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Cooperative and competitive negotiation in a supply chain model

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Abstract— **We present a negotiation paradigm for a simple Supply Chain (SC) model, to improve the performances in terms of bullwhip effect reduction, under a cooperative and a competitive scenario. In the case of a single frequency perturbation in the nominal demand, analytical results showed that cooperation among the sites is beneficial for the SC performances. In the case of multiple frequency perturbations this has been described through numerical examples, which confirm the analytical findings of the single frequency case.**

Keywords: Supply chains, multi-agent systems, bullwhip effect.

I. INTRODUCTION

In supply chain (SC) management, one of the most problematic issues to be faced is the bullwhip effect, the increase in order variance going to the upstream levels of the chain [10], [14], [5], [6]. Satisfactory control laws at the local level (in the sense that they reduce the variance of the inventory of a site considered as isolated) may cause the insurgence of this pathological behavior at the global level, with a negative repercussion on the overall system, and a modification of the order policy must be sought (see e.g. [5], [6]).

Despite the complexity of real SC systems, simplified models have provided insights on the development of this phenomenon and on the possible control strategies to avoid it (see e.g. [13], [7], [5], [10], [14]). In particular [14] introduced a suitable metric for the measure of the bullwhip effect which was the criterion in [1] to derive an optimal worst case control paradigm, based on a H_∞ approach, avoiding the bullwhip phenomenon if applied at the global level of the SC or if each site implemented it under an altruistic constraint (specified in [1]). The necessity of an altruistic behavior to optimize the performance of the whole SC has been proved in [8], in a slightly different scenario. As for the global level implementation, it has been actually recognized in the literature (see e.g. among others [4], [3]) how better would perform a SC if it could be governed by a centralized controller who has information on the whole chain. If, as often happens, the SC cannot be governed by a centralized controller and the information on the various echelons is not shared in the whole chain, a decentralized control scheme with partial information must be considered. One possibility to design a decentralized control scheme is based on a Multi-Agent control paradigm, which appears more reasonable in comparison to the possibility of applying in a real context the altruistic constraint pointed out in [1]. The literature about the SC seen as a multi-agent system is vast, see for example the survey [15], and the references therein; among those works, [9], [12], [16] show how virtuous behaviors may emerge by playing competing games, with a structure of revenues/costs reflecting the management of the SC at the operative level, whereas in [8], [11] it is shown how the bullwhip effect can be reduced by introducing cooperation and/or exchange of information among the sites of the chain.

In this paper we develop a negotiation mechanism for the reduction of the bullwhip effect, directly considering the costs related to its insurgence, hence according to a performance defined in terms of the gain between the demand and the inventory signals. The focus of this study is not on solving the local optimal control problem (for this see e.g., [1], [5], [10]) rather on analyzing the consequences of encouraging altruistic (even if under a cost compensation, subject of the negotiation) behaviors. For this reason, the local policy considered here to set up the negotiation is a simplified version of the order-up-to policy (see e.g., [5],[10]). Notice that similar negotiation schemes could be developed by the choice of other local control policies.

The multi-agent idea considered in this paper was firstly introduced in [2]. The contribution of this paper with respect to [2] consists in the definition of a cooperative and a competitive negotiation among sites and in an extension of some analytical results which allows, here, to compare the performance of the whole SC under the cooperative and the competitive negotiation paradigms.

II. NOTATION AND PROBLEM FORMULATION

A SC comprising N sites is considered, where site i receives raw materials from site $i-1$, and ships finished products to site $i+1$ (see Fig. 1). Let x_i and b_i be the levels of stocked finished goods and backlogged demand of site i, respectively, and denote by $s_i(k)$, the shipment of goods from site i to site $i+1$, and by $d_i(k)$ the demand of raw materials to site i, at time k. Here, site 0 can be considered as an infinite reservoir of raw materials while site N faces a demand signal d_N, made by a final customer. Then, the dynamics of site i is given by:

$$x_i(k+1) = x_i(k) + s_{i-1}(k-\lambda_i) - s_i(k) \quad (1)$$
$$b_i(k+1) = b_i(k) + d_i(k) - s_i(k) \quad (2)$$

where $\lambda_i \in \mathbb{Z}^+$ is the sum of the production lead time of site i and the transportation time from site $i-1$ to site i. All the products requested by site $i+1$ are shipped if there is enough stock at site i, otherwise site i will send its whole stock. Hence the shipment can be expressed as follows:

$$s_i(k) = \min\{x_i(k) + s_{i-1}(k-\lambda_i), b_i(k) + d_i(k)\} \quad (3)$$

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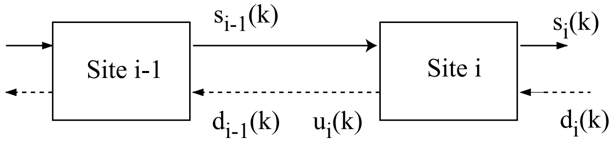

Fig. 1. The supply chain model: solid (dashed) arrows represent flow of material (information)

Notice that according to this dynamics, at each time k, only one between x and b is positive and the other is 0. Denote the backlog/inventory level by $\chi_i := x_i - b_i$ (more details on this model can be found in [1]).

In some real production enterprises the customer provides the supplier the orders relative to a long time period ahead, and these orders are confirmed or possibly modified shortly before their due date. This behavior can be characterized in terms of perturbations δd_N about a nominal, known in advance, demand signal $\bar{d}_N$, so that $d_N(\cdot) = \bar{d}_N(\cdot) + \delta d_N(\cdot)$. At each site i it is possible to compute the *nominal* order signal $\bar{d}_{i-1}(\cdot)$ which maintains a desired safety stock, if there is no perturbation on the nominal demand. When such desired stock level is large enough compared to the demand perturbations which may occur[1], Eq. (3) reduces to $s_i(k) = d_i(k)$, and the perturbed dynamics of the system is expressed by the linear relation $\delta x_i(k+1) = \delta x_i(k) + \delta d_{i-1}(k - \lambda_i) - \delta d_i(k)$, where δx_i is the deviation of the inventory from the nominal (desired) stock level. For simplicity of notation, as in the following we will always refer to the variational model, we will drop δ to indicate variated quantities; also, as δd_{i-1} represents a control variable for site i, we denote it by u_i; hence, the (variational) dynamics of site i is expressed as:

$$x_i(k+1) = x_i(k) + u_i(k - \lambda_i) - d_i(k), \tag{4}$$

with $x_i(k) = 0$ and $d_i(k) = 0$ (and $u_i(k)$ as well) for all $k \leq 0$. This kind of model has been considered in several papers (see e.g. [5] and [14]).

To define a cost index for this system, we consider the spectral representation of a signal $y(\cdot)$, and denote by $Y(f)$ the module of the component of frequency f of $y(\cdot)$; with this notation, the following cost index has been considered:

$$J = \sum_{i=1}^{N} Q_i \; \mathcal{L}\left[\frac{X_i(f)}{D_N(f)}\right] \tag{5}$$

where $Q_i > 0$ are weighing factors, $f \in (f_a, f_b)$ (with $0 \leq f_a \leq f_b \leq 1$ two frequency values) is the frequency component of the external demand, and $\mathcal{L}$ is a generic norm (on the frequency domain). For instance, the adoption of the $\mathcal{L}_\infty$ norm in (5) i.e., $\mathcal{L}_\infty[Y(f)] = \max_{f \in (f_a, f_b)} Y(f)$, amounts to define, as remarked in [5], [10], [14], a cost function that captures the insurgence of the bullwhip effect. This norm has been considered also in [1] where a H_∞ control policy has then been formulated for the local control of a site or to control the whole chain if this is governed by a centralized controller. When the $\mathcal{L}_1$ norm is used in (5), the cost index gives a measure of the maximum fluctuation experienced in the inventory and thus is related to holding costs. In the following, to simplify notation, we will take, in (5), $Q_i = 1$ for all i. This is in fact not essential in the development presented.

[1]A positive inventory for each site makes the dynamics of the chain coupled, as observed in [10], and is the most significant for the (possible) insurgence of the bullwhip effect (a negative $\bar{\chi}_i$ stops the propagation of the fluctuations).

The reference policy considered is a Proportional policy based on the Inventory Position (IP), whose dynamics is:

$$IP_i(k+1) = IP_i(k) + u_i(k) - d_i(k), \tag{6}$$

with $IP_i(0) = 0$ for all i. The above definition of IP is in terms of variational quantities, so that also this variable represents the deviation about a reference (time-varying) value. The considered policy is defined as follows:

$$u_i(k) = -\alpha_i IP_i(k) \tag{7}$$

and it is possible to show that it provides a stable behavior if and only if $\alpha_i \in (0, 2)$ for all i [10]. This policy, denoted in the following as P-IP policy, is similar, apart from the demand forecast term, to the generalized Order Up To (OUT) policy considered e.g. in [6], [5], [8] and corresponds to the one presented in [10] for a 0 set point level (in fact for the variational model the reference inventory level is 0).

In this paper we address the following problem. If α_i^*, $i = 1, \ldots, N$, denote the value of the α_i minimizing the index J in (5) for some specified norm $\mathcal{L}$ and J^* denotes the optimal cost, we want to evaluate how the cost obtained from the negotiation schemes defined below differs from J^*.

III. A DECENTRALIZED CONTROL BASED ON AGENTS

We will associate an agent to each site of the chain and assume that only neighboring sites may negotiate. Notice that due to linearity the inventory cost of a site i is proportional to the magnitude of the orders that it receives from its downstream site $i+1$. Hence, the negotiation among two sites i and $i+1$ arises when site i makes an offer to site $i+1$ asking it to change α_{i+1} in order to reduce the magnitude of the demand that site $i+1$ places to site i. The offer will be part of the saved cost. Then, roughly, the idea of the agent-based mechanism is the following one: each site i sets initially its own α_i to an *egoistic/isolated* value. Then, it will send a request to its downstream site to make it change the control coefficient α_{i+1} in order to reduce its own cost. The next two sections will discuss the case of external demand with a single frequency component $\bar{f}$. The performance index is then:

$$J = \sum_{i=1}^{N} \frac{X_i(\bar{f})}{D_N(\bar{f})} := \sum_{i=1}^{N} J_{xi} \tag{8}$$

with J_{xi} denoting the local performance of site i. If, under the considered P-IP policy, $\mathcal{V}_i(z)$ and $\mathcal{W}_i(z)$ denote respectively the transfer function from $D_i(z)$ to $U_i(z)$ and from $D_i(z)$ to $X_i(z)$, we have: $\mathcal{W}_i(\alpha_i, z) = \frac{1-\alpha_i - z + \alpha_i z^{-\lambda_i}}{(z-1)(z-1+\alpha_i)}$ and

$\mathcal{V}_i(\alpha_i, z) = \frac{\alpha_i}{z+\alpha_i-1}$. Then it is possible to write:

$$J_{xi} = W_i(\alpha_i, \bar{f}) \prod_{j=i+1}^{N} V_j(\alpha_j, \bar{f})$$

where $W_i(\alpha_i, f) := |\mathcal{W}_i(\alpha_i, e^{j2\pi f})|$ and $V_i(\alpha_i, f) := |\mathcal{V}_i(\alpha_i, e^{j2\pi f})|$. It can be seen that the function $V_i(\alpha_i, f)$ is increasing with respect to $\alpha \in (0, 2)$ for all $f \in (0, 1)$. As a consequence, it is clear that each site $i-1$ will try to make its downstream site i to decrease its α_i to 0. If this coincides with the local optimum (i.e. minimizes also $W_i(\alpha_i, \bar{f})$) then this will be the α_i selected, otherwise the two sites should negotiate to select a proper α_i. As a matter of fact, it is possible to see that the value of α_i minimizing $W_i(\alpha_i, \bar{f})$ (called α^{eg}, where eg stands for *egoistic*) is not 0 for all frequencies $\bar{f}$ and all delays λ.

The negotiation is defined according to a simple paradigm, by which the selection of the parameters at site i when negotiating with site $i-1$, only concerns the cost corresponding to these two sites. Notice however that each site i could make a selection of parameters considering at the same time all its costs: local inventory, offer to site $i+1$, offer from site $i-1$.

In the following, given a function $f(x)$, if $\arg\min_{x\in[x_a,x_b]} f(x)$ comprises more elements, we assume always to select the smallest of them. This is not essential in our derivations but simplifies matters. In the following discussion of the single frequency case, it will be understood that V and W are evaluated at a frequency value $\bar{f}$.

Algorithm 1: The negotiation algorithm.

Step 0. Let $m = 0$ and set for all $i = 1, \ldots, N$, $\alpha_i^{(0)} = \arg\min_{\alpha\in(0,2)} W_i(\alpha)$.

Step m+1. To each site i, $i = 2, \ldots, N$, is offered from its upstream site $i-1$ a fraction $P_{i-1} \in (0, 1)$ of what site $i-1$ saves when the downstream site sets its $\alpha_i^{(m+1)}$ to a certain α, that is, site $i-1$ offers to site i, $O_{i-1}D_i$, where $O_{i-1} := P_{i-1}W_{i-1}(\alpha_{i-1}^{(m)})(V_i(\alpha_i^{(0)}) - V_i(\alpha))$ and D_i is the amplitude of the input to site i. Hence site i selects the α minimizing $[(W_i(\alpha) - W_i(\alpha_i^{(0)})) - O_{i-1}]D_i$, i.e., it will set:

$$\alpha_i^{(m+1)} = \arg\min_{\alpha\in(0,2)} P_{i-1}W_{i-1}(\alpha_{i-1}^{(m)})V_i(\alpha) + W_i(\alpha) \quad (9)$$

The procedure terminates if $\alpha_i^{(m+1)} \equiv \alpha_i^{(m)}$ for all i. In this case, let $\alpha_i^L := \alpha_i^{(m)}$ denote the final value, else set $m = m + 1$ and return to Step $m + 1$. □

As proved in [2], the algorithm above, for any choice of the P_i, converges in $N-1$ steps and, if $P_i = 1$ for all i, the following relation characterizes the final parameters: $\alpha_i^L \geq \alpha_i^*$, for all i, with $\alpha_1^L = \alpha_1^*$ and $\alpha_2^L = \alpha_2^*$. What about the selection of the P_i? In [2] it was numerically observed in all the examples considered how the negotiation with $P_i = 1$ for all i provides the minimum value for the global cost, while it could be not optimal for the local costs. Before introducing some possible (cooperative or competitive) strategies to select this negotiation parameter (see Section IV), we prove that the larger are the P_i used in the negotiation algorithm, the smaller is the global cost achieved by negotiation. This introduces a partial order in the set of all possible choices of vector $P := (P_1, \ldots, P_{N-1})^T \in [0, 1]^{N-1}$. In the following, $J(P)$ will denote the cost (5), when the α_i are set as the final values α_i^L of the negotiation algorithm 1, with parameters P_i as in vector P. To proceed, we need the following lemma.

Lemma 1: Let $c_1 \geq c_2 > 0$ be constants, $f(x)$, $g(x)$ continuous functions with $f(x)$ increasing, where $x \in (0, B)$. Let $h_i(x) := c_i f(x) + g(x)$ and $x_i = \arg\min_{x\in(0,B)} h_i(x)$, $i = 1, 2$. Then, $x_1 \leq x_2$ and $g(x_1) \geq g(x_2)$.

Proof. If $c_1 = c_2$ the result trivially holds. Hence assume $c_1 > c_2$. Since x_2 is a minimum for $h_2(x)$, it follows that $h_2(x) \geq h_2(x_2)$ for all x and, in particular, for all $x > x_2$. Then, for all $x > x_2$,

$$\begin{gathered} h_1(x) - h_1(x_2) = c_1 f(x) + g(x) - c_1 f(x_2) - g(x_2) = \\ (c_1 - c_2)[f(x) - f(x_2)] + c_2[f(x) - f(x_2)] + g(x) - g(x_2) \\ = (c_1 - c_2)[f(x) - f(x_2)] + [h_2(x) - h_2(x_2)] > 0 \end{gathered}$$

because $f(x)$ is increasing. This implies that the minimum of $h_1(x)$ can not be larger than x_2, that is $x_1 \leq x_2$. Now, assume by contradiction that $g(x_1) < g(x_2)$. Then, since $f(x)$ is increasing, it follows: $c_2 f(x_1) + g(x_1) < c_2 f(x_2) + g(x_2)$; that is $h_2(x_1) < h_2(x_2)$ violating the assumption that x_2 is a minimum for $h_2(x)$. Hence, $g(x_1) \geq g(x_2)$. □

Theorem 1: Let P' and P'' be two choices of vector P and assume $0 \leq P'_i < P''_i \leq 1$, $\forall i$. Then, $J(P') \geq J(P'')$.

Proof. Let $\alpha_i^{L,P}$ denote the final value of α_i when applying the negotiation algorithm with a given vector P. Notice that for all $P \in [0, 1]^{N-1}$, we have $\alpha_1^{L,P} = \alpha_1^*$ (this follows from the fact that both the value of α_1 providing the global minimum of J and the limit value of the negotiation $\alpha_1^{L,P}$ minimize $W_1(\alpha)$). Consider now a two site SC ($N = 2$). In this case P' and P'' are scalar. We have:

$$\alpha_2^{L,P'} = \arg\min_{\alpha\in(0,2)} W_2(\alpha) + P'W_1(\alpha_1^*)V_2(\alpha) \quad (10)$$

$$\alpha_2^{L,P''} = \arg\min_{\alpha\in(0,2)} W_2(\alpha) + P''W_1(\alpha_1^*)V_2(\alpha) \quad (11)$$

Applying Lemma 1 with $c_1 = P''W_1(\alpha_1^*)$, $c_2 = P'W_1(\alpha_1^*)$, $x = \alpha$, $f = V_2$ and $g = W_2$, it follows $\alpha_2^{L,P'} \geq \alpha_2^{L,P''}$, $V_2(\alpha_2^{L,P'}) \geq V_2(\alpha_2^{L,P''})$ and $W_2(\alpha_2^{L,P'}) \leq W_2(\alpha_2^{L,P''})$. Now, let $a_1 := W_2(\alpha_2^{L,P'})$, $b_1 := V_2(\alpha_2^{L,P'})$, $a_2 := W_2(\alpha_2^{L,P''})$ and $b_2 := V_2(\alpha_2^{L,P''})$.

Equations (10)-(11) imply:

$$a_2 + P''b_2 \leq a_1 + P''b_1 \quad (12)$$

$$a_2 + P'b_2 \geq a_1 + P'b_1 \quad (13)$$

Notice that the expression $a_k + Pb_k$ is an affine function of P and, according to what said above, $a_1 \leq a_2$ and $b_1 \geq b_2$. So the two lines $a_1 + Pb_1$ and $a_2 + Pb_2$ intersect at some $\bar{P}$ and, according to (13), $P' \leq \bar{P}$, and, according to (12), $P'' \geq \bar{P}$. Since $1 \geq P''$ and $P'' \geq \bar{P}$, we have $1 \geq \bar{P}$. This implies that $a_1 + b_1 \geq a_2 + b_2$, which is exactly the result to be proved if $N = 2$ (being $a_1 + b_1 = J(P')$ and $a_2 + b_2 = J(P'')$). Consider now a three site SC ($N = 3$) and let for brevity $W_i^P := W_i(\alpha_i^{L,P})$ and $V_i^P := V_i(\alpha_i^{L,P})$, $i =$

$1,2,3$ the value of the functions at the end of the negotiation under a certain vector P. Considering the first two sites, we have, as mentioned above, $V_2^{P'} \geq V_2^{P''}$. Also, we know that $\alpha_1^{L,P'} \equiv \alpha_1^{L,P''} \equiv \alpha_1^*$. Considering sites 2 and 3, the same reasoning applied to the $N=2$ case (with $c_1 = P_2''W_2^{P''} \geq P_2'W_2^{P'} = c_2$) allows to show that: (i) $W_2^{P'}V_3^{P'} + W_3^{P'} \geq W_2^{P''}V_3^{P''} + W_3^{P''}$ and (ii) $V_3^{P'} \geq V_3^{P''}$. Then we have:

$$J(P') = \left(W_1^{P'}V_2^{P'} + W_2^{P'}\right)V_3^{P'} + W_3^{P'} =$$
$$W_1(\alpha_1^*)V_2^{P'}V_3^{P'} + \left(W_2^{P'}V_3^{P'} + W_3^{P'}\right) \geq$$
$$W_1^{P''}V_2^{P''}V_3^{P''} + \left(W_2^{P''}V_3^{P''} + W_3^{P''}\right) = J(P'').$$

The general N site case follows by induction: assuming the result holds up to $N-1$, the same reasoning of the $N=3$ case can be applied to the N case where in place of $W_1^{P'}$ and $W_1^{P''}$ we have to use, respectively, $J^{N-2}(P')$ and $J^{N-2}(P'')$ (where $J^{N-2}(P) := \sum_{i=1}^{N-2}\left[W_i^P \prod_{j=i+1}^{N-2} V_j^P\right]$ is the cost of the sub-chain composed by the first $N-2$ sites of the SC after a negotiation with vector P). Now, by the inductive assumption, $J^{N-2}(P') \geq J^{N-2}(P'')$, and $V_{N-1}^{P'} \geq V_{N-1}^{P''}$. Then, from the negotiation of the last two sites (with $c_1 = P_{N-1}''W_{N-1}^{P''}$, $c_2 = P_{N-1}'W_{N-1}^{P'}$ and $c_1 \geq c_2$ by the inductive assumption), we also have $W_{N-1}^{P'}V_N^{P'} + W_N^{P'} \geq W_{N-1}^{P''}V_N^{P''} + W_N^{P''}$ and $V_N^{P'} \geq V_N^{P''}$. This allows to conclude the proof as in the $N=3$ case:

$$J(P') = J^{N-2}(P')V_{N-1}^{P'}V_N^{P'} + \left(W_{N-1}^{P'}V_N^{P'} + W_N^{P'}\right) \geq$$
$$J^{N-2}(P'')V_{N-1}^{P''}V_N^{P''} + W_{N-1}^{P''}V_N^{P''} + W_N^{P''} = J(P'').\square$$

IV. Cooperative and competitive negotiation

According to the above negotiation paradigm, site $i-1$ and site i, $i = 2,\ldots,N$ involved in the negotiation at step m will incur respectively in the costs $\Gamma_{i-1}^{d,(m)}(\alpha_i, P_{i-1})$ and $\Gamma_i^{u,(m)}(\alpha_i, P_{i-1})$ described below (where the term $\prod_{j=i+1}^{N} V_j(\alpha_j^{(m)})$ has been omitted in all the expressions, being a common factor, hence uneffective in the negotiation algorithm).

$$\Gamma_{i-1}^{d,(m)}(\alpha_i, P_{i-1}) := J_{x,i-1}^{(m)}(\alpha_i) + O_{i-1}^{(m)}(\alpha_i, P_{i-1})$$

represents the cost corresponding to site $i-1$ at step m when considering the negotiation with its downstream site i (whence the apex d) with parameters to be negotiated $\alpha_i \in (0,2)$ and $P_{i-1} \in (0,1)$, with

$$J_{x,i-1}^{(m)}(\alpha_i) = W_{i-1}(\alpha_{i-1}^{(m)})V_i(\alpha_i) \quad (14)$$

the inventory cost of site $i-1$ and

$$O_{i-1}^{(m)}(\alpha_i, P_{i-1}) = P_{i-1}W_{i-1}(\alpha_{i-1}^{(m)})\left[V_i(\alpha_i^{eg}) - V_i(\alpha_i)\right] \quad (15)$$

the expense of site $i-1$ to make site i change its optimal local α_i^{eg} (minimizing $W_i(\alpha)$) to the value α_i. Similarly,

$$\Gamma_i^{u,(m)}(\alpha_i, P_{i-1}) := J_{x,i}(\alpha_i) - O_{i-1}^{(m)}(\alpha_i, P_{i-1})$$

represents the cost corresponding to site i at step m when considering the negotiation with its upstream site $i-1$ (whence the apex u) with parameters to be negotiated $\alpha_i \in (0,2)$ and $P_{i-1} \in [0,1]$, with $J_{x,i}(\alpha_i) = W_i(\alpha_i)$.

Now the question is: are the two sites really cooperative (hence the choice of P_{i-1} and of α_i can be made in collaboration) or are only negotiating, but remain competitive? The following two subsections present the consequences of the two different situations.

A. Cooperative negotiation

In this case actually the two sites behave like a single entity, in the selection of P_{i-1} and of α_i. One possibility is that the selection of these parameters is made in such a way that the two sites will incur in the same penalty with respect to their egoistic optimal situation (i.e. with respect to the situation where the selection is performed in order to minimize their own costs). However, as it is possible to verify, this may bring to a situation where the penalty is high for both the two sites, in the sense that an equilibrium is reached only with a choice of P_{i-1} and of α_i giving a bad performance of the couple. For this reason, it is more convenient that, in the cooperative case, each couple of sites, according to the fact that the minimum for the couple is obtained when $P_{i-1} = 1$ (see Theorem 1), directly selects:

$$\alpha_i^{(m+1)} = \arg\min_{\alpha \in (0,2)} \{W_i(\alpha) + W_{i-1}(\alpha_{i-1}^{(m)})V_i(\alpha)\}$$

(which corresponds to (9) with $P_{i-1} = 1$) with the agreement that all profits will be equally shared. This corresponds to the selection of α_i and P_{i-1} minimizing the sum of the inventory cost of the two sites, that is, minimizing $J_{x,i} + J_{x,i-1}$. Notice that this pairwise optimization will not bring in general to the global optimization, being the α_i selected only considering the upstream site $i-1$ and not all the upstream chain.

B. Competitive negotiation

In the competitive case, it is reasonable to assume that the negotiation parameter P_{i-1} is selected by site $i-1$ and its value can not be included in the negotiation, unless we assume that site i gives back a fraction of what is offered by site $i-1$ to convince it to change P_{i-1}. This actually corresponds to change the value of P_{i-1} to the original value that site $i-1$ wanted. Hence it is definitely site $i-1$ which decides the value of P_{i-1} corresponding to the part of the saving it is prepared to share. And site i acts in consequence, by selecting its α_i according to the proposed value p of P_{i-1}:

$$\alpha_i^{(m)}(p) := \arg\min_{\alpha \in (0,2)} J_{x,i}(\alpha) - O_{i-1}^{(m)}(\alpha, p). \quad (16)$$

In this case, site $i-1$ will select the p providing the minimum cost to it. Hence it will set at the step $m+1$ of the negotiation:

$$P_{i-1}^{(m+1)} = \arg\min_{p \in (0,1)} J_{x,i-1}^{(m)}(\alpha_i^{(m)}(p)) + O_{i-1}^{(m)}(\alpha_i^{(m)}(p), p) \quad (17)$$

Substituting (14) and (15) in (17), the $(m+1)$-th step of the negotiation among sites $i-1$ and i produces the following parameter update:

$$P_{i-1}^{(m+1)} = \arg\min_{p \in (0,1)} (1-p)V_i(\alpha_i^{(m)}(p)) + pV_i(\alpha_i^{eg}) \quad (18)$$

$$\alpha_i^{(m+1)} = \alpha_i^{(m)}(P_{i-1}^{(m+1)}) \quad (19)$$

[N, λ, f]	J^*	J^{coop}	J^{comp}	J^{eg}	α^*	α^{coop}	α^{comp}	α^{eg}	P_{comp}
[2, 3, 0.2]	.82	.82	.84	.89	[.47 .23]	[.47 .23]	[.47 .37]	.47	[.51]
[3, 3, 0.2]	.85	.86	.93	1.03	[.47 .23 .01]	[.47 .23 .16]	[.47 .37 .36]	.47	[.51 .54]
[4, 3, 0.2]	.85	.86	.96	1.1	[.47 .23 .01 .01]	[.47 .23 .16 .12]	[.47 .37 .36 .36]	.47	[.51 .54 .52]
[4, 4, 0.2]	0	0	0	0	[1 1 1 1]	[1 1 1 1]	[1 1 1 1]	1	[.45 .45 .45]
[4, 5, 0.2]	.86	.86	.86	3.51	[1.99 .01 .01 .01]	[1.99 .01 .01 .01]	[1.99 .01 .01 .01]	1.99	[.31 .23 .23]
[4, 5, 0.13]	1.26	1.28	1.4	1.59	[.35 .19 .03 .01]	[.35 .19 .15 .13]	[.35 .28 .272 .27]	.35	[.53 .54 .55]

TABLE I

GLOBAL OPTIMUM AND PERFORMANCE OF THE SC WITH COOPERATIVE AND COMPETITIVE NEGOTIATION, UNDER DIFFERENT SETTINGS.

Notice that the P_{i-1} and α_i selected depend on the particular $\bar{f}$ considered, on the sites $i-1$ and i (that is on the delays λ_{i-1} and λ_i) but also on the step m of the iteration (notice in fact that this dependence is due to the presence, through the term $O_{i-1}^{(m)}(\alpha, p)$, of $\alpha_{i-1}^{(m)}$ in the expression of $\alpha_i^{(m)}(p)$ given in (16)). If, as here, we deal with a unique frequency $\bar{f}$, actually the same convergence property (mentioned above) for the case of P_i fixed holds and the terminal values P_i^L and α_i^L of the P_i and the α_i verify the following equations:

$$\alpha_1^L = \alpha_1^{eg} = \arg\min_{\alpha\in(0,2)} W_1(\alpha).$$

$$P_1^L = \arg\min_{p\in(0,1)} \left\{(1-p)V_2(\alpha_2^L(p)) + pV_2(\alpha_2^{eg})\right\}$$

where $\alpha_i^L(p) = \arg\min_{\alpha\in(0,2)} W_i(\alpha) + pW_{i-1}(\alpha_{i-1}^L)V_i(\alpha)$. So:

$$\alpha_2^L = \arg\min_{\alpha\in(0,2)} \left\{W_2(\alpha) + P_1^L W_1(\alpha_1^L)V_2(\alpha)\right\} \tag{20}$$

$$P_2^L = \arg\min_{p\in(0,1)} \left\{(1-p)V_3(\alpha_3^L(p)) + pV_3(\alpha_3^{eg})\right\} \tag{21}$$

$$\alpha_3^L = \arg\min_{\alpha\in(0,2)} \left\{W_3(\alpha) + P_2^L W_2(\alpha_2^L)V_3(\alpha)\right\} \tag{22}$$

And so on. In this case, being the P_i selected different in general from one, the global cost will be not smaller in general than the one obtained with the cooperative negotiation (by Theorem 1). Remembering that $P_i = 0$ for all i corresponds to the egoistic situation where no negotiation arises and each site selects its isolated α_i, Theorem 1 allows to establish the following intuitive result, illustrated in the example section, which shows how the egoistic objectives negatively impact on the behavior of the overall chain.

Corollary 1: Let J^{eg} be the global cost corresponding to the case each site selects its egoistic α, J^{comp} and J^{coop} the global cost corresponding to the competitive and the cooperative negotiation (after convergence), respectively. Then $J^{eg} \geq J^{comp} \geq J^{coop} \geq J^*$.

V. THE MULTI FREQUENCY CASE

The case comprising two or more frequencies in the disturbance signal is much more involved since the optimal α_i also depends on downstream nodes, unlike the single frequency case. To easily see this, consider a two site system perturbed by a two frequency signal and take hereafter $\mathcal{L} = \mathcal{L}_1$. The optimal α_i^*, $i = 1, 2$ are the ones minimizing $\sum_{k=1}^{2}[W_1(\alpha_1, f_k)V_2(\alpha_2, f_k) + W_2(\alpha_2, f_k)]$. While for the single frequency case α_1^* was the value minimizing $W_1(\alpha_1, \bar{f})$, which is independent of the downstream sites, now, since each frequency is weighed differently due to the effect of downstream sites (one is weighed by $V_2(\alpha_2, f_1)$ the other by $V_2(\alpha_2, f_2)$), the optimal α_1^* depends on the downstream sites. To capture this dependence, we have decided to modify the negotiation algorithm by considering the gain of the downstream site (this would have been useless in the single frequency case). So the selection of α_i at Step $m+1$ will be performed not according to (9) but as follows:

$$\alpha_i^{(m+1)} = \arg\min_{\alpha\in(0,2)} \sum_{f_k\in\mathcal{F}} \Big[P_{i-1}W_{i-1}(\alpha_{i-1}^{(m)}, f_k)$$

$$V_i(\alpha, f_k) + W_i(\alpha, f_k)\Big] V_{i+1}(\alpha_{i+1}^{(m)}, f_k)$$

where $\mathcal{F} := \{f_1, \ldots, f_K\}$ is the set of frequencies present in the perturbation signal. At step 0 we simply assign α_i according to: $\alpha_i^{(0)} = \arg\min_{\alpha\in(0,2)} \sum_{f_k\in\mathcal{F}} W_i(\alpha, f_k)$. Notice that in this case the convergence is not assured (the value of α_i has an impact not only on the choice of site $i+1$ as it was in the single frequency case, but also on the choice of site $i-1$, which makes the interaction of sites bidirectional). However, such a bidirectional interaction seems to facilitate the algorithm in approaching the global optimum, as observed in the numerical examples (see Section VI-B). A cooperative and a competitive version of the algorithm can be set up similarly to the single frequency situation: in the first case each pair of sites minimizes the sum of the two inventory costs (selecting $P_i = 1$, for all i); in the second case each site i selects the P_i with the objective of maximizing its gain in the negotiation with site $i+1$.

VI. NUMERICAL EXAMPLES

A. Single frequency case

We report in Table I the α_i and the corresponding total cost J under the different control policies for different systems. We have reported only systems with homogeneous delays ($\lambda_i = \lambda$ for all i). The SC is then characterized by the number of sites N, the delay λ, the frequency of the disturbance $\bar{f}$. In the table, P_{comp} represents the values of the P_i selected at the sites of the SC under the competitive negotiation (i.e. according to (18)). Notice that if $\lambda = 4$ and $\bar{f} = 0.2$, since $W(\alpha, \bar{f}) = 0$ when $\alpha = 1$, we get a 0 cost for any method used. The α^{eg} is the same for all the sites, being the SC homogeneous with respect to the delay, hence it is reported as a scalar. It is also possible to see from the first 3 cases of the table (which only differ for the number of the sites in the SC) how the first $N-1$ elements of vector α are like the α of the

$[N, \lambda, (f_1, f_2)]$	J^*	J^{coop}	J^{comp}	J^{eg}	Convergence Coop. [Period]	Convergence Comp. [Period]	P_{comp}
$[2, 3, (.02, .15)]$	9.61	9.61	9.61	11.91	Yes	Yes	[.65]
$[3, 3, (.02, .15)]$	9.92	9.92	9.92	18.34	Yes	Yes	[.03 .65]
$[4, 3, (.02, .15)]$	10.02	[10.02 - 10.3]	10.3	25.14	No [4]	Yes	[.62 .03 .65]
$[4, 4, (.02, .15)]$	10.1	10.1	10.1	26.24	Yes	Yes	[.85 .85 .52]
$[4, 5, (.02, .15)]$	10.17	10.17	10.17	26.04	Yes	Yes	[.54 .54 .48]
$[4, 3, (.02, .4)]$	9.44	[9.44 - 9.71]	9.71	19.74	No [4]	Yes	[.63 1 .87]
$[4, 5, (.02, .4)]$	9.6	9.6	9.6	27.72	Yes	Yes	[.51 .51 .32]

TABLE II

MULTIPLE FREQUENCY CASE: GLOBAL OPTIMUM AND PERFORMANCE OF THE SC WITH COOPERATIVE AND COMPETITIVE NEGOTIATION.

previous case. The same happens for P. This depends on the fact that, in the case of a single perturbation frequency, the selection of each α_i only impacts on the upstream sites (so the situation of the first k sites of a chain with N sites is the same for all $N \geq k$ for all the control methods considered). For this reason in the other three cases (lines 4-6 of the table) we have only reported the $N = 4$ case.

From the table it is evident the relation between the global cost under the different methods established in the paper (Corollary 1) and how under the cooperative negotiation the first two parameters (α_1 and α_2) attain their global optimum, according to what mentioned above.

B. Multiple frequency case

As mentioned above, in the multi frequency case the convergence of the algorithm is not assured (although we observed that the value of the global cost achieved is often near to the global optimum). A modification of the negotiation algorithm could be considered to obtain convergence, where the modification consists in an incremental update of the parameters toward the computed value. This actually may improve the convergence of the algorithm but may reduce its capacity in approaching the global optimum. This has been observed in some simulation runs not reported in this paper.

In Table II we give other examples: when there is no convergence we report the minimum and the maximum of the performance and the period of the limit cycle. As before, P_{comp} denotes the set of P_i selected in the competitive negotiation scheme. We observed how this negotiation paradigm, although very simple, improves the performance of the whole SC toward the global optimum J^*. This fact, which has not been analytically justified, is possibly due to the upstream and downstream propagation of information in the negotiation paradigm for the multiple frequency case, which couples the dynamics of the whole chain.

VII. CONCLUSIONS

We investigated the effect of simple cooperative and competitive negotiation paradigm among the sites of a SC model. Some analytical results in the case of a single frequency perturbation in the nominal demand allowed to establish a relation among the performances achieved by the different versions of the negotiation algorithm, showing that the larger is the cooperation degree in the chain, the better the chain will perform. Under multiple frequency perturbations this has been described through numerical examples. In the simplified setting discussed, cooperation or simply an exchange among sites (like in the competitive case) show improvements of the performance of the whole SC. This may provide useful guidelines in the definition of control strategies for more complex and realistic problems.

REFERENCES

[1] M. Boccadoro, F. Martinelli and P. Valigi, "H-infinity control of a Supply Chain model", IEEE Conf. on Decision and Control (CDC) 2006

[2] M. Boccadoro, F. Martinelli and P. Valigi, "A multi-agent control scheme for a supply chain model", Asian Journal of Control, Vol. 10, n. 2, 2008

[3] F. Chen, "Echelon Reorder Points, Installation Reorder Points, and the Value of Centralized Demand Information," *Management Science*, vol. 44, (12, part 2), S221-S234, 1998.

[4] F. Chen and Y.-S. Zheng, "Near-Optimal Echelon-Stock (R,nQ) Policies in Multi-Stage Serial Systems," *Operations Research* , vol. 46, (4), 592-602, 1998.

[5] J. Dejonckheere, S. Disney, M. Lambrecht, D. Towill, "Measuring and avoiding the bullwhip effect: A control theoretic approach," *European Journal of Operational Research*, pp. 567–590, 2003.

[6] G. Gaalman, "Bullwhip reduction for ARMA demand: The proportional order-up-to policy versus the full-state-feedback policy," *Automatica*, 42, 1283–1290, 2006.

[7] D. Helbing, "Modeling and optimization of production processes: Lessons from traffic dynamics," in: G. Radons and R. Neugebauer (eds.)*Nonlinear Dynamics of Production Systems*, Pages 85-105 (Wiley-VCH, Weinheim), 2004.

[8] T. Hosoda, S. M. Disney, "The governing dynamics of supply chains: The impact of altruistic behavior," *Automatica*, vol. 42, pp. 1301–1309, 2006.

[9] S.O. Kimbrough, D.J. Wu, F. Zhong, "Computers play the beer game: can artificial agents manage supply chains?", Decision Support Systems, 33, 323–333, 2002

[10] P.-H. Lin, D.S.-H. Wong, S.-S. Jang, S.-S. Shieh and J.-Z. Chu, "Controller design and reduction of bullwhip for a model supply chain system using z-transform analysis", Journal of Process Control, vol. 14, pp. 487–499, 2004

[11] Li, Jingquan; Sikora, Riyaz; Shaw, Michael J.; Woo Tan, Gek "A strategic analysis of inter organizational information sharing", Decision Support Systems, vol. 42, pp. 251–266, 2006

[12] Mullen, T.; Shuang Sun; Avasarala, V.; Bagby, J.; Yen, J.; Levi, M.; "Implications of agent-based supply chain games", 2004 IEEE Conf. on Systems, Man and Cybernetics

[13] T. Nagatani and D. Helbing, "Stability analysis and stabilization strategies for linear supply chains," *Physica A*, vol. 335, pp. 644–660, 2004.

[14] Y. Ouyang and C. F. Daganzo, "Some properties of decentralized supply chains," Institute of Transportation Studies Research Report, UCB-ITS-RR-2005-3, 2005.

[15] Jiang Tian, Tianfield, H.; "Literature Review Upon Multi-Agent Supply Chain Management", 2006 International Conference on Machine Learning and Cybernetics, Dalian, August 13-16, 2006

[16] D.J. Wu, Yanjun Sun, "Cooperation in multi-agent bidding", Decision Support Systems, vol. 33, pp. 335–347, 2002