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Multi-valued robust control technique for uncertain systems

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Abstract— **A variety of plants with high parametric uncertainties are usually controlled with signals that may assume only a finite number of values, both to simplify actuator's construction and minimize the operation cost. The design of multi-valued control laws which provide a control signal that is discontinuous in time and quantized in magnitude is then of particular interest in many practical applications.**

This paper presents a new technique for robust control design in order to force a SISO linear plant, subject to disturbances and parametric uncertainties, to track a given sufficiently regular reference trajectory. The proposed approach is based on Lyapunov method and allows designing a control law which guarantees to follow the reference trajectory with prefixed values of the tracking error and of its derivatives up to the $n-1$-th, where n is the order of the plant. Moreover, the control law is quite robust and guarantees the convergence of the error in a prefixed time.

The technique is applied to design controllers characterized by control signals that may assume only a finite number of values. In this case, the control law can be seen as a generalization of the traditional relay control laws and of the sliding mode ones, with a relatively low switching frequency. Finally, a simple example shows the advantages of the control law obtained with the proposed design methodology with respect to the classical approaches.

I. Introduction

In many practical applications plants are controlled with signals that may assume only a finite number of values, the main reason being the choice of utilizing simple and reliable actuators with a relatively low cost and highly performing operation modes. This aspect originates the demand for developing new techniques in order to analyze, design and implement multi-valued controllers, i.e. systems which provide a control signal that is discontinuous in time and quantized in magnitude.

This paper presents a new method for controller synthesis, characterized by the request for a control signal that may assume only a finite number of values. The method allows designing a controller which is able to force a SISO linear plant, belonging to a class of sufficiently general plants and subject to disturbances and parametric uncertainties, to follow a given sufficiently regular reference trajectory.

In [16] and [17] controllers with control signals without amplitude constraints, but constant in assigned time intervals, are presented. In [14], [20], [21] and [22] sliding mode control laws with two or an infinite number of levels and with an infinite switching frequency are proposed. Various authors have studied problems concerning quantized control (see, for example, [7], [2], [8] [18], [1], [10], [11]). In [2] the authors deal with feedback stabilization problems for LTI control systems with saturating quantized measurements. The use of logarithmic quantizers in order to stabilize a discrete system is described in [8]. On the other hand, a uniformly quantized control set is used in [18]. Problems related to the structure of the reachable set for systems whose input sets are quantized are addressed in [1]. In [10] the authors propose some stabilization methods for scalar linear systems by means of static quantized feedback controls, depending on the amount of information flow in the feedback loop. In [11] the authors analyze the stabilization problem for discrete time linear systems with multidimensional state and one-dimensional input using quantized feedbacks with a memory structure.

The control law proposed in this paper solves a general tracking problem, defined in terms of practical tracking, for stable and unstable plants, imposing constraints only on the minimum and maximum values of the control signal, which depend both from the plant and from the amplitude and variability of the reference trajectory. In [6] a similar problem is treated but the method proposed in the work is not very robust and does not allow satisfying specifications about the error derivatives because of a severe limitation in the Lyapunov function used in the control law design.

The proposed control law allows using intermediate levels, which consent reducing the amplitude of the control signal and the average switching frequency. The theory of the practical stability is used to design the controller, with reference not only to the output error but also to its derivatives; this approach often allows satisfying process's vital specifications; in thermal processes, for example, small but fast temperature variations with respect to the reference can generate defects in the manufactured objects (see [19], [15] and [9]).

The paper is organized as follows. Section II introduces the control problem. Section III presents preliminary lemmas, which are used to derive the main result of the paper, presented in Section IV. A discussion regarding the characteristics of the proposed control law is provided in Section V. The multi-valued controller is proposed in Section VI, and some examples show the advantages of the proposed control law. Section VII draws final conclusions.

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II. Problem statement

Consider the continuous-time SISO linear plant

$$y^{(n)} = \sum_{i=1}^{n} a_i(p(t),t)y^{(n-1)} + b(p(t),t)u + d(p(t),t) \quad (1)$$

where: $t \in \wp_t \subseteq \mathbb{R}$ is the time; $u \in \wp_u \subset \mathbb{R}$ is the control input; $y \in \mathbb{R}$ is the output to be controlled; $d \in \wp_d \subset \mathbb{R}$ is the effect of disturbances acting on the plant; $p(t) \in \wp_p \subset \mathbb{R}^\gamma$, $t \in \wp_t$ is a vector of uncertain parameters;

$$a_1(p,t) \quad a_2(p,t) \quad \dots \quad a_n(p,t) \ \in \wp_a \subset \mathbb{R}^n, \quad (2a)$$
$$b(p,t) \cdot sgn(b(p,t)) \in \wp_p \subset \mathbb{R}^+, \quad (2b)$$
$$d(p,t) \in \wp_d, \forall t \in \wp_t,\ \forall p \in \wp_p, \quad (2c)$$
$$\wp_a, \wp_b, \wp_d \text{ compact sets.} \quad (2d)$$

Let $\hat{y}(t)$ be the trajectory that the plant (1) must track, with bounded n-th derivative. The equation of the tracking error vector

$$\epsilon = \begin{pmatrix} \epsilon_1 & \epsilon_2 & \dots & \epsilon_n \end{pmatrix}^T,$$
$$\epsilon_1 = \hat{y} - y,\ \epsilon_i = \epsilon_1^{(i-1)},\ i = 2, \dots, n,$$

can be rewritten as

$$\dot{\epsilon} = E\epsilon - Bw \quad (3)$$

where:

$$E = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \cdot & \cdot & \cdot & \dots & \cdot \\ 0 & 0 & 0 & \dots & 1 \\ -k_1 & -k_2 & -k_3 & \dots & -k_n \end{pmatrix},$$

$$B = \begin{pmatrix} 0 \\ 0 \\ \cdot \\ 0 \\ 1 \end{pmatrix},$$

$$k_i \in \mathbb{R}, i = 1, \dots, n,$$

$$w = b(p,t)u - \sum_{i=1}^{n} (a_i(p,t) + k_i)\epsilon_i + d(p,t) + \\ + \left[\sum_{i=1}^{n} a_i(p,t)\hat{y}^{(i-1)} - \hat{y}(n) \right]. \quad (4)$$

In order to define the control problem, we first introduce the following definition.

Definition 1 (Practical stabilization): Given a reference trajectory $\hat{y}(\cdot)$, a region T_0 (containing the origin of $\mathbb{R}^n$), a region $T_\rho \subset T_0$, $t_0 \in \wp_t$ and $t_c > 0$, we say that system (3)-(4) is practical stabilizable with respect to (t_0, t_c, T_0, T_ρ) if and only if, for all functions $p : \wp_t \to \wp_p$ and $\forall \epsilon_0 \in T_0$, there exists a control law $u(t, \epsilon) : \wp_t \times T_0 \to \wp_u$ such that the solution of the system (3)-(4), denoted by $\epsilon\left(t, t_0, \epsilon_0, u_{[t_0,t]}, p\right)$, is bounded and $\forall t > t_0 + t_c$ satisfies the condition $\epsilon\left(t, t_0, \epsilon_0, u_{[t_0,t]}, p\right) \in T_\rho$.

The general tracking problem is stated as follows.

Problem 1 (Practical tracking problem): Given the plant (1), a reference trajectory $\hat{y}$, a region T_0 (containing the origin of $\mathbb{R}^n$) of admissible initial errors ϵ_0 at time t_0 and a region $T_\rho \subset T_0$ of tolerable errors after the time t_c, design a control law with values in $\wp_u$ that practical stabilizes the associate error system (3)-(4) with respect to (t_0, t_c, T_0, T_ρ).

Imposing the practical stabilization of the error system implies that we aim at bound both the tracking error $\hat{y} - y$ and its derivatives up to the order of the plant minus one. This constraint is often required in practical applications where not only the error needs to be bounded, but also a slow variation of the state trajectory around the reference trajectory is necessary (see, for instance, the problem of ceramic kiln control in [4]).

III. Preliminary results

For the solution of Problem 1 we introduce the following lemmas.

Lemma 1: Let $S_\rho = \{\epsilon \in \mathbb{R}^n : \|\epsilon\|_P \le \rho, \rho > 0\}$ - where $\|\epsilon\|_P = \sqrt{\epsilon^T P \epsilon}$ and $P \in \mathbb{R}^{n \times n}$ is a symmetric and positive definite (p.d.) matrix - be an hyper-ellipsoid of $\mathbb{R}^n$ and T_ρ be the smallest hyper-rectangle including S_ρ and with it faces orthogonal to the coordinated axis.Then the semi-length of the edges of T_ρ parallel to the i-th axes is

$$\bar{\epsilon}_i = \rho\sqrt{p_{ii}^{inv}}, \quad i = 1, \dots, n, \quad (5)$$

where p_{ii}^{inv} denotes the (i,i)-element of the matrix P^{-1}.

Proof: The proof can be found in [3]. ■

Lemma 2: Let $E \in \mathbb{R}^{n \times n}$ be a matrix with v distinct real eigenvalues λ_i, $i = 1, \dots, n$, and $2l = n - v$ distinct complex conjugate eigenvalues $\lambda_{i\pm} = \alpha_i \pm j\omega_i, i = 1, \dots, l$. Moreover, let u_i, $i = 1, \dots, n$ and $u_{i\pm} = u_{ai} \pm ju_{bi}$, $i = 1, \dots, l$ be the corresponding eigenvectors. Then, denoting with Z^* the complex conjugate transposed matrix of $Z \in \mathbb{C}^{n \times n}$, the matrices:

$$P = (ZZ^*)^{-1} = \quad (6)$$
$$= \left[\sum_{i=1}^{v} u_i u_i^T + 2\sum_{i=1}^{l} (u_{ai}u_{ai}^T + u_{bi}u_{bi}^T) \right]^{-1}$$

$$Q = -(Z^*)^{-1}(\Lambda + \Lambda^*)Z^{-1} = \quad (7)$$
$$= -\left[\frac{1}{2}\sum_{i=1}^{v} \frac{1}{\lambda_i} u_i u_i^T + \sum_{i=1}^{l} \frac{1}{\alpha_i}(u_{ai}u_{ai}^T + u_{bi}u_{bi}^T) \right]^{-1}$$

with

$$Z = \begin{pmatrix} u_1 & \dots & u_v & u_{a1} + ju_{b1} & u_{a1} - ju_{b1} & \dots \\ \dots & u_{al} + ju_{bl} & u_{al} - ju_{bl} \end{pmatrix}$$

$$\Lambda = diag\begin{pmatrix} \lambda_1 & \dots & \lambda_v & \lambda_{1+} & \lambda_{1-} & \dots & \lambda_{l+} & \lambda_{l-} \end{pmatrix}$$

satisfy the Lyapunov equation

$$E^T P + PE = -Q. \quad (8)$$

Moreover, if the eigenvalues of E have negative real part, then the matrices P and Q are both p.d. and

$$\lambda_{max}(Q^{-1}P) = -\frac{1}{2\max_{i=1,\ldots,n}\Re(\lambda_i)} = \frac{1}{2}\tau_{max}(E), \tag{9}$$

where $\lambda_{max}(Q^{-1}P)$ denotes the maximum eigenvalue of the matrix $Q^{-1}P$ and $\tau_{max}(E)$ denotes the maximum time constant of the modes of the system $\dot{\epsilon} = E\epsilon$.

Proof: The proof can be found in [3]. ■

Lemma 3: Consider the system (3) with $\epsilon_0 \in T_0$, where all the eigenvalues of $E \in \mathbb{R}^{n\times n}$ are distinct and with negative real part, $B \in \mathbb{R}^{n\times 1}$ and P is given by (6). Let us define the linear function of the tracking error and of its derivatives $v = B^T P\epsilon$ and two subsets S_σ and S_ρ of $\mathbb{R}^n$ such that

$$S_\sigma = \{\epsilon \in \mathbb{R}^n : \|\epsilon\|_P \le \sigma, \sigma > 0\} \supseteq T_0,$$
$$S_\rho = \{\epsilon \in \mathbb{R}^n : \|\epsilon\|_P \le \rho, 0 < \rho < \sigma\} \subseteq T_\rho.$$

If

$$v \cdot w \ge 0 \quad \forall \epsilon \notin \mathring{S}_\rho, \tag{10}$$

where $\mathring{S}_\rho$ denotes the interior of S_ρ, then the system (3) is finite-time practically stable with respect to (t_0, t_c, T_0, T_ρ) for every $t \in \wp_t$ and

$$t_c \ge \tau_{max}(E)\ln\frac{\sigma}{\rho}. \tag{11}$$

Proof: The Lyapunov function $V(\epsilon) = \epsilon^T P\epsilon$ for the system (3) is chosen. Taking into account (8), it results $-\dot{V}(\epsilon) = \epsilon^T Q\epsilon + 2vw$. Using (10) it follows that

$$\frac{\dot{V}(\epsilon)}{V(\epsilon)} \le -\inf_{\epsilon}\frac{\epsilon^T Q\epsilon}{\epsilon^T P\epsilon} \quad \forall \epsilon \notin \mathring{S}_\rho.$$

Since

$$\frac{\epsilon^T Q\epsilon}{\epsilon^T P\epsilon} \ge \frac{1}{\lambda_{max}(Q^{-1}P)} \quad \forall \epsilon \ne 0,$$

for any symmetric and p.d. matrices P and Q (see, for example [13]), and by Lemma 2 it results

$$\frac{\dot{V}(\epsilon)}{V(\epsilon)} \le -\frac{1}{\tau_{max}(E)} \quad \forall \epsilon \notin \mathring{S}_\rho,$$

and then

$$\|\epsilon(t)\|_P \le \|\epsilon(t_0)\|_P \exp\left(-(t-t_0)/\tau_{max}(E)\right). \tag{12}$$

From last inequality it follows that ϵ converges into the hyper-ellipsoid S_ρ in a time not greater than

$$t_{c0} = \tau_{max}(E)\ln\frac{\|\epsilon(t_0)\|_P}{\rho}.$$

Since $\epsilon_0 \in S_\sigma$, the proof easily follows. ■

Remark 1: It is important to note that the matrix P given by (6) is optimal with respect to the estimation of the convergence velocity, according to the Lyapunov approach, of the system $\dot{\epsilon} = E\epsilon$. This is due to the fact that the time constant of $\|\epsilon(t)\|_P$ coincides with the maximum time constant of E (see (12)).

Remark 2: The Lemma 3 can be extended to the case where sliding mode occurs on the surface $v = 0$ for $\epsilon \notin \mathring{S}_\rho$. In this case, the time derivative of the Lyapunov function is to be computed along the sliding surface governed by the differential equation $\dot{\epsilon} = E\epsilon - Bw_{eq}$, where $w_{eq} = (p_{nn})^{-1}B^T PE\epsilon$, derived according to the equivalent control method [20]. The proof follows considering that on that surface the derivative of the Lyapunov function results $\dot{V}(\epsilon) = -\epsilon^T Q\epsilon$.

IV. Control law synthesis

It is now possible to state the following main result.

Theorem 1: Given the plant (1), a reference trajectory $\hat{y}$ with bounded n-th derivative, a region T_0 (containing the origin of $\mathbb{R}^n$) of admissible initial errors ϵ_0 at time t_0 and a region $T_\rho \subset T_0$ of tolerable errors after a prefixed time t_c.

Then it is possible to solve the practical tracking problem with respect to (t_0, t_c, T_0, T_ρ) choosing:

- σ, ρ and the values k_i, $i = 1, \ldots, n$ such that:
  - the eigenvalues of E are distinct and with negative real part and such that t_c in (11) is less or equal to the prefixed one;
  - the region S_σ contains T_0;
  - the region S_ρ is contained in T_ρ;
- the control law (see Fig. 1) $u(t,\epsilon) : \wp_t \times T_0 \to \wp_u$:
  - if $\epsilon \notin \mathring{S}_\rho$, equals to:

$$u = \begin{cases} \lceil U \rceil, & \text{if } \quad v \cdot b(p,t) \ge 0 \\ \lfloor U \rfloor, & \text{if } \quad v \cdot b(p,t) < 0 \end{cases} \tag{13}$$

where:

$$v = B^T P\epsilon,\ P \text{ is defined in (6)}$$
$$U = \frac{\left[\hat{y}^{(n)} - \sum_{i=1}^{n} a_i(p,t)\hat{y}^{(i-1)}\right] - d(p,t)}{b(p,t)} + \frac{\sum_{i=1}^{n}(a_i(p,t)+k_i)\epsilon_i}{b(p,t)}$$
$$\lfloor U \rfloor = max\{u \in \wp_u :\ u < U\ \forall p \in \wp_p\}$$
$$\lceil U \rceil = min\{u \in \wp_u :\ u \ge U\ \forall p \in \wp_p\}$$

  - if $\epsilon \in \mathring{S}_\rho$, equals to the last value assumed on the S_ρ boundary.

(14)

Proof: Consider the differential equation (3) which describes the closed loop system composed by the linear plant and the controller. By applying the Theorems 2, 4 and 5 in [12], it is possible to verify that the solution to (3) exists everywhere in $\mathbb{R}^n$, and for every initial condition.

For the hypothesis (2) the control u computed with (13) provides a signal w, given by (4), which satisfies condition

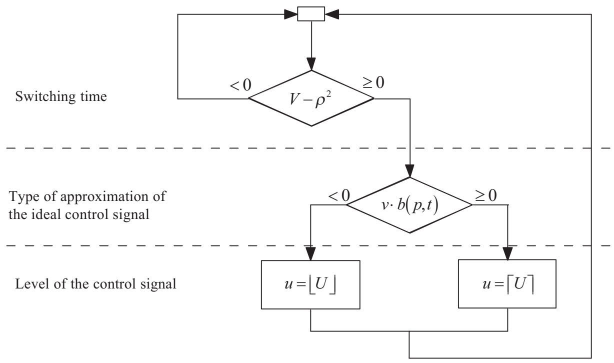

Fig. 1. Control Algorithm (CA).

(10). Then the proof of the theorem easily follows from Lemma 3. ∎

As regards the tracking error, we state and prove the following theorem.

Theorem 2: If the values k_i, $i = 1, \ldots, n$ in the control law of Theorem 1 are chosen such that the eigenvalues λ_i, $i = 1, \ldots, n$ of E are distinct, with negative real part and satisfy

$$\sum_{j=1}^{n} \lambda_j^{i-1} \bar{\lambda}_j^{i-1} \leq \left(\frac{\bar{\epsilon}_i}{\rho}\right)^2 \quad \bar{\epsilon}_i \in \mathbb{R} \quad \forall i = 1, \ldots, n,$$

then the tracking error ϵ converges into the region

$$T_\rho = \left\{\epsilon \in \mathbb{R}^n : |\epsilon_i| \leq \bar{\epsilon}_i \in \mathbb{R}^+ \quad \forall i = 1, \ldots, n\right\}.$$

Proof: Since the matrix E is in reachability canonical form, the matrix of its eigenvectors is

$$Z = \begin{pmatrix} 1 & 1 & \ldots & 1 \\ \lambda_1 & \lambda_2 & \ldots & \lambda_n \\ . & . & \ldots & . \\ \lambda_1^{n-1} & \lambda_2^{n-1} & \ldots & \lambda_n^{n-1} \end{pmatrix}$$

From (5) and (6) it follows that the semi-length $\bar{\epsilon}_i$ of the edges of the hyper-rectangle T_ρ, for Lemma 1, are

$$\rho \sqrt{\sum_{j=1}^{n} \lambda_j^{i-1} \bar{\lambda}_j^{i-1}} \quad \forall i = 1, \ldots, n,$$

and then the proof. ∎

Corollary 1: If the eigenvalues of E are distinct, with negative real part and with magnitude $\|\lambda_i\| = M, \forall i = 1, \ldots, n$, and it is desired to assign $\bar{\epsilon}_1$ and $\bar{\epsilon}_2$, a non-conservative choice of ρ and M is

$$\rho = \frac{\bar{\epsilon}_1}{\sqrt{n}} \quad M = \frac{\bar{\epsilon}_2}{\bar{\epsilon}_1}.$$

Furthermore, if the eigenvalues of E have magnitude M and relative phase shift of π/n (Butterworth eigenvalues) then ϵ converges into T_ρ in a time not greater than

$$t_c = \ln \frac{\sigma}{\rho} \left(M \cos \frac{\pi(n-1)}{2n}\right)^{-1} \tag{15}$$

V. Discussion

The control algorithm provides the following characteristics

- It guarantees the plant's output to practically track a given sufficiently regular reference trajectory with prefixed maximum values of the tracking error and its derivatives up to the order of the plant minus one.
- It is robust with respect to disturbances and uncertain parameters, and then the knowledge of the plant and of the disturbance does not need to be accurate.

This is obtained choosing a control signal depending on three quantities (see Fig. 1):

1) the value V of a suitable Lyapunov function, in order to decide the switching time;
2) the value v of a linear function of the tracking error and of its derivatives, in order to decide if the level must be the nearest admissible level to the nominal control for excess or defect;
3) the value w of deviation from the ideal error model, in order to decide the level of the control signal.

Remark 3: If the coefficients $a_i(p,t)$, $b(p,t)$ and the disturbance $d(p,t)$ dependence on the parameter p is multi-linear and $\wp_p$ is an hyper-rectangle, then $\lfloor U \rfloor$ and $\lceil U \rceil$ in (13) will be always in correspondence of vertices of $\wp_p$ (see [5]).

Remark 4: It should be noted that there can be sliding motion over $\mathcal{S} = \{\epsilon \in \mathbb{R}^n : v = 0\}$, but it will stop as soon as ϵ touches S_ρ. This event will happen in a finite time since the derivative of the Lyapunov function V is negative along $\mathcal{S}$ (see Remark 2).

Remark 5: In the hypothesis of possible sliding on $\mathcal{S}$, if the initial part of the reference trajectory is chosen such that $\epsilon(0) \in S_\rho$ then the evolution of ϵ will be always contained in S_ρ and therefore the control signal will never chatter. If, for example, the plant has the following initial conditions

$$y(0) = y_0,\ y^{(i)}(0) = 0 \quad \forall i = 1, \ldots, n-1,$$

it is possible to avoid the chattering by choosing the initial part of the reference signal such that

$$|\hat{y}(0) - y_0| < \frac{\rho}{\sqrt{p_{11}}}, \quad \hat{y}^{(i)}(0) = 0 \quad \forall i = 1, \ldots, n-1,$$

where p_{11} is the $(1,1)$-element of P.

VI. Multi-valued control

The Theorem 1 is valid even if the control signal may assume only a finite number l of levels

$$u_- = u_1 < u_2 < \cdots < u_l = u_+$$

and, in particular, also only two levels (the classical levels of the relay controller). As regards the steady-state tracking error and the convergence velocity, it is possible to use levels "greater" than the ones provided by (13), e.g. only the extreme levels. The intermediate levels are useful to reduce the amplitude of the control signal (and often the power peaks) and the average switching frequency. This is due to the fact that using the levels provided by (13), the escape velocity from S_ρ diminishes.

Moreover, note that the control signal's amplitude and switching frequency increase as the parameters uncertainties increase (see (13)). Such inconvenient can be reduced identifying the plant parameters.

Remark 6: It is easy to prove that, if the plant has order one and $\wp_u = \{U_-, U_+\}$, the controller of Theorem 1 becomes a classical relay control with hysteresis ρ.

A. Numerical examples

Consider the nominal linear plant

$$\ddot{y} + \dot{y} + y = u$$

with a control input that may assume only values in

$$\wp_u = \{\ -1.2 \quad -0.6 \quad 0.0 \quad +0.6 \quad +1.2\ \}.$$

We want to impose a steady-state tracking error

$$|\epsilon_1| \le \bar{\epsilon}_1 = 0.05, \quad |\epsilon_2| \le \bar{\epsilon}_2 = 0.05.$$

Following the Theorem 1, we designed a controller by selecting Butterworth eigenvalues for E, with $M = 1$ and $\rho = 0.05/\sqrt{2}$. With this controller we consider two cases illustrating the theory.

1) Reference signal $\hat{y}(t) = 1$ and initial conditions $y(0) = 0.5$, $\dot{y}(0) = 0$: In Figs. 2 and 3 the output y, the control u and the error ϵ are shown. It can be noted that there is an infinitely fast switching of the control signal in the transient because ϵ slides on $v(\epsilon) = 0$. However, when ϵ enters S_ρ the control signal switches only whenever ϵ reaches a boundary point.

Consequently, the proposed control law performs better than the classical sliding mode control, since the phenomenon of chattering is disallowed after the convergence time t_c is reached; such time can be imposed in advance by using the (15). Observe, moreover, that the t_c value given by (15) results 3.75s and it results a good estimation of the real value 3.44s.

2) Reference signal $\hat{y}(t) = cos(0.5t)$ and initial conditions $y(0) = 0.97$, $\dot{y}(0) = 0$: Figs. 4 and 5 show that there is not infinitely fast switching of the control signal because ϵ is always contained in S_ρ. This is in accordance with Remark 5.

Moreover, it is interesting to note that, considering only the extreme values of the control

$$\wp_u = \{\ -1.2 \quad +1.2\ \},$$

the output remains practically identical to the one shown in Fig. 4, while there is a consistent increase in the average switching frequency of the control signal (see Fig. 6). This last experiment shows how using intermediate values of the control signal allows reducing the switching frequency and the power peaks.

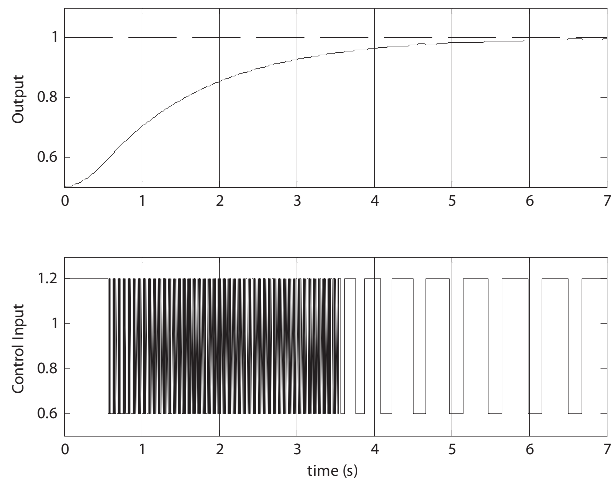

Fig. 2. Output and control signals. Case A.

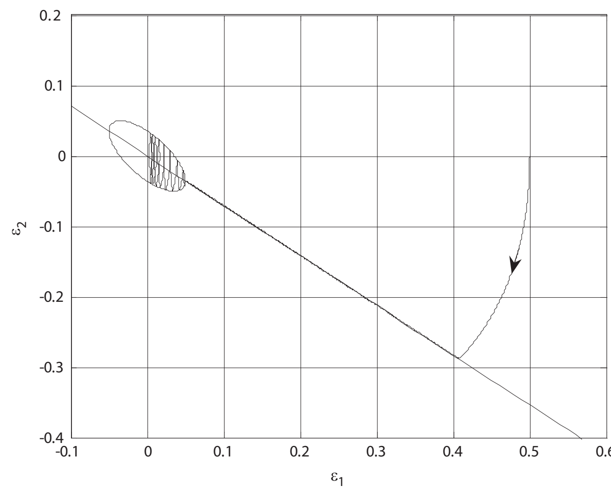

Fig. 3. Tracking error. Case A.

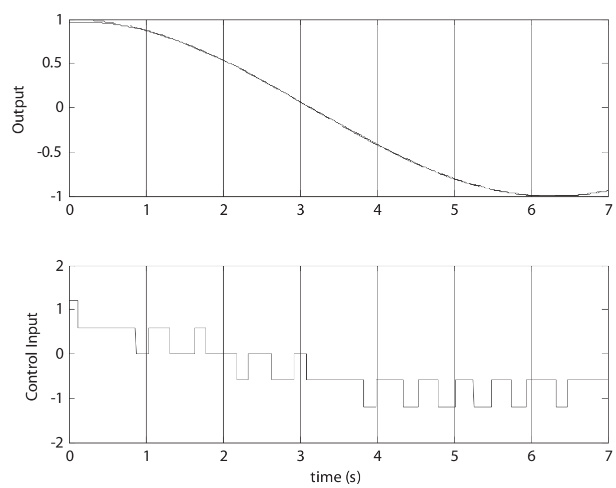

Fig. 4. Output and control signals. Case C.

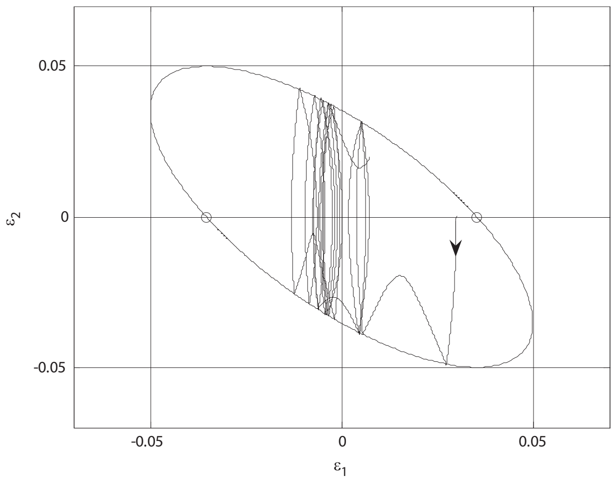

Fig. 5. Tracking error. Case C.

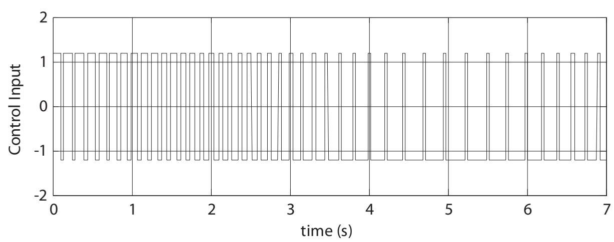

Fig. 6. Control signal if there are available only the extreme values. Case C.

VII. Conclusion

In this paper a new methodology for the design of control laws with multi valued control signals has been presented. This methodology allows designing controllers which guarantee the practical tracking of sufficiently regular reference trajectories for SISO linear plants subject to disturbances and parametric uncertainties.

The formulated theorems allow imposing prescribed maximum limits at the convergence time, the tracking error and its derivatives, limiting or deleting the sliding mode.

A simple example has been presented to put on evidence the advantages obtained using the proposed method, compared to either sliding mode and classical relay control approaches. Microcontroller implementations of the control law have been developed and applied in different practical applications, such as the ceramic kiln control described in [4].

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