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Equiangular Navigation and Guidance of a Wheeled Mobile Robot Using Range-only Measurements

Hamid Teimoori Andrey V. Savkin

Abstract— **We consider the problems of a wheeled mobile robot navigation and guidance towards an unknown stationary or maneuvering target using range-only measurements. We propose and study several methods for navigation and guidance termed Equiangular Navigation Guidance (ENG) laws. We give mathematically rigorous proofs of convergence and stability of the proposed guidance laws. The performance is confirmed with computer simulations and experiments with ActivMedia Pioneer 3-DX wheeled robots.**

I. Introduction

Navigation and guidance of mobile robots towards steady or moving objects (targets) is one of the most important areas of robotics that has attracted a lot of attention in recent decades (see e.g. [3], [10], [6] and references therein. However, in most of existing methods, both the line-of-sight angle (bearing) and the relative distance (range) are assumed to be available for navigation and guidance algorithms. There is also a relatively large body of research on navigation and guidance with bearings-only measurements. In contrast, few results on problems of navigation and guidance using range-only measurements were published.

Various problems of navigation, guidance, location estimation and target tracking based on range-only measurements often arise in new wireless networks related applications. In these problems, the only available information about the target is strength measurement of its radio signal [17], [12], [11]. The received radio signal strength approach is a natural low cost solution in these problems whereas Global Positioning Systems fail in indoor navigation where GPS signals cannot be reliably received. Furthermore, in some applications Video or IR based guidance is impossible because the target is either too small to appear in an image frame or located behind an obstacle in indoor applications or too far from the robot in outdoor applications. Such problems become more important with the growing use of mobile robots for deployment and localization of nodes in sensor networks [14], [11] or for localization of various small devices [2]. The received radio signal strength is a function of the range, hence, the range can be estimated using the robust extended Kalman filtering [11], [16]. Furthermore, range-only guidance problems may appear as an effort to reduce the cost of active target tracking or in robust tracking in highly noisy environment where angle measurements are often missing especially in naval applications (see e.g. [15]).

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H. Teimoori, A. V. Savkin, School of Electrical Engineering and Telecommunications, The University of New South Wales, Sydney 2052, Australia h.teimoori@student.unsw.edu.au, a.savkin@unsw.edu.au

In this paper, we propose several similar guidance algorithms for approaching and following both steady and moving targets. The proposed guidance methods have the property that the trajectory of the controlled robot is close to a certain curve called equiangular spiral. Therefore, we call the proposed algorithms Equiangular Navigation Guidance (ENG) laws. We give theoretical analysis of the proposed ENG laws with mathematically rigorous proof of their stability and performance. The applicability and performance of the proposed method is also confirmed by computer simulations and robotic experiments with ActivMedia Pioneer 3-DX wheeled robots. The videos of our experiments can be found on internet, [4]. We consider the case of wheeled mobile robots, however, the obtained results are obviously applicable to many other mechanical systems described by similar kinematics equations such as unmanned aerial vehicles (UAVs), missiles, space robots, underwater vehicles.

The proofs of all the presented results will be given in the full version of this paper.

II. Problem Description

Let us consider a three-wheeled, non-holonomic mobile robot of Dubin's car type, which moves in a horizontal plane. In a two-dimensional space, the position of the robot can be represented by a triplet $P_R = (X_R, Y_R, \theta_R)$ where (X_R, Y_R) is the location of the middle of the wheel base and θ_R is the heading angle with respect to the reference line. Let V_R be the linear velocity and ω_R the angular velocity of mobile robot. A rolling-without-slippage model is assumed for the robot. The kinematics model is classically given by:

$$\begin{aligned} \dot{X}_R(t) &= V_R(t)cos(\theta_R(t)) \\ \dot{Y}_R(t) &= V_R(t)sin(\theta_R(t)) \\ \dot{\theta}_R(t) &= \omega_R(t) \end{aligned} \tag{2.1}$$

with $U(t) = [V_R(t)\ \omega_R(t)]^T$ as the control vector of the mobile robot, $U(t) \in [V_{min}, V_{max}] \times [-\omega_{max}\ \omega_{max}]$ with $V_1 > 0, V_2 > 0, \omega_{max} > 0$. The equations (2.1) may also describe the kinematics of tactical missiles, space robots or UAVs; see e.g. [19], [7], [8].

The target may be stationary or moving in any direction with the velocity $V_T(t)$. We assume that the robot and the target are moving on a smooth horizontal surface and in an obstacle-free environment. The only available information about the target is the relative distance between the robot and the target. No information about target motion model is available. In particular, the target may be another nonholonomic mobile robot with its position and orientation (X_T, Y_T, θ_T)

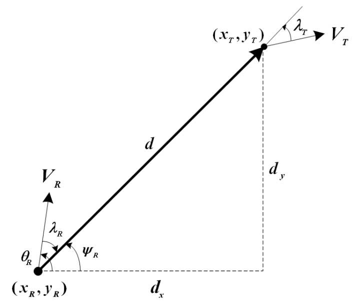


Fig. 1. Robot position and orientation with respect to target

and the same kinematic equation (2.1). We assume that the robot maximum linear speed is greater than the target maximum linear speed. It is obvious that if this assumption does not hold then for any guidance law there exists a target motion such that

$$|X_T(t) - X_R(t)| + |Y_T(t) - Y_R(t)| \to \infty$$

as $t \to \infty$. On the other hand, it should be pointed out that the target may be more maneuvering that the robot, for example, the target may have a smaller minimum turning radius than the controlled robot.

Given the robot position and orientation with respect to the target position in the polar coordination system, we define the relative distance between the robot and target, d, and the angle between the front-direction and the target direction, λ, as shown in Fig. (1)

$$\begin{aligned} d &= \sqrt{d_X^2 + d_Y^2} \\ \lambda_R &= \psi_R - \theta_R \\ \lambda_T &= \psi_R - \theta_T \end{aligned} \tag{2.2}$$

where θ_R and θ_T are the robot and target heading angles, respectively. ψ_R is the line-of-sight angle and $|\lambda_R| \leq \pi$, $|\lambda_T| \leq \pi$. The robot-target motions is expressed by

$$\dot{d} = -V_R cos(\lambda_R) + V_T cos(\lambda_T) \tag{2.3a}$$

$$\dot{\lambda}_R = -\omega_R + \frac{V_R}{d} sin(\lambda_R) - \frac{V_T}{d} sin(\lambda_T) \tag{2.3b}$$

Note that the kinematic equations (2.3) are only valid for non-zero values of the LOS-range, since λ_R is undefined for $d = 0$.

The objective is to design a guidance law that uses only measurements of the relative distance $d(t)$ between the robot and the target and allows the robot to approach a stationery or follow a maneuvering target while keeping a certain distance from the target.

III. Equiangular Navigation Guidance (ENG) Laws

We assume that the distance $d(t)$ to the target and its derivative $\dot{d}(t)$ are both available to the robot controller. Furthermore, we assume that the robot linear velocity is constant:

$$V_R(t) \equiv V_{R0} > 0. \tag{3.4}$$

In this case, the minimal turning radius of the robot is

$$R_{min} = \frac{V_{R0}}{\omega_{max}}. \tag{3.5}$$

We wish to introduce a robot guidance law of the form:

$$\omega_R(t) = \mathscr{F}\left(d(\cdot)\,|_0^t, \dot{d}(\cdot)\,|_0^t\right). \tag{3.6}$$

Of course, our guidance law (3.6) must satisfy the constraint:

$$-\omega_{max} \leq \omega_R(t) \leq \omega_{max}. \tag{3.7}$$

A. Steady Targets

In this subsection, we will consider the case of a steady target:

$$X_T(t) \equiv X_{T0}, \qquad Y_T(t) \equiv Y_{T0}.$$

Definition 3.1: A guidance law of the form (3.6) is said to be encircling if it satisfies (3.7) and for any steady target location the robot (1) guided by this law after a certain finite time moves along a circle of the minimal turning radius (3.5) such that the steady target lies inside this circle.

. We suppose that the following assumption holds:

$$d(0) > 4R_{min}. \tag{3.8}$$

Let L be a given constant such that

$$0 < L < V_{R0}. \tag{3.9}$$

We will use the notation $v(t-0)$ for the limit of the function $v(t)$ at the time t from the left, i.e.,

$$v(t-0) := \lim_{\varepsilon>0, \varepsilon\to 0} v(t-\varepsilon).$$

Also introduce the sign function as follows:

$$sgn(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases} \tag{3.10}$$

Introduce a symbolic variable $S(t)$ that takes values in the set of three symbols $S(t) \in \{A, B, C\}$, and a real variable g that takes values in the set $\{-1, 1\}$. We introduce the following guidance law:

$$S(0) := A;$$

$$\big(S(t) := B \quad \textbf{and} \quad g := sgn(L + \dot{d}(0))\big)$$
$$\textbf{if}\,\big(S(t-0) = A \quad \textbf{and} \quad \dot{d}(t) = -L\big);$$

$$S(t) := C \qquad \textbf{if}$$
$$\left(S(t-0) = B \quad \textbf{and} \quad \frac{V_{R0}}{d(t)}\sqrt{1 - \frac{L^2}{V_{R0}^2}} = \omega_{max}\right);$$

$$\omega_R = \begin{cases} \omega_{max} & S(t) = A \\ g\frac{V_{R0}}{d(t)}\sqrt{1 - \frac{L^2}{V_{R0}^2}} & S(t) = B \\ g\omega_{max} & S(t) = C. \end{cases} \tag{3.11}$$

Remark 3.1: The guidance law (3.11) is based on switching between three modes corresponding to the values A, B and C of the symbolic variable $S(t)$. A robotic system with such guidance law belongs to the class of so-called hybrid

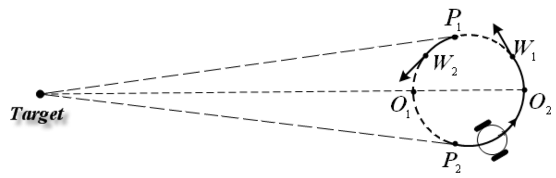


Fig. 2. Geometry of switching role in the first part of the trajectory

dynamical systems, see e.g. [13] and [9] and references therein.

Now we are in a position to present the following theorem.

Theorem 3.1: Let L be a given number. Suppose that assumptions (3.4), (3.9) and (3.8) hold. Then, the guidance law (3.11) is encircling.

The proof of this theorem will be given in the full version of this paper.

Remark 3.2: It follows from the proof of Theorem 3.1 that $\dot{d}(t) = -L$ during the second part of the trajectory when $S(t) = B$. Therefore, if $d(0) >> R_{min}$, the time before "encircling the target" is close to $\frac{d(0)}{L}$. Hence, with a larger L the robot approaches the target with a higher speed. On the other hand, a smaller L makes the guidance law more robust subject to noise and uncertainties in measurements of $d(t)$ and $\dot{d}(t)$. Therefore, the choice of L is determined by a trade-off between the convergence speed and robustness.

We will also present a robust modification of the guidance law (3.11). We introduce the following definition which is applicable to both steady and moving targets

Definition 3.2: Let $D > 0$ be a given constant. A guidance law of the form (3.6) is said to be $D-$approaching for a target if it satisfies (3.7) and there exists a time t_* such that the distance $d(t)$ between the robot and the target satisfies $d(t_*) = D$.

Introduce a symbolic variable $S(t)$ that takes values in the set of three symbols $S(t) \in \{A, B, \mathbf{stop}\}$, and a real variable g that takes values in the set $\{-1, 1\}$. Our guidance law is defined over finite time, and $S(t_*) = \mathbf{stop}$ means that the requirement $d(t_*) = D$ is achieved and the robot stops or switches to another guidance law. Let $k > 1$ be a given constant. We propose the following modification of the guidance law (3.11):

$$S(0) := A;$$
$$\big(S(t) := B \quad \textbf{and} \quad g := sgn(L + \dot{d}(0))\big)$$
$$\textbf{if}\,\big(S(t-0) = A \quad \textbf{and} \quad \dot{d}(t) = -L\big);$$
$$S(t) := \textbf{stop} \quad \textbf{if}$$
$$\left(S(t-0) = B \quad \textbf{and} \quad \frac{kV_{R0}}{d(t)}\sqrt{1 - \frac{L^2}{V_{R0}^2}} = \omega_{max}\right);$$
$$\omega_R = \begin{cases} \omega_{max} & S(t) = A \\ -kgsgn(L + \dot{d}(t))\frac{V_{R0}}{d(t)}\sqrt{1 - \frac{L^2}{V_{R0}^2}} & S(t) = B \end{cases} \quad (3.12)$$

Now we are in a position to present the following analysis result for the guidance law (3.12).

Theorem 3.2: Let L and $k > 1$ be given numbers. Moreover, introduce

$$D := k\sqrt{1 - \frac{L^2}{V_{R0}^2}}R_{min}. \quad (3.13)$$

Suppose that assumptions (3.4), (3.9) and (3.8) hold. Then, the guidance law (3.12) is $D-$approaching.

The proof of this theorem will be given in the full version of this paper. We now introduce the following simple guidance law:

$$\omega_R(t) = \omega_{max} sgn(L + \dot{d}(t)). \quad (3.14)$$

We will need the following assumption:

$$\lambda_R(0) \neq -\arccos\left(\frac{L}{V_R}\right). \quad (3.15)$$

The following theorem gives a mathematical analysis of the guidance law (3.14).

Theorem 3.3: Let L be a given number. Suppose that assumptions (3.4), (3.9), (3.8) and (3.15) hold. Then, the guidance law (3.14) is encircling.

The proof of this theorem will be given in the full version of this paper.

Remark 3.3: It follows from the proofs of Theorems , 3.1, 3.2 and 3.3 that during the most important part of our robot trajectories with the guidance laws (3.11), (3.12) and (3.14) corresponding to the sliding mode with the surface $\dot{d}(t) = -L$, the angle λ_R between the robot heading and the direction to target remains constant. That is why we call all the guidance laws presented in this paper Equiangular Navigation Guidance (ENG) Laws. The geometry of motion with a constant angle between the heading and the direction to the steady target is described by the so-called equiangular spiral; see e.g. [5]). Considering the steady target at the origin, the equiangular spiral is a spiral whose polar equation is given by

$$d = d_o e^{-b\gamma} \quad (3.16)$$

where d is the distance to the origin, and d_0 is the initial distance. Moreover, $b = \cot(\lambda_0)$, where $0 < |\lambda_0| < \frac{\pi}{2}$ is the approaching angle, and γ is the angle between the x-axis and the line the target and the robot, see Fig. (3).

Remark 3.4: Due to the symmetry properties, the guidance law

$$\omega_R = -\omega_{max} sgn(L + \dot{d}) \quad (3.17)$$

has similar performance and characteristics as the guidance law (3.14).

B. Moving Targets

In this subsection, we consider the case of a moving target. The target is another wheeled robot with kinematics described by the equation of the form (2.1). Unlike the controlled robot, the target linear velocity V_T may be time-varying. We assume that that there exists a known constant $V_{T0} > 0$ such that

$$V_T(t) \leq V_{T0} \quad \forall t \geq 0. \quad (3.18)$$

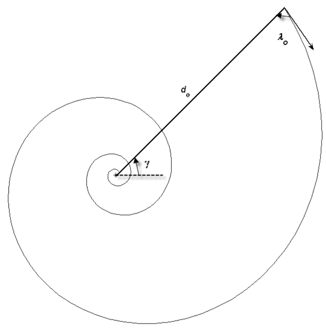


Fig. 3. An equiangular spiral

Furthermore, we suppose that the following assumption holds:

$$V_{R0} > 3V_{T0}. \tag{3.19}$$

We will need a constant L satisfying the constraints:

$$2V_{T0} < L < V_{R0} - V_{T0} \tag{3.20}$$

(such constants do exist due to (3.19)). Furthermore, the gain constant k is required to satisfy the following assumption:

$$k > \frac{\sqrt{1 - \frac{(L-V_{T0})^2}{V_{R0}^2}}}{\sqrt{1 - \frac{L^2}{V_{R0}^2}}}. \tag{3.21}$$

Moreover, since the target is moving we need a stronger version of the assumption (3.8):

$$d(0) > 6R_{min}. \tag{3.22}$$

Now we present an analysis of the guidance law (3.12) for the case of a moving target.

Theorem 3.4: Let L and k be given numbers. Suppose that assumptions (3.4), (3.18), (3.20), (3.21) and (3.22) hold. Then, the guidance law (3.12) is $D-$approaching with D defined by (3.13).

The proof of this theorem will be given in the full version of this paper.

Now we present an analysis of the guidance law (3.14) for the case of a moving target.

Theorem 3.5: Let L be a given number. Suppose that assumptions (3.4), (3.18), (3.20) and (3.22) hold. Moreover, assume that $\lambda_R(0)$ does not belong to the interval

$$\left[-\arccos\left(\frac{L-V_{T0}}{V_{R0}}\right), -\arccos\left(\frac{L+V_{T0}}{V_{R0}}\right)\right].$$

Then, the guidance law (3.14) is $D-$approaching with $D = \frac{4}{3}R_{min}$.

The proof of this theorem will be given in the full version of this paper.

IV. Computer Simulation

In this section, we present computer simulation results for steady and maneuvering targets. For simulation purposes and also throughout the experiment with real robots, we use the guidance law (3.14), due to its simplicity and applicability.

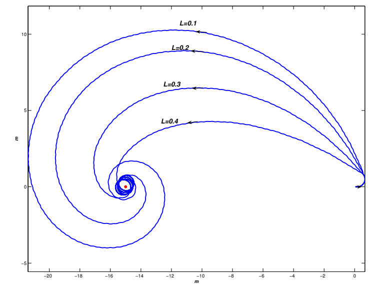


Fig. 4. Approaching a stationary target with different values of L

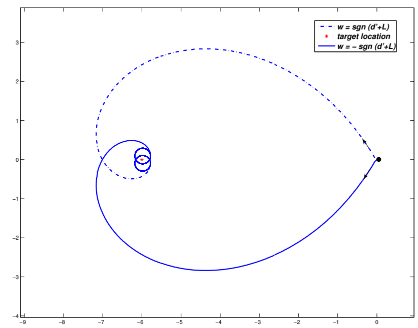


Fig. 5. Symmetry of two trajectories

A. Encircling a steady target

Fig.(4) shows the robot trajectory with different values of L when it moves towards a stationary target with the guidance law (3.14). Since the value of λ_R is nearly constant along the trajectory, the robot trajectory towards the target is a semi-equiangular spiral. The parameter b and hence, the arc length and the curvature change with L where $0 < L < V_R$. Different paths can be generated for different values of L and since it is real, an infinite number of paths is possible. As $L \to 0$, $\lambda_R \to \frac{\pi}{2}$ and as a result $b \to 0$, the robot path becomes more curved and spiral approaches a circle. With smaller values of L, the robot trajectory converges to a circle and as $L \to V_R$, $\lambda_R \to 0$ and the trajectory converges to a straight line.

B. Symmetry of trajectories

Applying the proposed steering controls (3.14) and (3.17), the robot approaches the target in symmetric trajectories. Having applied (3.17), the robot approaches the target with a negative approaching angle λ_R and the spiral's turning direction is clockwise. While, λ_R is positive with (3.14) and the spiral turns counterclockwise. Fig. (5) displays the robot trajectories for steering controls (3.14) and (3.17) with the same control parameter L.

C. Following a maneuvering target

In this simulation, the robot is supposed to approach a moving target with a smaller linear velocity. The target linear and angular velocities are $V_T = 0.1\ m/s$ and $\omega_T = 0.05cos(.001t+1)$, respectively. The robot has a constant

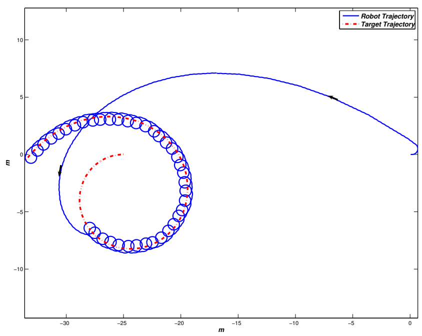
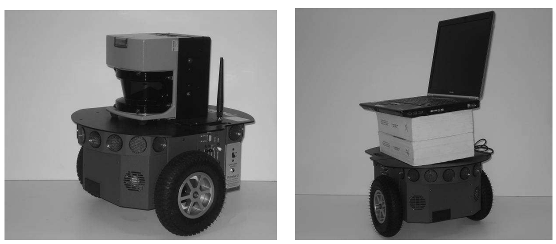


Fig. 6. Moving target following with constant linear velocity



(a) Robot (b) Target

Fig. 7. The pursuer robot is equipped with a SICK laser rangefinder and the target which is connected to the laptop

linear velocity $V_R = 0.5\,m/s$ and we have $\omega_{max} = 1\ rad/sec$. Choosing $L = 0.32$ with respect to the constraint (3.20), the result of simulation has been shown in Fig. (6). Having applied (3.14), the robot approaches the target and eventually goes into a circular trajectory around it.

V. Experimental Results

To verify validity and study the performance of ENG, various tests have been conducted on ActivMedia Pioneer 3-DX robots. The pursuer robot is equipped with a Sick laser scanner which gives the distant to the target if it is inside the field of view of the scanner and is controlled using its on-board PC. The target is connected and controlled via a laptop PC and carries two books to easily be recognized by laser sensor, see Fig. (7). Both pursuer and moving target robots move in an obstacle-free horizontal plane. To implement the guidance algorithm, we use C++ and Active Media Robotics Interface Application (ARIA), which is an Object Oriented C++ library for controlling ActivMedia mobile robots, [1]. Software development environment running in the Linux operating system.

Remark 5.1: Using the laser scanner, although we can also get the angle between the moving direction and target direction, in our experiments, we only use the relative distance between the robot and target.

Remark 5.2: In order to reject the measurement noise effects in generating LOS-range variation, since the scanning frequency is much faster compare to the robot speed, we take the mean of each 10 readings, as the LOS-range at the current time $d(t)$, and use it afterwards to produce $\dot{d}(t)$.

A. Approaching a Stationary target

In this experiment, applying the guidance law (3.14), the robot approaches a stationary target. The robot linear velocity is constant $V_R = 0.3\,m/s$ and $\omega_{max} = 1\ rad/sec$. Fig. (8) shows some snapshots of this experiment, at which the robot approaches the steady target along an equi-angular spiral with $L = 0.1$, and eventually goes into a circular trajectory around the target.

B. Approaching a Moving target

In the second experiment, we consider another mobile robot as the moving target. The parameters we used throughout this experiment, are the same as those we considered during the simulation with moving target in subsection IV-C. Four different snapshots of this experiment are shown in Fig. (9). The pursuer robot and the moving target trajectories are depicted with dotted and dashed lines, respectively. The robot approaches the target and goes into a circular trajectory around it. In order to prevent any collision, the target stops moving when $d < \frac{4}{3}R_{min}$.

Remark 5.3: Experimental results including two videos that demonstrate the application of ENG on approaching stationary and maneuvering targets can be found on Internet [4].

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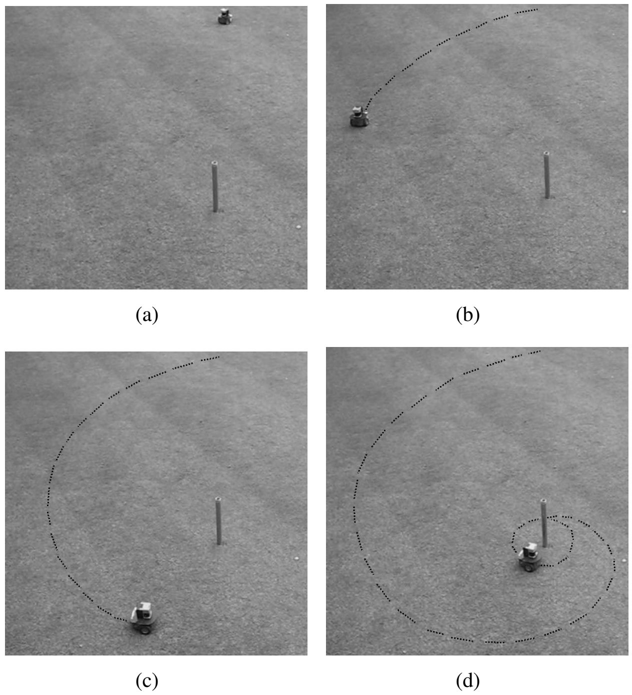

Fig. 8. Approaching a steady target

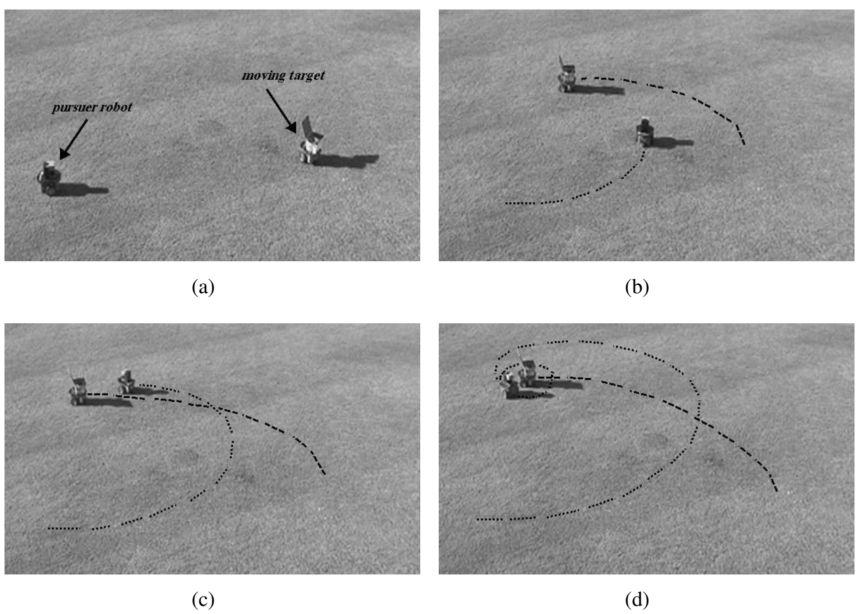

Fig. 9. Approaching a moving target

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