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ARGA Loop Pairing Criteria for Multivariable Systems

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***Abstract*—An important aspect in the control of multivariable (MIMO) systems is the pairing of the input and output variables for decentralized control. This paper addresses the problem of pairing if performance of the system is degraded, or modified, as changes or abrupt variations occur in the interacting loops. Typical cases are stability of the process in the face of one or more loop failures – the integrity problem, or if one or more control loops are switched to 'manual' by operators for maintenance and all other loops are maintained operating in stable conditions. Therefore a nonlinear, eventually time varying component is introduced inside each control loop to model such operations and the problem is solved in the framework of absolute stability theorems, mainly by using the Cook circle theorems. A global index, called Absolute Relative Gain Array (ARGA), is proposed for solving both integrity problems and robustness to parameter variations and nonlinearity effects. ARGA index has been tested on several examples from the literature.**

I. Introduction

An important issue in multivariable (MIMO) control in industrial systems is the pairing of the input and output variables for decentralized multi-loop SISO controllers [1]–[3]. The problem of loop pairing between controlled and manipulated variables is usually solved by the Relative Gain Array (RGA) method, introduced in 1966 by Bristol [4] and its several extensions.

Many improved approaches, RGA-like, have been proposed and described in all process control textbooks, for defining different measures of dynamic loop interactions.

In [5] a global index, called Relative Omega Array (ROmA), was proposed. The logic behind ROmA index was to measure interactions in MIMO systems, capturing information from critical frequencies variation in the passage from open loop to closed loop.

A decentralized structure is usually preferred for large scale industrial processes, since it is simple and requires few parameters to tune; moreover especially in case of sensor or actuator failures a process engineer can easily modify the controller parameters in order to counteract the abnormal operating condition.

This paper analyzes the problem of the integrity of the process, i.e., its stability robustness in the face of one or more loop failures. From an operating point of view, failure tolerance with respect to sensor or actuator failures is a critical condition in the life of an industrial plant, but often non-nominal conditions have to be considered, i.e. cases where one or more control loops are switched to 'manual' by operators for maintenance and all other loops are maintained operating in stable conditions.

Based on RGA-like methods many authors proposed solutions for eliminating unstable pairings under failure conditions, usually considering only the static problem (see [6] and references therein).

The main objective of this paper is to provide a method for solving both integrity problems and robustness to parameter variations and nonlinearity effects; the technique proposed relies on the theoretical framework of nonlinear MIMO circle theorems.

The key step in the development of this method is the characterization of the abnormal conditions due to sensor or actuator failures or to the presence of nonlinear constraints (e.g.: saturations on the actuators) as a nonlinear, eventually time varying component inside each control loop. The nonlinear component can be time-dependent and even history dependent, but its response is confined within a certain sector ([7], [8]).

This method retains the characteristics of ROmA index, able to capture information about dynamic interactions and extends such index including integrity problems, in the framework of absolute stability theorems mainly by using the MIMO circle theorems, as proposed by Cook [9].

The new index, called ARGA (Absolute Relative Gain Array), solves the pairing problem taking into account the bounds of the absolute stability in the presence of dynamical interactions.

II. ARGA index: definition

The first step for defining ARGA index is the characterization of the nonlinear components. A failure condition in a loop is modeled by the diagonal matrix N of nonlinear terms as in the hypotheses of the circle theorems.

Therefore the nonlinear components are described via an algebraic input/output function within a limited sector.

Note that this approach includes all the cases due to loop changes originated by saturations in the loop or by manual exclusions of single loops for maintenance purposes.

A loop shut down can be viewed as the transition from a

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condition of a short circuit (output equals input) to a condition of an open circuit (output equals zero) in an electrical circuit. Therefore if a loop has a ‘soft’ or ‘hard’ disconnection, it can be represented by a non linear function confined within a sector [0, k].

Consider now a MIMO system, reachable, observable and open loop stable, described by the n x n transfer function matrix $G(s) = \{g_{ij}(s)\}$.

For each element $g_{ij}(s)$ of the transfer function matrix the critical frequency $\omega_{\pi,ij}$ and the limit gain for absolute stability k_{ij} are evaluated in the hypothesis of non interacting loop and of nonlinearities confined within a sector [0, k].

This choice stems from the observation that in single-input single-output systems the critical frequency $\omega_{\pi,ij}$ (rad/s) remains unchanged in the passage from open loop to closed loop. This property holds also for MIMO systems, whenever perfect decoupling occurs. Then we use critical frequencies in the passage from open loop to closed loop, for measuring interactions in MIMO systems.

Difficulties in critical frequencies computation may eventually arise, since $\omega_{\pi,ij}$ does not necessarily exist or is computable. In such cases, as shown in [5], an additional time delay may be inserted in all channels.

A preliminary evaluation of the critical frequency $\omega_{\pi,ij}$ and of the limit gain k_{ij} in case of circle criterion may be estimated from the Nyquist plot of the frequency response $g_{ij}(j\omega)$, as shown in Fig.1.

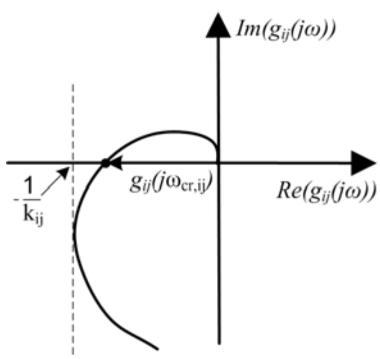

Fig.1. Critical frequency and stability bound in case of circle criterion and nonlinearity belonging to sector [0, k_{ij}]

Bringing process control fundamentals into practice, Harriott [10] in 1964 noted that the performance is proportional to ω_π for a well designed control loop, where ω_π is the pulsation where the phase diagram of frequency response equals $-\pi$.

Moreover an interesting result [11] was that performance indices of the indicial response (both IAE and ISE indices) show an inverse dependence on ω_π. After extensive tests it was verified that the best controller performance is achievable above all by a suitable reshaping of the frequency response.

Therefore the selection of ω_π as a meaningful parameter for describing the dynamic behavior of the process with performance indices is consistent and consequential.

A second step of the method estimates the interactions of the other loops on the i-j channel. In the frequency domain the method of Cook [9] is considered. It employs the symmetric Gershgorin bands, for quantifying interactions. Gershgorin bands are superimposed on Cook circles centered on the appropriate point of each diagonal locus of the linear system $G(j\omega)$.

Absolute stability is guaranteed if the Gershgorin bands do not contain or intersect critical circles in the Rosenbrock sense, i.e., in case of open loop stable systems each critical circle must be external to the Gershgorin band.

Cook circles are built to verify a dominance condition defined as

$$R_{ij}(\bar{\omega}) = \frac{1}{2}\left\{ \sum_{\substack{k=1 \\ k\neq j}}^{n} \left|g_{ik}(j\bar{\omega})\right| + \sum_{\substack{k=1 \\ k\neq i}}^{n} \left|g_{kj}(j\bar{\omega})\right| \right\} \tag{1}$$

where $\forall\bar{\omega}$ a mean row and column dominance is considered. Note that only the mean dominance case allows one to use the same critical circles as would appear in the original circle theorem, applied to each loop separately [9]. Cases of row, or column dominance would require enlarging critical circles by stability multipliers not a priori known, making absolute stability a difficult task to be fulfilled.

A new performance index is therefore introduced as:

$$a_{ij} = \omega_{\pi,ij} \cdot k_{ij} \tag{2}$$

A new evaluation of the critical frequency $\bar{\omega}_{\pi,ij}$ and of the limit gain $\bar{k}_{ij}$ is then performed.

A graphical interpretation of the Cook method for extracting the limit gain is shown in Fig. 2.

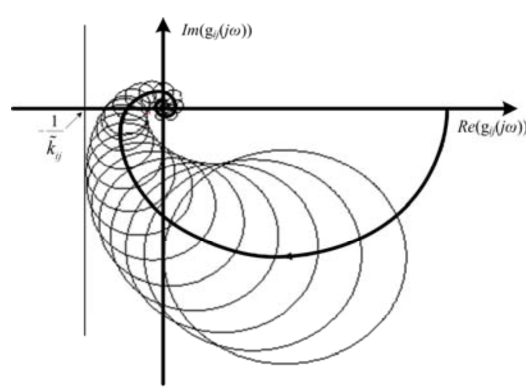

Fig.2. Stability bound in case of Cook circles and nonlinearity belonging to sector [0, $\bar{k}_{ij}$]

Note that the critical frequency $\bar{\omega}_{\pi,ij}$ is evaluated as the frequency for which the corresponding Cook circle intersects the negative real axis most on the left (see Fig.3).

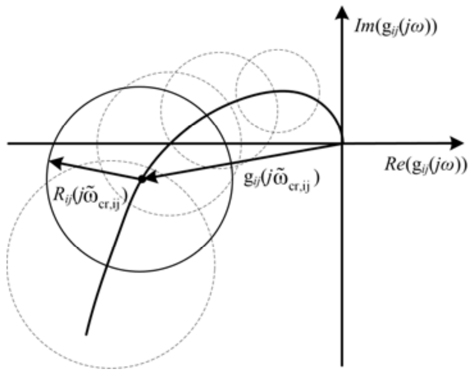

Fig.3. Evaluation of the critical frequency $\bar{\omega}_{cr,ij}$ in case of Cook circles

This result is quite conservative, and it represents the worst case, when the interaction of the other loops on stability is maximum: this consideration leads to the safest choice of pairings.

A product based on the interaction measurement is:

$$f_{ij} = \bar{\omega}_{\pi,ij} \cdot \bar{k}_{ij} \tag{3}$$

By mimicking the RGA procedure, the products (3) are considered for creating a new matrix $F = \{f_{ij}\}$ and the pairings can be easily verified introducing the matrix ARGA, in a way analogous to the RGA definition:

$$\Psi = F \otimes F^{-T} \tag{4}$$

where $\otimes$ denotes Hadamard (element-by-element) product.

Note that the ARGA matrix retains all the properties of the RGA matrix and is purely numerical.

The most important rules for pairings can be summarized as:

1. elements of ARGA matrix closest to 1 suggest the preferred pairings
2. all elements of ARGA matrix chosen for pairings must be positive
3. elements of ARGA matrix with values much greater than 1 should be considered indices of incorrect pairing

Note that, as in RGA or in ROmA indices, ARGA index is invariant if we consider $G(s)\cdot e^{-s\theta}$ instead of $G(s)$, where θ is an arbitrary small time delay. Indeed for a given matrix of nominal transfer functions it is possible that oscillation doesn't occur.

In practice, if the control is networked or is remote, we must always take into account some delay. Difficulties in computations of the closed loop bandwidth may arise, since $\omega_{\pi,ij}$s do not necessarily exist or are computable. In such cases, as an application of the invariance property, an additional time delay may be inserted in all channels: the simplest choice is to select the minimal θ useful for ARGA computation.

III. ARGA INDEX: EXAMPLES OF APPLICATION

ARGA method has been tested on several examples from the literature: to illustrate the method 4 cases varying from 2 x 2 systems up to 4 x 4 systems are proposed.

Example 1. Wood and Berry process [12]:

$$G(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix}$$

The steady-state RGA is:

$$\Lambda = \begin{bmatrix} 2.0094 & -1.0094 \\ -1.0094 & 2.0094 \end{bmatrix}$$

The ROmA index is:

$$\Psi = F \otimes F^{-T} = \begin{bmatrix} 1.1133 & -0.1133 \\ -0.1133 & 1.1133 \end{bmatrix} = ROmA$$

The matrix F is:

$$F = \begin{bmatrix} 0.2405 & 0.0355 \\ 0.0029 & 0.0891 \end{bmatrix}$$

and ARGA index is:

$$\Psi = F \otimes F^{-T} = \begin{bmatrix} 1.005 & -0.005 \\ -0.005 & 1.005 \end{bmatrix} = ARGA$$

Pairings suggested by the ROmA matrix are in perfect agreement with the RGA rule and both of them suggest the use of a diagonal pairing (y_1-u_1, y_2-u_2) in good agreement with the physical behavior of the process.

ARGA index confirms such pairing also in terms of integrity and of absolute stability.

Example 2 Process described by the following matrix transfer function [13]:

$$G(s)=\begin{bmatrix}\frac{e^{-s}}{1+s} & \frac{1}{1+s}\\ \frac{-1}{1+s} & \frac{e^{-2s}}{1+s}\end{bmatrix}$$

The steady-state RGA is:

$$\Lambda=\begin{bmatrix}0.5 & 0.5\\ 0.5 & 0.5\end{bmatrix}$$

The ROmA index is:

$$\Psi=F\otimes F^{-T}=\begin{bmatrix}-0.0461 & 1.0461\\ 1.0461 & -0.0461\end{bmatrix}=\text{ROmA}$$

By comparing this matrix with the traditional RGA matrix it may be observed that the suggested pairings is the off-diagonal one: y_1-u_2, y_2-u_1. This result is in good agreement with the results of Meeuse [13], in a critical case where the steady-state RGA does not suggest any preferential pairing.
The matrix F is:

$$F=\begin{bmatrix}0.7778 & 1.7663\\ 1.7663 & 0.4977\end{bmatrix}$$

and ARGA index is:

$$\Psi=F\otimes F^{-T}=\begin{bmatrix}-0.1417 & 1.1417\\ 1.1417 & -0.1417\end{bmatrix}$$

ARGA index again confirms the off-diagonal pairing suggested by ROmA also in terms of integrity and of absolute stability.

Example 3 Process described by the following matrix transfer function [14]:

$$G(s)=\begin{bmatrix}\frac{2.5e^{-5s}}{(1+15s)(1+2s)} & \frac{1}{1+4s}\\ \frac{1}{1+3s} & \frac{-4e^{-5s}}{1+20s}\end{bmatrix}$$

The steady-state RGA is:

$$\Lambda=\begin{bmatrix}0.9091 & 0.0909\\ 0.0909 & 0.9091\end{bmatrix}$$

It suggests the use of a diagonal pairing (y_1-u_1, y_2-u_2), but a practical implementation [14] leads to off-diagonal pairing (y_1-u_2, y_2-u_1).

In this example the static RGA doesn't suggest the correct pairing.

Matrix ROmA is:

$$\Psi=F\otimes F^{-T}=\begin{bmatrix}-0.0012 & 1.0012\\ 1.0012 & -0.0012\end{bmatrix}$$

An off-diagonal pairing is suggested, opposite to the inaccurate pairing given by the steady-state RGA.
The matrix F is:

$$F=\begin{bmatrix}0.1236 & 3.6628\\ 4.7134 & 0.1466\end{bmatrix}$$

and ARGA index is:

$$\Psi=F\otimes F^{-T}=\begin{bmatrix}-0.0011 & 1.0011\\ 1.0011 & -0.0011\end{bmatrix}$$

ARGA index again confirms the off-diagonal pairing suggested by ROmA also in terms of integrity and of absolute stability.

In the following figures Nyquist diagrams including Cook circles which define the Gershgorin bands are represented, for helping the reader to visualize the dominance properties.

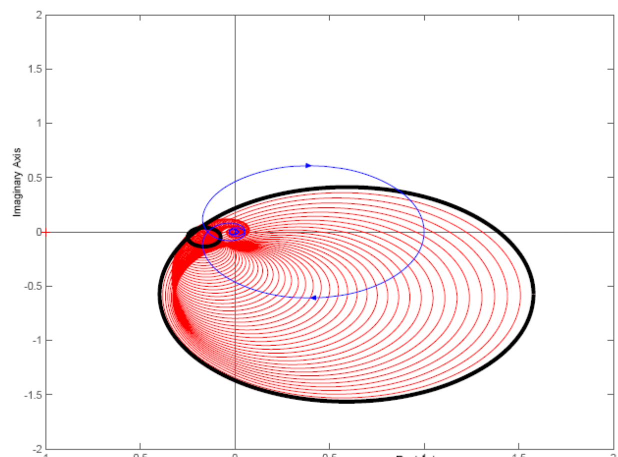

Fig. 4. Nyquist plot of element 1-2 (pairing 1-2 / 2-1) .

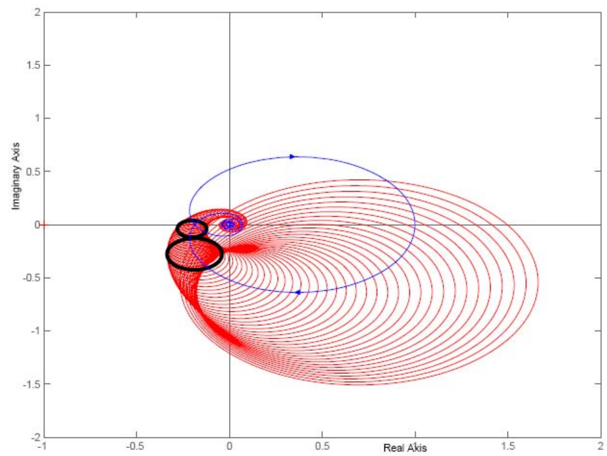

Fig. 5. Nyquist plot of element 2-1 (pairing 1-2 / 2-1).

From Fig. 4 and Fig.5 it can be shown how large are the margins of absolute stability in both loops chosen for pairing in case of example 3.

In bold line are represented the limit cases for the evaluation of the critical parameters $\bar{\omega}_{\pi,ij}$ and $\bar{k}_{ij}$.

The sectors of absolute stability are: [0, 2.5141] for the element 1-2 and [0, 2.974] for the element 2-1.

In Fig. 6 and Fig.7 the case of incorrect diagonal pairing is represented.

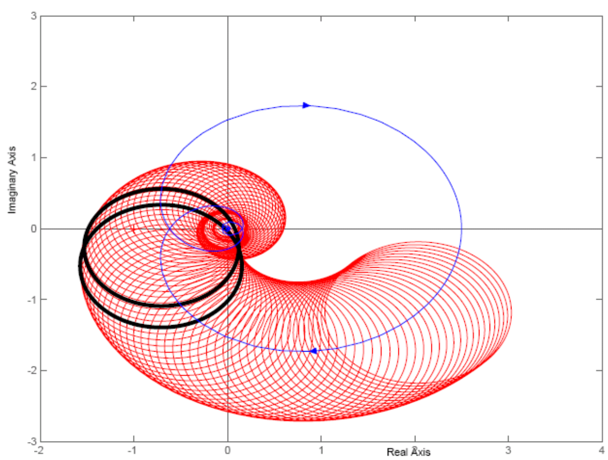

Fig. 6. Nyquist plot of element 1-1 (pairing 1-1 / 2-2).

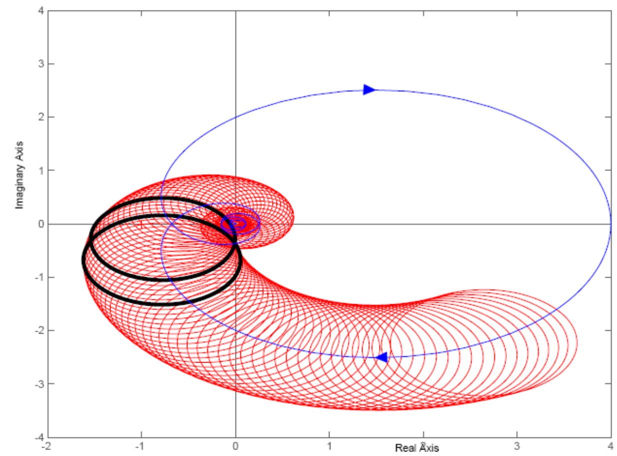

Fig. 7. Nyquist plot of element 2-2 (pairing 1-1 / 2-2)

The sectors of absolute stability are: [0, 0.635] for the element 1-1 and [0, 0.617] for the element 2-2.

Comparing the sectors in case of off-diagonal and diagonal pairings, it can be noted that the off-diagonal pairing is the correct choice in terms of absolute stability integrity and therefore of integrity to abnormal operating conditions.

Example 4 Process described by the following matrix transfer function [15]:

$$G(s)=\frac{1}{(s+1)(2s+1)^2(0.5s+1)}\begin{bmatrix}0.5 & -0.6 & 0.1\\ 0.2 & 0.8 & 0.3\\ -1.0 & 0.1 & 1.0\end{bmatrix}$$

The steady-state RGA is:

$$\Lambda=\begin{bmatrix}0.5020 & 0.3911 & 0.1069\\ 0.1591 & 0.6258 & 0.2151\\ 0.3390 & -0.0169 & 0.6780\end{bmatrix}$$

It suggests the use of a diagonal pairing (y_1-u_1, y_2-u_2, y_3-u_3).

The matrix F is:

$$F=\begin{bmatrix}0.5769 & 0.9343 & 0.0771\\ 0.0764 & 1.2283 & 0.1088\\ 0.8242 & 0.0531 & 0.9826\end{bmatrix}$$

and ARGA index is:

$$\Psi=F\otimes F^{-T}=\begin{bmatrix}1.1019 & 0.0217 & -0.1236\\ -0.1110 & 0.9831 & 0.1279\\ 0.0091 & -0.0048 & 0.9957\end{bmatrix}$$

ARGA index again confirms the diagonal pairing in terms of integrity and of absolute stability.

Example 4 Process described by the following matrix transfer function [16]:

$$G(s)=\begin{bmatrix} \frac{-9.811e^{-1.59s}}{11.36s+1} & \frac{0.374e^{-7.75s}}{22.22s+1} & \frac{-2.368e^{-27.33s}}{33.3s+1} & \frac{-11.3e^{-3.79s}}{(21.74s+1)^2} \\ \frac{5.984e^{-2.24s}}{14.29s+1} & \frac{-1.986e^{-0.71s}}{66.67s+1} & \frac{0.422e^{-8.72s}}{(250s+1)^2} & \frac{5.24e^{-60s}}{400s+1} \\ \frac{2.38e^{-0.42s}}{(1.43s+1)^2} & \frac{0.0204e^{-0.59s}}{(7.14s+1)^2} & \frac{0.513e^{-s}}{s+1} & \frac{-0.33e^{-0.68s}}{(2.38s+1)^2} \\ \frac{-11.3e^{-3.79s}}{(21.74s+1)^2} & \frac{-0.176e^{-0.48s}}{(6.9s+1)^2} & \frac{15.54e^{-s}}{s+1} & \frac{4.48e^{-0.52s}}{11.11s+1} \end{bmatrix}$$

The steady-state RGA is:

$$\Lambda=\begin{bmatrix} 0.1264 & -0.1013 & -0.0314 & 1.0063 \\ 0.0107 & 1.0935 & 0.0003 & -0.1045 \\ 0.7264 & 0.0025 & 0.1630 & 0.1081 \\ 0.1366 & 0.0054 & 0.8680 & -0.0099 \end{bmatrix}$$

It suggests the use of a pairing (y_1-u_4, y_2-u_2, y_3-u_1, y_4-u_3).

The matrix F is:

$$F=\begin{bmatrix} 0.2757 & 0.0021 & 0.0014 & 0.0079 \\ 0.0165 & 0.2189 & 0.0002 & 0.0008 \\ 0.0833 & 0.0347 & 0.0237 & 0.0309 \\ 0.0032 & 0.0069 & 0.2687 & 0.0402 \end{bmatrix}$$

and ARGA index is:

$$\Psi=F\otimes F^{-T}=\begin{bmatrix} 1.0910 & -0.0006 & 0.0024 & -0.0928 \\ 0.0023 & 1.0024 & 0.0002 & -0.0048 \\ -0.0935 & -0.0018 & -0.1414 & 1.2368 \\ -0.0003 & 0.0001 & 1.1389 & -0.1392 \end{bmatrix}$$

ARGA index suggest a pairing (y_1-u_1, y_2-u_2, y_3-u_4, y_4-u_3) different from static RGA in terms of integrity and of absolute stability.

IV. Conclusion

In this paper a new approach for the selection of the pairings between input and output variables in decentralized MIMO control schemes has been presented.

ARGA index has the peculiarity to address some important items in loop pairing:

- it retains all properties of ROmA index, including information about dynamic interactions with the choice of ω_π the critical frequency as a meaningful parameter for describing the dynamic behavior of the process;
- it introduces nonlinearities inside each loop for modeling changes originated by saturations in the loop or by manual operations of single loops for maintenance purposes or in case of failures. Therefore ARGA index includes the limit gains for ensuring the absolute stability inside a sector including nonlinearities.

ARGA index gives the best pairing in terms of integrity to loop changes: absolute stability is the theoretical framework leading to the safest choice of pairing.

To authors' knowledge, the results obtained could not easily be established in any other way.

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