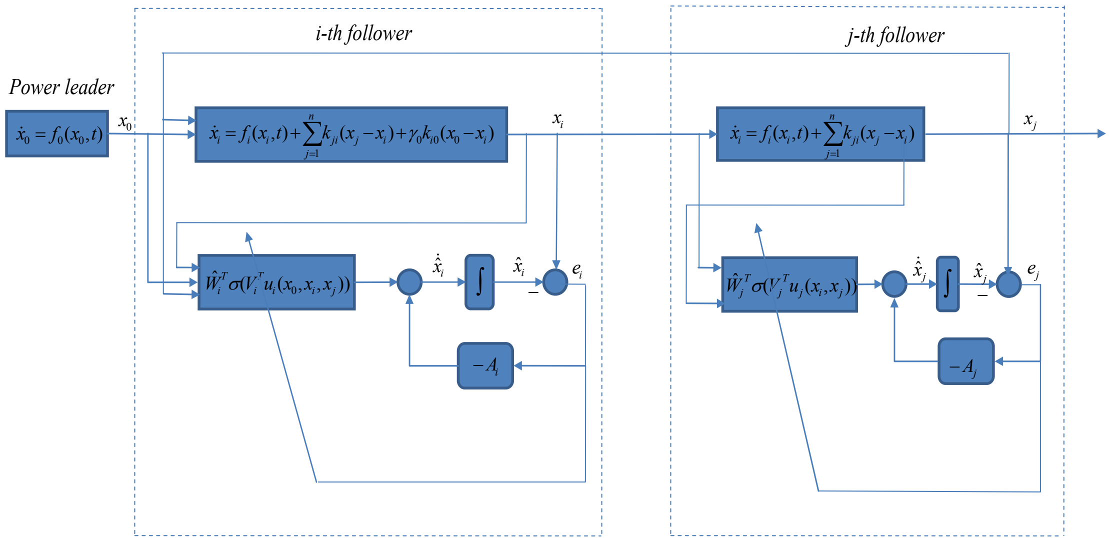

Fig. 2. A coupled leader-follower network with follower observers.

where $A_{i\min}$ is a minimum eigenvalue of A_i. The approximation error term ε_i is bounded by $\|\varepsilon_i\| < \varepsilon_{ic}$ [7]. The ideal NN weights W_i are also bounded such that $\|W_i\| \le W_{ic}$. Therefore, one has

$$\dot{L}_i \le \|e_i\| \left[-h_i \left(\|\tilde{W}_i\| - \frac{1}{2} W_{ic} \right)^2 + h_i \frac{W_{ic}^2}{4} + \varepsilon_{ic} - A_{i\min} \|e_i\| \right] . \quad (10)$$

The derivative of Lyapunov function $\dot{L}_i$ is negative semi-definite if the following inequality conditions are satisfied

$$\|e_i\| > \frac{h_i \frac{W_{ic}^2}{4} + \varepsilon_{ic}}{A_{i\min}} , \quad (11)$$

$$\|\tilde{W}_i\| > \sqrt{\frac{h_i \frac{W_{ic}^2}{4} + \varepsilon_{ic}}{h_i}} + \frac{1}{2} W_{ic} . \quad (12)$$

Inequalities (11) and (12) provide the explicit error bound of NN observer for i-th follower under healthy (nominal) system conditions, i.e.

$$e_{iB} = \frac{h_i \frac{W_{ic}^2}{4} + \varepsilon_{ic}}{A_{i\min}} \quad (13)$$

$$\tilde{W}_{iB} = \sqrt{\frac{h_i \frac{W_{ic}^2}{4} + \varepsilon_{ic}}{h_i}} + \frac{1}{2} W_{ic} . \quad (14)$$

The state estimation error bound can be reduced by increasing $A_{i\min}$. However, extremely large values of $A_{i\min}$ can result in amplifying noise and disturbance effects [6]. The small values of $A_{i\min}$ also result in a singularity of (13).

The bound on the state estimation error will be used as a threshold for fault detection at the i-th follower. The fault detection and isolation for the whole dynamic network depends on the parameters of individual observers. Knowing that $\|\hat{W}_i\| - \|W_i\| \le \|\tilde{W}_i\|$, the bound on actual NN weights can be given by

$$\|\hat{W}_i\| \le \sqrt{\frac{h_i \frac{W_{ic}^2}{4} + \varepsilon_{ic}}{h_i}} + \frac{3}{2} W_{ic} = \hat{W}_{iB} . \quad (15)$$

D. Modified NN Weight Tuning Law

It is assumed that the learning of the system dynamics is completed during an initial (healthy) phase of the system operation. Theorem 1 guarantees NN weight bounds while the system is healthy. In case of a power leader/follower fault, the NN weights will be limited by the following algorithm modification that is equivalent to saturation-based tuning laws

$$\dot{\hat{W}}_i = \begin{cases} C_i \sigma(V_i^T u_i) e_i^T - h_i C_i \|e_i\| \hat{W}_i, for \hat{W}_i \le \hat{W}_{iB} \\ 0, for \hat{W}_i > \hat{W}_{iB} \end{cases} . \quad (16)$$

Assumption 2. (Power-leader Fault Occurred After Learning Phase) A fault in a nonlinear system (1) has occurred after the NN identification error has settled below the bounds given by Theorem 1.

This is a natural requirement that the system must be healthy during the learning phase in order to be able to detect the potential fault. The identification is performed during the healthy phase of the system.

IV. Power Leader Fault Detection by Network Followers

In this section, the proposed fault detection (FD) approach will be described and analyzed with reference to the special case where the full state is available for measurement. It is assumed that the power leader has a fault in its dynamics which can be represented by

$$\begin{aligned} \dot{x}_0 &= f_0(x_0,t) + \Delta(x_0,t)\beta(t-t_0) \\ \dot{x}_i &= f_i(x_i,t) + \sum_{j=1}^{n} k_{ji}(x_j - x_i) + \gamma_0 k_{i0}(x_0 - x_i) \end{aligned} . \quad (17)$$

where $\Delta(x_0,t)$ is a fault in the power leader. The function $\beta(t-t_0)$ and t_0 are a step function and an unknown fault occurrence time, respectively. The power leader fault model corresponds to the model presented in [19].

A. *Fault detectability condition of each follower*

In this section, the fault detectability condition for an individual follower is derived. Consider the simplest network configuration with the power leader and two followers connected to it, i.e. one immediate follower (one hop away from the leader) and a two-hop away follower as described in Figure 3. Denoting the *i*-th follower as the one connected to the power leader directly, while the *j*-th follower is connected to the *i*-th follower.

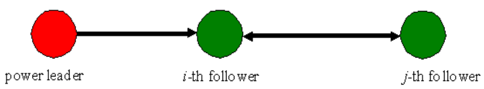

Fig. 3. The simple network with the power leader, *i*-th follower, and *j*-th follower.

1) *Fault Propagation through Network*

Assume that a fault exists in the power leader. Upon the fault occurrence we are interested in fault propagation through power leader dynamics as well as the network dynamics. The fault in the power leader might cause system trajectories to be deviated from nominal system trajectories, i.e.

$$\tilde{x}_0 = x_0 - \bar{x}_0 \,, \tag{18}$$

where state $\bar{x}_0$ and $\tilde{x}_0$ represent a nominal state of a healthy system and a deviation term, respectively. For this simple network case, the dynamics of the *i*-th follower is given by

$$\dot{x}_i = f_i(x_i,t) + k_{ji}(x_j - x_i) + \gamma_0 k_{i0}(\bar{x}_0 - x_i) + \gamma_0 k_{i0}\tilde{x}_0 \,. \tag{19}$$

Equation (19) shows the fault propagation through the power leader dynamics and the first-neighbor followers.

Now, let study how the fault of the power leader is propagated towards the *j*-th follower.

Rearranging (19) in terms of x_i yields

$$\begin{aligned} x_i &= \frac{1}{k_{ij}+\gamma_0 k_{i0}}\left[f_i(x_i,t) + k_{ji}x_j + \gamma_0 k_{i0}\bar{x}_0 - \dot{x}_i\right] + \frac{1}{k_{ij}+\gamma_0 k_{i0}}\gamma_0 k_{i0}\tilde{x}_0, \\ &= F_i(x_i,\dot{x}_i,x_j,\bar{x}_0,t) + \frac{1}{k_{ij}+\gamma_0 k_{i0}}\gamma_0 k_{i0}\tilde{x}_0. \end{aligned} \tag{20}$$

The *j*-th follower dynamics is given by,

$$\dot{x}_j = f_j(x_j,t) - k_{ij}x_j + k_{ij}x_i = F_j(x_j,t) + k_{ij}x_i \,. \tag{21}$$

Substituting (20) into (21),

$$\dot{x}_j = F_j(x_j,x_k,t) + k_{ij}F_i(x_i,\dot{x}_i,x_j,\bar{x}_0,t) + \frac{k_{ij}}{k_{ij}+\gamma_0 k_{i0}}\gamma_0 k_{i0}\tilde{x}_0 \,. \tag{22}$$

The effect of the fault generated by the power leader is diminished by the factor $\dfrac{k_{ij}}{k_{ij}+\gamma_0 k}$ at the *j*-th follower.

2) *Fault Detectability Condition*

Lemma 1. (The Fault Detectability Condition of the i-th Follower) A sufficient condition for a power leader fault detection at an immediate, one-hop away neighbor, at time T_i is given by the following condition,

$$\left\|\int_{t_0}^{T_i}\exp\left(-A_i(T_i-\tau)\right)\gamma_0 k_{i0}\tilde{x}_0\,d\tau\right\| \ge \left[2e_{iB} + \frac{(\widetilde{W}_{iB}+\varepsilon_{ic})}{A_{i\min}}(1-\exp(A_{i\min}(t_0-T_i)))\right]. \tag{23}$$

Proof: The *i*-th follower observer error is given by

$$\begin{aligned} e_i(t) &= \int_{t_0}^{t}\exp\left(-A_i(t-\tau)\right)\left(\widetilde{W}_i^T\sigma(V_i^T u_i) + \varepsilon_i + \gamma_0 k_{i0}\tilde{x}_0\right)d\tau \\ &\quad + \exp\left(-A_i(t-t_0)\right)e_i(t_0). \end{aligned} \tag{24}$$

Knowing that $\|e_i(t_0)\| \le e_{iB}$ and using the triangle inequality from (24) follows that the power leader fault detectability condition at the *i*-th follower is given by

$$\left\|\int_{t_0}^{T_i}\exp\left(-A_i(T_i-\tau)\right)\gamma_0 k_{i0}\tilde{x}_0\,d\tau\right\| - \left\|\int_{t_0}^{T_i}\exp\left(-A_i(T_i-\tau)\right)\left(\widetilde{W}_i^T\sigma(V_i^T u_i)+\varepsilon_i\right)d\tau\right\| > 2e_{iB} \tag{25}$$

$$\left\|\int_{t_0}^{T_i}\exp\left(-A_i(T_i-\tau)\right)\gamma_0 k_{i0}\tilde{x}_0\,d\tau\right\| - \frac{(\widetilde{W}_{iB}+\varepsilon_{ic})}{A_{i\min}}(1-\exp(A_{i\min}(t_0-T_i))) > 2e_{iB}\,, \tag{26}$$

thus yielding to (23). Inequality (23) provides the fault detectabilty condition that the discrepancy $\tilde{x}_0$ between the healthy state and the faulty state of a power leader is sufficient enough to exceed to terms related to NN estimator parameters and the fault detection time. The following result specifies sufficient fault detectability conditions for followers two or more hops away from the power leader. It is assumed that only the power leader fault has occurred, i.e., followers are in the healthy state.

Lemma 2. (The Fault Detectability Condition of the j-th Follower) A sufficient condition for the power leader fault detection at the *j*-th follower at time T_j is given by the following condition

$$\begin{aligned} &\left\|\int_{t_0}^{T_j}\exp(-A_j(T_j-\tau))\left[\frac{k_{ij}}{k_{ij}+\gamma_0 k_{i0}}\gamma_0 k_{i0}\tilde{x}_0\right]d\tau\right\| \\ &\ge 2e_{jB} + \frac{(\widetilde{W}_{jB}+\varepsilon_{jc})}{A_{j\min}}(1-\exp(A_{j\min}(t_0-T_j))). \end{aligned} \tag{27}$$

Proof: The *j*-th follower observer error vs. time is given by

$$\begin{aligned} e_j(t) &= \int_{t_0}^{t}\exp(-A_j(t-\tau)\left[\widetilde{W}_j^T\sigma(V_j^T u_j) + \varepsilon_j + \frac{k_{ij}}{k_{ij}+\gamma_0 k_{i0}}\Delta_{10}\right]d\tau \\ &\quad + e_j(t_0)\exp(-A_j(t_0-t)). \end{aligned} \tag{28}$$

Taking the norm of (28) and substituting the fault detection time T_j into *t*,

$$\begin{aligned} \|e_j(T_j)\| = &\left\|\int_{t_0}^{T_j}\exp(-A_j(t-\tau)\left[\widetilde{W}_j^T\sigma(V_j^T u_j) + \varepsilon_j + \frac{k_{ij}}{k_{ij}+\gamma_0 k_{i0}}\Delta_{10}\right]d\tau\right. \\ &\left. + e_j(t_0)\exp(-A_j(t_0-T_j))\right\|. \end{aligned} \tag{29}$$

Using the triangular inequality yields to (27).

B. *Sequence of Fault Detection in Followers*

It is difficult to identify the source of a potential fault in a coupled network, because the failure of a power leader or one of the followers can affect the entire network operation. Thus, the NN observer for each follower might detect the fault occurred from unknown sources including the follower itself and/or other followers.

In this paper we address a problem of the power leader using the followers' observers and when this type of fault can be detected.

Consider the case when a fault can be detected by both *i*-th follower and *j*-th follower. It is desired that the fault is detected sooner and closer to the real source of the fault, i.e., $T_i \le T_j$, as illustrated in Figure 4.

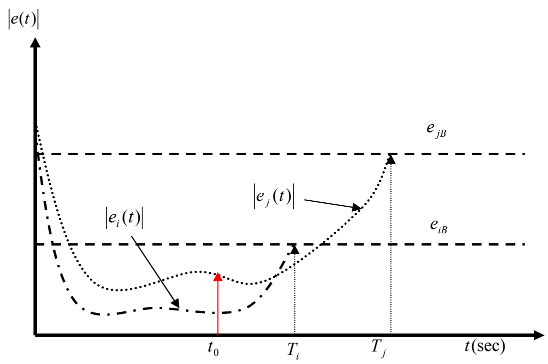

Fig. 4. Fault detection of a power leader using *i*-th follower and *j*-th follower.

Lemma 3. Choose NN observer bounds such that $e_{jB} > e_{iB}$. Let the observer parameters satisfy

$$\frac{(\tilde{W}_{jB}+\varepsilon_{jc})}{(\tilde{W}_{iB}+\varepsilon_{ic})} < \frac{A_{j\min}}{A_{i\min}} < 1 . \tag{30}$$

Then the fault detection time of the *i*-th follower is less than the detection time of the *j*-th follower, i.e. $T_i < T_j$.

Proof: For the *i*-th follower, using the property of norm, the upper inequality of e_{iB} is given by,

$$e_{iB} \le \int_{t_0}^{T_i} \exp(-A_{i\min}(T_i-\tau))\left[\left\|\tilde{W}_i^T\sigma(V_i^T u_i)\right\|+\|\varepsilon_i\|+\|\Delta_{10}\|\right]d\tau + \left\|e_i(t_0)\exp(-A_i(t_0-T_i))\right\| \tag{31}$$

Furthermore(31) is equivalent to

$$\begin{aligned}&\int_{t_0}^{T_i} \exp(-A_{i\min}(T_i-\tau))\left[\left\|\tilde{W}_i^T\sigma(V_i^T u_i)\right\|+\|\varepsilon_i\|+\|\Delta_{10}\|\right]d\tau \\ &+\left\|e_i(t_0)\exp(-A_i(t_0-T_i))\right\| \le \frac{(\tilde{W}_{iB}+\varepsilon_{ic})}{A_{i\min}}(1-\exp(A_{i\min}(t_0-T_i))) \\ &+\left\|e_i(t_0)\exp(-A_i(t_0-T_i))\right\| + \left\|\int_{t_0}^{T_i}\exp(-A_{i.\min}(T_i-\tau))\Delta_{10}\,d\tau\right\|.\end{aligned} \tag{32}$$

Taking its norm and using the triangle inequality on the error expression for the *j*-th follower yields,

$$\begin{aligned}&\left\|\int_{t_0}^{T_j}\exp(-A_j(T_j-\tau))\left[\tilde{W}_j^T\sigma(V_j^T u_j)+\varepsilon_j\right]d\tau\right\| - \left\|\int_{t_0}^{T_j}\exp(-A_j(T_j-\tau))\frac{k_{ij}}{k_{ij}+\gamma_0 k_{i0}}\Delta_{10}\,d\tau\right\| \\ &-\left\|e_j(t_0)\exp(-A_j(t_0-\tau))\right\| \le e_{jB} .\end{aligned} \tag{33}$$

To satisfy the condition $e_{jB} > e_{iB}$ it is required to have

$$\begin{aligned}&\frac{(\tilde{W}_{iB}+\varepsilon_{ic})}{A_{i\min}}(1-\exp(A_{i\min}(t_0-T_i))) + \left\|e_i(t_0)\exp(-A_i(t_0-\tau))\right\| \\ &+\left\|\int_{t_0}^{T_i}\exp(-A_{i\min}(T_i-\tau))\Delta_{10}\,d\tau\right\| < \left\|\int_{t_0}^{T_j}\exp(-A_j(T_j-\tau))\left[\tilde{W}_j^T\sigma(V_j^T u_j)+\varepsilon_j\right]d\tau\right\| \\ &-\left\|\int_{t_0}^{T_j}\exp(-A_j(T_j-\tau))\frac{k_{ij}}{k_{ij}+\gamma_0 k_{i0}}\Delta_{10}\,d\tau\right\|.\end{aligned} \tag{34}$$

Inequality (34) is equivalent to

$$\frac{(\tilde{W}_{iB}+\varepsilon_{ic})}{A_{i\min}}(1-\exp(A_{i\min}(t_0-T_i))) < \left\|\int_{t_0}^{T_j}\exp(-A_j(T_j-\tau))\left[\tilde{W}_j^T\sigma(V_j^T u_j)+\varepsilon_j\right]d\tau\right\| . \tag{35}$$

The right side of (35) is bounded by

$$\left\|\int_{t_0}^{T_j}\exp(-A_j(T_j-\tau))\left[\tilde{W}_j^T\sigma(V_j^T u_j)+\varepsilon_j\right]d\tau\right\| \le \frac{(\tilde{W}_{jB}+\varepsilon_{jc})}{A_{j\min}}(1-\exp(A_{j\min}(t_0-T_j))) \tag{36}$$

From (35) and (36) one has

$$1-\exp(A_{i\min}(t_0-T_i)) < \frac{(\tilde{W}_{jB}+\varepsilon_{jc})}{(\tilde{W}_{iB}+\varepsilon_{ic})}\frac{A_{i\min}}{A_{j\min}}(1-\exp(A_{j\min}(t_0-T_j))) \tag{37}$$

Applying condition (30) and using $A_{i\min} > A_{j\min}$ implies $T_j > T_i$.

V. SIMULATION EXAMPLE

The simulation was conducted to illustrate the fault detection in coupled networks using NNs. The numerical simulation program was constructed in Matlab and Simulink. Consider the network model consisting of a power leader and three followers as shown in Figure 5.

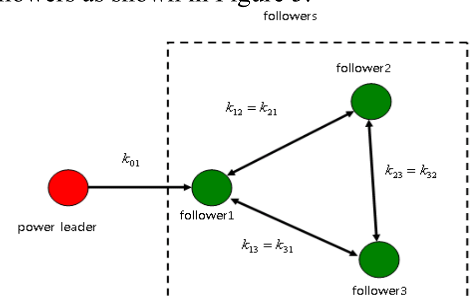

Fig. 5. Coupled network dynamic model containing a power leader and three followers.

The power leader dynamics is given by

$$\dot{x}_0 = f_0(x_0,t) = -3x_0^3 - 2\dot{x}_0 + 2(x_0+1)\sin(0.3t) + \Delta(x_0,t)\beta(t_0-t) \tag{38}$$

Assume that the follower dynamics is given by

$$\begin{aligned}\dot{x}_1 &= -5x_1^3 - 2\dot{x}_1 + x_1 + k_{21}(x_2-x_1) + k_{31}(x_3-x_1) + \gamma_{01}k_{01}(x_0-x_1) \\ \dot{x}_2 &= -4x_2^3 - 2\dot{x}_2 + 0.5x_2 + k_{12}(x_1-x_2) + k_{32}(x_3-x_2) \\ \dot{x}_3 &= -4x_3^3 - 2\dot{x}_3 + x_3 + k_{13}(x_1-x_3) + k_{23}(x_2-x_3)\end{aligned} \tag{39}$$

with constants $k_{01} = k_{12} = k_{21} = 1.25$, $k_{13} = k_{31} = 0.75$, and $k_{23} = k_{32} = 1$.

The observer NNs have 3, 6, and one neurons at the input, hidden, and output layers, respectively. The minimum eigenvalues $A_{i\min}$ for follower1, follower2, and, follower3 are selected as 3.0, 2.8, and 2.8, respectively. Parameters h_i are chosen as 7.3, 13, and 13.5, respectively. These design parameters $A_{i\min}$ and h_i are selected according to Lemma 3.

The power leader is first assumed healthy. Figure 6 and Figure 7 show observer states of followers and the error norm, respectively. From Figure 7, the error boundary of the followers' observer is estimated as $e_{1B} \approx 0.046$, $e_{2B} \approx 0.066$, and $e_{3B} \approx 0.065$. The NNs weight bounds and approximation error bounds are estimated as $W_{1c} = 0.125$ $W_{2c} = 0.175$, $W_{3c} = 0.168$, $\varepsilon_{1c} = 0.11$, $\varepsilon_{2c} = 0.08$, and $\varepsilon_{3c} = 0.09$.

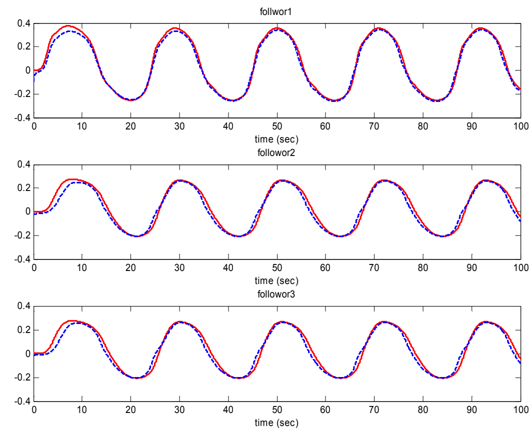

Fig. 6. Follower states (full line) and observer states (dotted line) in healthy system.

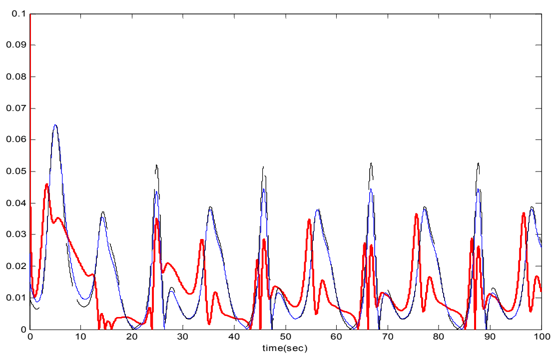

Fig. 7. Norm of errors follower1 (thick full line), follower2 (full line), and follower3 (dotted line).

The bounds on NNs weight errors of followers are estimated yielding inequalities

$$\tilde{W}_{1B}+\varepsilon_{1c}>\tilde{W}_{2B}+\varepsilon_{2c}>\tilde{W}_{3B}+\varepsilon_{3c}\ . \tag{40}$$

In this section, we assume that a fault in the power leader at t=100 seconds is $\Delta(x_0,t)=4\sin(0.3t)$ and the simulation results are given in Figure 8. The full straight line and the dotted straight line in Figure 8 indicate e_{1B} and $e_{2B}\approx e_{3B}$, respectively. The fault detection times are estimated as $T_1=117.1$, $T_2=128.5$, and $T_3=128.9$, respectively. The fault is detected by follower1 after 17.1 seconds.

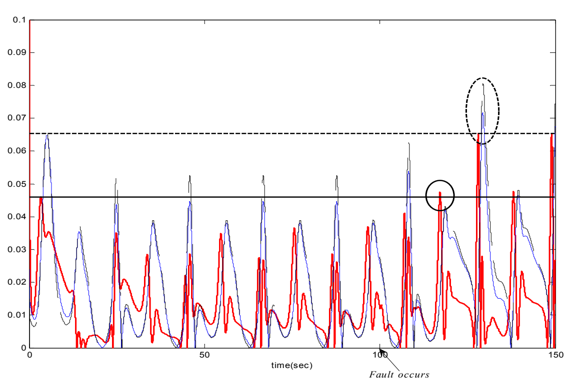

Fig. 8. Norm of errors for follower1 (thick full line), follower2 (full line), and follower3 (dotted line).

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