Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

Adaptive Observer Design for the Bottomhole Pressure of a Managed Pressure Drilling System

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Abstract— **In this paper a reduced order observer that adapts to unknown friction and density, and estimates the bottomhole pressure in a well during drilling, is presented. The design is based on a newly developed third order nonlinear model with a nonlinear output equation containing a product between an unknown parameter and unmeasured state. Based on a Lyapunov approach the pressure estimate is shown to converge to the true pressure under reasonable conditions. Application of the observer to real data from a North Sea oil well demonstrates promising behaviour.**

Index Terms— **Drilling, nonlinear observer, adaptive observer, pressure estimation.**

I. INTRODUCTION

As an introduction to drilling consider the drill rig set-up illustrated in Fig. 1. The figure illustrates a jacket platform performing offshore drilling. At the top of the derrick the drill string is attached to the topdrive which is a motor that turns the drill string. The drill string can move up and down inside the derrick as the topdrive is attached to a hook that can be lowered or raised. As the drilling progresses the top of the drill string sinks towards the drill floor. After approximately $27m$ a new stand of drill pipe is connected to the top and drilling resumes. This procedure is referred to as a pipe connection. For a typical rate of penetration of $15\frac{m}{hr}$ a pipe connection is performed roughly every two hours.

During drilling, down hole cuttings need to be transported out of the bore hole. This is done by using a mud circulation system. On board the rig, tanks filled with drilling mud feed the main mud pump which pumps the drilling fluid through the topdrive and into the drill string. The mud then flows down through the bit and up through the annulus carrying the cuttings along before the flow exits through a choke. After exiting, the fluid is recycled and returned to the mud tanks.

The example illustrated in Fig. 1 has a rotating control device which seals off the annulus from the outside while a choke controls the flow of mud out from the annulus.

The main reason for pressure control is to maintain the annulus pressure profile within its margins, i.e., above the pore pressure of the reservoir or the collapse pressure of the bore hole, and below the fracturing pressure of the bore hole. Another important reason for pressure control is to prevent uncontrolled reservoir influx which in the worst case scenario can lead to a surface blowout with large financial losses, environmental damage and possible loss of lives.

The pressure in the annulus is mainly affected by the hydrostatic weight and the pressure due to friction losses [1]. In addition, if the annulus is closed off, the pressure at the top of the annulus induced by choking will significantly affect the pressure in the well.

There are several operational procedures that affect the pressure in the annulus. Pipe connection affects the pressure as the main pump must be disconnected to attach a new section of drill pipe, this leads to zero flow and loss of pressure due to friction. Moving the drill string all the way in/out of the well (tripping) changes the volume in the annulus. Tripping out pipe causes reduced pressure in the annulus, and tripping in pipe creates a surge in the pressure. Similar effects can be experienced due to wave-induced motion (heave) when drilling from a floater.

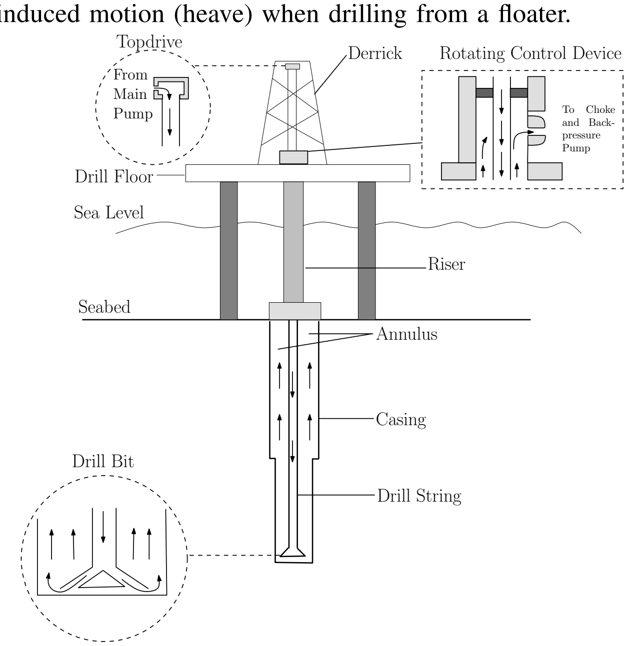

Fig. 1. Offshore drilling from a jacket platform. Drill mud flows from the main pump through the drill string, drill bit and out through the choke. The mud transports cuttings out of the wellbore and helps to maintain the desired pressure in the borehole.

A. Pressure Control

As described in the previous section there is a demand for accurate control of the annulus pressure. As a response to these demands a fairly new (for offshore drilling) technology for pressure control has emerged [2]. It is named Managed Pressure Drilling (MPD) and is defined by the IADC Underbalanced Operations Committee as: *"Managed*

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978-1-4244-3124-3/08/$25.00 ©2008 IEEE

Pressure Drilling is an adaptive drilling process used to precisely control the annular pressure profile throughout the well bore. The objectives are to ascertain the down hole pressure environment limits and to manage the annular hydraulic pressure profile accordingly" [3].

In many cases the bottomhole pressure (the pressure at the bit) is used as the variable to control [4], [5], [6], [7]. The bottomhole pressure is measured, but the signal is usually transmitted by mud-pulse telemetry which is powered by a mud flow turbine. It is therefore hampered with slow sampling and no signal when the circulation is low, e.g., during pipe connection procedures. Since the measurement is unreliable the pressure needs to be estimated, which is non-trivial due to uncertainties in friction and density.

B. Pressure Estimation

Some existing pressure estimation schemes are found in the literature. The multiphase flow dynamics of a well can be described fairly accurately by a set of partial differential equations derived from mass balance equations and a simplified momentum balance known as the drift-flux formulation [9], [10]. The PDEs can be discretized and implemented for simulation as as a large set of ordinary differential equations that can be used to predict the pressure in the well provided all parameters are known and inputs (such as pump flows and choke flows) are measured and fed into the simulator. Existing schemes like these do not use the estimation error to adjust the future estimate and hence they are non-robust to modeling errors. In [11] a new MPD concept which uses a modified version of OLGA 2000 to provide an estimate of the pressure profile in the annulus is presented. OLGA 2000 is a powerful multiphase flow simulator developed for the petroleum industry [12]. The robustness of complex estimation schemes like these in conjunction with a control system is hard to analyze in a rigid manner. For these simulators, verification by extensive Monte Carlo simulations or trials is the only method to guarantee proper functionality.

The complexity of such schemes is increased by the fact that many of the parameters in such models are uncertain/unknown and possibly slowly changing, which implies that they would need to be tuned as operating conditions change. This tuning can be done by an experienced operator or by using automatic tuning methods such as parameter estimation algorithms. In [13] an unscented Kalman filter is used to update the friction estimate in both the drill string and the annulus. The scheme uses a measurement of the bottomhole pressure to update the parameters every 30 seconds. Although no formal proofs are shown the estimation scheme shows promising behavior with better estimates of the bottomhole pressure than without the unscented Kalman filter, and fairly accurate estimation of the friction factors.

Previous attempts at using low order models for control and estimation of the bottomhole pressure can be found in [14] and [5]. In [14] nonlinear model predictive control (NMPC) was used together with an unscented Kalman filter to control the bottomhole pressure. A third order nonlinear model was used as the basis for the control and estimation design. The Kalman filter was used to estimate the states, and the friction and choke coefficients. The estimated parameters showed unwanted and unexplained spikes and oscillations during and after a pipe connection procedure. The bottomhole pressure was kept fairly stable. In [5] it is shown that pressure variations in the bottomhole pressure during surge and swab can be suppressed by controlling the choke and main pump. The control is based on a fourth order model and assumes that all parameters and the bottomhole pressure is known, hence there is no estimation scheme involved. While the present paper focuses on nonlinear adaptive observer design and testing on real data, the related paper [7] uses a similar observer in conjuction with a nonlinear controller to stabilize the bottomhole pressure. Simulation results in [7] show that this observer-controller combination applied to a state-of-the-art drilling simulator successfully stabilizes the bottomhole pressure in the presence of common disturbances such as drill string movement and main pump flow variations.

II. MODEL

To facilitate the design of an observer a dynamic model for the circulation system shown in Fig. 1 has been developed [8]. The model only considers fluid phase flow. For modeling purposes the well is divided into two separate compartments. Fig. 2 shows the two control volumes considered, one control volume for the drill string and one for the annulus. The volumes are connected through the drill bit.

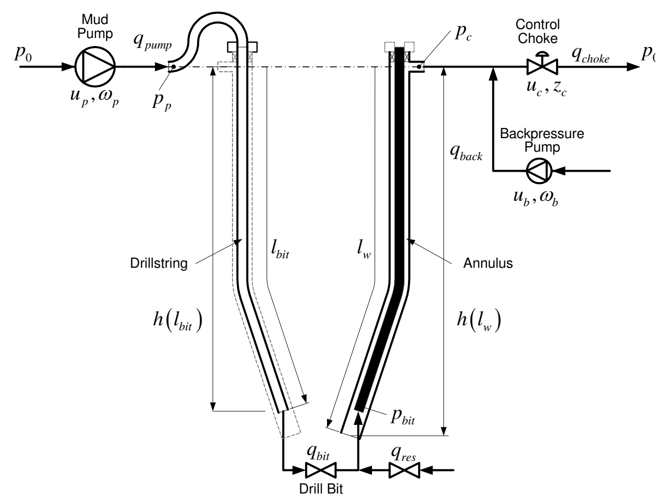

Fig. 2. The drilling system can be viewed as two control volumes connected through the drill bit.

The pressure dynamics are derived based on mass balance and are

$$\frac{V_d}{\beta_d}\dot{p}_p = q_{pump} - q_{bit} \tag{1}$$

$$\frac{V_a}{\beta_a}\dot{p}_c = q_{bit} + q_{back} - q_{choke} + q_{res} - \dot{V}_a, \tag{2}$$

where p_p is the pump pressure and p_c is the choke pressure. V_d is the volume in the drill string, V_a is the volume in the annulus. β_d and β_a are the bulk moduli of the fluid in the drill string and the annulus respectively. q_{pump} is the volume flow through the mud pump. q_{bit} is the volume flow through the bit. q_{back} is the flow through the back pressure pump, q_{choke} is the flow through the choke and q_{res} is the influx from the reservoir.

The volume flow dynamics is derived from a momentum balance and is governed by

$$\begin{aligned} M\dot{q}_{bit} &= p_p - p_c - F_d|q_{bit}|q_{bit} \\ &\quad - F_a|q_{bit} + q_{res}|(q_{bit} + q_{res}) + (\rho_d - \rho_a)gh_{bit}. \end{aligned} \quad (3)$$

Here F_d and F_a are the friction factors in the drill string and the annulus, respectively, ρ_d and ρ_a are the average densities in the drill string and the annulus, $g = 9.81\frac{m}{s^2}$ and h_{bit} is the vertical depth of the bit, see Fig 2. Furthermore $M = M_a + M_d$ with

$$M_a = \rho_a \int_0^{l_w} \frac{1}{A_a(x)} \mathrm{d}x, \;\; M_d = \rho_d \int_0^{L_{dN}} \frac{1}{A_d(x)} \mathrm{d}x, \quad (4)$$

where l_w is the length of the annulus, L_{dN} is the total length of the drill string, and $A_a(x)$ and $A_d(x)$ are the cross sectional areas of the annulus and the drill string, respectively.

The pressure at the bit depends on the choke pressure, pressure due to rate of change in q_{bit}, friction pressure and hydrostatic pressure, and is given as

$$p_{bit} = p_c + M_a\dot{q}_{bit} + F_a|q_{bit} + q_{res}|(q_{bit} + q_{res}) + \rho_a g h_{bit}. \quad (5)$$

Substituting (3) into (5) gives

$$\begin{aligned} p_{bit} &= \frac{M_a}{M}p_p + \frac{M_d}{M}p_c + (\frac{M_d}{M}\rho_a + \frac{M_a}{M}\rho_d)gh_{bit} \\ &\quad + (\frac{M_d}{M}F_a - \frac{M_a}{M}F_d)|q_{bit} + q_{res}|(q_{bit} + q_{res}). \end{aligned} \quad (6)$$

Using the notation $a_1 = \frac{\beta_d}{V_d}$, $b_1 = \frac{\beta_d}{V_d}$, $a_2 = \frac{1}{M}$, $a_5 = \beta_a$, $u_p = q_{pump}$, $u = q_{back} - q_{choke}$, $v_1(t) = V_a(t)$, $v_2(t) = -\dot{V}_a$ and $v_3(t) = h_{bit}(t)$ equations (1) – (3) can be written more compactly as

$$\dot{p}_p = -a_1 q_{bit} + b_1 u_p \quad (7)$$

$$\begin{aligned} \dot{q}_{bit} &= a_2(p_p - p_c) - \frac{F_d}{M}|q_{bit}|q_{bit} \\ &\quad - \frac{F_a}{M}|q_{bit} + q_{res}|(q_{bit} + q_{res}) + \frac{(\rho_d - \rho_a)g}{M}v_3 \end{aligned} \quad (8)$$

$$\dot{p}_c = \frac{a_5}{v_1}(q_{bit} + q_{res} + u + v_2). \quad (9)$$

III. OBSERVER

As the friction factor in the annulus depends on several uncertain parameters such as viscosity of the fluid, pipe roughness and flow regime, it is assumed unknown. The density in the annulus is also encumbered with uncertainty as the amount of cuttings in the drill mud affects it, and is also assumed unknown. Let the unknown parameters be denoted as

$$\theta_1 = \frac{F_d + F_a}{M} > 0 \Rightarrow F_a = M\theta_1 - F_d \quad (10)$$

$$\theta_2 = \frac{(\rho_d - \rho_a)g}{M} \Rightarrow \rho_a = \rho_d - \frac{M}{g}\theta_2. \quad (11)$$

In the choice of the unknown parameter θ_2 certain assumptions have been made. The reason for choosing θ_2 as an unknown is because ρ_a is encumbered with uncertainty. From (4) one can see that M_a is linearly dependent on ρ_a, which implies that $M = M_d + M_a$ will also depend on ρ_a. Neglecting this dependency can be justified by two observations. Firstly, M_a affects only transients in the flow dynamics, which are fast compared to the dominating pressure dynamics. Secondly, the sensitivity of M w.r.t. changes in ρ_a is small as M_d is greater than M_a. Treating M as a known constant considerably reduces observer complexity. We will furthermore assume zero reservoir influx, hence $q_{res} = 0$. In view of these assumptions, (8) can be simplified to

$$\dot{q}_{bit} = a_2(p_p - p_c) - \theta_1|q_{bit}|q_{bit} + \theta_2 v_3, \quad (12)$$

and (6) can be written as

$$\begin{aligned} p_{bit} &= p_c + M_a(a_2(p_p - p_c) - \theta_1|q_{bit}|q_{bit} + \theta_2 v_3) \\ &\quad + (M\theta_1 - F_d)|q_{bit}|q_{bit} + (\rho_d g - M\theta_2)v_3. \end{aligned} \quad (13)$$

The goal for this section is to design an observer that estimates p_{bit} and adapts to the unknown parameters θ_1 and θ_2. The estimated states and estimated parameters will be denoted with a hat. Before continuing the following assumptions regarding boundedness and knowledge of signals will be made:

Assumption 1: All signals in (7) – (8) are bounded $\Leftrightarrow$ $p_p, p_c, q_{bit}, u_p, v_3 \in \mathcal{L}_\infty$ and $\dot{v}_3 \in \mathcal{L}_\infty$.
Considering that the system is stable and v_3 is the vertical depth of the well this assumption is mild.

Assumption 2: The following signals are assumed known: $p_p, p_c, u_p, v_3, \dot{v}_3$.
Standard top side measurements include p_p and p_c. The pump flow u_p can be estimated accurately by using the known pump speeds (ω_p), the number of pistons (N_p) and volume per stroke per piston (V_p) according to $u_p = N_p V_p 2\pi\omega_p$. The depth of the bit ($v_3$) and its rate of change ($\dot{v}_3$), are given indirectly by the known geometry of the well path and the topside measurement of the block (top drive) position.

Assumption 3: $\dot{\theta}_1 = \dot{\theta}_2 = 0$.
Both parameters vary slowly therefore the assumption is valid.

A. Error Dynamics

Motivated by [15], define the following change of coordinates

$$\xi = q_{bit} + l_1 p_p, \quad (14)$$

where l_1 is a feedback gain. From (7) and (12), the dynamics of ξ is

$$\dot{\xi} = -l_1 a_1 q_{bit} - \theta_1|q_{bit}|q_{bit} + \theta_2 v_3 + a_2(p_p - p_c) + l_1 b_1 u_p. \quad (15)$$

An observer for q_{bit} is

$$\dot{\widehat{\xi}} = -l_1 a_1 \widehat{q}_{bit} - \widehat{\theta}_1 |\widehat{q}_{bit}| \widehat{q}_{bit} + \widehat{\theta}_2 v_3 + a_2(p_p - p_c) + l_1 b_1 u_p, \quad (16)$$

$$\widehat{q}_{bit} = \widehat{\xi} - l_1 p_p. \quad (17)$$

Noticing that

$$\theta_1 |q_{bit}| q_{bit} - \widehat{\theta}_1 |\widehat{q}_{bit}| \widehat{q}_{bit} = \theta_1(|q_{bit}| q_{bit} - |\widehat{q}_{bit}| \widehat{q}_{bit}) + \tilde{\theta}_1 |\widehat{q}_{bit}| \widehat{q}_{bit}, \quad (18)$$

and from (14) and (17) that $\tilde{\xi} = \xi - \widehat{\xi} = \tilde{q}_{bit}$, the dynamics of the state estimation error becomes

$$\dot{\tilde{\xi}} = -l_1 a_1 \tilde{q}_{bit} - \theta_1(|q_{bit}| q_{bit} - |\widehat{q}_{bit}| \widehat{q}_{bit}) - \tilde{\theta}_1 |\widehat{q}_{bit}| \widehat{q}_{bit} + \tilde{\theta}_2 v_3. \quad (19)$$

Let the parameter errors and the regressor be denoted as

$$\tilde{\theta} = \begin{bmatrix} \tilde{\theta}_1 \\ \tilde{\theta}_2 \end{bmatrix}, \qquad \phi(\widehat{q}_{bit}, v_3) = \begin{bmatrix} -|\widehat{q}_{bit}| \widehat{q}_{bit} \\ v_3 \end{bmatrix}. \quad (20)$$

Using (20) and $\tilde{\xi} = \tilde{q}_{bit}$, (19) can be rewritten as

$$\dot{\tilde{\xi}} = -l_1 a_1 \tilde{\xi} - \theta_1(|q_{bit}| q_{bit} - |\widehat{q}_{bit}| \widehat{q}_{bit}) + \tilde{\theta}^T \phi. \quad (21)$$

B. Lyapunov Analysis

For the error system $(\tilde{\xi}, \tilde{\theta})$ with $\tilde{\xi}$ dynamics described by (21) and $\tilde{\theta}$ dynamics to be found, consider the candidate Lyapunov function

$$U(\tilde{\xi}, \tilde{\theta}) = \frac{1}{2}\tilde{\xi}^2 + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}, \quad (22)$$

where $\Gamma = \Gamma^T > 0$ is the adaptation gain matrix. Using (21), the time derivative of U is

$$\dot{U} = -l_1 a_1 \tilde{\xi}^2 - \theta_1(|q_{bit}| q_{bit} - |\widehat{q}_{bit}| \widehat{q}_{bit})\tilde{\xi} + \tilde{\theta}^T(\phi \tilde{\xi} + \Gamma^{-1} \dot{\tilde{\theta}}). \quad (23)$$

Choosing the $\tilde{\theta}$ dynamics to be

$$\dot{\tilde{\theta}} = -\Gamma \phi \tilde{\xi}, \quad (24)$$

gives

$$\dot{U} = -l_1 a_1 \tilde{\xi}^2 - \theta_1(|q_{bit}| q_{bit} - |\widehat{q}_{bit}| \widehat{q}_{bit})\tilde{\xi}. \quad (25)$$

Since $\theta_1 > 0$ and $\tilde{\xi} = \tilde{q}_{bit}$, it follows that $\theta_1(|q_{bit}| q_{bit} - |\widehat{q}_{bit}| \widehat{q}_{bit})\tilde{\xi} \geq 0$, so

$$\dot{U} \leq -l_1 a_1 \tilde{\xi}^2. \quad (26)$$

Since $a_1 > 0$, choosing $l_1 > 0$ gives $\dot{U}(\tilde{\xi}, \tilde{\theta}) \leq 0$. Noticing that $\tilde{\xi} = \tilde{\theta} = 0$ is an equilibrium point for the system defined by (21) and (24), and that the system is locally Lipschitz in $(\tilde{\xi}, \tilde{\theta})$, uniformly in t under Assumption 1, the LaSalle-Yoshizawa Theorem [16] can be invoked to conclude that all solutions to (21) and (24) are uniformly bounded and that

$$\lim_{t \to \infty} -l_1 a_1 \tilde{\xi}^2 = 0. \quad (27)$$

This implies that $\widehat{q}_{bit}$ and $\widehat{\theta}$ are bounded, and that $\widehat{q}_{bit} \to q_{bit}$ as $t \to \infty$. There is no guarantee that the parameter estimates converge to their true values. The results derived hold for all $(\tilde{\xi}, \tilde{\theta}) \in \mathbf{R}^3$.

C. Adaptive Law

In (24) $\tilde{\xi}$ is unknown which implies that the adaptive law $\dot{\widehat{\theta}} = -\dot{\tilde{\theta}}$ cannot be implemented in this form. This problem will be dealt with now. Define

$$\sigma = \theta + \eta(\widehat{q}_{bit}, v_3), \quad (28)$$

where η is a function of known/measured signals that is to be designed to assign σ the desired dynamics. Differentiating σ with respect to time (remembering that $\dot{\theta} = 0$) gives

$$\dot{\sigma} = \frac{\partial \eta}{\partial \widehat{q}_{bit}} \dot{\widehat{q}}_{bit} + \frac{\partial \eta}{\partial v_3} \dot{v}_3. \quad (29)$$

Substituting for $\dot{\widehat{q}}_{bit}$ by differentiating (17) w.r.t. time and using (7) gives

$$\dot{\sigma} = \frac{\partial \eta}{\partial \widehat{q}_{bit}} (\dot{\widehat{\xi}} - l_1 \dot{p}_p) + \frac{\partial \eta}{\partial v_3} \dot{v}_3, \quad (30)$$

$$= -l_1 \frac{\partial \eta}{\partial \widehat{q}_{bit}} (-a_1 q_{bit} + b_1 u_p) + \frac{\partial \eta}{\partial \widehat{q}_{bit}} \dot{\widehat{\xi}} + \frac{\partial \eta}{\partial v_3} \dot{v}_3, \quad (31)$$

where $\dot{\widehat{\xi}}$ is known from (16). From Assumption 2, only q_{bit} in (31) is unknown. To deal with this an estimate $\dot{\widehat{\sigma}}$ is used

$$\dot{\widehat{\sigma}} = -l_1 \frac{\partial \eta}{\partial \widehat{q}_{bit}} (-a_1 \widehat{q}_{bit} + b_1 u_p) + \frac{\partial \eta}{\partial \widehat{q}_{bit}} \dot{\widehat{\xi}} + \frac{\partial \eta}{\partial v_3} \dot{v}_3, \quad (32)$$

$$\widehat{\theta} = \widehat{\sigma} - \eta(\widehat{q}_{bit}, v_3). \quad (33)$$

Since $\tilde{\theta} = \tilde{\sigma}$ the dynamics of the estimation error is obtained from (31)–(32) as

$$\dot{\tilde{\theta}} = \dot{\tilde{\sigma}} = l_1 a_1 \frac{\partial \eta}{\partial \widehat{q}_{bit}} \tilde{\xi}, \quad (34)$$

where the fact $\tilde{\xi} = \tilde{q}_{bit}$ has been used. Comparing (34) to (24) suggests that η should be chosen such that

$$-l_1 a_1 \frac{\partial \eta(\widehat{q}_{bit}, v_3)}{\partial \widehat{q}_{bit}} = \Gamma \phi. \quad (35)$$

Using (20) and integrating (35) w.r.t $\widehat{q}_{bit}$ gives

$$\eta(\widehat{q}_{bit}, v_3) = \Gamma \begin{bmatrix} \frac{|\widehat{q}_{bit}|^3}{3 l_1 a_1} \\ -\frac{v_3 \widehat{q}_{bit}}{l_1 a_1} \end{bmatrix}. \quad (36)$$

The partial derivatives of $\eta(\widehat{q}_{bit}, v_3)$ needed in (32) are

$$\frac{\partial \eta}{\partial \widehat{q}_{bit}} = \Gamma \begin{bmatrix} \frac{|\widehat{q}_{bit}| \widehat{q}_{bit}}{l_1 a_1} \\ -\frac{v_3}{l_1 a_1} \end{bmatrix}, \quad \frac{\partial \eta}{\partial v_3} = \Gamma \begin{bmatrix} 0 \\ -\frac{\widehat{q}_{bit}}{l_1 a_1} \end{bmatrix}. \quad (37)$$

D. Initial Conditions

There are two initial conditions that need to be set. One is $\widehat{\xi}(0)$ in (16) and the other is $\widehat{\sigma}(0)$ in (32). The initial conditions should be constructed by using the relationships

$$\widehat{\xi}(0) = \widehat{q}_{bit}(0) + l_1 p_p(0) \quad (38)$$

$$\widehat{\sigma}(0) = \widehat{\theta}(0) + \eta(\widehat{q}_{bit}(0), v_3(0)), \quad (39)$$

where $p_p(0)$ and $v_3(0)$ are known since they are measured. The user can now come up with initial estimates of $\widehat{q}_{bit}(0)$ and $\widehat{\theta}(0)$ and then use relations (38) and (39) to compute the corresponding $\widehat{\xi}(0)$ and $\widehat{\sigma}(0)$.

E. Convergence of $\tilde{p}_{bit}$

The goal of Section III is to design an observer so that the estimated pressure at the bit $\widehat{p}_{bit}$ tracks p_{bit}. In Sections III-A – III-D an observer for the unmeasured state q_{bit} has been designed. In this section convergence properties of $\tilde{p}_{bit} = p_{bit} - \widehat{p}_{bit}$ will be proved. Motivated by (13) an estimate of p_{bit} is

$$\widehat{p}_{bit} = p_c + M_a(a_2(p_p - p_c) - \widehat{\theta}_1|\widehat{q}_{bit}|\widehat{q}_{bit} + \widehat{\theta}_2 v_3) \quad (40)$$

$$+ (M\widehat{\theta}_1 - F_d)|\widehat{q}_{bit}|\widehat{q}_{bit} + (\rho_d g - M\widehat{\theta}_2)v_3. \quad (41)$$

Using (13) the error in the estimate can be expressed as

$$\begin{aligned}\tilde{p}_{bit} &= M_a(-(\theta_1|q_{bit}|q_{bit} - \widehat{\theta}_1|\widehat{q}_{bit}|\widehat{q}_{bit}) + \tilde{\theta}_2 v_3)\\ &\quad - F_d(|q_{bit}|q_{bit} - |\widehat{q}_{bit}|\widehat{q}_{bit})\\ &\quad + M(\theta_1|q_{bit}|q_{bit} - \widehat{\theta}_1|\widehat{q}_{bit}|\widehat{q}_{bit}) - M\tilde{\theta}_2 v_3. \quad (42)\end{aligned}$$

Using $M = M_a + M_d$ and (18), (42) can be rewritten as

$$\begin{aligned}\tilde{p}_{bit} &= M_d(\theta_1|q_{bit}|q_{bit} - \widehat{\theta}_1|\widehat{q}_{bit}|\widehat{q}_{bit}) - M_d\tilde{\theta}_2 v_3\\ &\quad - F_d(|q_{bit}|q_{bit} - |\widehat{q}_{bit}|\widehat{q}_{bit})\\ &= M_d(\theta_1(|q_{bit}|q_{bit} - |\widehat{q}_{bit}|\widehat{q}_{bit}) + \tilde{\theta}_1|\widehat{q}_{bit}|\widehat{q}_{bit} - \tilde{\theta}_2 v_3)\\ &\quad - F_d(|q_{bit}|q_{bit} - |\widehat{q}_{bit}|\widehat{q}_{bit})\\ &= M_d(\theta_1(|q_{bit}|q_{bit} - |\widehat{q}_{bit}|\widehat{q}_{bit}) - \tilde{\theta}^T\phi)\\ &\quad - F_d(|q_{bit}|q_{bit} - |\widehat{q}_{bit}|\widehat{q}_{bit}). \quad (43)\end{aligned}$$

From the error equation (43) and remembering that $(q_{bit} - \widehat{q}_{bit}) \to 0$ from the previous Lyapunov analysis it can be seen that if $\tilde{\theta}^T\phi \to 0$ then $\tilde{p}_{bit} \to 0$. In view of (21), $\widehat{q}_{bit} \to q_{bit}$ and $\tilde{\xi} \to 0$, the convergence $\tilde{\theta}^T\phi \to 0$ follows directly from the extended Barbalat's Lemma [17, Lemma 1], provided $\tilde{\theta}^T\phi$ is uniformly continuous. This is the case if $\tilde{\theta}$, ϕ, $\dot{\tilde{\theta}}$ and $\dot{\phi}$ are bounded. The previous analysis has established that $\tilde{\theta}, \phi, \dot{\tilde{\theta}} \in \mathcal{L}_\infty$, so it remains to show that $\dot{\phi} \in \mathcal{L}_\infty$. Using (17) we obtain

$$\dot{\phi} = \begin{bmatrix} -2|\widehat{q}_{bit}|\dot{\widehat{q}}_{bit} \\ \dot{v}_3 \end{bmatrix} = \begin{bmatrix} -2|\widehat{q}_{bit}|(\dot{\widehat{\xi}} - l_1\dot{p}_p) \\ \dot{v}_3 \end{bmatrix}. \quad (44)$$

From (16) we can conclude that $\dot{\widehat{\xi}} \in \mathcal{L}_\infty$ as $\widehat{q}_{bit}, \phi, \widehat{\theta}, p_p, p_c, u_p \in \mathcal{L}_\infty$ from the previous Lyapunov analysis and Assumption 1. Similarly from (7) and Assumption 1 we can conclude that $\dot{p}_p \in \mathcal{L}_\infty$. Finally $\dot{v}_3 \in \mathcal{L}_\infty$ by Assumption 1.

F. Summary Adaptive Observer

The adaptive observer is summarized in Table I, and has the following properties:

- All solutions to (21), (24) are uniformly bounded.
- $\lim_{t\to\infty} \tilde{q}_{bit} = 0$
- $\lim_{t\to\infty} \dot{\tilde{\theta}} = 0$
- $\lim_{t\to\infty} \tilde{\theta}^T\phi = 0$
- $\lim_{t\to\infty} \tilde{p}_{bit} = 0$

TABLE I

SUMMARY OF ADAPTIVE OBSERVER BASED ON NONLINEAR MODEL

Plant	$\dot{p}_p = -a_1 q_{bit} + b_1 u_p$ $\dot{q}_{bit} = a_2(p_p - p_c) - \theta_1\lvert q_{bit}\rvert q_{bit} + \theta_2 v_3$ $p_{bit} = p_c + M_a(a_2(p_p - p_c) - \theta_1\lvert q_{bit}\rvert q_{bit} + \theta_2 v_3)$ $+(M\theta_1 - F_d)\lvert q_{bit}\rvert q_{bit} + (\rho_d g - M\theta_2)v_3$ p_p, p_c, u_p and v_3 are measured
Observer	$\widehat{p}_{bit} = p_c + M_a(a_2(p_p - p_c) - \widehat{\theta}_1\lvert\widehat{q}_{bit}\rvert\widehat{q}_{bit} + \widehat{\theta}_2 v_3)$ $+(M\widehat{\theta}_1 - F_d)\lvert\widehat{q}_{bit}\rvert\widehat{q}_{bit} + (\rho_d g - M\widehat{\theta}_2)v_3$ $\widehat{q}_{bit} = \widehat{\xi}_1 - l_1 p_p$ $\dot{\widehat{\xi}}_1 = -l_1 a_1\widehat{q}_{bit} - \widehat{\theta}_1\lvert\widehat{q}_{bit}\rvert\widehat{q}_{bit} + \widehat{\theta}_2 v_3 + a_2(p_p - p_c) + l_1 b_1 u_p$ $\widehat{\xi}_1(0) = \widehat{q}_{bit}(0) + l_1 p_p(0)$
Adaptive law	$\widehat{\theta} = \widehat{\sigma} - \eta(\widehat{q}_{bit}, v_3)$ $\dot{\widehat{\sigma}} = -l_1\frac{\partial\eta}{\partial\widehat{q}_{bit}}(-a_1\widehat{q}_{bit} + b_1 u_p) + \frac{\partial\eta}{\partial\widehat{q}_{bit}}\dot{\widehat{\xi}}_1 + \frac{\partial\eta}{\partial v_3}\dot{v}_3$ $\widehat{\sigma}(0) = \widehat{\theta}(0) + \eta(\widehat{q}_{bit}(0), v_3(0))$ $\eta(\widehat{q}_{bit}, v_3) = \Gamma\begin{bmatrix}\frac{\lvert\widehat{q}_{bit}\rvert^3}{3l_1 a_1}\\ -\frac{v_3\widehat{q}_{bit}}{l_1 a_1}\end{bmatrix}$ $\frac{\partial\eta}{\partial\widehat{q}_{bit}} = \Gamma\begin{bmatrix}\frac{\lvert\widehat{q}_{bit}\rvert\widehat{q}_{bit}}{l_1 a_1}\\ -\frac{v_3}{l_1 a_1}\end{bmatrix}$ $\frac{\partial\eta}{\partial v_3} = \Gamma\begin{bmatrix}0\\ -\frac{\widehat{q}_{bit}}{l_1 a_1}\end{bmatrix}$
Design variables	Observer gain $l_1 > 0$ Adaption gain: $\Gamma = \Gamma^T > 0$ Initial conditions: $\widehat{q}_{bit}(0)$ and $\widehat{\theta}(0)$

IV. SIMULATION

The proposed observer has been tested on real data from the Grane field in the North Sea. The model (7), (12)– (13) was manually fitted to steady state data, resulting in the parameter values given in Table II. The depth of the bit was constant at $v_3 = 1825m$. Fig. 3 shows the measured data p_{bit}, p_p, p_c and u_p (in solid lines) and the resulting fit denoted $p_{bit_{bit}}$ and $p_{p_{fit}}$ (in dashed lines). Note that the p_{bit} measurement is lost in the time interval $t \approx 1hr$ to $t \approx 1hr10min$ as the flow is too low for the mud pulse telemetry system to function. The parameter values in Table II, with the exception of the ones assumed uncertain, are used in the observer test that now follows.

TABLE II

PARAMETER VALUES GRANE DATA

Parameter	Value	Description
V_d	42	Volume drill string (m^3)
β_d	14000	Bulk modulus drill string (bar)
ρ_a	0.0121	Density annulus ($10^5 \times \frac{kg}{m^3}$)
ρ_d	0.0121	Density drill string ($10^5 \times \frac{kg}{m^3}$)
F_d	0.16	Friction factor drill string ($10^6 \times \frac{bar\ s^2}{m^6}$)
F_a	0.003	Friction factor annulus ($10^6 \times \frac{bar\ s^2}{m^6}$)
M_a	1.5	($10^8 \times \frac{kg}{m^4}$)
M_d	4.2	($10^8 \times \frac{kg}{m^4}$)
$v_3 = h_{bit}$	1825	Vertical depth (m) of bit

The observer design parameters were set to $l_1 = 0.5$, $\Gamma = \begin{bmatrix} 5 \times 10^{-10} & 0 \\ 0 & 10^{-9} \end{bmatrix}$, $\widehat{\theta}_1(0) = (F_d + 1.5 \times F_a)/M$ corresponding to a 50% error in the friction factor F_a, $\widehat{\theta}_2(0) = \frac{(\rho_d - 1.2\times\rho_a)g}{M}$ corresponding to a 20% error in the density ρ_a, $\widehat{q}_{bit}(0) = 16.67\frac{liter}{s}$.

From Fig. 4 it can be seen that the observer estimates the pressure at the bit p_{bit} well after initial transients. The

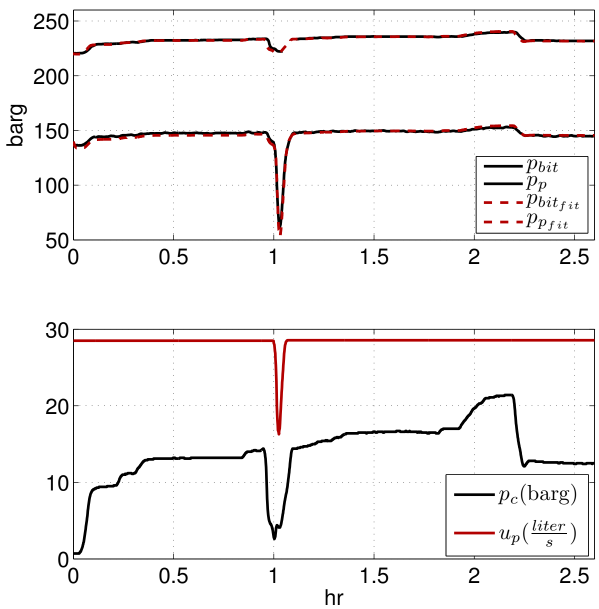

Fig. 3. The low order model (7), (12)–(13) fitted to data from the Grane field (top figure). Choke pressure p_c and pump flow u_p (bottom figure).

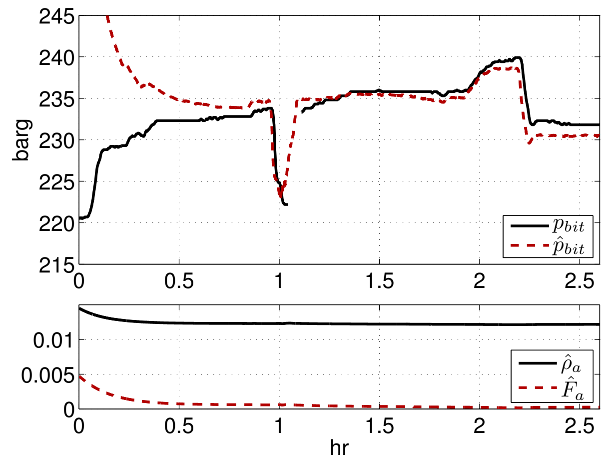

Fig. 4. Estimated bit pressure and measured bit pressure (top figure). Note that the measured bit pressure is lost in the time interval $t \approx 1hr$ to $t \approx 1hr10min$. Estimated density $\widehat{\rho}_a$ and friction factor $\widehat{F}_a$ (bottom figure).

estimation error is usually less than 2 bar, although slightly higher during the transient at $t = 1hr$. The estimated parameters, $\widehat{F}_a = M\widehat{\theta}_1 - F_d$ and $\widehat{\rho}_a = \rho_d - \frac{M}{g}\widehat{\theta}_2$ from (10)–(11), settle at approximately 0.000068 and 0.0122 which gives a small error in the density estimate $\widehat{\rho}$ while the friction factor estimate suffers as F_d is approximately 50 times larger than F_a which means that $\widehat{F}_a$ is very sensitive to inaccuracies in F_d.

V. CONCLUSIONS

Based on a newly developed nonlinear model an observer that estimates bit pressure during drilling has been presented. Through Lyapunov analysis the estimation error is shown to converge to zero. The proposed observer adapts to unknown friction and density in the annulus. Performance of the observer has been verified using data from the Grane field in the North Sea showing good results.

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