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Fault Detection for Discrete-Time Markov Jump Linear Systems with Partially Known Transition Probabilities

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Abstract— **In this paper, the fault detection problem for a class of discrete-time Markov jump linear system (MJLS) with partially known transition probabilities is investigated. The proposed class of systems is more general, which relaxes the traditional assumption in Markov jump systems that all the transition probabilities must be completely known. A residual generator is constructed and the corresponding fault detection and isolation (FDI) problem is formulated as an H_∞ filtering problem by which the error between residual and fault are minimized in the H_∞ sense. The LMI-based sufficient conditions for the existence of FDI filter are derived. A numerical example is given to illustrate the effectiveness and potential of the developed theoretical results.**

I. INTRODUCTION

The past two decades have witnessed the prosperous research on the stochastic hybrid systems, where the so-called Markov jump systems have kept being a hot topic due to their widely practical applications in manufacturing system, power systems, aerospace systems and networked control system (NCS), etc [1], [3], [7], [11]. For Markov jump systems with completely known transition probabilities, many problems have been addressed, see [1], [3]. Some recent burgeoning extensions considered the uncertain transition probabilities, which aimed to utilize robust methodologies to deal with the norm-bounded or polytopic uncertainties presumed in the transition probabilities, see for example, [4], [9]. Unfortunately, the structure and "nominal" terms of the considered uncertain transition probabilities have to be known *a priori*.

The ideal assumption on the transition probabilities inevitably limits the application of the traditional Markov jump systems theory. In fact, the likelihood to obtain the complete knowledge on the transition probabilities is questionable and the cost is probably high. A typical example could be found in networked control systems (NCS), where the time-varying delays and random packet loss induced by communication channels can be modeled as Markov chains, and accordingly the resulting closed-loop system can be studied by means of Markov jump systems theory, see for example, [5], [7], [11]. However, either the variation of delays or the packet dropouts in almost all types of communication networks are vague in different running period of networks, all or part of the elements in the desired transition probabilities matrix are hardly or costly to obtain. The same problems may arise in other practical systems with Markovian jumps. Thus, rather than the large complexity to measure or estimate all the transition probabilities, it is significant and necessary from control perspectives to further study more general Markov jump systems with partially known transition probabilities.

On another research front line, fault detection and isolation (FDI) for many dynamic systems have been much investigated, see for example [2], [6]. The key point of FDI is to construct the residual generator, determine the residual evaluation function and the threshold, then make judgment whether an alarm of fault is generated by comparing the values of the evaluation function with the prescribed threshold. Usually, the residual generator is realized by formulating FDI as a filtering problem with some performance index, such as H_∞ filtering, and therefore the FDI filter gains and the optimal H_∞ performance index for the augmented systems can be obtained by some optimization methods, such as LMI [8], [12], [13]. Recently, some attention have been drawn to Markov jump linear systems, with uncertainties [13] or with time delays [8], [13]. However, unfortunately, the obtained results are obtained still based on the traditional assumption of complete knowledge on transition probabilities. Thus it is more practical and challenging to find a FDI filter for the underlying systems with partially known transition probabilities, which motivates us for this study.

In this paper, the problem of fault detection for a class of discrete-time Markov jump linear systems (MJLS) with partially known transition probabilities is investigated. More precisely, the considered systems relax the assumption that all the transition probabilities have to be completely known, and cover the traditional MJLS as a special case. The residual generator is constructed and the FDI problem is formulated as an H_∞ filtering problem such that the error between residual and fault are minimized in the H_∞ sense. Sufficient conditions for the existence of the FDI filter for the underlying systems are derived via LMIs. A numerical example is presented to show the validness and potential of the developed theoretical results.

Notation: The notation used in this paper is fairly standard. The superscript "T" stands for matrix transposition, $\mathbb{R}^n$ denotes the n dimensional Euclidean space, the notation $|\cdot|$ refers to the Euclidean vector norm. $l_2[0,\infty)$ is the space of square summable infinite sequence and for $w = \{w(k)\} \in l_2[0,\infty)$, its norm is given by $\|w\|_2 = \sqrt{\sum_{k=0}^{\infty} |w(k)|^2}$. For notation $(\Omega, \mathcal{F}, \mathcal{P})$, Ω represents the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $E[\cdot]$

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stands for the mathematical expectation and for sequence $e = \{e(k)\} \in l_2((\Omega, \mathcal{F}, \mathcal{P}), [0, \infty))$, its norm is given by $\|e\|_{E_2} = \sqrt{E\left[\sum_{k=0}^{\infty} |e(k)|^2\right]}$. In addition, in symmetric block matrices or long matrix expressions, we use * as an ellipsis for the terms that are introduced by symmetry and $diag\{\cdots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. The notation $P > 0$ (≥ 0) means P is a symmetric and positive (semi-positive) definite matrix. I and 0 represent respectively, identity matrix and zero matrix.

II. Problem Formulation and Preliminary Results

Fix the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and consider the following class of discrete-time Markov jump linear systems:

$$\begin{aligned} x(k+1) &= A(r_k)x(k) + B(r_k)u(k) \\ &\quad + E(r_k)d(k) + F(r_k)f(k) \\ y(k) &= C(r_k)x(k) + D(r_k)d(k) + G(r_k)f(k) \end{aligned} \quad (1)$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^u$ is the known input, $d(k) \in \mathbb{R}^d$ is the unknown input, $f(k) \in \mathbb{R}^f$ is the fault to be detected. $u(k)$, $d(k)$ and $f(k)$ aare assumed to belong to $l_2[0, \infty)$. $y(k) \in \mathbb{R}^y$ is the output vector, $\{r_k, k \geq 0\}$ is a discrete-time homogeneous Markov chain, which takes values in a finite set $\mathcal{I} \triangleq \{1, ..., N\}$ with a transition probabilities matrix $\Lambda = \{\pi_{ij}\}$ namely, for $r_k = i, r_{k+1} = j$, one has

$$\Pr(r_{k+1} = j | r_k = i) = \pi_{ij}$$

where $\pi_{ij} \geq 0 \ \forall \ i, \ j \in \mathcal{I}$, and $\sum_{j=1}^{N} \pi_{ij} = 1$. When $r_k = i \in \mathcal{I}$, the system matrices of the ith mode are denoted by A_i, B_i, C_i, D_i, E_i, F_i and G_i, which are considered here to be known real constant with appropriate dimensions.

In addition, the transition probabilities of the jumping process $\{r_k, k \geq 0\}$ in this paper are assumed to be partially accessible, i.e., some elements in matrix $\mathbf{\Lambda}$ are unknown. For instance, for system (1) with 5 operation modes, the transition probability matrix may be as:

$$\begin{bmatrix} \pi_{11} & ? & \pi_{13} & ? & \pi_{15} \\ ? & ? & ? & \pi_{24} & \pi_{25} \\ \pi_{31} & \pi_{32} & \pi_{33} & ? & ? \\ ? & ? & \pi_{43} & \pi_{44} & ? \\ ? & \pi_{52} & ? & \pi_{54} & ? \end{bmatrix}$$

where "?" represents the unaccessible elements. For notation clarity, $\forall i \in \mathcal{I}$, we denote that

$$\mathcal{I}_{\mathcal{K}}^i \triangleq \{j : \pi_{ij} \text{ is known}\}, \quad \mathcal{I}_{\mathcal{UK}}^i \triangleq \{j : \pi_{ij} \text{ is unknown}\}, \quad (2)$$

Also, we denote $\pi_{\mathcal{K}}^i \triangleq \sum_{j \in \mathcal{I}_{\mathcal{K}}^i} \pi_{ij}$ throughout the paper.

Remark 1: In literature, the transition probabilities of the Markov chain $\{r_k, k \geq 0\}$ are generally assumed to be completely known ($\mathcal{I}_{\mathcal{UK}}^i = \emptyset$, $\mathcal{I}_{\mathcal{K}}^i = \mathcal{I}$) or completely unknown ($\mathcal{I}_{\mathcal{K}}^i = \emptyset$, $\mathcal{I}_{\mathcal{UK}}^i = \mathcal{I}$). Therefore, the transition probability matrix considered in this paper is more natural to the Markov jump systems and hence covers the previous two cases. Note that the transition probabilities with polytopic or norm-bounded uncertainties can still be viewed as completely known in the sense of this paper.

Here, we are interested in designing a FDI filter for the underlying system, its desired structure is considered to be:

$$\begin{aligned} x_F(k+1) &= A_F(r_k)x_F(k) + B_F(r_k)y(k) \\ r(k) &= C_F(r_k)x_F(k) + D_F(r_k)y(k) \end{aligned} \quad (3)$$

where $x_F(k) \in \mathbb{R}^n$, $r(k) \in \mathbb{R}^f$ is the so-called residual, and $A_F(r_k), B_F(r_k), C_F(r_k)$ and $D_F(r_k)$, $\forall r_k \in \mathcal{I}$ are the matrices with compatible dimensions to be determined. The FDI filter with the above structure is assumed to jump synchronously with the modes in system (1).

Denoting $\tilde{x}(k) \triangleq [\ x^T(k) \quad x_F^T(k)\ ]^T$, $e(k) \triangleq r(k) - f(k)$ and augmenting the model of (1) to include (3), we can obtain the following augmented system:

$$\begin{aligned} \tilde{x}(k+1) &= \tilde{A}(r_k)\tilde{x}(k) + \tilde{B}(r_k)w(k) \\ e(k) &= \tilde{C}(r_k)\tilde{x}(k) + \tilde{D}(r_k)w(k) \end{aligned} \quad (4)$$

where $w(k) = \left[u^T(k)\ d^T(k)\ f^T(k)\right]^T$ and

$$\begin{aligned} \tilde{A}(r_k) &= \begin{bmatrix} A(r_k) & 0 \\ B_F(r_k)C(r_k) & A_F(r_k) \end{bmatrix}, \\ \tilde{B}(r_k) &= \begin{bmatrix} B(r_k) & E(r_k) & F(r_k) \\ 0 & B_F(r_k)D(r_k) & B_F(r_k)G(r_k) \end{bmatrix}, \\ \tilde{C}(r_k) &= \begin{bmatrix} D_F(r_k)C(r_k) & C_F(r_k) \end{bmatrix}, \\ \tilde{D}(r_k) &= \begin{bmatrix} 0 & D_F(r_k)D(r_k) & D_F(r_k)G(r_k) - I \end{bmatrix}. \end{aligned}$$

Obviously, the resulting system (4) is also a Markov jump linear system with partially known transition probabilities (2). Now, to present the main objective of this paper more precisely, we also introduce the following definitions for system (4), which are essential for the later development.

Definition 1: System (4) is said to be stochastically stable if for $w(k) \equiv 0, k \geq 0$ and every initial condition $\tilde{x}_0 \in \mathbb{R}^n$ and $r_0 \in \mathcal{I}$, the following holds:

$$E\left\{\sum_{k=0}^{\infty} \|\tilde{x}(k)\|^2 |\tilde{x}_0, r_0\right\} < \infty$$

Definition 2: Given a scalar $\gamma > 0$, system (4) is said to be stochastically stable and has an H_∞ performance index γ if it is stochastically stable and under zero initial condition, $\|e\|_{E_2} < \gamma \|w\|_2$ holds for all nonzero $w(k) \in l_2[0, \infty)$.

Therefore, our objective in this paper is to determine matrices $\{A_F(r_k), B_F(r_k), C_F(r_k), D_F(r_k)\}$ of the FDI filter such that the augmented system (4) is stochastically stable and has a guaranteed H_∞ performance index. Note that the original system (1) will be assumed to be stable in the sequel, see [13], [8], as an usual precondition in the fault detection problems. Furthermore, as commonly adopted in literature, the fault $f(k)$ can be deteted by the following steps.

- Select a residual evaluation function $J(r) = \sqrt{\sum_{k=k_0}^{k_0+L} r^T(k)r(k)}$, where k_0 denotes the initial evaluation time instant and L denotes the evaluation time steps.
- Select a threshold $J_{th} = \sup_{d \in l_2, f=0} E\left[J(r)\right]$.

- Test:

$$J(r) > J_{th} \Longrightarrow \text{with faults} \Longrightarrow alarm \quad (5)$$
$$J(r) < J_{th} \Longrightarrow \text{no faults} \quad (6)$$

Before ending the section, we give the following lemma for system (4), which will be used in the proof of our main results.

Lemma 1: [3] System (4) is stochastically stable with a prescribed H_∞ performance index $\gamma > 0$ if and only if there exists a set of symmetric and positive-definite matrices $P_i, i \in \mathcal{I}$ satisfying

$$\Xi_i \triangleq \begin{bmatrix} -\bar{\mathcal{P}}_i & 0 & \bar{\mathcal{P}}_i\tilde{A}_i & \bar{\mathcal{P}}_i\tilde{B}_i \\ * & -I & \tilde{C}_i & \tilde{D}_i \\ * & * & -P_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (7)$$

where $\bar{\mathcal{P}}_i \triangleq \sum_{j\in\mathcal{I}} \pi_{ij}P_j$.

III. Main Results

In this section, based on Lemma 1, we will first give two H_∞ bounded real lemmas (BRLs) for the underlying augmented system (4), and further give the design of the FDI filter for system (1).

A. H_∞ BRLs:

The following lemma presents a bounded H_∞ performance criterion for system (4) with the partially known transition probabilities (2).

Lemma 2: Consider system (4) with partially known transition probabilities (2) and let $\gamma > 0$ be a given constant. If there exist matrices $P_i > 0$, $\forall i \in \mathcal{I}$ such that

$$\begin{bmatrix} -\mathcal{P}^i_{\mathcal{K}} & 0 & \mathcal{P}^i_{\mathcal{K}}\tilde{A}_i & \mathcal{P}^i_{\mathcal{K}}\tilde{B}_i \\ * & -\pi^i_{\mathcal{K}} I & \pi^i_{\mathcal{K}}\tilde{C}_i & \pi^i_{\mathcal{K}}\tilde{D}_i \\ * & * & -\pi^i_{\mathcal{K}}P_i & 0 \\ * & * & * & -\pi^i_{\mathcal{K}}\gamma^2 I \end{bmatrix} < 0, \quad (8)$$

$$\begin{bmatrix} -P_j & 0 & P_j\tilde{A}_i & P_j\tilde{B}_i \\ * & -I & \tilde{C}_i & \tilde{D}_i \\ * & * & -P_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0, \forall j \in \mathcal{I}^i_{\mathcal{UK}} \quad (9)$$

where $\mathcal{P}^i_{\mathcal{K}} \triangleq \sum_{j\in\mathcal{I}^i_{\mathcal{K}}} \pi_{ij}P_j$, then the augmented system (4) is stochastically stable with an H_∞ performance index γ.

Proof. Note that (7) can be rewritten as

$$\begin{aligned} \Xi_i &= \begin{bmatrix} -\sum_{j\in\mathcal{I}^i_{\mathcal{K}}}\pi_{ij}P_j & 0 & \sum_{j\in\mathcal{I}^i_{\mathcal{K}}}\pi_{ij}P_j\tilde{A}_i & \sum_{j\in\mathcal{I}^i_{\mathcal{K}}}\pi_{ij}P_j\tilde{B}_i \\ * & -\left(\sum_{j\in\mathcal{I}^i_{\mathcal{K}}}\pi_{ij}\right)I & \sum_{j\in\mathcal{I}^i_{\mathcal{K}}}\pi_{ij}\tilde{C}_i & \sum_{j\in\mathcal{I}^i_{\mathcal{K}}}\pi_{ij}\tilde{D}_i \\ * & * & -\sum_{j\in\mathcal{I}^i_{\mathcal{K}}}\pi_{ij}P_i & 0 \\ * & * & * & -\left(\sum_{j\in\mathcal{I}^i_{\mathcal{K}}}\pi_{ij}\right)\gamma^2 I \end{bmatrix} \\ &\quad + \sum_{j\in\mathcal{I}^i_{\mathcal{UK}}}\pi_{ij}\begin{bmatrix} -P_j & 0 & P_j\tilde{A}_i & P_j\tilde{B}_i \\ * & -I & \tilde{C}_i & \tilde{D}_i \\ * & * & -P_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} \\ &= \begin{bmatrix} -\mathcal{P}^i_{\mathcal{K}} & 0 & \mathcal{P}^i_{\mathcal{K}}\tilde{A}_i & \mathcal{P}^i_{\mathcal{K}}\tilde{B}_i \\ * & -\pi^i_{\mathcal{K}} I & \pi^i_{\mathcal{K}}\tilde{C}_i & \pi_i\tilde{D}_i \\ * & * & -\pi^i_{\mathcal{K}}P_i & 0 \\ * & * & * & -\pi^i_{\mathcal{K}}\gamma^2 I \end{bmatrix} \\ &\quad + \sum_{j\in\mathcal{I}^i_{\mathcal{UK}}}\pi_{ij}\begin{bmatrix} -P_j & 0 & P_j\tilde{A}_i & P_j\tilde{B}_i \\ * & -I & \tilde{C}_i & \tilde{D}_i \\ * & * & -P_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} \end{aligned}$$

Therefore, inequalities (8) and (9) guarantee $\Xi_i < 0$ (obviously, no knowledge on $\pi_{ij}, \forall j \in \mathcal{I}^i_{\mathcal{UK}}$ is required in (8) and (9)), this completes the proof. □

Remark 2: Although Lemma 2 gives a bounded real lemma (BRL) for the MJLS with partially known transition probabilities, it is hard to apply it to obtain the desired reduced-order model here due to the cross coupling of matrix product terms among different system operation modes, as shown in (8) and (9). To overcome this difficulty, the technique using slack matrix developed in [10] can be adopted here to obtain the following improved criterion for system (4).

Lemma 3: Consider system (4) with partially known transition probabilities (2) and let $\gamma > 0$ be a given constant. If there exist matrices $P_i > 0$, and R_i, $\forall i \in \mathcal{I}$ such that

$$\begin{bmatrix} \mathbf{\Upsilon}_j - R_i - R_i^T & 0 & R_i\tilde{A}_i & R_i\tilde{B}_i \\ * & -I & \tilde{C}_i & \tilde{D}_i \\ * & * & -P_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (10)$$

where if $\pi^i_{\mathcal{K}} = 0, \mathbf{\Upsilon}_j \triangleq P_j,\ j \in \mathcal{I}^i_{\mathcal{UK}}$ otherwise,

$$\begin{cases} \mathbf{\Upsilon}_j \triangleq \frac{1}{\pi^i_{\mathcal{K}}}\mathcal{P}^i_{\mathcal{K}}, & \forall j \in \mathcal{I}^i_{\mathcal{K}} \\ \mathbf{\Upsilon}_j \triangleq P_j, & \forall j \in \mathcal{I}^i_{\mathcal{UK}} \end{cases} \quad (11)$$

and $\mathcal{P}^i_{\mathcal{K}} = \sum_{j\in\mathcal{I}^i_{\mathcal{K}}} \pi_{ij}P_j$, then the augmented system (4) is stochastically stable with an H_∞ performance index γ.

Proof. First of all, by Lemma 3, we conclude that system (4) is stochastically stable with an H_∞ performance index γ if inequalities (8) and (9) hold. Notice that if $\pi^i_{\mathcal{K}} = 0$, the conditions (8)-(9) will be reduced to (9). Then, for $\pi^i_{\mathcal{K}} \neq 0$, (8) can be rewritten as:

$$\begin{bmatrix} -\frac{1}{\pi^i_{\mathcal{K}}}\mathcal{P}^i_{\mathcal{K}} & 0 & \frac{1}{\pi^i_{\mathcal{K}}}\mathcal{P}^i_{\mathcal{K}}\tilde{A}_i & \frac{1}{\pi^i_{\mathcal{K}}}\mathcal{P}^i_{\mathcal{K}}\tilde{B}_i \\ * & -I & \tilde{C}_i & \tilde{D}_i \\ * & * & -P_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0. \quad (12)$$

On the other hand, for an arbitrary matrix $R_i, \forall i \in \mathcal{I}$, we have the following facts:

$$\begin{aligned} \left(\frac{1}{\pi^i_{\mathcal{K}}}\mathcal{P}^i_{\mathcal{K}} - R_i\right)^T \left(\frac{1}{\pi^i_{\mathcal{K}}}\mathcal{P}^i_{\mathcal{K}}\right)^{-1} \left(\frac{1}{\pi^i_{\mathcal{K}}}\mathcal{P}^i_{\mathcal{K}} - R_i\right) &\geq 0, \\ (P_j - R_i)^T P_j^{-1}(P_j - R_i) &\geq 0, \end{aligned}$$

then by using (11), one has

$$\mathbf{\Upsilon}_j - R_i - R_i^T \geq -R_i^T\mathbf{\Upsilon}_j^{-1}R_i.$$

Furthermore, from (10), we can obtain that

$$\begin{bmatrix} -R_i^T\Upsilon_j^{-1}R_i & 0 & R_i\tilde{A}_i & R_i\tilde{B}_i \\ * & -I & \tilde{C}_i & \tilde{D}_i \\ * & * & -P_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0$$

Performing now a congruence transformation using $diag\{R_i^{-1}\Upsilon_j, I, I, I\}$ yields (12) and (9) for $j \in \mathcal{I}_{\mathcal{K}}^i$ and $j \in \mathcal{I}_{\mathcal{UK}}^i$, respectively (note that R_i is invertible if it satisfies (10)). This completes the proof. □

B. H_∞ *FDI filter design:*

As an application of Lemma 3, the following Theorem presents sufficient conditions for the existence of an admissible H_∞ FDI filter with the form of (3).

Theorem 1: Consider system (1) with partially known transition probabilities (2) and let $\gamma > 0$ be a given constant. If there exist matrices $P_{1i} > 0$, $Y_i > 0$ and $P_{3i} > 0, \forall i \in \mathcal{I}$, and matrices P_{2i}, X_i, Z_i, A_{fi}, B_{fi}, C_{fi}, D_{fi}, $\forall i \in \mathcal{I}$, such that

$$\begin{bmatrix} \Upsilon_{1j} & \Upsilon_{2j} & 0 & \Upsilon_{4i} & A_{fi} & \Upsilon_{6i} \\ * & \Upsilon_{3j} & 0 & \Upsilon_{5i} & A_{fi} & \Upsilon_{7i} \\ * & * & -I & D_{fi}C_i & C_{fi} & \Upsilon_{8i} \\ * & * & * & -P_{1i} & -P_{2i} & 0 \\ * & * & * & * & -P_{3i} & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (13)$$

where

$$\begin{aligned} \Upsilon_{4i} &\triangleq X_iA_i + B_{fi}C_i \\ \Upsilon_{5i} &\triangleq Z_iA_i + B_{fi}C_i \\ \Upsilon_{6i} &\triangleq \begin{bmatrix} X_iB_i & X_iE_i + B_{fi}D_i & X_iF_i + B_{fi}G_i \end{bmatrix} \\ \Upsilon_{7i} &\triangleq \begin{bmatrix} Z_iB_i & Z_iE_i + B_{fi}D_i & Z_iF_i + B_{fi}G_i \end{bmatrix} \\ \Upsilon_{8i} &\triangleq \begin{bmatrix} 0 & D_{fi}D_i & D_{fi}G_i - I \end{bmatrix} \end{aligned}$$

and if $\pi_{\mathcal{K}}^i = 0$, $\Upsilon_{1j} \triangleq P_{1j} - X_i - X_i^T$, $\Upsilon_{2j} \triangleq P_{2j} - Y_i - Z_i^T$, $\Upsilon_{3j} \triangleq P_{3j} - Y_i - Y_i^T$, $j \in \mathcal{I}_{\mathcal{UK}}^i$, otherwise,

$$\begin{cases} \Upsilon_{1j} \triangleq \frac{1}{\pi_{\mathcal{K}}^i}\mathcal{P}_{\mathcal{K}}^{1i} \triangleq \frac{1}{\pi_{\mathcal{K}}^i}\sum\limits_{j\in\mathcal{I}_{\mathcal{K}}^i}\pi_{ij}P_{1j} - X_i - X_i^T \\ \Upsilon_{2j} \triangleq \frac{1}{\pi_{\mathcal{K}}^i}\mathcal{P}_{\mathcal{K}}^{2i} = \frac{1}{\pi_{\mathcal{K}}^i}\sum\limits_{j\in\mathcal{I}_{\mathcal{K}}^i}\pi_{ij}P_{2j} - Y_i - Z_i^T \\ \Upsilon_{3j} \triangleq \frac{1}{\pi_{\mathcal{K}}^i}\mathcal{P}_{\mathcal{K}}^{3i} = \frac{1}{\pi_{\mathcal{K}}^i}\sum\limits_{j\in\mathcal{I}_{\mathcal{K}}^i}\pi_{ij}P_{3j} - Y_i - Y_i^T \end{cases} \quad (14)$$

$$\begin{cases} \Upsilon_{1j} \triangleq P_{1j} - X_i - X_i^T \\ \Upsilon_{2j} \triangleq P_{2j} - Y_i - Z_i^T \\ \Upsilon_{3j} \triangleq P_{3j} - Y_i - Y_i^T \end{cases}, \ \forall\ j \in \mathcal{I}_{\mathcal{UK}}^i \quad (15)$$

then, there exists a FDI filter such that the resulting system (4) is stochastically stable with an H_∞ performance index γ. Moreover, if a feasible solution exists, the gains of an admissible FDI filter in the form of (3) are given by

$$A_{Fi} = Y_i^{-1}A_{fi}, B_{Fi} = Y_i^{-1}B_{fi}, C_{Fi} = C_{fi}, D_{Fi} = D_{fi}. \quad (16)$$

Proof. Consider system (4) and assume the matrices P_i, and R_i in Lemma 3 to have the following forms:

$$P_i \triangleq \begin{bmatrix} P_{1i} & P_{2i} \\ * & P_{3i} \end{bmatrix}, \ R_i \triangleq \begin{bmatrix} X_i & Y_i \\ Z_i & Y_i \end{bmatrix}$$

then we have

$$\begin{aligned} \mathcal{P}_{\mathcal{K}}^i &\triangleq \sum\nolimits_{j\in\mathcal{I}_{\mathcal{K}}^i}\pi_{ij}P_j = \sum\nolimits_{j\in\mathcal{I}_{\mathcal{K}}^i}\pi_{ij}\begin{bmatrix} P_{1j} & P_{2j} \\ * & P_{3j} \end{bmatrix} \\ &\triangleq \begin{bmatrix} \mathcal{P}_{\mathcal{K}}^{1i} & \mathcal{P}_{\mathcal{K}}^{2i} \\ * & \mathcal{P}_{\mathcal{K}}^{3i} \end{bmatrix} \end{aligned}$$

Further define matrix variables

$$A_{fi} = Y_iA_{Fi}, \ B_{fi} = Y_iB_{Fi}, \ C_{fi} = C_{Fi}, \ D_{fi} = D_{Fi}$$

$$\Upsilon_j \triangleq \begin{bmatrix} \Upsilon_{1j} & \Upsilon_{2j} \\ * & \Upsilon_{3j} \end{bmatrix}$$

where Υ_{1j}, Υ_{2j} and Υ_{3j} are defined in (14) and (15) for $j \in \mathcal{I}_{\mathcal{K}}^i$ and $j \in \mathcal{I}_{\mathcal{UK}}^i$, respectively, one can readily obtain (13) by replacing $\tilde{A}_i, \tilde{B}_i, \tilde{C}_i, \tilde{D}_i, \Upsilon_j, P_i$ and R_i into (10), namely, if (13) holds, the augmented system (4) will be stochastically stable with an H_∞ performance. Meanwhile, if a solution of (13) exists, the parameters of admissible filter are given by (16). This completes the proof. □

Remark 3: By setting $\delta = \gamma^2$ and minimizing δ subject to (13), we can obtain the optimal H_∞ performance index γ^* (by $\gamma = \sqrt{\delta}$) and the corresponding filter gains as well. Also, it can be deduced from (13) that, given different degree of unknown elements in the transition probabilities matrix, the optimal γ^* achieved for system (4) and the corresponding filter should be different, which we will illustrate via a numerical example in the next section.

IV. Numerical Example

Consider the MJLS (1) with four operation modes and the following data:

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.1 & 0 & 1 & 0 \\ 0 & 0.1 & 0 & 0.5 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 0.3 & 0 & -1 & 0 \\ -0.1 & 0.2 & 0 & -0.5 \\ 0 & 0 & -0.2 & 0 \\ 0 & 0 & 0 & -0.5 \end{bmatrix}, \\ A_3 &= \begin{bmatrix} 0.2 & 0 & 1 & 0 \\ 0 & 0.3 & 0 & 0.4 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & -0.1 \end{bmatrix}, \\ A_4 &= \begin{bmatrix} 0.1 & 0 & -1 & 0 \\ -0.2 & 0.2 & 0 & -0.1 \\ 0 & 0 & -0.2 & 0 \\ 0 & 0 & 0 & -0.3 \end{bmatrix}, \\ B_1 &= B_2 = B_3 = B_4 = \begin{bmatrix} 0.5 & 0.1 & 0.2 & 0.3 \end{bmatrix}^T, \\ C_1 &= C_2 = C_3 = C_4 = \begin{bmatrix} 1 & 0.1 & 0 & 1 \\ 0 & 0.8 & 1 & 0 \end{bmatrix}, \\ E_1 &= E_2 = E_3 = E_4 = \begin{bmatrix} 0.08 & 0.1 & 0.5 & 0.3 \end{bmatrix}^T, \end{aligned}$$

TABLE I

DIFFERENT TRANSITION PROBABILITIES MATRICES

Completely known

	1	2	3	4
1	0.3	0.2	0.1	0.4
2	0.3	0.2	0.3	0.2
3	0.1	0.1	0.5	0.3
4	0.2	0.2	0.1	0.5

Partly unknown (case I)

	1	2	3	4
1	0.3	0.2	0.1	0.4
2	?	?	0.3	0.2
3	0.1	0.1	0.5	0.3
4	0.2	?	?	?

Partly unknown (case II)

	1	2	3	4
1	0.3	?	0.1	?
2	?	?	0.3	0.2
3	?	0.1	?	0.3
4	0.2	?	?	?

Completely unknown

	1	2	3	4
1	?	?	?	?
2	?	?	?	?
3	?	?	?	?
4	?	?	?	?

TABLE II

COMPUTATION RESULTS FOR DIFFERENT CASES

Transition probabilities	γ^*	L
Completely known	0.1406	4
partially known (Case I)	0.2133	4
partially known (Case II)	0.3234	6
Completely unknown	0.4430	7

$$
\begin{aligned}
D_1 &= D_2 = D_3 = D_4 = \begin{bmatrix} 0.8 \\ 0.4 \end{bmatrix}, \\
F_1 &= F_2 = F_3 = F_4 = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}^T, \\
G_1 &= G_2 = G_3 = G_4 = \begin{bmatrix} -1 \\ 0.9 \end{bmatrix}.
\end{aligned}
$$

The four cases for the transition probability matrix will be considered in this example as shown in Table 1.

Here, for $k = 0, 1, ..., 300$, the unknown input $d(k)$ is simulated by white noise signal with amplitude less than 0.5 (given in Fig. 1). The known input $u(k)$ is given by step signal with amplitude 0.3. The fault signal is set up as

$$
f(k) = \begin{cases} 0.8, & \text{for } k = 100, 101, ..., 200 \\ 0, & \text{others} \end{cases}
$$

By solving the convex problem in (13), the optimal H_∞ performance indices and the corresponding FDI filter are obtained for the four different transition probability cases. The corresponding results are listed in Table 2. We omit the filter gains for simplicity.

Now, consider the transition probability matrix with completely known elements as the practical one for other three cases in Table 1, we can generate a possible evolution of system modes as shown in Fig. 2. Accordingly, Fig. 3 shows the generated residual signals $r(k)$, and Fig. 4 presents the evolution of $J(r) = \sqrt{\sum_{l=0}^{k} r^T(l) r(l)}$ for both faulty case and fault-free case, respectively, for four different transition probability matrices in Table 1. Then, based on the path in Fig. 2 and the selected threshold $J_{th} = \sup_{d \in l_2, f=0} E\left[\sqrt{\sum_{k=0}^{300} r^T(k) r(k)}\right]$, the time steps L for the fault detection by the evaluation function $J(r) = \sqrt{\sum_{k=0}^{L} r^T(k) r(k)}$ and test (5)-(6) can be calculated and also given in Table 2. Obviously, it is seen from Table 2 that the more transition probability knowledge we have, the better H_∞ performance index the augmented system can achieve and the less times are needed for the fault detection. Therefore, a tradeoff can be actually built in practice between the complexity to obtain transition probabilities and the performance benefits and efficience of detection by means of our ideas and approaches.

V. CONCLUSIONS

The fault detection problem for discrete-time MJLS with partially known transition probabilities is investigated in this paper. The underlying systems are more general than the traditional MJLS, where all the transition probabilities are assumed to be completely known. The LMI-based sufficient conditions of FDI filter is obtained, and a tradeoff can be observed between the complexity of obtaining all the transition probabilities and the time steps to detect the fault.

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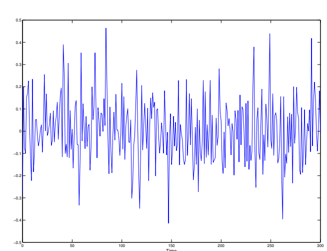

Fig. 1. Unknown input $d(k)$

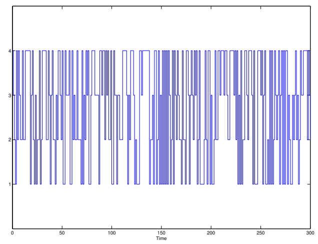

Fig. 2. Modes evolution

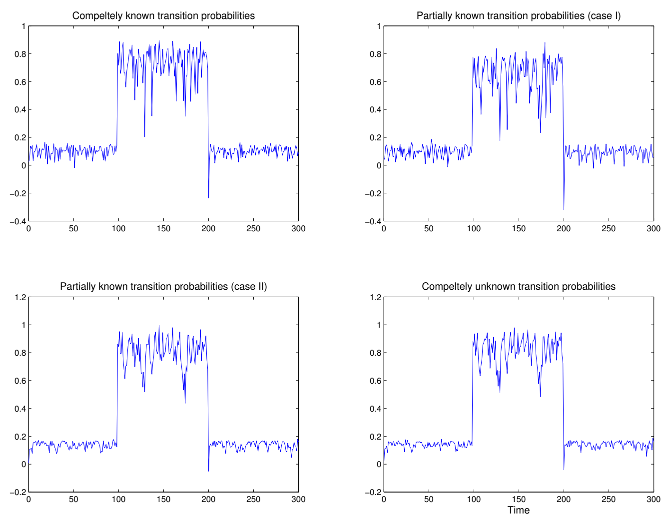

Fig. 3. Generated residual $r(k)$

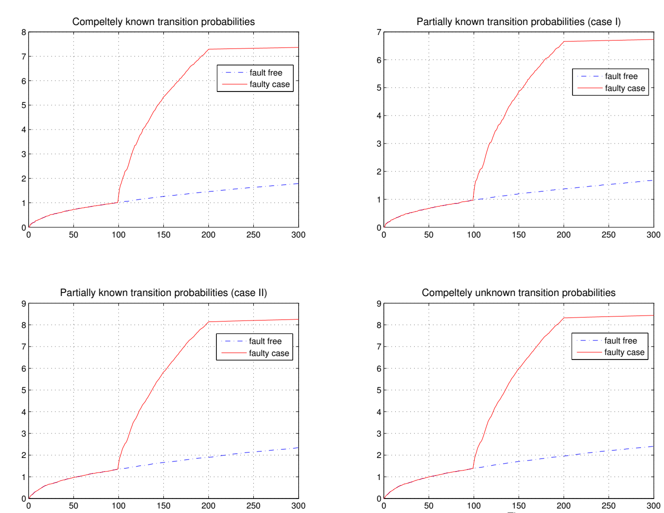

Fig. 4. Evolution of $J(r) = \sqrt{\sum_{l=0}^{k} r^T(l) r(l)}$