

# Prediction of Vertical Motions for Landing Operations of UAVs

Xilin Yang, Hemanshu Pota, Matt Garratt and Valery Ugrinovskii

**Abstract**—This paper outlines a novel and feasible procedure to predict vertical motions for safe landing of unmanned aerial vehicles (UAVs) during maritime operations. In the presence of stochastic sea state disturbances, dynamic relationship between an observer and a moving deck is captured by the proposed identification model, in which system order is specified by a new order-determination principle based on Bayes Information Criterion (BIC). In addition, the resulting system model is extended to develop accurate multi-step predictors for estimation of vertical motion dynamics. Simulation results demonstrate that the proposed prediction approach substantially reduces the model complexity and exhibits excellent prediction performance, making it suitable for integration into ship-helicopter approaches and landing guidance systems.

## I. INTRODUCTION

The present research is part of efforts devoted to develop a feasible procedure for landing an UAV on moving platforms in typical sea states. Our objective is firstly to predict dynamics of pitch and heave motions, as efficiently as possible, to trigger the optimal landing operation. In this way, minimizing relative velocity can be achieved.

Various maritime operations require efficient prediction of ship dynamics, such as cargo transfer between vessels in an oscillating environment and emergent crew rescue in danger. In our project, which aims at safe landing of an autonomous helicopter, vertical motions, including pitch and heave motions, are particularly important. For control purposes, accurate prior knowledge of pitch and heave motions will improve efficiency of planning the optimal landing trajectory, and facilitate the process of controller design to realise it. However, the uncertainty and randomness of environmental disturbances in rough seas greatly complicate attempts to obtain satisfactory prediction results. The difficulty mainly comes from complex coupling effects among six motions of vessels caused by fluctuating waves.

The choice of prediction algorithm will depend on desired specifications which the algorithm can achieve and to what extent effect of random disturbances can be attenuated. There are two mainstream approaches to prediction of vertical motions: The first one is to develop a proper model able to capture main system features, such as uncertain stochastic processes (e.g. wind gust, sea wave), characteristics of unknown ship motion behavior, and random unmodeled

dynamics, which requires an in-depth and comprehensive analysis of the above factors. Hence, system characteristics can be contained distinctly, and available prediction methods depend greatly on the fidelity of the model. In addition, system dynamics can be treated as an unknown box, and approached by an approximate model which captures system dynamics implicitly. In this way, measured data can be input into the model, which outputs the prediction results.

The main challenge of ship motion prediction is to develop an appropriate prediction model, resulting from complicated wave-excitation coupling dynamics caused by the local stochastic sea states such as barometric pressure, wind speed and wave heights [1]. Also, the accumulation of prediction error of landing position due to variations of relative motion between a helicopter and a ship deck exacerbates the difficulty of designing an accurate predictor. Furthermore, in situations where an automated landing must be made urgently and without warning (e.g. unexpected weather, mechanical failure), a safe landing necessitates the incorporation of an efficient and rapidly converging estimation algorithm into the flight control system.

It has been pointed out that ship motion dynamics are not so remarkably affected by local sea states as a result of the narrowband feature of their power spectra around the central frequency [2], and it is reasonable to represent sea wave dynamics as a superposition of sinusoidal forms covering a wide range of wave frequencies by abnegating high-frequency components [2], [3]. However, in these cases, such sinusoidal superpositions are obtained from experimental results under particular conditions. Therefore, these conclusions are subject to question as to whether they can be valid for other maritime situations.

The approach to ship motion model identification using time series theory has received limited attention in the literature. The problems of establishing an appropriate model arise when a time series model with unknown system parameters is adopted to complete a prediction task, which are mainly concerned with determination of system order and corresponding coefficients. Clearly, the prediction results can be significantly improved when the real system parameters are accurately approached. Recently, Ma et al. [4] suggested an Auto-Regressive (AR) fitting model, in which the system order was verified using the Akaike Information Criterion (AIC). This method lacks long-term prediction capability and also suffers from the inconsistency feature of the AIC. In contrast, ship motion prediction using state-space approaches has been subject to extensive investigation in a considerable number of papers, and significant efforts, including theoretical analysis and experimental research, have been made

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to deal with different practical problems in ship motion prediction. Triantafyllou et al. [5] addressed Kalman filtering techniques for prediction of six motions of vessels using a precise state-space model, which requires tremendous efforts in that the transfer functions between ship dynamics and sea elevation are irrational nonminimum-phase functions. Also, how to develop a proper state-space model for prediction still remains a question. Lainiotis et al. [6] focused on deriving a state-space model based on a sufficient knowledge of ship motion dynamics, which suffers from the dependency on available information. Ra et al. [2] regarded the ship motion as a particular sinusoidal form, and obtained a recursive robust least squares frequency estimator by assuming that the ship motion frequency changed slowly. Therefore, real-time application of the suggested algorithm in prediction course is difficult in cases where the frequency changes rapidly. An initial prediction algorithm using Minor Component Analysis developed by Zhao et al. [7] requires substantial computation efforts for updating identifying coefficients, which compromises its practicality in real-time prediction.

The present study is aimed at effective prediction of pitch and heave motions. In the proposed ARX model, a novel information criterion is proposed to obtain optimal system order based on the Bayes Information Criterion. The criterion considers prediction capability, model complexity, and error accumulation, achieving a tradeoff among the three important factors. Next, an ARX prediction model is derived, with system order defined by a new criterion. Finally, the model coefficients identified from the Recursive Least-Square (RLS) method are employed to predict vertical motions of the vessel. Simulation results demonstrate the suggested algorithm can efficiently predict both motions with acceptable accuracy.

## II. IDENTIFICATION OF SYSTEM PARAMETERS

Vertical motions of the helideck are of vital significance for landing tasks, since the helideck is normally built at prow or buttock part of a vessel. The intense jounce of the helideck mainly results from heave and pitch motions excited by sea waves, and is especially significant for oversized ships.

Due to symmetric features of the vessel, vertical motions are not coupled with the group of sway, roll and yaw motions [5]. Also, the previous experiments [8] show that linear models match well with heave and pitch motions. Therefore, it is reasonable to employ linear theories for the prediction problem.

The proposed methodology is inspired by phase-lead networks after investigation of the dynamic relationship between the true and the predicted vertical motion data. Here, the measured and the predicted data are considered as input  $u(t)$  and output  $y(t)$ , respectively. A phase-lead network constructed properly, with a large phase lead, means a reasonable prediction can be obtained as early as possible. The predictor with phase lead feature, in discrete domain, has the transfer function in the form of

$$\frac{Y(z)}{U(z)} = \frac{b_{(n,0)} + b_{(n,1)}z^{-1} + \dots + b_{(n,n-1)}z^{-(n-1)}}{1 + \bar{a}_{(m,1)}z^{-1} + \dots + \bar{a}_{(m,m)}z^{-m}}. \quad (1)$$

Numerous wave spectra analysis methods [6], [9] suggest that the sea wave excitation can be treated as a white noise. Hence, we consider the vertical dynamics described by  $y(t)$  as a stationary Gaussian process driven by a stochastic disturbance  $e(t)$  with normal distribution  $N(0, \sigma_e^2)$ . According to (1), we describe the relationship between current and previous ship dynamics by the following model

$$y(t) = A(q^{-1})y(t) + B(q^{-1})u(t) + e(t), \quad (2)$$

$$u(t) = q^{-m-1}y(t), \quad (3)$$

$$A(q^{-1}) = \sum_{i=1}^m a_{(m,i)}q^{-i}, m \in N, \quad (4)$$

$$B(q^{-1}) = \sum_{j=0}^{n-1} b_{(n,j)}q^{-j}, n \in N, n < m. \quad (5)$$

Here  $q^{-1}$  is the forward shift operator,  $a_{(m,i)} = -\bar{a}_{(m,i)}$ ,  $i = 1, \dots, m$ , and  $b_{(n,j)}$ ,  $j=0, \dots, n-1$  denote system coefficients to be determined,  $m$  is order of  $A(q^{-1})$ , and  $n$  indicates order of  $B(q^{-1})$ .

Without loss of generality, it is assumed that ordered pairs  $(m, n)$  lie within the following bounds:

$$m \in V_1 = \{m | 1 \leq m \leq m_{max}, m \in N\}, \quad (6)$$

$$n \in V_2 = \{n | 1 \leq n \leq n_{max}, n \in N\}, \quad (7)$$

where  $m_{max}$  and  $n_{max}$  are upper bounds on the output order and input order, respectively. For the purpose of determining an optimal output order  $m^*$  and an input order  $n^*$ , reasonable bounds on the system order  $(m_{max}, n_{max})$  should be assigned in advance. Smaller upper bounds on the system order will lead to a simplistic identification model unable to represent vertical dynamics accurately. Hence, upper bounds on the system order should be large enough to guarantee an acceptable accuracy of identification model. Meanwhile, the selection of upper bounds  $(m_{max}, n_{max})$  has a significant influence on the complexity of the system model, i.e., excessively large upper bounds would increase the complexity of an identification model and aggravate computational burden. Based on empirical results, a feasible selection scheme proposed from our experience is to select  $(m_{max}, n_{max})$  such that:

$$m_{max} = O(\sqrt{T}), n_{max} = O(\sqrt{T}/2), \quad (8)$$

here,  $T$  denotes the number of measured data. The selection principle (8) constrains the searching scope for the optimal system order selection by avoiding too simplistic models and excessive computational burden. By introducing the vector of lagged input-output data

$$\varphi^T(t) = [y(t-1), \dots, y(t-m), u(t), \dots, u(t-n+1)] \quad (9)$$

and the following notation

$$\theta^T(m, n, t) = [a_{m,1}(t), \dots, a_{m,m}(t), \\ b_{n,0}(t), \dots, b_{n,n-1}(t)], \quad (10)$$

we can write from (2)

$$y(t) = \theta^T(m, n, t)\varphi(t) + e(t). \quad (11)$$

Vector  $\theta^T(m, n, t)$  can be effectively estimated via the RLS algorithm. Using the quadratic criteria function [10]

$$J(\theta) = \sum_{j=1}^t [y(j) - \theta^T(m, n, j)\varphi(j)]^2 \quad (12)$$

leads to the following estimates for the identification coefficients:

$$\hat{\theta}(m, n, t) = \left[ \sum_{j=1}^t \varphi(m, n, j)\varphi^T(m, n, j) \right]^{-1} \\ \cdot \left[ \sum_{j=1}^t \varphi(m, n, j)y(j) \right]. \quad (13)$$

The latter can be computed recursively by

$$\hat{\theta}(m, n, t+1) = \hat{\theta}(m, n, t) + M(m, n, t+1) \\ \cdot [y(t+1) - \varphi^T(t+1)\hat{\theta}(m, n, t)], \quad (14)$$

$$M(m, n, t+1) = P(m, n, t)\varphi(t+1) \\ \cdot [1 + \varphi^T(t+1)P(m, n, t)\varphi(t+1)]^{-1}, \quad (15)$$

$$P(m, n, t+1) = P(m, n, t) \\ - M(m, n, t+1)\varphi^T(t+1)P(m, n, t), \quad (16)$$

$$\hat{\theta}(m, n, 0) = 0, \quad P(m, n, 0) = \alpha I, \quad \alpha = 10000. \quad (17)$$

Define the prediction error as

$$\xi(m, n, t+1) = y(t+1) - \varphi^T(m, n, t+1)\hat{\theta}(m, n, t) \quad (18)$$

and compute the maximum likelihood estimate of the error covariance until time  $T$

$$\hat{\sigma}^2(m, n, T) = \frac{1}{T - m - n} \sum_{m+n+1}^T \xi^2(m, n, t). \quad (19)$$

The error covariance  $\hat{\sigma}^2(m, n, T)$  will be used subsequently for optimal order determination. Some available methods to specify system order are the AIC [11], the BIC [12], and the Feedback Control System Information Criterion (CIC) [13]:

$$AIC(m, n, T) = \log \hat{\sigma}^2(m, n, T) + \frac{2(m+n)}{T}, \quad (20)$$

$$BIC(m, n, T) = \log \hat{\sigma}^2(m, n, T) + \frac{(m+n)\log T}{T}, \quad (21)$$

$$CIC(m, n, T) = \sum_{m+n+1}^T \xi^2(m, n, t) + (m+n)(\log T)^2. \quad (22)$$

For an ARX model, the AIC is not recommended since the consistency feature of the AIC cannot be guaranteed [14]. For the CIC, if the magnitude of the error accumulation is much smaller compared with the second term, the variation tendency of the CIC would be obliterated as the second term plays a decisive role, which leads to failure to determine optimal system order. Such phenomena arise when identification coefficients are determined very accurately by the RLS at the initial computation stage, thus preventing finding optimal system order. Additionally, the CIC also requires sufficient available information to assign the initial system order, which is almost inaccessible in ship motion prediction.

It follows from the strong consistency of the BIC that the unique system order can be obtained when the BIC value reaches a minimum. In our case, an ARX requires the joint determination of  $m$  and  $n$ . For every given input order  $n \leq n_{max}$ , the BIC value changes convexly. Thus, the minimum BIC value corresponds to optimal output order for a given input order, which results in the difficulty of selecting the desired system order in the global sense. In our case, selection of the optimal pairs  $(m^*, n^*)$  should include a tradeoff among prediction ability, accumulated prediction error, and model complexity.

In the ship motion estimation problem, our main concern is the prediction capability. Meanwhile, the accumulated prediction error and identification model complexity should be considered.

The following three important aspects should be analyzed:

- 1) How can ordered pairs  $(m, n)$ ,  $m \in V_1, n \in V_2$  be determined to maximize the prediction horizon?
- 2) How to reduce the model complexity to reduce the computational burden?
- 3) How can the accumulated prediction error be contained within the acceptable range?

Regarding the first question, a tradeoff should be achieved between the seemingly incompatible aspects. When recursive prediction models are considered, prediction capability should come first. Our main purpose is to increase prediction horizon with acceptable prediction error as large as possible. The proposed selection principle begins with computing the candidate output order series  $m_i^* = \arg\{\min(BIC(j, i, T))\}$ ,  $j = 1, \dots, m_{max}$  for every

$$i = 1, \dots, n_{max}, \quad (23)$$

then it selects the largest output order  $m^*$  in the candidate output order series

$$m^* = \max\{m_i^*\}. \quad (24)$$

For the  $m^*$ , there usually exist several input order  $n_1, n_2, \dots, n_r, n_r \leq n_{max}$ . One possible method is to select optimal input order  $n^*$  such that

$$n^* = \arg\{\min(\frac{m^* + n_k}{m^*})\}, k = 1, 2, \dots, r. \quad (25)$$

Equation (25) seeks to reduce the model complexity in consideration of long-term prediction requirement, i.e., the identification model with the smallest system order while

achieving satisfactory prediction ability is obtained. After optimal system order in the sense of Eq. (23)-(25) is determined, we would like to check the accumulated prediction error in the following sections.

### III. PREDICTION ALGORITHM FOR VERTICAL SHIP MOTIONS

After the optimal output order  $m^*$ , input order  $n^*$  and corresponding coefficients of the model are calculated from the RLS, the efforts will be focused on the prediction of ship motion dynamics. Suppose the prediction step is  $L$ . Rewrite the ARX model as follows

$$[1 - \hat{A}(q^{-1})]y(t) = \hat{B}(q^{-1})u(t) + C(q^{-1})e(t), \quad (26)$$

and note the identity [15]

$$F(q^{-1})[1 - \hat{A}(q^{-1})] + q^{-L}G(q^{-1}) = C(q^{-1}). \quad (27)$$

In our case,  $C(t) = 1$  and

$$F(q^{-1}) = \sum_{i=0}^{L-1} f_i q^{-i}, \quad f_i = \sum_{j=0}^{i-1} f_j a_{(m^*, i-j)}, \quad (28)$$

$$f_0 = 1, \quad i = 1, \dots, L-1,$$

$$G(q^{-1}) = \sum_{i=0}^{m^*-1} g_i q^{-i}, \quad g_i = \sum_{j=0}^{L-1} f_j a_{(m^*, i+L-j)}, \quad (29)$$

$$i = 0, \dots, m^* - 1.$$

Substituting Eq. (27)-(29) into Eq. (26) yields

$$y(t) = \frac{\hat{B}(q^{-1})}{1 - \hat{A}(q^{-1})}u(t) + \frac{F(q^{-1})[1 - \hat{A}(q^{-1})] + q^{-L}G(q^{-1})}{1 - \hat{A}(q^{-1})}e(t), \quad (30)$$

since  $F(q^{-1}) = 1$ , Eq. (30) is converted to

$$y(t+L) = \frac{\hat{B}(q^{-1})}{1 - \hat{A}(q^{-1})}u(t+L) + \frac{G(q^{-1})}{1 - \hat{A}(q^{-1})}e(t) + e(t+L). \quad (31)$$

Replacing  $e(t)$  in Eq. (31) with Eq. (26) gives

$$y(t+L) = e(t+L) + G(q^{-1})[\xi(t) + \hat{y}(t|t-L)] + \hat{B}(q^{-1})u(t+L). \quad (32)$$

Here,  $\hat{y}(t|t-L)$  is the estimated value of  $y(t)$  based on the measured data up to time  $t-L$ , and  $\xi(t)$  the estimation error. Meanwhile,

$$y(t+L) = \xi(t+L) + \hat{y}(t+L|t) = \xi(t+L) + \{[1 - \hat{A}(q^{-1})] + q^{-L}G(q^{-1})\}\hat{y}(t+L|t). \quad (33)$$

It follows from Eq. (32) and Eq. (33) that

$$\xi(t+L) = G(q^{-1})\xi(t) + \hat{B}(q^{-1})u(t+L) - [1 - \hat{A}(q^{-1})]\hat{y}(t+L|t) + e(t+L). \quad (34)$$

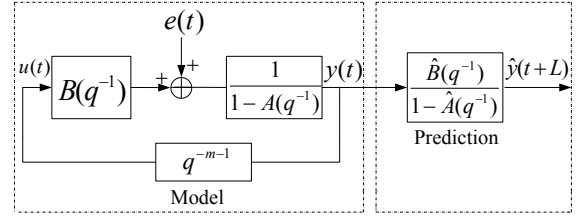


Fig. 1. Architecture of the proposed prediction method

The prediction error covariance

$$V = E\{\xi(t)^2\} \quad (35)$$

is minimized if the following equation holds

$$G(q^{-1})\xi(t) + \hat{B}(q^{-1})u(t+L) - [1 - \hat{A}(q^{-1})]\hat{y}(t+L|t) = 0, \quad (36)$$

which indicates the predictor is in the form of

$$\hat{y}(t+L|t) = \hat{A}(q^{-1})\hat{y}(t+L|t) + G(q^{-1})\xi(t) + \hat{B}(q^{-1})u(t+L). \quad (37)$$

In our case, the inputs are assumed to be the measured data, then  $u(t+L)$  can be replaced by  $y(t)$ . If we wish to predict further, the compensation term  $G(q^{-1})$  can be removed. Thus, the applicable predictor is

$$\hat{y}(t+L|t) = \hat{A}(q^{-1})\hat{y}(t+L|t) + \hat{B}(q^{-1})y(t). \quad (38)$$

To explain explicitly, the prediction procedure involves the determination of  $\hat{A}(q^{-1})$  and  $\hat{B}(q^{-1})$ , which are obtained using the identification procedure based on the process model depicted on the left in the diagram shown in Fig. 1. Afterwards,  $\hat{A}(q^{-1})$  and  $\hat{B}(q^{-1})$  are employed to derive the predictor  $\hat{y}(t+L|t)$ .

### IV. SIMULATION RESULTS

The performance of the proposed predictor is demonstrated in this section. The vertical motion data were generated from the FREYDYN 8.0 software package for an 8,500-ton LPA class amphibious platform. The vertical motion data were sampled at every 0.25s at sea state 3 which had a typical wave height of 0.5m.

The data were divided into two segments: the first group of  $N_T$  points were used for training and another of  $N_P$  points as a test. We chose  $N_T$  and  $N_P$  large enough in the sense that  $N_T$  points could capture vertical motion feature and  $N_P$  could be utilized for testing. We chose  $N_T = 500, 1000, 1500, 2000$  for training, and every time  $N_P = N_T - L$  points with combination of white noise to check the prediction results. Numerous simulations were carried out for  $N_T = 1000$ . For pitch motion, the predicted and the true pitch motion data versus time are plotted in Fig. 2 (20-step-ahead and 30-step-ahead), and for heave motion in Fig. 3 (20-step-ahead and 30-step-ahead). It is seen that the prediction results produced by the proposed algorithm match pretty well with the true data of pitch and heave motions.

For pitch motion, the lead phase margin is 107.85 degree for 20-step-prediction, and 81.34 degree for 30-step-prediction. For heave motion, the lead phase margin is 136.86 degree for 20-step-prediction, and 65.57 degree for 30-step-prediction. With the increase in prediction points, it can be seen that the prediction error for posterior points is not necessarily worse than previous ones.

## V. COMPARATIVE STUDIES

To test the validity of the new method, we compared our algorithm with other conventional predictors. A brief description of those predictors is listed below.

### A. Order-predefined ARX predictor

This comparison aimed to check performance of the proposed order determination method. From the classical control viewpoint, it is usually preferred to choose a phase-lead network with small system orders, and here a second-order predictor was adopted

$$\frac{Y(z)}{U(z)} = \frac{b_0 + b_1z^{-1} + b_2z^{-2}}{1 + a_1z^{-1} + a_2z^{-2}}. \quad (39)$$

### B. Autoregressive model predictor (AR)

Based on the previous measured data, the forecasts of an AR process with system order  $p$  can be obtained by iterating on

$$\hat{y}(t+j|t) = a_1\hat{y}(t+j-1|t) + \dots + a_p\hat{y}(t+j-p|t) \quad (40)$$

for  $j = 1, \dots, L$ . The key to prediction is to define the system order  $p$ . To avoid the inconsistency of the AIC, the BIC is used. Several AR predictors are required with the first one producing a one-step-ahead prediction, the second one producing a two-step-ahead, so on and so forth.

### C. Performance comparison among three predictors

In this investigation, we used  $N_T$  points to obtain system order, and another group of  $N_P$  points to check prediction results. Besides, a zero-mean Gaussian random noise was added to vertical motion data in order to represent the sea wave dynamics. The peak amplitude percentage rate of the white noise to the measured data is 10%. The mean squared prediction error  $\Phi$  was employed as the measurement of overall performance:

$$\Phi = \frac{1}{N_P} \sum_{i=T+1}^{T+N_P} [y(i) - \hat{y}(i)]^2. \quad (41)$$

The maximum prediction error for  $N_P$  points was evaluated by

$$\Psi = \max_i |y(i) - \hat{y}(i)|, \quad (42)$$

where  $y(i)$  and  $\hat{y}(i)$  were the true and the predicted data. To find the variations of  $\Phi$ , we employed the index

$$I = 20 \log_{10} \frac{\sqrt{\Phi}}{|y_{max}|} \quad (43)$$

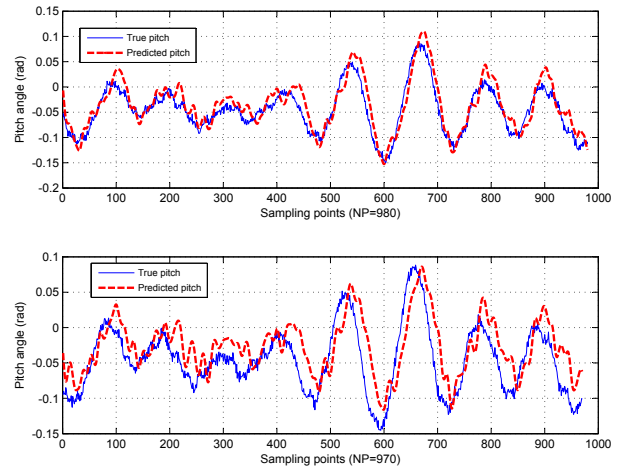


Fig. 2. Pitch motion prediction (20-step-ahead and 30-step-ahead)

to investigate the average trend of prediction results. Therefore, proper prediction horizon would be accessible in consideration of  $\Psi$  and  $I$ . As is shown in Fig. 4, the index  $I$  remains less than -20dB until 25 steps for pitch motion, i.e., the prediction error of pitch motion is within 10% of the true data can be obtained up to 25-step-ahead with acceptable maximum prediction error 0.0723. This is assumed to be acceptable in the considered application. Meanwhile, Fig. 4 indicates that precise 40-step-ahead prediction can be obtained with acceptable prediction error. Table 1 summarizes the experimental results on the  $\Phi$  and  $\Psi$  of three predictors for pitch motion, each taking four groups of  $N_P$  points and predicting 20 and 40 steps ahead, respectively. For 20-step-ahead prediction, the proposed algorithm gives consistently acceptable performance even when  $N_P$  is much larger, whereas the order-predefined and AR predictors produce greater  $\Phi$ . The order-predefined and AR predictors suffer from much inaccuracy when we predict 40 steps. For 40-step-prediction, our algorithm predicts with acceptable  $\Phi$  while producing larger  $\Psi$ , which indicates the new method sacrifices  $\Psi$  to compensate for overall performance. Fortunately, there is just a few number of such points, and general trends of vertical motions can be captured. Since our algorithm focuses on prediction capability, it cannot always achieve the smallest accumulated prediction error. However, mean squared prediction error  $\Phi$  is within a relatively acceptable range. For heave motion, as is shown in table 2, satisfactory prediction results are available up to 40 steps, which is acceptable for our landing task.

## VI. CONCLUSION

In this paper we concentrate on building prediction models for vertical motion dynamics. A feasible principle is utilized to solve the problem of system order selection. Based on determination of optimal system order and associated identification coefficients, a multi-step self-tuning predictor is employed for prediction. Simulation results demonstrate that the proposed prediction approach exhibits satisfactory

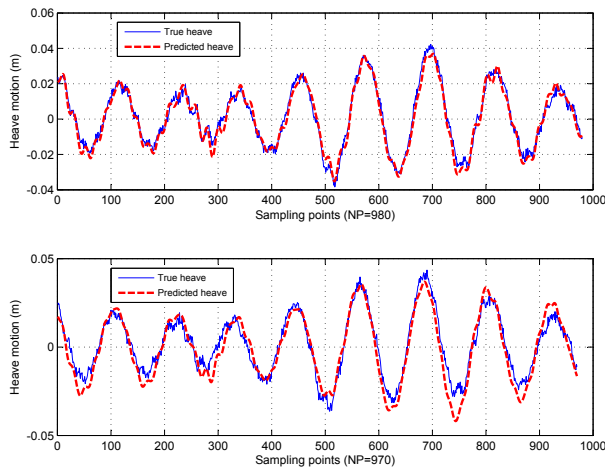


Fig. 3. Heave motion prediction (20-step-ahead and 30-step-ahead)

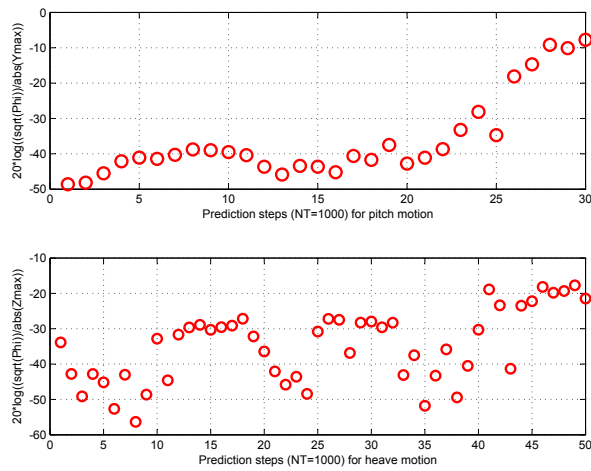


Fig. 4. Accumulated prediction error for different prediction steps for pitch and heave motions

prediction performance. Furthermore, the proposed procedure facilitates the accurate prediction of vertical motion dynamics in long distance circumstances for use in ship-helicopter flight operations. Future work will be aimed at increasing prediction precision when more prediction steps are expected in high sea states.

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Table 1: Performance comparison for three predictors for pitch motion

Prediction methods	NP	System Orders (m, n)	20-step-ahead		40-step-ahead	
			MSPE	SUP	MSPE	SUP
The proposed predictor	500	(18,2)	7.8368*E-5	0.06831	0.0012	0.1487
	1000	(35,3)	1.3523*E-6	0.06614	6.6912*E-4	0.3652
	1500	(36,1)	4.2983*E-4	0.06548	1.3672*E-4	0.1449
	2000	(39,2)	4.5836*E-4	0.06897	6.5211*E-4	0.1339
Order-predefined predictor	500	(2,3)	3.2252*E-5	0.09133	0.00136	0.1484
	1000	(2,3)	5.5225*E-5	0.07173	0.00659	0.1215
	1500	(2,3)	8.0312*E-4	0.0767	1.0652*E-4	0.1365
	2000	(2,3)	0.00214	0.08329	0.0023	0.1460
AR predictor	500	5	5.4278*E-4	0.0714	0.0012	0.1432
	1000	7	1.7516*E-4	0.0645	0.0154	0.1298
	1500	8	8.3272*E-4	0.0698	3.0945*E-4	0.1315
	2000	6	0.0026	0.0722	0.0021	0.1456

Table 2: Performance comparison for three predictors for heave motion

Prediction methods	NP	System Orders (m, n)	20-step-ahead		40-step-ahead	
			$\Phi$	$\Psi$	$\Phi$	$\Psi$
The proposed predictor	500	(9,2)	3.5247*E-4	0.0209	2.8301*E-4	0.0311
	1000	(10,3)	7.3685*E-8	0.0203	2.9597*E-8	0.0221
	1500	(12,3)	1.1891*E-7	0.0179	6.3819*E-5	0.0348
	2000	(17,3)	2.8638*E-6	0.0137	8.3508*E-5	0.0221
Order-predefined predictor	500	(2,3)	5.5506*E-4	0.0297	3.9019*E-4	0.0329
	1000	(2,3)	9.0747*E-4	0.0861	2.2624*E-5	0.0312
	1500	(2,3)	8.7189*E-5	0.0201	1.2478*E-4	0.0388
	2000	(2,3)	1.0380*E-4	0.0182	2.9997*E-4	0.0232
AR predictor	500	8	8.0541*E-4	0.0518	0.0011	0.0607
	1000	11	3.3044*E-4	0.0628	5.5053*E-4	0.0597
	1500	10	7.0795*E-4	0.0583	8.3241*E-4	0.0568
	2000	14	7.9786*E-4	0.0701	0.001	0.0636

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