Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

On the passivity properties of a new family of repetitive (hyperbolic) controllers

P.G. Hernández-Briones, G. Escobar and R. Ortega

***Abstract*— This paper studies the passivity properties of three recently reported repetitive schemes [1], [2]. They are referred as negative feedback, positive feedback and $6\ell \pm 1$ repetitive compensators. The first two controllers are composed of a feedback array of a single delay line, while the third controller comprises the feedback array of two delay lines. As most repetitive schemes, these three schemes are intended for the compensation or tracking of period signals, which are composed of harmonic components of a fundamental frequency. In particular, the negative feedback scheme is aimed for the compensation of odd-harmonic components, the positive feedback scheme for the compensation of all harmonics, and the $6\ell \pm 1$ scheme for the compensation of $6\ell \pm 1$ ($\ell = 0, 1, 2, ..., \infty$) harmonics. It is shown here that all three schemes have also equivalent expressions in terms of hyperbolic functions. The main contribution of the present work is to show that these three schemes are discrete-time positive real and thus passive. Moreover, it is shown that, after a a modification, motivated by practical issues, these schemes become strictly passive.**

I. Introduction

Recently, two novel repetitive schemes have been proposed in [1] aimed to compensate for selected harmonics of periodic disturbances. These schemes, in contrast to conventional repetitive schemes [3]-[7], include a feedforward path aimed to enhance their selectivity. Moreover, it was also introduced a negative feedback scheme, which differs from the usual positive feedback used in conventional schemes. A similar scheme using negative feedback can also be found in [8]. An interesting observation in the present work is that these schemes, owning such a particular structure, have equivalent representations in terms of hyperbolic functions. In particular, the negative feedback plus the feedforward scheme can be expressed as a hyperbolic tangent function, while the positive feedback scheme can be expressed as a hyperbolic cotangent function.

In [2] a new scheme aimed for the compensation of the $6\ell \pm 1$ ($\ell = 0, 1, 2, ..., \infty$) harmonics has been proposed. This scheme is composed of a feedback array of two delay lines plus a feedforward path. As in the previous cases, this scheme has also an equivalent representation in terms of hyperbolic functions.

As most repetitive schemes, the three schemes studied here, are intended for the tracking or rejection of periodic signals, that is, for the compensation of harmonic components of the fundamental frequency, which is referred along the paper as ω_0. It has been shown in [1] that, based on the fact that the negative feedback (hyperbolic tangent) scheme creates imaginary poles at odd multiples of the fundamental, then it is able to compensate odd harmonics only (see also [8]). It is shown also that the positive feedback scheme (hyperbolic cotangent) creates poles at every single multiple of the fundamental frequency, therefore, it is able to compensate every (even and odd) harmonics. On the other hand, it is shown in [2] that the third repetitive scheme, creates poles at every $6\ell \pm 1$ ($\ell = 0, 1, 2, ..., \infty$) multiples of ω_0, and thus, it is able to compensate the $6\ell \pm 1$ ($\ell = 0, 1, 2, ...\infty$) harmonics only. This scheme is of particular interest in industrial applications where many processes involve the use of six pulse converters producing harmonics at multiples $6\ell \pm 1$ ($\ell = 0, 1, 2, ...\infty$) of the fundamental line frequency [13]. It has been shown also that the introduction of the feedforward path in the above schemes creates an infinite set of zeros which are located between every two consecutive poles. The last has the advantage of enhancing the controller selectivity, a characteristic very well appreciated in harmonic distortion compensation.

This paper studies the passivity properties of these three schemes, each scheme is studied in a separate section. It is shown that all of three scheme are discrete-time positive real (PR) and thus passive. Moreover, it is shown that all these scheme become strictly passive after a modification that allows, in principle, a more practical implementation.

II. Passivity properties of the negative feedback (hyperbolic tangent) compensator

Before proceeding with the study of the passivity properties of this scheme, it is important to remark that we are faced with infinite-dimensional delay-differential equations to which the standard tools cannot be applied directly. In [9] the authors show that, several reported statements of positive-real (PR) discrete-time transfer functions are not completely correct, and then presented a lemma that gave the correct conditions for a system to be discrete-time passive, or equivalently discrete-time PR. This lemma is recalled here below for completeness, as well as a lemma taken from [10] which are the basis for our stability study.

Lemma II.1 (Discrete-time PR) Consider an LTI discrete-time system

$$y(k\tau_d) + \sum_{i=1}^{n_D} D_i y(k\tau_d - i\tau_d) = \sum_{\ell=0}^{n_N} N_\ell u(k\tau_d - \ell\tau_d)$$

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978-1-4244-3124-3/08/$25.00

with $\tau_d \in \mathbb{R}_+$, $k \in \mathbb{Z}_+$, D_ℓ, $N_\ell \in \mathbb{R}$, $n_N \leq n_D$. Assume the associated discrete-time transfer function

$$H(e^{\tau_d s}) = \frac{\sum_{\ell=0}^{n_N} N_\ell e^{-\ell\tau_d s}}{1+\sum_{i=1}^{n_D} D_i e^{-i\tau_d s}} \quad (1)$$

is discrete-time PR, that is, it satisfies
(i) $H(e^{\tau_d s})$ is analytic in $|e^{\tau_d s}| > 1$.
(ii) All poles of $H(e^{\tau_d s})$ on $|e^{\tau_d s}| = 1$ are simple.
(iii) $Re\{H(e^{j\theta})\} \geq 0$ for all $\theta \in \mathbb{R}$ at which $H(e^{j\theta})$ exists.
(iv) If $e^{j\theta_0}$, $\theta_0 \in \mathbb{R}$ is a pole of $H(e^{\tau_d s})$, and if r_0 is the residue of $H(e^{\tau_d s})$ at $e^{\tau_d s} = e^{j\theta_0}$, then $e^{-j\theta_0} r_0 \geq 0$.

Then the system is discrete-time passive, that is, there exists $\beta_0 \in \mathbb{R}$ such that

$$\sum_{k=0}^{N} y(k\tau_d)u(k\tau_d) \geq \beta_0$$

for all input sequences $u(k\tau_d) \in \mathcal{L}_2$ and all $N \in \mathbb{Z}_+$ □

Lemma II.2 (Passivity of continuous-time delayed syst.) Consider an LTI continuous-time system described by the delay equation

$$y(t) + \sum_{i=1}^{n_D} D_i y(t - i\tau_d) = \sum_{\ell=0}^{n_N} N_\ell u(t - \ell\tau_d)$$

with $\tau_d, t \in \mathbb{R}_+$, D_ℓ, $N_\ell \in \mathbb{R}$, $n_N \leq n_D$. Assume the discrete-time transfer function (1) is discrete-time PR. Then, the system is passive, that is, there exists $\beta_1 \in \mathbb{R}$ such that

$$\int_0^t y(\tau)u(\tau)d\tau \geq \beta_1$$

for all input functions $u(t) \in \mathcal{L}_2$ and all $t \in \mathbb{R}_+$. □

The following expressions for the negative feedback (hyperbolic tangent) compensator have been taken from [1]

$$\tanh\left(\frac{s\pi}{2\omega_0}\right) = \frac{\cosh(\frac{s\pi}{2\omega_0})}{\sinh(\frac{s\pi}{2\omega_0})} = \frac{e^{\frac{s\pi}{2\omega_0}} + e^{-\frac{s\pi}{2\omega_0}}}{e^{\frac{s\pi}{2\omega_0}} - e^{-\frac{s\pi}{2\omega_0}}} = \frac{1 - e^{-\frac{s\pi}{\omega_0}}}{1 + e^{-\frac{s\pi}{\omega_0}}} \quad (2)$$

The block diagram of the previous expression involving a single delay line is presented in Fig. 1. This scheme is referred as the *negative feedback plus feedforward repetitive compensator*.

This repetitive scheme has the following equivalent expression that exhibits its infinite dimensionality.

$$\tanh\left(\frac{s\pi}{2\omega_0}\right) = \frac{\frac{s\pi}{2\omega_0}\prod_{k=1}^{\infty}\left(\frac{s^2}{(2k)^2\omega_0^2}+1\right)}{\prod_{k=1}^{\infty}\left(\frac{s^2}{(2k-1)^2\omega_0^2}+1\right)} \quad (3)$$

Notice that, the poles are located at odd multiples of ω_0, while the zeros are located at the even multiples. Therefore, this scheme is also referred as the *odd harmonics compensator*. The frequency response is composed of resonant peaks of infinite magnitude located at the odd multiples of ω_0 due to the poles, and notches at even multiples due to the zeros.

Moreover, this scheme has also the following expression in the form of an infinite bank of harmonic oscillators tuned at odd harmonics of ω_0

$$\tanh\left(\frac{s\pi}{2\omega_0}\right) = \frac{\omega_0}{\pi}\sum_{\ell=1}^{\infty}\frac{4s}{s^2+(2\ell-1)^2\omega_0^2} \quad (4)$$

The delay time required for the implementation of this scheme is given by $\tau_d = \frac{\pi}{\omega_0}$, which will be used on the rest of this section to reduce the notation.

Proposition II.3 The hyperbolic tangent scheme given by (2) is discrete-time PR and thus passive. □

Proof: Rewriting (2) in terms of the delay time τ_d yields

$$H(e^{\tau_d s}) = \tanh\left(\frac{\tau_d s}{2}\right) = \frac{1-e^{-\tau_d s}}{1+e^{-\tau_d s}} = \frac{e^{\tau_d s}-1}{e^{\tau_d s}+1}$$

The partial fraction expansion of this expression gives

$$H(e^{\tau_d s}) = 1 - \frac{2}{e^{\tau_d s}+1}$$

hence the transfer function satisfies conditions (i) and (ii) of Lemma II.1. The residue associated with the fixed pole at $e^{-j\theta_0} = -1$ is $r_0 = -2$, and thus condition (iv) is satisfied. Finally, some simple computations prove that, $Re\{H(e^{j\theta})\} = Re\{\frac{j\sin(\theta)}{1+\cos(\theta)}\} = 0$, thus fulfilling condition (iii). This proves that the hyperbolic tangent scheme is discrete-time PR and, according to Lemma II.2, it is passive. ■

In [1] a gain K is included as follows.

$$\frac{1-Ke^{-\tau_d s}}{1+Ke^{-\tau_d s}} \quad (5)$$

The aim of this practical modification is to prevent high gains in the resonance peaks and to enhance the robustness with respect to frequency variations. In fact, the peaks, originally of infinite magnitude, reach a maximum magnitude of $\frac{1+K}{1-K}$ while the notches reach a minimum magnitude of $\frac{1-K}{1+K}$.

This modification can also be seen as a frequency shifting process $\tilde{H}(s) = H(s+a)$. Direct application of this shifting process to the exponential term results in $e^{-\tau_d(s+a)} = e^{-\tau_d a}e^{-\tau_d s}$. In other words, by proposing a gain factor $K = e^{-\tau_d a}$ we obtain

$$\begin{aligned}\frac{1-Ke^{-\tau_d s}}{1+Ke^{-\tau_d s}} &= \frac{1-e^{-\tau_d(s+a)}}{1+e^{-\tau_d(s+a)}} = \frac{e^{-\frac{\tau_d(s+a)}{2}} - e^{-\frac{\tau_d(s+a)}{2}}}{e^{-\frac{\tau_d(s+a)}{2}} + e^{-\frac{\tau_d(s+a)}{2}}} \\ &= \frac{\sinh\left(\frac{\tau_d(s+a)}{2}\right)}{\cosh\left(\frac{\tau_d(s+a)}{2}\right)} = \tanh\left(\frac{\tau_d(s+a)}{2}\right)\end{aligned}$$

Therefore, if a gain $K > 1$ is proposed, the poles and zeros move to the right, while if $0 < K < 1$ is proposed then they move to the left.

The following definition has been extracted from [11] and is used here to prove that the modified proposed scheme is strictly positive real (SPR).

Definition II.4 A transfer function $G(s)$ is SPR if and only if there exists some $\varepsilon > 0$ such that $G(s-\varepsilon)$ is PR.

Proposition II.5 The modified scheme (5) with $0 < K < 1$ is SPR and thus strictly passive. □

Proof: According to Definition II.4, it should be proved that, there exists an $\varepsilon > 0$ such that

$$\frac{1 - Ke^{-\tau_d(s-\varepsilon)}}{1 + Ke^{-\tau_d(s-\varepsilon)}} \tag{6}$$

is positive real ($\in \{PR\}$).

First, let us select $\varepsilon = a$ where $a = -\tau_d \ln(K)$. Notice that, $a > 0$ as far as $0 < K < 1$. Second, consider $K = e^{-\tau_d a}$, as defined above, which, after direct substitution, reduces expression (6) to $\tanh\left(\frac{\tau_d s}{2}\right)$. The proof is completed by recalling that, $\tanh\left(\frac{\tau_d s}{2}\right)$ is PR according to Proposition II.3. ■

Figure 2 shows that, the Nyquist plot of scheme (5) is a circle with radius $\frac{2K}{1-K^2}$ and center $\frac{1+K^2}{1-K^2}$, which is strictly contained in the right hand side of the complex plane ($Re\{s\} > 0$). The last can be easily seen from the rectangular representation of $\tanh(\frac{j\omega\tau_d}{2})$ given by

$$\tanh\left(\frac{j\omega\tau_d}{2}\right) = \frac{1 - Ke^{-j\omega\tau_d}}{1 + Ke^{-j\omega\tau_d}} =$$

$$\frac{1-K^2}{1+K^2+2K\cos(\omega\tau_d)} + j\frac{2K\sin(\omega\tau_d)}{1+K^2+2K\cos(\omega\tau_d)}$$

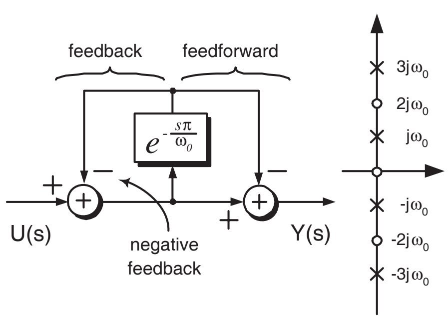

Fig. 1. Continuous-time model and poles/zeros location of the negative feedback (odd harmonics) repetitive compensator with feedforward.

Notice that, the maximum $\frac{1+K}{1-K}$ and minimum $\frac{1-K}{1+K}$ magnitudes occur whenever the imaginary part is zero, and subsequently, with zero phase shift, that is, at $\omega\tau_d = (2\ell - 1)\pi$ and $\omega\tau_d = 2\ell\pi$ ($\ell = 1, 2, 3, ..., \infty$), respectively, which correspond to $\omega = (2\ell-1)\omega_0$ (odd multiples) and $\omega = 2\ell\omega_0$ (even multiples). Figure 3 shows the frequency response of this modified scheme for different values of K. Notice that, the introduction of gain K does not cause a shifting of the resonance peaks nor of the notches.

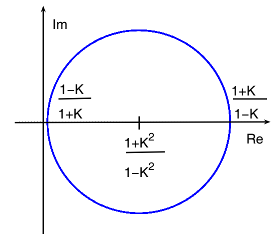

Fig. 2. Nyquist plot of the negative feedback (odd harmonics) compensator including a modification with a gain K, i.e., $\left(1 - Ke^{-\tau_d s}\right) / \left(1 + Ke^{-\tau_d s}\right)$.

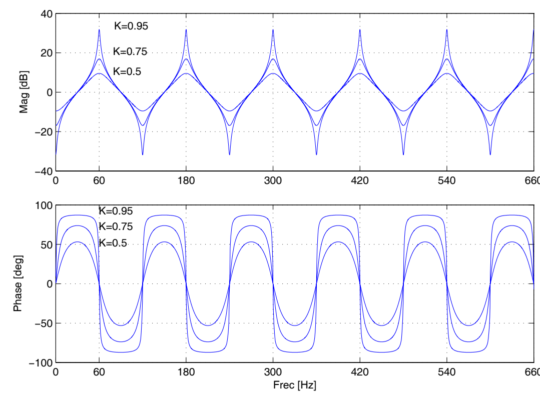

Fig. 3. Bode plot of the negative feedback (odd harmonics) modified compensator, i.e., $\left(1 - Ke^{-\tau_d s}\right) / \left(1 + Ke^{-\tau_d s}\right)$ for different values of gain K: 0.5, 0.75 and 0.95.

III. Passivity properties of the positive feedback (hyperbolic cotangent) repetitive compensator

In what follows we consider the positive feedback (hyperbolic cotangent) repetitive scheme, which is described by the following expressions taken from [1]

$$\operatorname{cotanh}\left(\frac{s\pi}{\omega_0}\right) = \frac{\cosh(\frac{s\pi}{\omega_0})}{\sinh(\frac{s\pi}{\omega_0})} = \frac{e^{\frac{s\pi}{\omega_0}} + e^{-\frac{s\pi}{\omega_0}}}{e^{\frac{s\pi}{\omega_0}} - e^{-\frac{s\pi}{\omega_0}}} = \frac{1 + e^{-\frac{2s\pi}{\omega_0}}}{1 - e^{-\frac{2s\pi}{\omega_0}}} \tag{7}$$

Its block diagram is presented in Fig. 4. This scheme is referred as the *positive feedback plus feedforward repetitive compensator*. This scheme has the following equivalent expression involving an infinite number of poles and zeros.

$$\operatorname{cotanh}\left(\frac{s\pi}{\omega_0}\right) = \frac{\prod_{\ell=1}^{\infty}\left(\frac{s^2}{\left(\frac{2\ell-1}{2}\right)^2\omega_0^2} + 1\right)}{\frac{s\pi}{\omega_0}\prod_{\ell=1}^{\infty}\left(\frac{s^2}{\ell^2\omega_0^2} + 1\right)} \tag{8}$$

Notice that, the poles located at every single multiple of ω_0, while the zeros are located exactly in the middle point between every two consecutive poles as shown in Fig. 4. Therefore, this scheme is also referred as *all harmonics repetitive compensator*.

Similar to the odd harmonics compensator, damping can be added in the form of gain $0 < K < 1$ to limit the resonance peak gains. The Bode plot for different values of K is shown in Fig. 5. The Nyquist plot for the positive compensator is the same as for the negative feedback compensator. See figure 6.

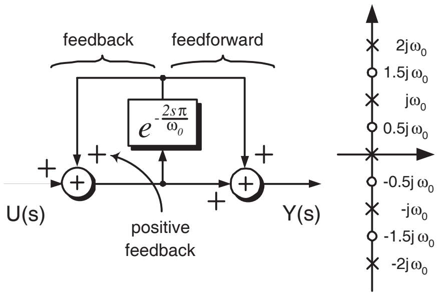

Fig. 4. Continuos-time model and poles/zeros location of the positive feedback (all harmonics) compensator with feedforward.

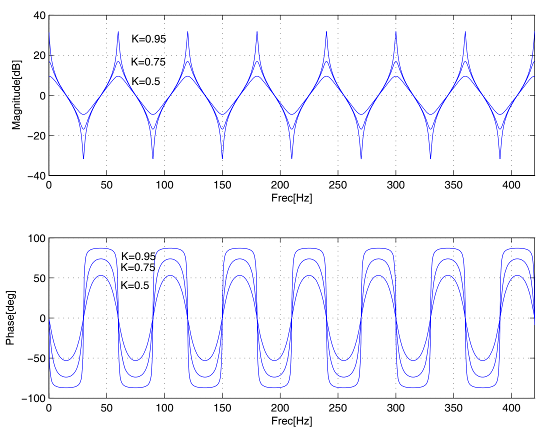

Fig. 5. Bode plot of the positive feedback (all harmonics) modified compensator, i.e., $\left(1+Ke^{-\tau_d s}\right)/\left(1-Ke^{-\tau_d s}\right)$ for different values of gain K: 0.5, 0.75 and 0.95..

In [12] it is shown that, $Z(s)$ is PR if and only if $1/Z(s)$ is PR. And also that, $Z(s)$ is SPR iff $1/Z(s)$ is SPR. According to this statements and based on the fact that $\cot\text{anh}(\cdot) = 1/\tanh(\cdot)$, it is straightforward to establish the validity of the following corollaries. The time delay required for the implementation of this scheme is given by $\tau_d = \frac{2\pi}{\omega_0}$, which will be used on the rest of this section to reduce the notation.

Corollary III.1 The hyperbolic cotangent scheme given by (7) is PR and thus passive.

□

Corollary III.2 The modified scheme

$$\frac{1+Ke^{-\tau_d s}}{1-Ke^{-\tau_d s}} \tag{9}$$

with $0<K<1$ is SPR and thus strictly passive.

□

In fact the modified scheme can also be written as $\text{cotanh}\left(\frac{\tau_d(s+a)}{2}\right)$ with $a = -\frac{2}{\tau_d}\ln(K)$, and taking $0<K<1$ the poles and zeros are shifted to the left of the imaginary axis in the complex plane $(Re\{s\}<0)$.

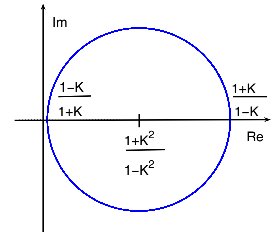

Fig. 6. Nyquist plot of the positive feedback (all harmonics) compensator including a modification with a gain K, i.e., $\left(1+Ke^{-\tau_d s}\right)/\left(1-Ke^{-\tau_d s}\right)$.

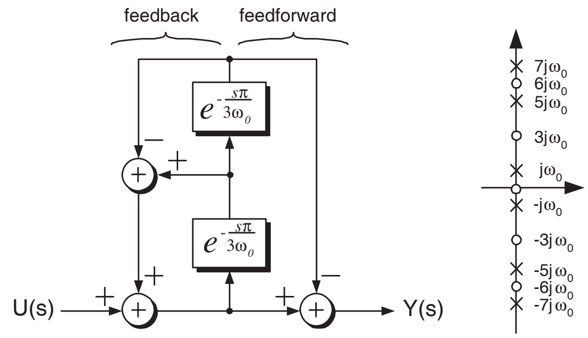

Fig. 7. Continuous-time model and poles/zeros location of the $6\ell\pm1(\ell = 0,1,2,3,...)$ repetitive compensator with feedforward.

IV. Passivity properties of the $6\ell\pm1$ compensator

Finally, the $6\ell\pm1$ $(\ell = 0,1,2,3,...,\infty)$ scheme is studied. This last compensator is described by the following expression

$$H(s) = \frac{1-e^{-\frac{2s\pi}{3\omega_0}}}{1+e^{-\frac{2s\pi}{3\omega_0}}-e^{-\frac{s\pi}{3\omega_0}}} \tag{10}$$

Its block diagram is presented in Fig. 7. This scheme is referred as the $6\ell\pm1$ *repetitive compensator*. The scheme has the following equivalent expression which involves an infinite number of poles and zeros.

$$\begin{aligned} H(s) &= \frac{e^{\frac{s\pi}{3\omega_0}}-e^{-\frac{s\pi}{3\omega_0}}}{e^{\frac{s\pi}{3\omega_0}}+e^{-\frac{s\pi}{3\omega_0}}-1} = \frac{2\,\sinh(\frac{s\pi}{3\omega_0})}{2\,\cosh(\frac{s\pi}{3\omega_0})-1} = \\ &= \frac{\frac{s\pi}{3\omega_0}\prod_{\ell=1}^{\infty}(\frac{s^2}{(3l)^2\omega_0^2}+1)}{\prod_{\ell=-\infty}^{\infty}(\frac{s^2}{(6l+1)^2\omega_0^2}+1)} \end{aligned}$$

where it is easy to see that the transfer function comprises an infinite number of poles located at $\pm j(6\ell+1)\omega_0$ and $\pm j(6\ell-1)\omega_0$ $(\ell = 0,1,2,3,...,\infty)$. Moreover, it also contains an infinite number of zeros located at $\pm j3\ell\omega_0$.

Proposition IV.1 The $6\ell\pm1$ repetitive scheme given by (10) is discrete-time PR and thus passive. □

Proof: Rewriting (10) in terms of the time delay $\tau_d = \frac{\pi}{3\omega_0}$ (used along this section to simplify the notation) yields

$$H(e^{\tau_d s}) = \frac{1-e^{-2\tau_d s}}{1+e^{-2\tau_d s}-e^{-\tau_d s}} = \frac{e^{2\tau_d s}-1}{e^{2\tau_d s}-e^{\tau_d s}+1} \quad (11)$$

The partial fraction expansion of this expression gives

$$\begin{aligned} H(e^{\tau_d s}) &= 1+\frac{e^{\tau_d s}-2}{e^{2\tau_d s}+1-e^{\tau_d s}} \\ &= 1+\frac{1}{2}\left(\frac{1+\sqrt{3}i}{e^{\tau_d s}-e^{\frac{i\pi}{3}}}+\frac{1-\sqrt{3}i}{e^{\tau_d s}-e^{\frac{-i\pi}{3}}}\right) \end{aligned} \quad (12)$$

Hence, the transfer function satisfies conditions (i) and (ii) of Lemma II.1. For (iii) it is found that $Re\{H(e^{j\theta})\} = Re\{\frac{j2\sin(\theta)}{-1+2\cos(\theta)}\} = 0$. For the last condition, the two residues are given by $r_1 = \frac{1}{2}+\frac{\sqrt{3}}{2}i$ and $r_2 = \frac{1}{2}-\frac{\sqrt{3}}{2}i$, and the poles corresponding to each residue are $\theta_1 = \frac{\pi}{3}$ and $\theta_2 = -\frac{\pi}{3}$, respectively. Then $e^{-j\theta_1}r_1 = 1$ and $e^{-j\theta_2}r_2 = 1$ and the condition is fulfilled. This proves that, the $6\ell \pm 1$ scheme is discrete-time PR and, according to Lemma II.2, it is passive. ■

Similar to previous cases, with the aim to prevent high gains in the resonance peaks and to enhance the robustness with respect to frequency variations, a gain K is included multiplying each delay line. This yields the following modified expression

$$H(e^{\tau_d s}) = \frac{1-K^2e^{-2\tau_d s}}{1+K^2e^{-2\tau_d s}-Ke^{-\tau_d s}} \quad (13)$$

In fact, the peaks, originally of infinite magnitude, reach a maximum magnitude of

$$M_1 = M_2 = \sqrt{\frac{6K^2+2\sqrt{3}\sqrt{1+K^4+K^8}}{3(-1+K^2)^2}} \quad (14)$$

and the notches reach either of the following two minimum magnitudes

$$m_1 = \frac{1-K^2}{1+K+K^2} \quad (15)$$

$$m_2 = \frac{1-K^2}{1-K+K^2} \quad (16)$$

This modification can also be seen as a frequency shifting process of the form $\tilde{H}(s) = H(s+a)$. Direct application of this shifting process to the exponential term results in $e^{-\tau_d(s+a)} = e^{-\tau_d a}e^{-\tau_d s}$. In other words, by proposing a gain factor $K = e^{-\tau_d a}$ we obtain

$$\frac{1-K^2e^{-2\tau_d s}}{1+K^2e^{-2\tau_d s}-Ke^{-\tau_d s}} = \frac{1-e^{-2\tau_d(s+a)}}{1+e^{-2\tau_d(s+a)}-e^{-\tau_d(s+a)}} = \frac{e^{\tau_d(s+a)}-e^{-\tau_d(s+a)}}{e^{\tau_d(s+a)}+e^{-\tau_d(s+a)}-1} = \frac{2\sinh(\tau_d(s+a))}{2\cosh(\tau_d(s+a))-1}$$

Therefore, if a gain $K>1$ is proposed, the poles and zeros move to the right, while if $0<K<1$ is proposed then they move to the left.

Proposition IV.2 The modified scheme (13) with $0<K<1$ is SPR and thus strictly passive. □

Proof: According to Definition II.4, it should be proved that, there exists an $\varepsilon>0$ such that

$$\frac{1-K^2e^{-2\tau_d(s-\varepsilon)}}{1+K^2e^{-2\tau_d(s-\varepsilon)}-Ke^{-\tau_d(s-\varepsilon)}} \quad (17)$$

is positive real ($\in \{PR\}$).

First, let us select $\varepsilon = a$ where $a = -\tau_d \ln(K)$. Notice that, $\varepsilon = a > 0$ as far as $0 < K < 1$. Second, consider $K = e^{-\tau_d a}$ as defined above, which, after direct substitution, reduces expression (17) to $\frac{2\sinh(\tau_d s)}{2\cosh(\tau_d s)-1}$. The proof is completed by recalling that, $\frac{2\sinh(\tau_d s)}{2\cosh(\tau_d s)-1}$ is PR according to Proposition IV.1.

■

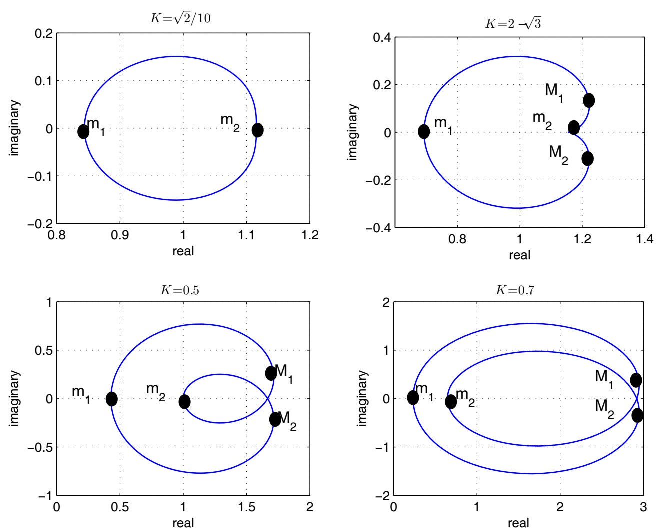

Fig. 8. Nyquist plot of the $6\ell \pm 1(\ell = 0, 1, 2, ..)$, harmonics compensator.

Figure 8 shows that the Nyquist plot of scheme (13) goes from a flattened circle for $0 < K < \frac{\sqrt{2}}{10}$ to a cardioid for $\frac{\sqrt{2}}{10} < K < 2-\sqrt{3}$. Then, for $2-\sqrt{3} < K < 1$ the Nyquist plot becomes a limaon that approaches a circle of arbitrarily large radium as K gets closer to 1. It is clear that the range of interest lies in values of K slightly smaller than 1, i.e., when the Nyquist plot corresponds to a limaon. Notice that, the Nyquist plot is strictly contained in the right hand side of the complex plane ($Re\{s\} > 0$), as established by (15), that is, $m_1 > 0$ and $m_2 > 0$ as far as $0<K<1$.

It is important to remark that, the maximum magnitudes (in the resonance peaks) does not occur exactly when the imaginary part equals zero, as a consequence, a small phase shift appears as it can be observed in Fig. 9. Another effect of the introduction of gain K is that the resonance peak occurs at $\omega = (6n\pm1)\omega_0 \pm \left[\frac{\sqrt{3}}{\pi}\frac{(K-1)^2}{2}\right]\omega_0$. which has been slightly shifted with respect to $\omega = (6n\pm1)\omega_0$. However, it should be noticed that this difference between the expected frequency, and the one obtained after introduction of gain K, tends to zero as K gets closer to one.

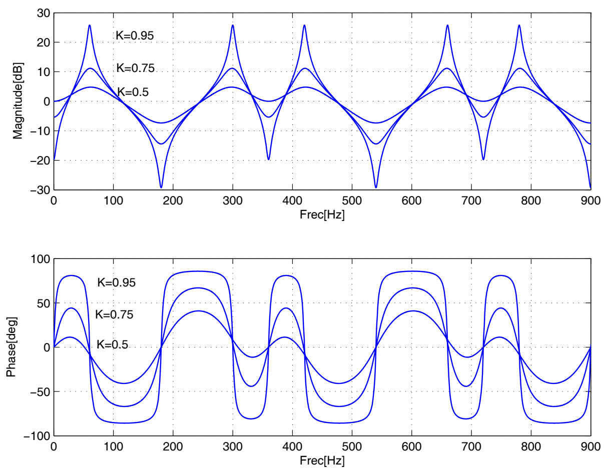

Fig. 9. Bode plot of the modified $6\ell \pm 1$ compensator, i.e., $\left(1 - K^2 e^{-2\tau_d s}\right) / \left(1 + K^2 e^{-2\tau_d s} - K e^{-\tau_d s}\right)$ for different values of gain K: 0.5, 0.75 and 0.95.

In the case of the zeros, all minimum magnitudes occur when the imaginary part is zero, and subsequently, with zero phase shift. Moreover, the minimums occur exactly at $\omega\tau_d = \ell\pi$, which corresponds to $\omega = 3\ell\omega_0$ $(\ell = 0, 1, 2, ..., \infty)$.

V. CONCLUDING REMARKS

This paper presented a study of the passivity properties of three repetitive schemes recently reported in literature. These schemes were referred as negative feedback (odd harmonics) compensator, positive feedback (all harmonics) compensator, and $6\ell \pm 1$ harmonics compensator. It was shown that all of them are discrete time positive real, and thus passive. Moreover, it was shown that introducing a suitable damping, which turned out to be equivalent to a pole/zero shifting, all these schemes become strictly passive. Frequency responses are presented that showed the harmonic compensation capabilities of these schemes.

VI. ACKNOWLEDGMENTS

The authors gratefully acknowledge the financial support of the Laboratory Franco-Mexican of Applied Control (LAFMAA) through project MOPOFA-II.

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