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Packet-loss Dependent Controller Design for Networked Control Systems via Switched System Approach

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Abstract— **The stabilization problem of networked control systems with bounded packet loss is addressed. We model such networked control systems as a class of switched systems, and present sufficient conditions for the stabilization by using a packet-loss dependent Lyapunov function. Moreover, different from existing results, we propose the design for packet-loss dependent stabilizing controllers for two types of packet-loss processes: one is a arbitrary packet-loss process, and the other is a Markovian packet-loss process. Several numerical examples and simulations are worked out to demonstrate the effectiveness of the proposed design technique.**

I. INTRODUCTION

Networked control systems(NCSs) are a class of feedback control systems with network channels, and have many industrial applications, such as large-scale distributed industrial processes, fieldbus systems and intelligent traffic systems, etc. Compared with the traditional point-to-point wiring, the use of the communication channels provides a control system with many advantages such as lower costs of cables and power, simpler installation and maintenance of the whole system, and higher reliability. NCSs have received increasing attentions in recent years [1]-[9] and the references therein. Among these results, [1] introduced the tryonce-discard protocol for multiple-input-multiple-output NCSs, and provided firstly an analytic proof of global exponential stability for both the new protocol and the more commonly used access methods; Both state and output feedback were considered in [2] where knowledge of the plant dynamics was used to reduce the usage of the network, and necessary and sufficient conditions for stability were derived. Based on the theories for the discrete-time switched systems, an H_∞ control problem for a remotely controlled system over a shared network was discussed in [3]. [4] studied the optimal LQG control problem of a class of NCSs with packet dropout, and designed optimal linear quadratic Gaussian controllers by using a separation principle. [7] discussed the stabilization problem of NCSs with network-induced delay, and sufficient conditions expressed in the form of linear matrix inequalities(LMIs) have been derived by viewing the time-varying delay as time-varying parameter uncertainties; In order to reduce the amount of the data sent to the network, [8] presented a framework for stabilization of NCSs with limited data rates by only sending a nonsingular transform of the state vector at each time instants; In [9], switching impulse was considered in order to reduce the error between theory and application for an NCS, and a sufficient condition for exponential stabilization of networked control systems under a given switching rule was derived by multiple Lyapunov-like functions.

However, the use of communication network in the feedback control loop causes that many ideal assumptions made in the traditional control theory can not be applied to NCSs directly, for example, the standard assumption on the state measurement that all information can be obtained by the controller. Therefore, the analysis and design for an NCS is more complex due to the use of communication network than that for a traditional control system. In an NCS, communication capacity depends not only on the protocol, but also on the topology of the network. One of the major issues raised in NCSs is the packet loss which is a potential source of instability and poor performance of NCSs, therefore, it is very essential to construct a feedback controller using the most fresh information to stabilize an NCS with packet dropout. To our best knowledge, two effective approaches have been used in existing results in existing results: one is delayed system approach [7][10] where NCSs were modelled as delayed systems, and the other is switched system approach [11][13] where NCSs were modelled as a class of switched systems with several subsystems. In addition, in these two approaches, two types of packet loss processes have been considered: one regards the data packet dropout as a arbitrary process [11]-[13], and the other is a Markovian process [14] [15].

The advantage of the switched system approach is that the controllers can make full use of the previous information to stabilize NCSs when the current state measurements are not available from the network. Based on the bounded packet loss, NCSs were modelled as a class of switched systems in [11], and many existing switched system theories [16]-[20] can be used. [11] presented the design for output feedback controllers by using the feasible solutions of some LMIs. [12] presented an extension of the results in [11] into a nonlinear NCS. Recently, [13] considered the same stabilization problem as [11], and designed state feedback

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controllers by introducing the packet-loss dependent Lyapunov function. In the field of control, if a single controller fails to solve a control problem, a multiple of controllers might be used in the hope that the problem may be solved by switching among these controllers. However, it is noticed that most of the existing results concerned the design for a single controller, even when NCSs are modelled as switched systems. By the delayed system approach, delay-dependent controllers were designed in [15] where the packet dropout in the backward channel was regarded as a bounded delay. However, as for switched system approach, few results have concerned such a multiple of controllers. In this paper, we discuss the stabilization problem by using the switched system approach. Similar approach has been used in [11][13], but different from them, for the NCSs with the bounded packet loss, we present a new control design method, that is, we propose the design for packet-loss dependent state feedback controllers, and such design can provide the NCSs under study with better performances. Moreover specifically, two types of packet-loss processes are considered: one is the arbitrary packet-loss process, and the other is the Markovian packet-loss process. For both cases, sufficient conditions in the form of LMIs for stabilization are derived by using a packet-loss dependent Lyapunov function, and packet-loss dependent state feedback controllers are designed by solving some LMIs.

The paper is organized as follows: Section II introduces the mathematical model of NCSs under study, and some definitions and lemmas are also presented in this section. Section III deals with the stabilization problem for NCSs with the arbitrary packet-loss process, and stabilizing state feedback controllers are constructed by using the feasible solutions of some LMIs. Section IV discusses the stabilization for NCSs with the Markovian packet-loss process, and stabilizing state feedback controllers are derived by solving some LMIs. Some numerical examples demonstrating the effectiveness of the proposed design technique are given in Section V. The conclusion is provided in Section VI.

Notations. Throughout this paper, the following notations are used. $\|\cdot\|$ refers to the Euclidean norm for vectors and induced 2-norm for matrix; For any two positive integers i and j satisfying $j \geq i$, $[i,j] = \{i, i+1, \cdots, j\}$.

II. Problem Formulation

Consider the NCS with the bounded packet loss in both the channel between the sensor and the controller(the backward channel) and the channel between the actuator and the controller(the forward channel) illustrated in Fig. 1, where the sensor is clock-driven and the actuator is event-driven. We first consider the NCS setup with a clock-driven sensor, and both the controller and the actuator are combined into one event-driven node, that is, network communication only occurs form the sensor to the controller through a communication channel with finite bandwidth. The NCS with a time-varying controller is described as

$$\begin{array}{l} x(t+1) = Ax(t) + Bu(t), \\ u(t) = F(t)\bar{x}(t), \end{array} \quad (1)$$

where $t \in \mathbf{N}$, $x(t) \in \mathbf{R}^n$ is the plant state vector, $u(t) \in \mathbf{R}^m$ is the plant input vector. A, B are known real constant matrices with proper dimensions. $F(t) \in \mathbf{R}^{m\times n}$ is the state feedback gain matrix to be designed. $\bar{x}(t) \in \mathbf{R}^n$ is the state measurement that is successfully transmitted over the network. We suppose that a sensor data containing the

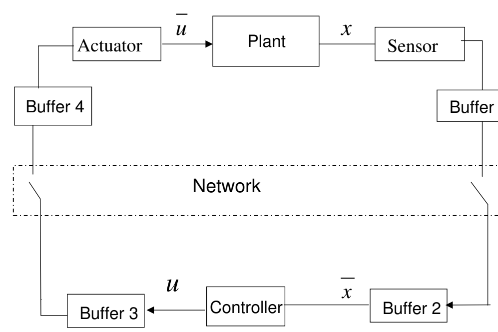

Fig. 1. Structure of a networked control system with packet losses.

state information will substitute the old data when it is successfully sent to the controller through the communication channel, and the updated data is denoted by $\bar{x}(t)$. The controller reads out the content of $\bar{x}(t)$ and utilizes it to compute the new control input, which will be applied to the controller. Further, we suppose that the update instants of $\bar{x}(t)$ is numerable and the set of successive update instants $\{t_0 = 0, t_1, \cdots, t_k, \cdots\}$ is a subset of $\mathbf{N}$, and the update instants of $\bar{x}(t)$ is described as

$$\bar{x}(t) = \begin{cases} x(t), & \text{if the packet containing } x(t) \\ & \text{ is transmitted successfully;} \\ \bar{x}(t-1), & \text{otherwise.} \end{cases}$$

Here, we consider NCS (1) by using the switched system approach [11] and discuss the stabilization of (1) by modelling the NCSs as a class of switched systems which are different from the switched systems in [11]. In what follows we describe our model using the switched system approach.

Without loss of generality, we assume that the packet containing $x(0)$ is transmitted to the controller successfully, that is $\bar{x}(0) = x(0)$, then $x(1) = (A + BF(0))x(0)$. In the next time instant, if the data packet containing $x(1)$ is transmitted to the controller successfully, then

$$x(2) = (A + BF(1))x(1),$$

otherwise,

$$\begin{aligned} x(2) &= Ax(1) + BF(1)x(0) \\ &= (A(A + BF(0)) + BF(1))x(0). \end{aligned}$$

We refer to the time interval between t_k and t_{k+1} as one transmission interval. In this pattern of transmission, the states of NCS (1) at the successive update instants can be described as follows:

$$x(t_{k+1}) = (A^{t_{k+1}-t_k} + \sum_{l=0}^{t_{k+1}-t_k-1} A^l B \times F(t_{k+1}-l-1))x(t_k),\ k \in \mathbf{N}.$$

Define

$$z(0) = x(0),\ z(1) = x(t_1), \cdots,\ z(k) = x(t_k), \cdots,$$

and

$$A(k) = A^{t_{k+1}-t_k} + \sum_{l=0}^{t_{k+1}-t_k-1} A^l BF(t_{k+1}-l-1),$$

we can obtain

$$z(k) = A(k)z(k-1). \tag{2}$$

We assume that the maximum transmission interval is d, therefore the upper bound of the dropped data packets is $d-1$. With a set of candidate gains $\{F_1, F_2, \cdots, F_d\}$ to be designed, we propose the following schedule algorithm to stabilize NCS (1):

Schedule Algorithm 1. For any t_k, assuming that there is a counter which records the length of the last transmission interval $[t_{k-1}, t_k)$, we take the packet-loss dependent feedback gain as $F_{t_k - t_{k-1}}$ in the following transmission interval, i.e.,

$$u(t) = F_{t_k - t_{k-1}} \bar{x}(t), t \in [t_k, t_{k+1}).$$

Without loss of generality, for any t_k, let $t_{k+1} - t_k = i, t_k - t_{k-1} = j$, and apply Schedule Algorithm 1 to NCS (1), we get

$$A(k) = \bar{A}_{ij} = A^i + \sum_{l=0}^{i-1} A^l BF_j. \tag{3}$$

Then the state evolution of NCS (1) at the transmission instants can be described as the following switched system

$$z(k+1) = \bar{A}_{\eta(k)} z(k),\ k \in \mathbf{N}, \tag{4}$$

where

$$\bar{A}_{\eta(k)} = A^{r(k)} + \sum_{l=0}^{r(k)-1} A^l BF_{r(k-1)} \in \bar{\Omega} = \{\bar{A}_{11}, \bar{A}_{12}, \cdots, \bar{A}_{1d}, \cdots, \bar{A}_{d1}, \bar{A}_{d2}, \cdots, \bar{A}_{dd}\},$$

and $r(k) = t_{k+1} - t_k$, $\eta(k) = (r(k), r(k-1)) \in [1,d] \times [1,d]$ with $\eta(1) = (r(1), 1)$, which means that $x(0)$ is transmitted to the controller successfully.

Remark 1: For the NCS with packet dropout in both the forward and the backward channels, we suppose that the set of successive update instants of plant input $\bar{u}(t)$ is $\{t_0 = 0, t_1, \cdots, t_k, \cdots\}$, which is a subset of $\mathbf{N}$. Similarly, by using Schedule Algorithm 1, we can obtain the same model as those above. Thus, the model is suitable for the NCS with packet dropout in both the forward and the backward channels.

Remark 2: Let $F_1 = F_2 = \cdots = F_d$. Then our problem under study reduce to stabilization problem via a single state feedback control law discussed in [11] [13]. In fact, suppose that $F_1 = F_2 = \cdots = F_d = F$, then (3) becomes $\bar{A}_i = A^i + \sum_{l=0}^{i-1} A^l BF$. It follows that (4) reduces to (4) in [13]. Thus, the stabilization problem discussed in [13] is a special case of ours.

Now, we present the following definitions and technical lemmas for later use.

Definition 1: [13] A packet-loss process $\{r(k) \in \mathbf{N} : r(k) = t_{k+1} - t_k\}$ is said to be arbitrary if it takes values in the interval $[1,d]$ arbitrarily.

Definition 2: A packet-loss process $\{r(k) \in \mathbf{N} : r(k) = t_{k+1} - t_k\}$ is said to be Markovian if it is a Markov chain whose probability matrix is $P = [p_{ij}] \in \mathbf{R}^{d \times d}$, with $p_{ij} = \Pr(r(t_{k+1}) = i | r(t_k) = j) \geq 0$ for any $(i,j) \in [1,d] \times [1,d]$, and $\sum_{i=1}^{d} p_{ij} = 1$.

Lemma 1: [21] Given the symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$, where S_{11} is $r \times r$, then the following three statements are equivalent:

a) $S < 0$,

b) $S_{11} < 0,\ S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$,

c) $S_{22} < 0,\ S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

III. STABILIZATION OF NCSS WITH ARBITRARY PACKET-LOSS PROCESS

In this section, we suppose that the packet loss of NCS (1) is arbitrary, and study the stabilization problem of the NCS via state feedback. Sufficient conditions in the form of LMIs for the stabilization are derived by using a packet-loss dependent Lyapunov function, and stabilizing state feedback controller are designed by solving some LMIs.

Definition 3: A function $\phi : \mathbf{R}^n \to \mathbf{R}_+$ is of class K if it is continuous, strictly increasing, and $\phi(0) = 0$.

Without loss of generality, we assume that 0 is an unique equilibrium of NCS (1), and the state response starts at $t_0 = 0$ with an initial condition $x(0)$. The following result ensures the uniformly asymptotic stability of NCS (1). It is a generalization of Lemma 1 in [11].

Lemma 2: If there exist a function $V : \mathbf{R}^n \to \mathbf{R}_+$ taking its value in a continuous function set $\Omega = \{V_1, V_2, \cdots, V_q\}$ with $V_l(0) = 0$ for all $l \in [1,q]$, and functions α, β, γ of class K such that for all $x \in B_r = \{x : \|x\| \leq r\}$,

$$\alpha(\|x\|) \leq V_l(x) \leq \beta(\|x\|),\ \forall l \in [1,q], \tag{5}$$

and

$$\begin{aligned} \Delta V(x(t_k)) &= V(x(t_{k+1})) - V(x(t_k)) \\ &\leq -\gamma(\|x(t_k)\|),\ \forall k \in \mathbf{N}, \end{aligned} \tag{6}$$

then NCS (1) is uniformly asymptotically stable.

Here, we omit the proof due to the limit of the length.

Theorem 1: If there exist symmetric positive definite matrices $X_1, X_2, \cdots, X_d$ and matrices $Y_1, Y_2, \cdots, Y_d$ such that

$$\begin{bmatrix} X_j & (A^i X_j + \sum_{l=0}^{i-1} A^l B Y_j)^T \\ A^i X_j + \sum_{l=0}^{i-1} A^l B Y_j & X_i \end{bmatrix} > 0, \quad \forall (i,j) \in [1,d] \times [1,d], \tag{7}$$

then NCS (1) is stabilizable via the state feedback control law

$$u(t) = Y_{r(k-1)} X_{r(k-1)}^{-1} \bar{x}(t),\ t \in [t_k, t_{k+1}),\ k \in \mathbf{N}.$$

We omit the proof due to the limit of the length.

IV. STABILIZATION OF NCSS WITH MARKOVIAN PACKET-LOSS PROCESS

In this section, we suppose that the packet loss of NCS (1) is a Markovian process defined in Definition 2, and discuss the mean square stabilization problem of (1). We derive sufficient conditions for the mean square stabilization via state feedback, and propose the design for packet-loss dependent stabilizing state feedback controllers by solving some LMIs.

Here, we still use Schedule Algorithm 1 to stabilize NCS (1). Based on the analysis in Section II, we can obtain the states of NCS (1) at the update instants

$$z(k+1) = \bar{A}_{\eta(k)} z(k), \tag{8}$$

where $\bar{A}_{\eta(k)} \in \bar{\Omega}$, and for all $k > 1$, $\eta(k) = (r(k), r(k-1)) \in [1,d] \times [1,d]$, $\eta(1) = (r(1), 1)$, and $r(k)$ is a Markovian chain as defined in Definition 2.

Definition 4: NCS (1) with the Markovian packet-loss process defined in Definition 2 is said to be mean square stable(MS) if $\lim_{t\to\infty} E[\|x(t)\|^2] = 0$ for any initial state x_0.

The following result is a simple generalization of Theorem 9 in [13]. Here, we omit its proof.

Lemma 3: NCS (1) with the Markovian packet-loss process defined in Definition 2 is MS if there exist positive definite matrices P_i with $i \in [1,d]$, such that

$$\sum_{i=1}^{s} [p_{ij}(A^i + \sum_{l=0}^{i-1} A^l B F_j) P_i (A^i + \sum_{l=0}^{i-1} A^l B F_j)] - P_j < 0. \tag{9}$$

Remark 3: From (9), we get that the arbitrary packet-loss stability implies the Markovian packet-loss stability for NCS (1) since $\sum_{i=1}^{d} p_{ij} = 1$.

Based on Lemma 3, we can get that the following result which proposes a sufficient condition for the MS of NCS (1) with the Markovian packet-loss process defined in Definition 2.

Theorem 2: NCS (1) with the Markovian packet-loss process defined in Definition 2 is MS if there exist symmetric positive definite matrices $X_1, X_2, \cdots, X_d$, and matrices $G_1, G_2, \cdots, G_d$,$Y_1, Y_2, \cdots, Y_d$ satisfying

$$\begin{bmatrix} \Lambda & Q_i^T \\ Q_i & X_i \end{bmatrix} > 0,\ \forall i \in [1,d], \tag{10}$$

where

$$\Lambda = diag(G_1 + G_1^T - X_1, \cdots, G_d + G_d^T - X_d),$$
$$Q_i = [\sqrt{p_{1i}}(AG_i + B_1 Y_i)^T \ \cdots \ \sqrt{p_{di}}(A^d G_i + B_d Y_i)^T],$$

with $B_j = \sum_{l=0}^{j-1} A^l B$.

Moreover,

$$u(t) = F_{r(k-1)} \bar{x}(t) = Y_{r(k-1)} G_{r(k-1)}^{-1} \bar{x}(t),\ t \in [t_k, t_{k+1}),$$

is a stabilizing control law.

V. NUMERICAL EXAMPLES

In this section, some numerical examples and simulations are given to demonstrate the effectiveness of the proposed design technique.

Example 1: Consider a third-order NCS presented in [13]

$$\begin{aligned} x(t+1) &= \begin{bmatrix} 0.6065 & 0 & -0.2258 \\ 0.3445 & 0.7788 & -0.0536 \\ 0 & 0 & 1.2840 \end{bmatrix} x(t) \\ &+ \begin{bmatrix} -0.0582 \\ -0.0093 \\ 0.5681 \end{bmatrix} u(t), \\ u(t) &= F_i \bar{x}(t), \forall i \in [1,d], \end{aligned} \tag{11}$$

where the state feedback gains $F_i, i \in [1,d]$ are to be designed. Here, we suppose that the maximum transmission interval $d = 3$. When the packet loss of NCS (11) is arbitrary,

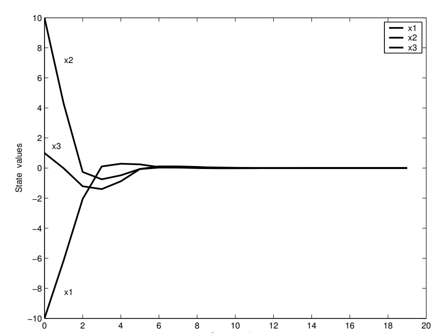

Fig. 2. State response (arbitrary packet loss).

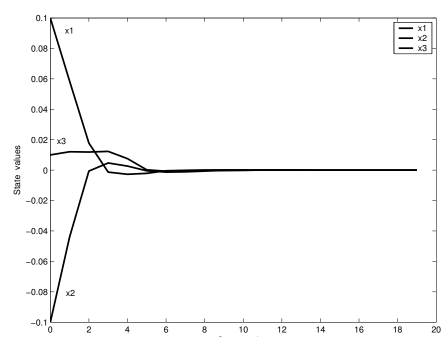

Fig. 3. State response (arbitrary packet loss).

we solve the LMIs in Theorem 1 by using the Matlab LMI Toolbox, and obtain the feedback gains:

$$F_1 = \left[\begin{array}{ccc} 0.2554 & 0.1473 & -1.2090 \end{array}\right],$$

$$F_2 = \left[\begin{array}{ccc} 0.2576 & 0.1514 & -1.2092 \end{array}\right],$$

$$F_3 = \left[\begin{array}{ccc} 0.2704 & 0.1589 & -1.2165 \end{array}\right].$$

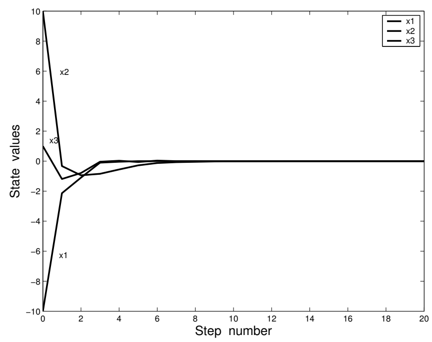

Fig. 4. State response (Markovian packet losses).

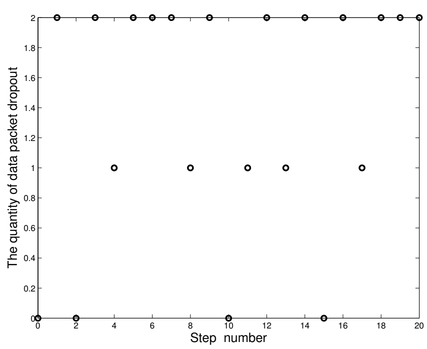

Fig. 5. Frequency of packet losses (Markovian packet losses).

Next, suppose that the distribution of transmission interval is $1, 2, 3, 1, 2, 3, \cdots$, and we give the simulations of the the state response under the state feedback above. When the initial state is $x_0 = [-10\ 10\ 1]^T$, the system state response is shown in Fig. 1. When the initial state is $x_0 = [0.1\ -0.1\ 0.01]^T$, the system state response is simulated in Fig. 3.

In addition, when the packet loss of NCS (11) is a Markovian process whose transition probability matrix is

$$P = \left[\begin{array}{ccc} 0.1 & 0.2 & 0.2 \\ 0.3 & 0.1 & 0.3 \\ 0.6 & 0.7 & 0.5 \end{array}\right]. \tag{12}$$

By solving the LMI in Theorem 2, the use of the Matlab LMI Toolbox yields the following feedback gains:

$$F_1 = \left[\begin{array}{ccc} 0.0773 & 0.0373 & -1.0776 \end{array}\right],$$

$$F_2 = \left[\begin{array}{ccc} 0.0762 & 0.0365 & -1.0635 \end{array}\right],$$

$$F_3 = \left[\begin{array}{ccc} -0.0045 & -0.0327 & -1.0704 \end{array}\right].$$

When the initial state is $x_0 = [-10\ 10\ 1]^T$, the system state response under the state feedback above and the time instants when $\bar{x}(t)$ updating its state are simulated in Fig. 1 and Fig. 5 separately. From all the analysis and simulations, we see that NCS (11) presented in [13] can be stabilized effectively by the designed packet-loss dependent controller.

Example 2: Consider a second-order NCS

$$\begin{aligned} x(t+1) &= \left[\begin{array}{cc} 1.166 & 0.209 \\ -0.123 & -1 \end{array}\right] x(t) + \left[\begin{array}{c} -1 \\ 1 \end{array}\right] u(t), \\ & u(t) = F_i \bar{x}(t), \forall i \in [1, d], \end{aligned} \tag{13}$$

where the state feedback gains F_i with $i \in [1, d]$ are to be designed. Notice that the open-loop system is unstable since since one of its poles 1.1541 is outside the unit disk. Next, we design packet-loss dependent controllers to stabilize the unstable system.

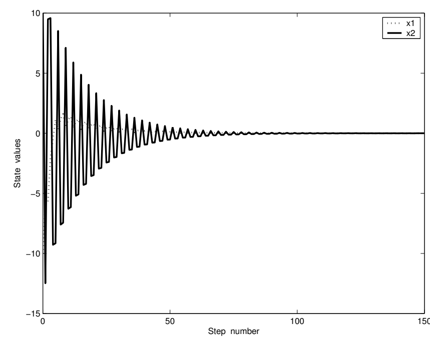

Fig. 6. State response (arbitrary packet loss).

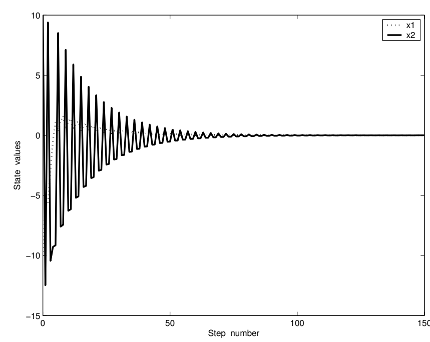

Fig. 7. State response (no packet loss).

We suppose that the maximum transmission interval $d = 3$ which means that 66% of the packets may be lost, and the initial state is $x_0 = [-10\ 10]^T$. When the packet loss of NCS (13) is arbitrary, we solve the LMI in Theorem 1 and the use of the Matlab LMI Toolbox yields the following feedback gains:

$$F_1 = \left[\begin{array}{cc} 0.4610 & 0.0894 \end{array}\right],$$

$$F_2 = \left[\begin{array}{cc} 0.3732 & 0.0732 \end{array}\right],$$

$$F_3 = \left[\begin{array}{cc} 0.4226 & 0.0960 \end{array}\right].$$

When the distribution of transmission interval is $1, 2, 3, 1, 2, 3, \cdots$, the state response of NCS (13) is shown in Fig. 6. It can be seen clearly from the figure that even in such a case that the controller can only obtain 33% of

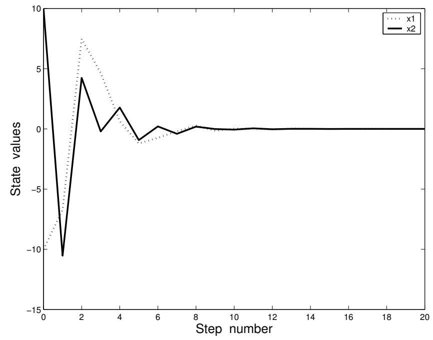

Fig. 8. State response (Markovian packet losses).

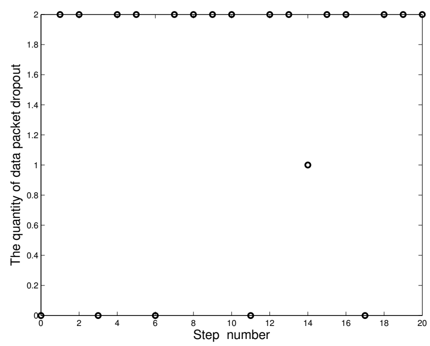

Fig. 9. Frequency of packet losses(Markovian packet losses).

the packets, the system can still be effectively stabilized via the packet-loss dependent state feedback controller. Fig. 7 depicts the trajectory of the system when no packet loss occurs. Compare Fig. 6 with Fig. 7, we can see that the state response under the packet-loss controller is very similar with that when there are no packet loss, which means that the effect of the packet loss to the stabilization is small when we use the packet-loss controller to stabilize the NCS.

Suppose that the packet loss of NCS (13) is a Markovian process with the transition probability matrix given by (12). We solve the LMI in Theorem 2 and the use of the Matlab LMI Toolbox yields the following feedback gains:

$$F_1 = \begin{bmatrix} 0.4422 & 0.2473 \end{bmatrix},$$
$$F_2 = \begin{bmatrix} 0.4274 & 0.2838 \end{bmatrix},$$
$$F_3 = \begin{bmatrix} 0.5072 & 0.2733 \end{bmatrix}.$$

When the initial state is $x_0 = [-10 \ 10]^T$, the state response is shown in Fig. 8. The small circles in Fig. 9 simulate the time instants when $\bar{x}(t)$ updating its states.

VI. Conclusion

This paper has discussed the stabilization problem of the NCSs with the bounded packet loss by modelled such NCSs as a class of switched systems. Two types of the packet-loss processes have been considered: one is the arbitrary packet-loss process, and the other is the Markovian packet-loss process. For both cases, we have derived the sufficient conditions in the form of LMIs for the state feedback stabilization by using packet-loss dependent Lyapunov functions, and presented a new control design method. More specially, based on the theories for the discrete-time switched systems, we have proposed the design for the packet-loss dependent stabilizing controllers which are easy to obtain by solving some LMIs via using the Matlab LMI Toolbox. Several examples and simulations have been worked out to demonstrate the effectiveness of the proposed design technique.

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