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Adaptive Output Feedback Control of a Managed Pressure Drilling System

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Abstract— **This paper presents a nonlinear observer-based control scheme to stabilize the annular pressure profile throughout the well bore continuously while drilling. A simple mechanistic model is presented that captures the dominant phenomena of the drilling system and forms the basis for model-based observer and control design. A new nonlinear adaptive observer is developed for state estimations. A new adaptive controller is designed to stabilize the annular pressure and achieve asymptotic tracking by applying feedback control of the choke valve opening and the main pumps.**

Index Terms— **Drilling, nonlinear observer, adaptive control, stabilization, tracking.**

I. Introduction

During well drilling, a drilling fluid (mud) is pumped into the drill string topside and through the drill bit at the bottomhole of the well [1], [2]. The mud then transports cuttings in the annulus side of the well (*i.e.* in the well bore outside the drill string) up to the drill rig, where a choke valve and a backpressure pump are used to control the annular pressure. A more elaborate description of the drilling process is given in [3].

The main objective is to precisely control the annular pressure profile throughout the well bore continuously while drilling, *i.e.* to maintain the annular pressure in the well above the pore or collapse pressure and below the fracture or sticking pressure. Usually, this amounts to stabilizing the downhole annular pressure at a critical depth within its margins, *i.e.* either at a particular depth where the pressure margins are small, or at the drill bit where conditions are the most uncertain.

Basically, two strategies for closed-loop control of the choke are used: indirect topside control and direct bottomhole control. Indirect topside control is to stabilize the bottomhole pressure indirectly by applying feedback control to stabilize the topside annulus pressure instead, where the pressure setpoint corresponding to a desired bottomhole pressure is calculated online using a steady-state model. This strategy is the most common and straightforward mainly due to the availability of high-frequency and robust topside pressure measurements. Direct bottomhole control is to stabilize the bottomhole pressure at the critical depth at a desired setpoint directly. Even though a bottomhole measurement usually exists, an estimate of the pressure is needed between samples because the transfer rate of the measurement usually is slow, or for additional safety because the sensor itself may be unreliable.

State-of-the-art solutions typically employ conventional PI control applied to the choke, using one of the above strategies. There are significant drawbacks with both strategies. One is that the control system based on conventional PI control will react slowly to fast pressure changes, which results from movements of the drill string. Another drawback, is the uncertainty in the modelled bottomhole pressure, due to uncertainties in the friction and mud compressibility parameters in both the drill string and annulus.

There is significant potential to improve existing algorithms, either the control law itself, or the observer used to estimate the critical downhole pressure. Model-based control enables improved compensation of pressure fluctuations during particularly critical drilling operations. Also, by using model-based compensation with adaptation of uncertain parameters rather than integral action in the controller, one typically enable faster reaction to changes in setpoints and disturbances. In the absence of full-state measurement, observer design is an effective way to control systems, such as in [4], [5], [6], [7]. In this paper, we will address nonlinear adaptive observer-based control of a drilling system in the presence of unknown parameters and unmeasured downhole pressure. A simple dynamic model developed for the observer and model-based control design, is further developed to better describe the liquid fluid flow behavior. A new nonlinear observer is developed by using Lyapunov techniques to estimate the unmeasured downhole pressure. Precise and robust estimation of the annular pressure during drilling allows for reduced pressure margins. Online adaptation of unknown model parameters can extract more information from the system. The adaptive controller is designed by using Lyapunov techniques and parameter estimation to stabilize the annular pressure at the desired setpoint. The stabilization of the dynamic system is demonstrated by the proposed control. It is shown that the proposed controller can guarantee asymptotic tracking. Simulation results are presented to illustrate the effectiveness of the proposed control scheme.

II. Model

In this section, we present a model developed in [8], which captures the dominant phenomena of the drilling system and forms the basis for model-based observer and control design. The model only considers fluid phase flow and the well is divided into two separate compartments. Figure 1 shows

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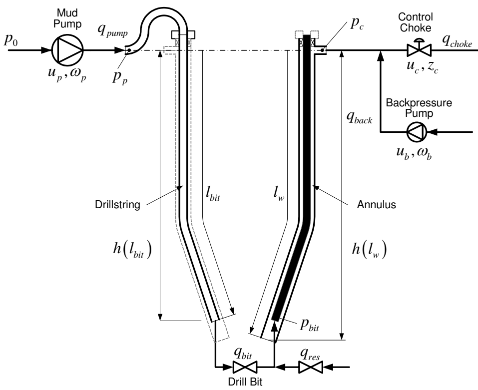

Fig. 1. A simplified schematical drawing of the drilling system.

the two control volumes considered, one control volume for the drill string and one for the annulus. The volumes are connected through the drill bit. The detailed derivation of the model is given in [8], where the dynamics of the drilling system is described by

$$\dot{p}_p = -a_1 q_{bit} + b_1 u_p \quad (1)$$

$$\dot{q}_{bit} = a_2(p_p - p_c) - \frac{F_d}{M}|q_{bit}|q_{bit} - \frac{F_a}{M}|q_{bit} + q_{res}|(q_{bit} + q_{res}) + \frac{(\rho_d - \rho_a)g}{M}v_3 \quad (2)$$

$$\dot{p}_c = \frac{a_5}{v_1}(q_{bit} + q_{res} + u + v_2). \quad (3)$$

The states p_p and p_c are the inlet mud pump pressure and outlet choke pressure (bar), q_{bit} is the flow rate through the drill bit (m^3/s), $u = q_{back} - q_{choke}$ is control input, u_p, q_{back} and q_{choke} are the flow rates through the mud pump, the back pressure pump and the choke valve, and q_{res} is the reservoir influx, v_1, v_2 and v_3 are the annulus volume, rate of change of the annulus volume and vertical depth of the bit, respectively. The rest of the quantities in (1)–(3) are constant parameters and can be explained as

- $a_1 = \frac{\beta_d}{V_d}$, $a_2 = \frac{1}{M}$, $a_5 = \beta_a$, $b_1 = \frac{\beta_d}{V_d}$, $M = M_a + M_d$;
- V_d: volume of the drill string;
- β_d and β_a: bulk modulus of the drill string and the annulus;
- M_d and M_a: density per meter of the drill string and the annulus;
- F_d and F_a: friction factor of the drill string and the annulus;
- ρ_d and ρ_a: density in the drill string and the annulus;
- g: gravity.

The parameters are known except as stated in the following assumptions.

Assumption 1: The reservoir influx q_{res} is an unknown constant.

Assumption 2: $\theta = \frac{F_a}{M} > 0$ is an unknown constant.

Assumption 3: The flow rate $q_{bit} > 0$ and $q_{bit} + q_{res} \geq 0$.

Using these assumptions and the notations $a_3 = \frac{F_d}{M}$ and $a_4 = \frac{(\rho_d - \rho_a)g}{M}$, the system (1)–(3) is rewritten as

$$\dot{p}_p = -a_1 q_{bit} + b_1 u_p \quad (4)$$

$$\dot{q}_{bit} = a_2(p_p - p_c) - a_3 q_{bit}^2 - \theta(q_{bit} + q_{res})^2 + a_4 v_3 \quad (5)$$

$$\dot{p}_c = \frac{a_5}{v_1}(q_{bit} + q_{res} + u + v_2). \quad (6)$$

The main variable of interest is the annular downhole pressure p_{bit} given by

$$p_{bit} = p_c + M_a \dot{q}_{bit} + F_a(q_{bit} + q_{res})^2 + \rho_a g v_3. \quad (7)$$

Our objective is to design a control law for the control input u which stabilizes p_{bit} at the desired set-point p_{ref}.

III. Nonlinear Adaptive Observer

A. Observer

Consider that p_p and p_c are measured and q_{bit} is unmeasured, where the parameter θ and q_{res} are unknown constants. The following change of coordinates is defined

$$\xi \triangleq q_{bit} + l_1 p_p, \quad (8)$$

where l_1 is a positive constant. This gives the dynamics

$$\begin{aligned} \dot{\xi} &= \dot{q}_{bit} + l_1 \dot{p}_p \\ &= -l_1 a_1 q_{bit} + l_1 b_1 u_p + a_2(p_p - p_c) - a_3 q_{bit}^2 \\ &\quad -\theta(q_{bit} + q_{res})^2 + a_4 v_3. \end{aligned} \quad (9)$$

Defining $\Theta = [\theta_1, \theta_2, \theta_3]^T, \theta_1 = \theta, \theta_2 = \theta q_{res}$, and $\theta_3 = \theta q_{res}^2$, the equation (9) can be written as

$$\begin{aligned} \dot{\xi} &= -l_1 a_1 q_{bit} + l_1 b_1 u_p + a_2(p_p - p_c) - a_3 q_{bit}^2 \\ &\quad -\Theta^T \phi(q_{bit}) + a_4 v_3, \end{aligned} \quad (10)$$

where $\phi(q_{bit}) = [q_{bit}^2, 2q_{bit}, 1]^T$ and Θ will be estimated in the observer design.

An adaptive observer for q_{bit} is developed as follows

$$\begin{aligned} \dot{\hat{\xi}} &= -l_1 a_1 \hat{q}_{bit} + l_1 b_1 u_p + a_2(p_p - p_c) \\ &\quad -a_3 \hat{q}_{bit}^2 - \hat{\Theta}^T \phi(\hat{q}_{bit}) + a_4 v_3, \end{aligned} \quad (11)$$

$$\hat{q}_{bit} = \hat{\xi} - l_1 p_p, \quad (12)$$

where $\hat{\Theta} = [\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3]^T$ and $\phi(\hat{q}_{bit}) = [\hat{q}_{bit}^2, 2\hat{q}_{bit}, 1]^T$. Firstly, we obtain the following error terms

$$\theta_2 q_{bit} - \hat{\theta}_2 \hat{q}_{bit} = \theta_2 \tilde{q}_{bit} + \tilde{\theta}_2 \hat{q}_{bit} \quad (13)$$

$$\begin{aligned} \theta_1 q_{bit}^2 - \hat{\theta}_1 \hat{q}_{bit}^2 &= \theta_1 q_{bit}^2 - (\theta_1 - \tilde{\theta}_1)\hat{q}_{bit}^2 \\ &= \theta_1(q_{bit}^2 - \hat{q}_{bit}^2) + \tilde{\theta}_1 \hat{q}_{bit}^2 \end{aligned} \quad (14)$$

$$q_{bit}^2 - \hat{q}_{bit}^2 = (q_{bit} + \hat{q}_{bit})\tilde{q}_{bit}, \quad (15)$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ and $\tilde{q}_{bit} = q_{bit} - \hat{q}_{bit}$, the error dynamics of $\tilde{\xi}$ becomes

$$\begin{aligned} \dot{\tilde{\xi}} &= -l_1 a_1 \tilde{q}_{bit} - (a_3 + \theta_1)(q_{bit}^2 - \hat{q}_{bit}^2) - \tilde{\theta}_1 \hat{q}_{bit}^2 \\ &\quad -2\theta_2 \tilde{q}_{bit} - 2\tilde{\theta}_2 \hat{q}_{bit} - \tilde{\theta}_3, \end{aligned} \quad (16)$$

and since $\tilde{\xi} = \xi - \hat{\xi} = \tilde{q}_{bit}$, we get

$$\begin{aligned}\dot{\tilde{\xi}} &= -l_1 a_1 \tilde{\xi} - (a_3+\theta_1)(q_{bit}+\hat{q}_{bit})\tilde{\xi} - 2\theta_2\tilde{\xi} \\ &\quad -\tilde{\Theta}^T\phi(\hat{q}_{bit}). \end{aligned} \tag{17}$$

B. Lyapunov Analysis

Consider the Lyapunov function

$$U\left(\tilde{\xi},\tilde{\Theta}\right) = \frac{1}{2}\tilde{\xi}^2 + \frac{1}{2}\tilde{\Theta}^T\Gamma^{-1}\tilde{\Theta}, \tag{18}$$

where Γ is the adaptation gain. Using (17), the time derivative of U is

$$\begin{aligned}\dot{U} &= -l_1 a_1 \tilde{\xi}^2 - (a_3+\theta_1)(q_{bit}+\hat{q}_{bit})\tilde{\xi}^2 - 2\theta_2\tilde{\xi}^2 \\ &\quad +\tilde{\Theta}^T\Gamma^{-1}\left(\dot{\tilde{\Theta}} - \Gamma\phi(\hat{q}_{bit})\tilde{\xi}\right). \end{aligned} \tag{19}$$

This suggests that we should choose an adaptation law satisfying

$$\dot{\tilde{\Theta}} = \Gamma\phi(\hat{q}_{bit})\tilde{\xi}, \tag{20}$$

giving

$$\dot{U} = -l_1 a_1\tilde{\xi}^2 - (a_3+\theta_1)(q_{bit}+\hat{q}_{bit})\tilde{\xi}^2 - 2\theta_1 q_{res}\tilde{\xi}^2. \tag{21}$$

If $q_{bit} > 0$, $\hat{q}_{bit} > 0$, and $q_{bit} > 2|q_{res}|$, the time-derivative of U satisfies

$$\dot{U} \leq -l_1 a_1 \tilde{\xi}^2. \tag{22}$$

Thus, $\tilde{\xi} \to 0$ as $t \to \infty$. Consider the following dynamic equation

$$\begin{aligned}\dot{\hat{q}}_{bit} &= -a_3\hat{q}_{bit}^2 - \hat{\theta}_1\hat{q}_{bit}^2 - 2\hat{\theta}_2\hat{q}_{bit} - \hat{\theta}_3 \\ &\quad +a_2(p_p - p_c) + a_4 v_3 + l_1 a_1 (q_{bit} - \hat{q}_{bit}). \end{aligned} \tag{23}$$

It can be shown that $\hat{q}_{bit} > 0$ if $\hat{q}_{bit}(0) > 0$, and

$$p_p > p_c - \frac{1}{a_2}a_4 v_3 + \frac{1}{a_2}max\{\hat{\theta}_3\}. \tag{24}$$

C. Adaptation Law

Note that (20) cannot be used for parameter estimation because $\tilde{\xi}$ is unavailable. We introduce a new variable

$$\sigma \triangleq \Theta + \eta\left(p_p, \hat{\xi}\right), \tag{25}$$

where $\eta(\cdot)$ is a vector function to be designed to assign σ a desired dynamics. Differentiating σ with respect to time, gives

$$\dot{\sigma} = \frac{\partial\eta}{\partial p_p}\dot{p}_p + \frac{\partial\eta}{\partial\hat{\xi}}\dot{\hat{\xi}}. \tag{26}$$

Let an estimate $\hat{\theta}$ of the parameter vector be given by

$$\dot{\hat{\sigma}} = \frac{\partial\eta}{\partial p_p}\left(-a_1\hat{q}_{bit} + b_1 u_p\right) + \frac{\partial\eta}{\partial\hat{\xi}}\dot{\hat{\xi}}, \tag{27}$$

$$\hat{\Theta} = \hat{\sigma} - \eta\left(p_p, \hat{\xi}\right). \tag{28}$$

The resulting estimation error is then governed by

$$\begin{aligned}\dot{\tilde{\Theta}} &= \dot{\Theta} - \dot{\hat{\Theta}} \\ &= \dot{\sigma} - \dot{\eta}\left(p_p,\hat{\xi}\right) - \left(\dot{\hat{\sigma}} - \dot{\eta}\left(p_p,\hat{\xi}\right)\right) \\ &= -a_1\frac{\partial\eta}{\partial p_p}\tilde{\xi}. \end{aligned} \tag{29}$$

Compared with (20), this suggests that η should be selected such that

$$-a_1\frac{\partial\eta}{\partial p_p} \triangleq \Gamma\phi(\hat{q}_{bit}) = \Gamma\begin{bmatrix}\hat{q}_{bit}^2 \\ 2\hat{q}_{bit} \\ 1\end{bmatrix}. \tag{30}$$

A solution $\eta(\cdot)$ can be found by integrating (30)

$$\eta = -\frac{1}{a_1}\int \Gamma\phi(\hat{\xi} - l_1 p_p)dp_p = \Gamma\begin{bmatrix}\frac{1}{3l_1 a_1}(\hat{\xi} - l_1 p_p)^3 \\ \frac{1}{l_1 a_1}(\hat{\xi} - l_1 p_p)^2 \\ -\frac{1}{a_1}p_p\end{bmatrix}. \tag{31}$$

The resulting partial derivatives become

$$\frac{\partial\eta}{\partial p_p} = -\frac{1}{a_1}\Gamma\begin{bmatrix}(\hat{\xi} - l_1 p_p)^2 \\ 2(\hat{\xi} - l_1 p_p) \\ 1\end{bmatrix}, \tag{32}$$

$$\frac{\partial\eta}{\partial\hat{\xi}} = \Gamma\begin{bmatrix}\frac{1}{l_1 a_1}(\hat{\xi} - l_1 p_p)^2 \\ \frac{2}{l_1 a_1}(\hat{\xi} - l_1 p_p) \\ 0\end{bmatrix}. \tag{33}$$

Lemma 1: With the application of the adaptive nonlinear observer (11)–(12), and the parameter update law (27)–(28), in the set

$$\begin{aligned}\mathcal{A} &= \Big\{(p_p, \hat{q}_{bit}, p_c) : q_{bit} > 2|q_{res}|, \hat{q}_{bit}(0) > 0, \\ &\quad p_p > p_c - \frac{1}{a_2}a_4 v_3 + \frac{1}{a_2}max\{\hat{\theta}_3\},\Big\}, \end{aligned} \tag{34}$$

the signals $\tilde{\xi}$ and $\tilde{\Theta}$ are bounded and the observation error converges to zero, i.e., $\lim_{t\to\infty}[q_{bit} - \hat{q}_{bit}] = 0$.

IV. Adaptive Controller Design

A. Annular pressure profile

The annular downhole pressure p_{bit}, which from (5) and (7), can be written as

$$\begin{aligned}p_{bit} &= M_a a_2 p_p + M_d a_2 p_c - M_a a_3 q_{bit}^2 \\ &\quad +M_d\Theta^T\phi(q_{bit}) + f_0, \end{aligned} \tag{35}$$

where $f_0 = (M_a a_4 + \rho_a g)v_3$.

B. Controller design

Based on the observer (11)–(12), the system (4)–(6) is rewritten as

$$\begin{aligned}\dot{p}_p &= -a_1\hat{q}_{bit} + b_1 u_p - a_1\tilde{\xi} \\ \dot{\hat{q}}_{bit} &= a_2(p_p - p_c) - a_3\hat{q}_{bit}^2 - \hat{\Theta}^T\phi(\hat{q}_{bit}) \qquad (36) \\ &\quad +a_4 v_3 + l_1 a_1\tilde{\xi} \qquad (37) \\ \dot{p}_c &= \frac{a_5}{v_1}\left(\hat{q}_{bit} + q_{res} + u + v_2 + \tilde{\xi}\right), \qquad (38)\end{aligned}$$

and the output

$$\begin{aligned} y\left(p_p, p_c, \hat{q}_{bit}\right) &= \hat{p}_{bit}\left(p_p, p_c, \hat{q}_{bit}\right) \\ &= M_a a_2 p_p + M_d a_2 p_c - M_a a_3 \hat{q}_{bit}^2 \\ &\quad + M_d \hat{\Theta}^T \phi(\hat{q}_{bit}) + f_0. \end{aligned} \tag{39}$$

Hence the system (36)–(38) with (39) has relative degree one. Our objective is to design a control law for the control input u which stabilizes the annular down-hole pressure p_{bit} at the desired set-point p_{ref}. Define the set point error as the following

$$e = y\left(p_p, p_c, \hat{q}_{bit}\right) - p_{ref}. \tag{40}$$

Computing the derivative of e from (36)–(40) gives

$$\begin{aligned} \dot{e} &= \dot{y}\left(p_p, p_c, \hat{q}_{bit}\right) \\ &= \frac{M_d a_2 a_5}{v_1} u + M_a a_2 \left(\dot{\hat{p}}_p - a_1 \tilde{\xi}\right) \\ &\quad + \frac{M_d a_2 a_5}{v_1}\left(\dot{\hat{p}}_c + \tilde{\xi} + q_{res}\right) \\ &\quad + 2\left(M_d \hat{\theta}_1 \hat{q}_{bit} + M_d \hat{\theta}_2 - M_a a_3 \hat{q}_{bit}\right)\left(\dot{\hat{q}} + l_1 a_1 \tilde{\xi}\right) \\ &\quad - M_d \phi^T(\hat{q}_{bit}) \Gamma \phi(\hat{q}_{bit}) \tilde{\xi} + \dot{f}_0, \end{aligned} \tag{41}$$

where

$$\dot{\hat{p}}_p = -a_1 \hat{q}_{bit} + b_1 u_p \tag{42}$$

$$\dot{\hat{p}}_c = \hat{q}_{bit} + v_2 \tag{43}$$

$$\dot{\hat{q}} = a_2\left(p_p - p_c\right) - a_3 \hat{q}_{bit}^2 - \hat{\Theta}^T \phi(\hat{q}_{bit}) + a_4 v_3. \tag{44}$$

Thus the control law is designed as

$$\begin{aligned} u &= -(\hat{q}_{bit} + v_2 + \hat{q}_{res}) \\ &\quad + \frac{v_1}{M_d a_2 a_5}\Big(- C_1 e - M_a a_2 \dot{\hat{p}}_p - \dot{f}_0 - k_1 B^2 e \\ &\quad - 2\left(M_d \hat{\theta}_1 \hat{q}_{bit} + M_d \hat{\theta}_2 - M_a a_3 \hat{q}_{bit}\right) \dot{\hat{q}}\Big), \end{aligned} \tag{45}$$

where C_1 and k_1 are positive constants, and

$$\begin{aligned} B &= \| -M_a a_1 a_2 - \frac{a_2 a_5}{v_1} M_d - M_a \phi^T(\hat{q}_{bit}) \Gamma \phi(\hat{q}_{bit}) \\ &\quad + 2 l_1 a_1 \left(M_d \hat{\theta}_1 \hat{q}_{bit} + M_d \hat{\theta}_2 - M_a a_3 \hat{q}_{bit}\right) \| \end{aligned} \tag{46}$$

$$k_1 > \frac{1}{4 k l_1 a_1}, \tag{47}$$

and the parameter adaptive law for q_{res} is given by

$$\dot{\hat{q}}_{res} = \frac{\gamma M_d a_2 a_5}{v_1} e, \tag{48}$$

where γ is a positive adaptation gain.

C. Lyapunov analysis

Consider the control Lyapunov function

$$V = kU + \frac{1}{2} e^2 + \frac{1}{2\gamma} \tilde{q}_{res}^2. \tag{49}$$

Using (22), (45) and (48), the derivative of V is

$$\begin{aligned} \dot{V} &\le -C_1 e^2 + B|e\tilde{\xi}| - k_1 B^2 e^2 - k l_1 a_1 \tilde{\xi}^2 \\ &\quad - \frac{1}{\gamma} \tilde{q}_{res}\left(\dot{\hat{q}}_{res} - \gamma \frac{M_d a_2 a_5}{v_1} e\right) \\ &\le -C_1 e^2 - k l_1 a_1 \epsilon_l \tilde{\xi}^2 - k l_1 a_1 (1 - \epsilon_l) \tilde{\xi}^2 \\ &\quad + B|e\tilde{\xi}| - k_1 B^2 e^2. \end{aligned} \tag{50}$$

Let

$$\epsilon_l = 1 - \frac{1}{4 k k_1 l_1 a_1}. \tag{51}$$

Note that $\epsilon_l \in (0,1)$ due to (47) and the fact that k, k_1, l_1 and a_1 are strictly positive constants. We obtain that

$$\begin{aligned} \dot{V} &\le -C_1 e^2 - k l_1 a_1 \epsilon_l \tilde{\xi}^2 - \frac{1}{4k_1} \tilde{\xi}^2 + B|e\tilde{\xi}| - k_1 B^2 e^2 \\ &\le -C_1 e^2 - k l_1 a_1 \epsilon_l \tilde{\xi}^2 - \left(\frac{1}{2\sqrt{k_1}} |\tilde{\xi}| - \sqrt{k_1} B |e|\right)^2 \\ &\le -C_1 e^2 - k l_1 a_1 \epsilon_l \tilde{\xi}^2, \end{aligned} \tag{52}$$

where Young's inequality was used. Since V is positive definite and $\dot{V}$ is negative semidefinite in $\mathcal{A}$, it proves that signals $e, \tilde{\xi}, \tilde{\Theta}, \tilde{q}_{res}$ are bounded. From the LaSalle-Yoshizawa Theorem in [9], it further follows that $e, \tilde{\xi} \to 0$ as $t \to \infty$. The signals $y, \tilde{\xi}, \hat{\Theta}, \hat{q}_{res}$ are bounded and asymptotic tracking is achieved, i.e.,

$$\lim_{t \to \infty} [\hat{p}_{bit} - p_{ref}] = 0. \tag{53}$$

Note that u in (45) includes the signals p_p, p_c and $\hat{q}_{bit}$. To make the control input u bounded and the system (38)–(39) stable, we will ensure that p_p and $\hat{q}_{bit}$ are bounded. Towards that end, we consider the dynamics of p_p and $\hat{q}_{bit}$, given as

$$\begin{aligned} \dot{\hat{q}}_{bit} &= a_2 p_p - a_3 \hat{q}_{bit}^2 + \frac{1}{M_d}\Big(- y + M_a a_2 p_p - M_a a_3 \hat{q}_{bit}^2 \\ &\quad + M_d \hat{\Theta}^T \phi(\hat{q}_{bit}) + f_0\Big) - \hat{\Theta}^T \phi(\hat{q}_{bit}) + a_4 v_3 + l_1 a_1 \tilde{\xi} \\ &= \frac{1}{M_d} p_p - \frac{M}{M_d} a_3 \hat{q}_{bit}^2 - \frac{1}{M_d} y + l_1 a_1 \tilde{\xi} + d(t) \end{aligned} \tag{54}$$

$$\dot{p}_p = -a_1 q_{bit} + b_1 u_p, \tag{55}$$

where $d(t) = (\frac{M_a a_4}{M_d} + \frac{\rho_a g}{M_d} + a_4) v_3(t)$ is a bounded term. Assuming that p_p is bounded, we obtain the boundedness of $\hat{q}_{bit}$ from the boundedness of $y, \tilde{\xi}, v_3, p_p$ in (54) and $\hat{q}_{bit} > 0$. Therefore, u is bounded from (45). Now we have a conclusion that the signals in the closed-loop system can be shown to be bounded, as stated in the following theorem.

Remark 1: In practice the pump will not be able to drive the pressure to infinity. Thus, we can assume that the pump speed signal will be such that p_p stays bounded.

Theorem 1: With the application of the adaptive nonlinear observer (11)–(12), the control law (45), and the parameter update law (27)–(28), in the set

$$\begin{aligned} \mathcal{A} &= \Big\{(p_p, \hat{q}_{bit}, p_c) : q_{bit} > 2|q_{res}|, \hat{q}_{bit}(0) > 0, \\ &\quad p_p > p_c - \frac{1}{a_2} a_4 v_3 + \frac{1}{a_2} max\{\hat{\theta}_3\}\Big\}, \end{aligned} \tag{56}$$

all signals $y, p_p, \hat{q}_{bit}, \hat{\Theta}$, and $\hat{q}_{res}$ are bounded and asymptotic tracking is achieved given as

$$\lim_{t\to\infty}[\hat{p}_{bit} - p_{ref}] = 0. \quad (57)$$

D. Tracking performance

We now introduce the following useful Lemma and proposition as in [10], [11], [12].

Lemma 2: (Micaelli and Samson 1993 [10], Lemma 1). Let $f : \Re_+ \to \Re$ be any differentiable function. If $f(t)$ converges to zero as $t \to \infty$ and its derivative satisfies

$$\dot{f}(t) = f_1(t) + \eta(t) \qquad t \geq 0$$

where f_1 is a uniformly continuous function and $\eta(t)$ tends to zero as $t \to \infty$, then $\dot{f}(t)$ and $f_1(t)$ tend to zero as $t \to \infty$.

Proposition 1: Suppose f_1 is differentiable on (a, b) and $f_1^{'}$ is bounded on (a, b). Then $f_1(t)$ is uniformly continuous on (a, b).

Considering the error dynamics (17), we have from Lemma1 and Theorem 1 that

$$\lim_{t\to\infty} \tilde{\xi} = 0 \quad (58)$$

$$\lim_{t\to\infty}\Big(-l_1a_1\tilde{\xi} - (a_3+\theta_1)(q_{bit}+\hat{q}_{bit})\tilde{\xi} - 2\theta_2\tilde{\xi}\Big) = 0. \quad (59)$$

Define $f_1(t) = \tilde{\Theta}^T\phi(\hat{q}_{bit})$. So f_1 is differentiable and

$$\begin{aligned}\dot{f}_1(t) &= \phi^T\Gamma\phi\tilde{\xi} + 2\left(\hat{q}_{bit}\tilde{\theta}_1^T + \tilde{\theta}_2\right)\Big(a_2\,(p_p - p_c) \\ &\quad -a_3\hat{q}_{bit}^2 - \hat{\Theta}^T\phi(\hat{q}_{bit}) + a_4v_3 + l_1a_1\tilde{\xi}\Big). \quad (60)\end{aligned}$$

From the boundedness of $p_p, p_c, \hat{q}_{bit}, \tilde{\xi}, \tilde{\Theta}, v_3$ in $\mathcal{A}$, we have $\dot{f}_1(t)$ is bounded in $\mathcal{A}$. Therefore $f_1(t)$ is uniformly continuous in $\mathcal{A}$ and $\lim_{t\to\infty}\tilde{\Theta}^T\phi(\hat{q}_{bit}) = 0$ by using Lemma 2 and Proposition 1. From (35) and (39), we have

$$\begin{aligned}p_{bit} - \hat{p}_{bit} &= -M_a a_3(q_{bit}^2 - \hat{q}_{bit}^2) \\ &\quad +M_d(\Theta^T\phi(q_{bit}) - \hat{\Theta}^T\phi(\hat{q}_{bit})) \\ &= -M_a a_3(q_{bit}^2 - \hat{q}_{bit}^2) + M_d\tilde{\Theta}^T\phi(\hat{q}_{bit}) \\ &\quad +M_d\Theta^T\begin{bmatrix}(q_{bit}^2 - \hat{q}_{bit}^2)\\ 2(q_{bit} - \hat{q}_{bit})\\ 0\end{bmatrix}. \quad (61)\end{aligned}$$

Since $\lim_{t\to\infty}[q_{bit} - \hat{q}_{bit}] = 0$, $\lim_{t\to\infty}[q_{bit}^2 - \hat{q}_{bit}^2] = 0$, and $\lim_{t\to\infty}\tilde{\Theta}^T\phi(\hat{q}_{bit}) = 0$ in $\mathcal{A}$, it follows that $\lim_{t\to\infty}[p_{bit} - \hat{p}_{bit}] = 0$, which in conjunction with Theorem 1 gives

Theorem 2: With the application of the adaptive nonlinear observer (11)–(12), the control law (45), and the parameter update laws (27)–(28), in the set

$$\begin{aligned}\mathcal{A} &= \Big\{(p_p, \hat{q}_{bit}, p_c) : q_{bit} > 2|q_{res}|, \hat{q}_{bit}(0) > 0, \\ &\quad p_p > p_c - \frac{1}{a_2}a_4v_3 + \frac{1}{a_2}max\{\hat{\theta}_3\}\Big\}, \quad (62)\end{aligned}$$

all signals $y, p_p, \hat{q}_{bit}, p_c, \hat{\Theta}$, and $\hat{q}_{res}$ are bounded and asymptotic tracking is achieved given as

$$\lim_{t\to\infty}[p_{bit} - p_{ref}] = 0. \quad (63)$$

V. Simulation results

In this section we test our proposed controller on model (1)–(3). When doing so, we need to distribute the control signal u from (45) to the two physical actuation devices, the backpressure pump and the choke opening, according to $u = q_{back} - q_{choke}$. We assume that the backpressure pump is set at a constant rate, while the choke opening is related to the choke flow by the standard valve equation

$$q_{choke} = K_c\sqrt{\frac{2}{\rho_a}(p_c - p_0)}\; z_c. \quad (64)$$

For simulation studies, the following values are selected for the system: $\beta_a = \beta_d = 14000$, $V_d = 28.3$, $V_a = 96.1$, $M_d = 5700$, $M_a = 1700$, $F_d = 165000$, $F_a = 20800$, $\rho_a = \rho_d = 1250\times10^{-5}$, $h_{bit} = 2000$, $g = 9.8$, $p_0 = 1$, $K_c = 0.004626$, $q_{res} = 0.001$, $\dot{V}_a = 0$, $q_{pump} = 0.01$, $q_{back} = 0.003$. The parameters F_a and q_{res} need not be known in the controller design. The design objective is to stabilize p_{bit} at the desired set point $p_{ref} = 310(bar)$. With the proposed adaptive observer and controller, we take the following set of design parameters: $l_1 = 10^{-5}, C_1 = 0.01, k_1 = 0.01, \Gamma = diag\{6.95\times10^3, 0.0226, 0.012\}, \gamma = 10^{-5}$. The initials are set as $p_p(0) = 120, p_c(0) = 70, q_{bit}(0) = 0.014, \hat{q}_{bit}(0) = 0$, $\hat{q}_{res}(0) = 1.2q_{res}$ and $\hat{F}_a(0) = 0.6F_a$, respectively. Figure 2 shows the annular downhole pressure p_{bit}, $\hat{p}_{bit}$ and p_{ref} and the choke opening z_c. Figure 3 shows the parameter estimations. Clearly, the annular pressure asymptotically tracks the pressure reference and parameter convergence is achieved.

The proposed nonlinear observer-based controller has been tested on WeMod, a simulator based on a distributed parameter model of the fluid dynamics in the well [13]. The model (1)–(3) was fitted to steady state data resulting in the parameter values in Table I and the initials $V_a(0) = 100$ and $\dot{V}_a(0) = 0$. We turn on the observer at $t = 5min$ with initials $\hat{q}_{bit}(0) = 1/600$, $\hat{\Theta}(0) = [0.8\times F_a/M; 0; 0]$ and design parameters $l_1 = 10^{-4}$ and $\Gamma = diag(10000, 0.001, 0.001)$. The controller starts at $t = 20min$ with design parameters $C_1 = 0.05$, $k_1 = 3*10^{-7}$ and $\gamma = 10^{-6}$. The set point is changed from 340 to $300(bar)$ at $t = 25min$. The pump is changed from 1500 to $500(l/min)$ at $t = 30min$. From $t = 50min$ to $t = 51min30s$ approximately $26m$ of the drill pipe is pulled out of the bore hole. Figure 4 shows the pump pressure p_p, the choke pressure p_c and the pump flow u_p, the annular downhole pressure p_{bit} and $\hat{p}_{bit}$, the flow through the bit q_{bit} and $\hat{q}_{bit}$, and the actual and desired choke opening z_c. From Figure 4 we can see that the controller is able to suppress the changes in downhole pressure with maximum deviation from the desired set point $300(bar)$ of approximately $5(bar)$. The desired choke opening (z_c) is calculated from (64), but there is a difference between the desired choke opening and the actual choke opening due to the additional actuator dynamics in WeMod. The simulation results show that the presented model can fit the data and the annular pressure can track the pressure reference well with the proposed controller.

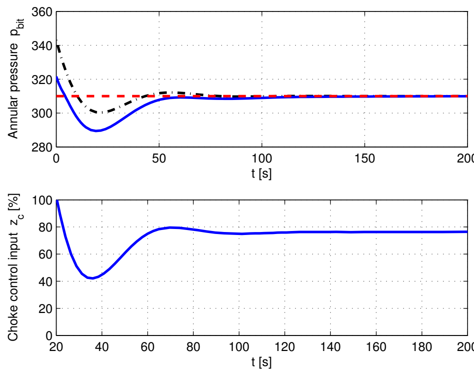

Fig. 2. Simulations of observed-based stabilization with adaptive controller. (p_{bit}(solid), $\hat{p}_{bit}$(dashed-dot) and p_{ref} (dashed))

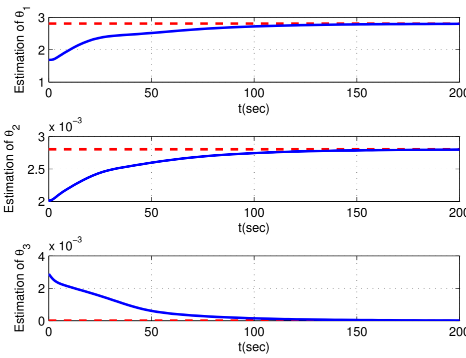

Fig. 3. Parameter estimations with adaptive nonlinear observer.

TABLE I

PARAMETER VALUES WITH WEMOD

Parameter	Value	Description
V_d	26.7	Volume drill string (m^3)
β_d	13000	Bulk modulus drill string (bar)
β_a	7300	Bulk modulus drill string (bar)
ρ_a	0.0125	Density annulus ($10^{-5}\frac{kg}{m^3}$)
ρ_d	0.0125	Density drill string ($10^{-5}\frac{kg}{m^3}$)
F_d	170000	Friction factor drill string ($\frac{bar\ s^2}{m^6}$)
F_a	16000	Friction factor annulus ($\frac{bar\ s^2}{m^6}$)
M_a	1600	Density per meter of annulus ($10^{-5}\frac{kg}{m^4}$)
M_d	6000	Density per meter of drill string ($10^{-5}\frac{kg}{m^4}$)
h_{bit}	2010	Vertical depth of bit (m)

VI. CONCLUSIONS

During well drilling, the annular downhole pressure should be precisely controlled throughout the well bore continuously while drilling. A choke valve and a back pressure pump is used to control the annular pressure. This paper presents a nonlinear adaptive observer control applied to stabilize the annular pressure. A simple model is used to capture the dominant phenomena of the drilling system and for the observer and model-based control design. A new nonlinear adaptive observer-based control is developed to stabilize the annular pressure and achieve asymptotic tracking. The

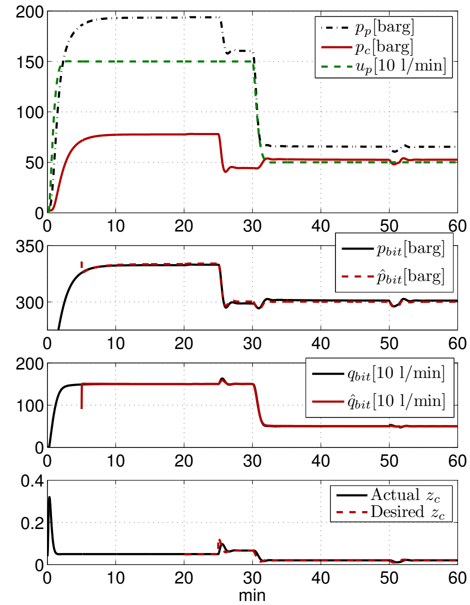

Fig. 4. Simulations of observed-based control with WeMod

simulation results are presented to illustrate the effectiveness of the proposed control scheme.

REFERENCES

[1] G. Nygaard, *Multivariable process control in high temperature and high pressure environment using non-intrusive multi sensor data fusio*. PhD Thesis, Norwegian University of Science and Technology, 2006.

[2] G. Nygaard and G. Nævdal, "Modelling two-phase flow for control design in oil well drilling," in *Proceedings of the 2005 IEEE Conference on Control Applications*, Toronto, Canada,, 2005, pp. 675–680.

[3] Ø. N. Stamnes, J. Zhou, G.-O. Kaasa, and O. M. Aamo, "Adaptive observer design for the bottomhole pressure of a managed pressure drilling system," in *Proceedings of the 47th IEEE Conference on Decision and Control*, 2008.

[4] Y. Tan, I. Kanellakopoulos, and Z. P. Jiang, "Nonlinear observer/controller design for a class of nonlinear systems," in *Proceedings of the 37th IEEE Conference on Decision and Control*, Florida U.S., 1998, pp. 2503–2508.

[5] B. Yao and L. Xu, "Observer-based adaptive robust control of a class of nonlinear systems with dynamic uncertainties," *International Journal of Robust and Nonlinear Control*, vol. 11, pp. 335–356, 2001.

[6] J. Zhou, G.-O. Kaasa, and O. M. Aamo, "Nonlinear adaptive observer control for a riser slugging system in unstable wells," in *Proceedings of American Control Conference*, USA, 2008, pp. 2951–2956.

[7] G.-O. Kaasa, V. Alstad, J. Zhou, and O. M. Aamo, "Attenuation of slugging in unstable oil wells by nonlinear control," in *Proceedings of the 17th IFAC World Congress*, Seoul, Korea, 2008, pp. 6251-6256.

[8] G.-O. Kaasa, "A Simple Dynamic Model of Drilling for Control," *Technical Report*, StatoilHydro Resarch Centre Porsgrunn, 2007.

[9] H. K. Khalil, *Nonlinear Systems*. U.S.: 3rd ed, Prentice-Hall, 2002.

[10] A. Micaelli and C. Samson, "Trajectory tracking for unicycle-type and two-steering-wheels mobile robots," *Rapport de recherche 2097, INRIA, Sophia-Antipolis, France*, 1993.

[11] A. A. J. Lefeber, *Tracking control of nonlinear mechanical systems*. PhD Thesis, Proefschrift Universiteit Twente, Enschede, 2000.

[12] E. Panteley, M. A. Lorıa, and A. Teel, "Relaxed persistency of excitation for uniform asymptotic stability," *IEEE Transactiosn on Automatic Control*, vol. 46, pp. 1874–1886, 2001.

[13] G. Nygaard and J. E. Gravdal, "Wemod for matlab user's guide," International Research Institute of Stavanger, Bergen, Norway, Tech. Rep. Publ. nr. 2007/234, 2007.