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On Robust Maximum Lifetime Routing in Wireless Sensor Networks*

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Abstract— **We consider the maximum lifetime routing problem in wireless sensor networks, which has been formulated as a linear programming problem in the literature (Chang and Tassiulas [1]). The optimal value and optimal solution of this problem provide optimal flows for the network and the corresponding predicted lifetime, respectively. We study the situation when there is uncertainty in various network parameters (available energy and energy depletion rates). We show that for specific, yet typical, network topologies the actual lifetime will reach the predicted value with a probability that converges to zero as the number of nodes grows large. We develop a series of alternative robust problem formulations, ranging from worst-case to optimistic. A set of parameters enable the tuning of the conservatism of the formulation to obtain network flows with a desirably high probability that the corresponding lifetime prediction will be achieved. We establish a number of properties for the robust network flows and provide an illustrative set of numerical results to highlight the trade-off between predicted lifetime and the probability it is achieved.**

I. Introduction

Wireless Sensor Networks (WSNETs) have emerged as an exciting new paradigm of inexpensive, easily deployable, completely untethered, self-organizing device networks that enable the automated and intelligent monitoring and control of physical systems. WSNET nodes can be equipped with a variety of sensors, have a built-in radio to communicate with each other, are powered by batteries, and have limited information storage and processing capabilities. WSNETs can be useful in a plethora of applications including industrial and building automation, health monitoring, wildlife monitoring, and asset and personnel tracking [2].

Battery technology (at small enough sizes appropriate for WSNETs) imposes very stringent limitations on what the nodes can do and on the lifetime of the network. In many applications a large number of sensor nodes have to be deployed; replacing batteries is not desirable and may even be impossible. Still, one would like to use the WSNET for long periods, often years. As a result, energy conservation is a primary concern and aggressive optimization of network operations becomes indispensable.

In this paper we focus on the problem of selecting an optimal routing strategy for routing packets from the data collecting sensor nodes to a set of gateways (or sinks) so as to minimize the rate at which energy is consumed, or equivalently, to maximize the lifetime of the network. Routing, of course, has received quite a bit of attention in WSNETs. Even the task of finding and maintaining correct routes to sinks is not trivial [3]. Optimization, primarily to minimize energy use, is a harder task. Most of the literature (e.g., [4–8]) focuses on finding a single path from an origin to a specific destination, often adapting to a changing topology and statistics on connectivity and node availability. A more static view is adopted in [1] which considers long-term average flows between nodes as the quantities that are subject to optimization. [9] devises a similar formulation and studies the routing problem in some regular planar topologies.

The starting point of our work is the flow optimizing formulation of [1]. Key data to solving this problem include the total available energy at the nodes and the rates at which packet transmissions and receptions consume energy. These quantities are hardly known with any degree of certainty or accuracy. Yet, they critically affect both the optimal solution – the network flows – and the corresponding optimal value – the predicted network lifetime. The optimal value of the problem will in fact be equal to the actual network lifetime if all problem data are known with certainty. Uncertainty though, has rather drastic consequences rendering the predicted lifetime overly optimistic. We show that for specific, yet typical, topologies including linear and two-dimensional grid-like networks, the actual lifetime will reach the predicted value with a probability that converges to zero as the number of nodes grows large. This suggests that the optimal value of the maximum lifetime formulation can not be used as a "quote" for the actual network lifetime. We also find that uncertainty impacts the optimal policy as well, and one needs to use a different set of flows to protect against uncertainty. To that end, we develop a series of alternative *robust problem formulations*, ranging from worst-case to optimistic. A set of parameters enable the tuning of the conservatism of the formulation to obtain network flows with a high probability that the corresponding lifetime prediction will be achieved – a *lifetime guarantee probability*. Our robust formulations are based on recent work in robust linear programming in [10, 11]. However, the problem we consider has special structure which we exploit to show a number of interesting properties of the robust solutions and establish the result on the optimism of the predicted lifetime.

The rest of the paper is organized as follows. In Sec. II we present the nominal and a series of robust problem formulations based on a parameter uncertainty model we introduce. Sec. III establishes key properties of optimal flows that maximize the network lifetime under the various formulations. Sec. IV considers a restricted uncertainty setting (when only node initial energies are uncertain) and

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establishes for specific topologies that the predicted network lifetime is achieved with a probability that converges to zero as the number of nodes increases. Sec. V contains numerical results. Conclusions are in Sec. VI.

II. Maximum lifetime routing formulations

We represent a WSNET as a directed graph $\mathscr{G}(\mathscr{N},\mathscr{A})$ where $\mathscr{N}$ is the set of all nodes and $\mathscr{A}$ is the set of all directed links (i,j) with $i,j\in\mathscr{N}$. Link (i,j) exists if and only if $j\in\mathscr{S}_i$, where $\mathscr{S}_i$ is the set of nodes "reachable" from i. Each node i has an initial battery energy of E_i. The transmission energy consumed at node i to transmit a data unit to node j is denoted by e^t_{ij} and the corresponding energy consumed at the receiver j is denoted by e^r_{ij}. We assume that every WSNET node is able to relay packets and to adjust the transmit power level to the minimum required in order to reach the intended receiver.

We define the set of *origin nodes* $\mathscr{O}$ containing all nodes i with a positive (constant) information generation rate Q_i, i.e., $\mathscr{O}=\{i|Q_i>0, i\in\mathscr{N}\}$. Let $\mathscr{D}$ be a set of *sink nodes*. We assume that $\mathscr{O}\cap\mathscr{D}=\emptyset$. We will refer to nodes in $\mathscr{N}\setminus\mathscr{D}$ simply as *sensor nodes*. Every node in $\mathscr{O}$ seeks to send each data unit generated to one of the nodes in $\mathscr{D}$, not necessarily the same one for each data unit. To that end, node i may use multiple other nodes as relays. Let q_{ij} be the transmission rate of information from node i to node j to be assigned by the routing algorithm. We will use $\mathbf{q}$ to denote the vector of all q_{ij}'s for all $i\in\mathscr{N}$ and $j\in\mathscr{S}_i$. [1] Note that the routing decision and the transmission power level selection are intrinsically connected since the power level is adjusted depending on the choice of the next hop node.

We consider only the energy consumed for transmissions and receptions. Additional energy consumption terms could be incorporated into e^t_{ij}, e^r_{ij}, and E_i, for instance a constant processing energy cost per received data unit can be easily incorporated into e^r_{ij}. We also assume that e^t_{ij} is monotonically increasing with the distance between two nodes i and j. Finally, the sink nodes are assumed to be powered by line power or have an infinite amount of initial energy.

The lifetime of a sensor node i under a given set of flows $\mathbf{q}$ is given by

$$T_i(\mathbf{q})=\frac{E_i}{\sum_{j\in\mathscr{S}_i} e^t_{ij}q_{ij}+\sum_{j|i\in\mathscr{S}_j} e^r_{ji}q_{ji}},\ \forall i\in\mathscr{N}\setminus\mathscr{D}.$$

We define the *network lifetime* under flow $\mathbf{q}$ as the minimum lifetime over all nodes, i.e.,

$$T_{net}(\mathbf{q})\triangleq\min_{i\in\mathscr{N}\setminus\mathscr{D}} T_i(\mathbf{q}).$$

The network lifetime is equivalent to the earliest time a sensor node runs out of energy. Let $\hat{q}_{ij}=q_{ij}T$; [1] observed that the maximum lifetime routing problem can be written as the following linear programming problem:

$$\begin{aligned} \max\ & T \qquad\qquad (1)\\ s.t.\ & \sum_{j|i\in\mathscr{S}_j}\hat{q}_{ji}+Q_iT=\sum_{j\in\mathscr{S}_i}\hat{q}_{ij},\quad \forall i\in\mathscr{N}\setminus\mathscr{D},\\ & \sum_{j\in\mathscr{S}_i} e^t_{ij}\hat{q}_{ij}+\sum_{j|i\in\mathscr{S}_j} e^r_{ji}\hat{q}_{ji}\le E_i,\quad \forall i\in\mathscr{N}\setminus\mathscr{D},\\ & \hat{q}_{ij}\ge 0,\quad \forall i\in\mathscr{N},\ \forall j\in\mathscr{S}_i,\\ & T\ge 0,\end{aligned}$$

[1] In general, we use bold letters to denote vectors and all vectors are assumed to be column vectors unless explicitly stated otherwise.

where the decision variables are T and the $\hat{q}_{ij}$'s. The first set of constraints correspond to flow conservation for sensor nodes. The second set of constraints follows from the definition of lifetime. We refer to problem (1) as the *nominal* problem. Note that it is always feasible if for every sensor node there exists a path to a sink node. We assume that this will always be the case.

The data for the nominal problem are the quantities e^t_{ij}, e^r_{ij}, and E_i and these affect both the optimal solution and the optimal value. To accommodate uncertainty, we model the data as symmetrically bounded nonnegative random variables with ranges given by: $e^t_{ij}\in[\bar{e}^t_{ij}-\Delta e^t_{ij},\bar{e}^t_{ij}+\Delta e^t_{ij}]$, $e^r_{ij}\in[\bar{e}^r_{ij}-\Delta e^r_{ij},\bar{e}^r_{ij}+\Delta e^r_{ij}]$, and $E_i\in[\bar{E}_i-\Delta E_i,\bar{E}_i+\Delta E_i]$. We will call $\bar{e}^t_{ij}$, $\bar{e}^r_{ij}$, and $\bar{E}_i$ the nominal values and assume that these are the means of the corresponding random variables. The values Δe^t_{ij}, Δe^r_{ij}, and ΔE_i are the maximum deviations from the mean and are defined so that all random variables we consider have positive support. We define the *uncertainty sets* $J^t_i\triangleq\{j|\Delta e^t_{ij}>0, j\in\mathscr{S}_i\}$ and $J^r_i\triangleq\{j|\Delta e^r_{ji}>0, i\in\mathscr{S}_j\}$, $\forall i\in\mathscr{N}\setminus\mathscr{D}$.

Due to the uncertainty of the data, the optimal solution of the nominal problem (1) may not be feasible. To guarantee feasibility for any realization of the data we consider the following worst-case formulation:

$$\begin{aligned} \max\ & T \qquad\qquad (2)\\ s.t.\ & \sum_{j|i\in\mathscr{S}_j}\hat{q}_{ji}+Q_iT=\sum_{j\in\mathscr{S}_i}\hat{q}_{ij},\quad \forall i\in\mathscr{N}\setminus\mathscr{D}\\ & \sum_{j\in\mathscr{S}_i}\bar{e}^t_{ij}\hat{q}_{ij}+\sum_{j\in J^t_i}\Delta e^t_{ij}\hat{q}_{ij}+\sum_{j|i\in\mathscr{S}_j}\bar{e}^r_{ji}\hat{q}_{ji}\\ & \quad+\sum_{j\in J^r_i}\Delta e^r_{ji}\hat{q}_{ji}\le\bar{E}_i-\Delta E_i,\quad \forall i\in\mathscr{N}\setminus\mathscr{D},\\ & \hat{q}_{ij}\ge 0,\quad \forall i\in\mathscr{N},\ \forall j\in\mathscr{S}_i,\\ & T\ge 0.\end{aligned}$$

We refer to the above problem as the *fat* routing problem. By construction, its optimal solution is feasible for any data realization but it may be overly conservative, that is, predicting a much smaller lifetime than what can actually be achieved. Intuitively, the probability that all data values take their "extreme" value should not be high in most cases. This motivates a less conservative formulation we present next.

We introduce the *uncertainty budget* $\Gamma^e_i\in[0,|J^t_i|+|J^r_i|]$ for every sensor node i and define the *restricted uncertainty set* $\mathscr{R}_i(\Gamma^e_i)$ as

$$\begin{aligned}\mathscr{R}_i(\Gamma^e_i)=\Big\{e^t_{ij},e^r_{ji}\ \Big|\ & e^t_{ij}\in[\bar{e}^t_{ij}-\Delta e^t_{ij},\bar{e}^t_{ij}+\Delta e^t_{ij}],\\ & e^r_{ij}\in[\bar{e}^r_{ij}-\Delta e^r_{ij},\bar{e}^r_{ij}+\Delta e^r_{ij}],\\ & \sum_{j\in J^t_i}\frac{|e^t_{ij}-\bar{e}^t_{ij}|}{\Delta e^t_{ij}}+\sum_{j\in J^r_i}\frac{|e^r_{ji}-\bar{e}^r_{ji}|}{\Delta e^r_{ji}}\le\Gamma^e_i\Big\}.\end{aligned}$$

The role of the uncertainty budget Γ^e_i is to limit the sum of the normalized deviations of e^t_{ij} and e^r_{ji}. One can also view the uncertainty budget constraint as an ℓ_1-norm constraint for the vector $\left(\left(\frac{e^t_{ij} - \bar{e}^t_{ij}}{\Delta e^t_{ij}} \right)_{j \in J^t_i}, \left(\frac{e^r_{ji} - \bar{e}^r_{ji}}{\Delta e^r_{ji}} \right)_{j \in J^r_i} \right)$. In short, $\mathscr{R}_i(\Gamma^e_i)$ is the set of all realizations of e^t_{ij}, e^r_{ji} that satisfy the uncertainty budget constraint. Similarly, we define $\Gamma^E_i \in [0,1]$ to be the uncertainty budget for E_i. In particular, $E_i \in [\bar{E}_i - \Gamma^E_i \Delta E_i, \bar{E}_i + \Gamma^E_i \Delta E_i]$ and we will also refer to this interval as the restricted uncertainty set for E_i. The following *robust* maximum lifetime routing problem is formulated so that we can guarantee feasibility for all data realizations in the restricted uncertainty sets:

$$
\begin{aligned}
\max\ & T \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (3)\\
s.t.\ & \sum_{j | i \in \mathscr{S}_j} \hat{q}_{ji} + Q_i T = \sum_{j \in \mathscr{S}_i} \hat{q}_{ij}, \quad \forall i \in \mathscr{N} \setminus \mathscr{D} \\
& \max_{e^t_{ij}, e^r_{ji} \in \mathscr{R}_i(\Gamma^e_i)} \Big\{ \sum_{j \in \mathscr{S}_i} e^t_{ij} \hat{q}_{ij} + \sum_{j | i \in \mathscr{S}_j} e^r_{ji} \hat{q}_{ji} \Big\} \\
& \qquad \leq \bar{E}_i - \Gamma^E_i \Delta E_i, \quad \forall i \in \mathscr{N} \setminus \mathscr{D}, \\
& \hat{q}_{ij} \geq 0, \quad \forall i \in \mathscr{N},\ \forall j \in \mathscr{S}_i, \\
& T \geq 0.
\end{aligned}
$$

Using duality it can be shown that the robust routing problem is equivalent to the following linear programming problem:

$$
\begin{aligned}
\max\ & T \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (4)\\
s.t.\ & \sum_{j | i \in \mathscr{S}_j} \hat{q}_{ji} + Q_i T = \sum_{j \in \mathscr{S}_i} \hat{q}_{ij}, \quad \forall i \in \mathscr{N} \setminus \mathscr{D} \\
& \sum_{j \in \mathscr{S}_i} \bar{e}^t_{ij} \hat{q}_{ij} + \sum_{j | i \in \mathscr{S}_j} \bar{e}^r_{ji} \hat{q}_{ji} + \sum_{j \in J^t_i} \omega_{ij} + \sum_{j \in J^r_i} \nu_{ji} \\
& \qquad + \Gamma^e_i p_i \leq \bar{E}_i - \Gamma^E_i \Delta E_i, \quad \forall i \in \mathscr{N} \setminus \mathscr{D}, \\
& p_i + \omega_{ij} \geq \Delta e^t_{ij} \hat{q}_{ij}, \quad \forall i \in \mathscr{N} \setminus \mathscr{D}, j \in J^t_i, \\
& p_i + \nu_{ji} \geq \Delta e^r_{ji} \hat{q}_{ji}, \quad \forall i \in \mathscr{N} \setminus \mathscr{D}, j \in J^r_i, \\
& \hat{q}_{ij} \geq 0, \quad \forall i \in \mathscr{N}, j \in \mathscr{S}_i, \\
& \omega_{ij} \geq 0, \quad \forall i \in \mathscr{N} \setminus \mathscr{D}, j \in J^t_i, \\
& \nu_{ji} \geq 0, \quad \forall i \in \mathscr{N} \setminus \mathscr{D}, j \in J^r_i, \\
& p_i \geq 0, \quad \forall i \in \mathscr{N} \setminus \mathscr{D} \\
& T \geq 0.
\end{aligned}
$$

This result is summarized in the following theorem; we omit the proof due to space limitations.

Theorem II.1 *The robust routing problem* (3) *is equivalent to the linear programming formulation* (4). *Furthermore, by solving* (4) *we obtain an optimal solution* $(\hat{\mathbf{q}}^R, T^R, \mathbf{p}^R, \boldsymbol{\omega}^R, \boldsymbol{\nu}^R)$ *so that* $(\hat{\mathbf{q}}^R, T^R)$ *is feasible for* (3) *and* T^R *is equal to the optimal value of* (3).

III. PROPERTIES OF THE OPTIMAL SOLUTIONS

In this section, we study the relationship between the formulations introduced earlier and establish certain properties of the corresponding optimal solutions. We also introduce a metric – the lifetime guarantee probability – to quantify how likely it is for the lifetime predicted by one of these formulations to be achieved.

A. Optimal Lifetime

Let T^*_N, T^*_F, T^*_R denote the optimal values of the nominal, fat, and robust routing problem, respectively. Let $\mathbf{\Gamma}^e = (\Gamma^e_1, \ldots, \Gamma^e_{|\mathscr{N} \setminus \mathscr{D}|})$ and $\mathbf{\Gamma}^E = (\Gamma^E_1, \ldots, \Gamma^E_{|\mathscr{N} \setminus \mathscr{D}|})$. Note that T^*_R depends on $\mathbf{\Gamma}^e$ and $\mathbf{\Gamma}^E$. To express this dependence, we write $T^*_R(\mathbf{\Gamma}^e, \mathbf{\Gamma}^E)$. Due to space limitations, we omitted the proofs for the following results.

Proposition III.1 $T^*_R(\mathbf{\Gamma}^e, \mathbf{\Gamma}^E)$ *is an non-increasing function of both* $\mathbf{\Gamma}^e$ *and* $\mathbf{\Gamma}^E$. *Furthermore,* $T^*_F \leq T^*_R(\mathbf{\Gamma}^e, \mathbf{\Gamma}^E) \leq T^*_N$.

Standard sensitivity analysis results from linear programming yield the following corollary.

Corollary III.2 $T^*_R(\mathbf{\Gamma}^e, \mathbf{\Gamma}^E)$ *is a concave function of* $\mathbf{\Gamma}^E$.

Noticing the structure of our three maximum lifetime routing formulations we observe that at optimality at least one of the energy constraints (2nd set of constraints in each formulation) will be active. This is stated in the following proposition. We will call *dead* the nodes that correspond to active constraints at optimality. The lifetime of a dead node equals the lifetime of the network.

Proposition III.3 *At optimality at least one of the energy constraints in each of the nominal, fat, and robust formulations will be active.*

B. Optimal Flows

Consider an optimal flow vector $\hat{\mathbf{q}}$ obtained by solving one of the three formulations. Recall that $\hat{\mathbf{q}}$ denotes total flow over the network lifetime and $\mathbf{q}$ the flow per unit of time. We associate a directed graph $\mathscr{G}_{\mathbf{q}} = (\mathscr{N}, \mathscr{A}_{\mathbf{q}})$ to $\mathbf{q}$ where $\mathscr{N}$ is the set of WSNET nodes and $\mathscr{A}_{\mathbf{q}}$ contains all directed links (i,j) such that $q_{i,j} > 0$. We will say that a flow $\mathbf{q}$ is *acyclic* if the associated graph $\mathscr{G}_{\mathbf{q}}$ contains no cycles and we will say that $\mathbf{q}$ is *cyclic* otherwise.

Theorem III.4 *For all three routing formulations there exist acyclic optimal flows.*

Proof: Suppose we have an optimal solution $(\mathbf{q}^*, T^*)$. Let $q^*_{i_1 i_2}$, $q^*_{i_2 i_3}$, $q^*_{i_3 i_4}$, $\ldots$, $q^*_{i_k i_1}$ form a cycle in $\mathscr{G}_{\mathbf{q}^*}$. Let $\delta q = \min\{q^*_{i_1 i_2}, q^*_{i_2 i_3}, \ldots, q^*_{i_k i_1}\}$. Subtract δq from all the flows on the cycle. At least one of $q^*_{i_1 i_2}, q^*_{i_2 i_3}, q^*_{i_3 i_4}, \ldots, q^*_{i_k i_1}$ becomes zero and all others remain non-negative. Because both the in-flow and out-flow at each node is reduced by the same amount, the flow conservation condition for all the nodes $i_1, \ldots, i_k$ still holds. Since the above operation only reduces flows all the energy constraints remain satisfied. Hence, the reduced flows remain optimal. We can repeat the same process to eliminate any other cycles. ∎

We omit the proofs of the following corollaries due to space limitations.

Corollary III.5 *For all three routing formulations there exists an optimal flow* $\mathbf{q}$ *which satisfies* $q_{ij} q_{ji} = 0$ *for all possible links* (i,j) *and* (j,i).

Corollary III.6 *For all three routing formulations let* $\mathbf{q}$ *be a cyclic optimal flow. There exists at least one dead node that does not participate in any cycle.*

Corollary III.7 *For all three routing formulations there exists an optimal flow* $\mathbf{q}$ *satisfying* $q_{ij} = 0$, $\forall i \in \mathscr{D}$.

C. Lifetime Guarantee Probability

Consider one of our three formulations and let $\mathbf{q}^*, T^*$ be an optimal flow vector and the corresponding optimal value (lifetime), respectively. We will refer to the probability

$$\mathbf{P}\Big[\min_{i\in\mathscr{N}\setminus\mathscr{D}} \frac{E_i}{\sum_{j\in\mathscr{S}_i} e^t_{ij}q^*_{ij} + \sum_{j|i\in\mathscr{S}_j} e^r_{ji}q^*_{ji}} \geq T^*\Big],$$

evaluated under the distributions of the random variables E_i,e^t_{ij},e^r_{ji}, as the *lifetime guarantee probability*. This is the probability that the actual lifetime obtained by applying the optimal flow $\mathbf{q}^*$ achieves the predicted lifetime. We denote by P_N, P_F, P_R the lifetime guarantee probabilities for the nominal, fat, and robust formulations, respectively.

The following result states that the fat formulation provides an "absolute" guarantee. Again, proofs are omitted for brevity.

Theorem III.8 *It holds* $P_F = 1$.
The straightforward observation is that when $\Gamma^e_i \to |J^t_i| + |J^r_i|$ and $\Gamma^E_i \to 1$, $\forall i \in \mathscr{N}\setminus\mathscr{D}$, then $P_R \to P_F$, while when $\Gamma^e_i \to 0$ and $\Gamma^E_i \to 0$, $\forall i \in \mathscr{N}\setminus\mathscr{D}$, then $P_R \to P_N$.

Let now $\mathscr{A}^N$ be the set of active energy constraints at optimality in the nominal formulation. We establish the following upper bound on P_N.

Theorem III.9 *If E_i,e^t_{ij},e^r_{ji} are independent uniformly distributed random variables then* $P_N \leq (\frac{1}{2})^{|\mathscr{A}^N|}$.

IV. Uncertainty in Initial Energies

Next, we study a simplified version of the maximum lifetime routing problem where uncertainty appears only in the available initial energies E_i at every sensor node.

We define an overall *robustness budget* $\Gamma = \sum_{\forall i\in\mathscr{N}\setminus\mathscr{D}}\Gamma_i$ and incorporate the allocation of Γ to individual Γ_i into the following robust formulation:

$$\begin{aligned}
\max\ & T \qquad\qquad\qquad\qquad\qquad\qquad (5)\\
s.t.\ & \sum_{j\in\mathscr{S}_i} e^t_{ij}\hat{q}_{ij} + \sum_{j|i\in\mathscr{S}_j} e^r_{ji}\hat{q}_{ji} \leq \bar{E}_i - \Gamma_i\Delta E_i,\\
& \qquad \forall i\in\mathscr{N}\setminus\mathscr{D},\\
& \sum_{j|i\in\mathscr{S}_j}\hat{q}_{ji} + TQ_i = \sum_{j\in\mathscr{S}_i}\hat{q}_{ij}, \forall i\in\mathscr{N}\setminus\mathscr{D}\\
& \sum_{\forall i\in\mathscr{N}\setminus\mathscr{D}}\Gamma_i = \Gamma,\\
& \hat{q}_{ij}\geq 0, \qquad \forall i\in\mathscr{N},\ \forall j\in\mathscr{S}_i,\\
& 0\leq\Gamma_i\leq 1, \qquad \forall i\in\mathscr{N}\setminus\mathscr{D},\\
& T\geq 0,
\end{aligned}$$

where the decision variables are T, $\hat{q}_{ij}$'s, and Γ_i's. The proofs of the following two propositions are omitted in the interest of space.

Proposition IV.1 *The optimal value T^*_R of* (5) *is monotonically nonincreasing with the robustness budget* Γ.

The following upper bound on P_N will be useful in proving the key results in this section. Recall that $\mathscr{A}^N$ denotes the set of active constraints at optimality in the nominal formulation.

Proposition IV.2 *Assuming that E_i are i.i.d. we have* $P_N \leq \prod_{i\in\mathscr{A}^N}\mathbf{P}\big[E_i \geq \bar{E}_i\big]$.

Note that if E_i are uniformly distributed then $P_N \leq (\frac{1}{2})^{|\mathscr{A}^N|}$ which is consistent with Thm. III.9.

Next we study two regular network topologies — linear arrays and square arrays. Linear arrays appear, for instance, in pipeline monitoring applications and square arrays in environmental monitoring applications.

A. Linear Arrays

We consider the *linear array* like the L_1 depicted in Fig. 1 where the sink node is at the center and an equal number of sensor nodes are aligned one by one on both sides of the sink. The distance between neighboring nodes is d. Assume that the radio range is less than $3d$, which means that every node can only communicate with the very next four neighbors.

We can form a large linear network by lining up multiple segments. A network with two such segments is shown in Fig. 1. The motivation for growing the network in this particular way is that typically one would need a sink per given number of sensor nodes.

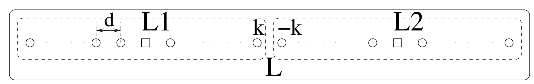

Fig. 1. A linear array consisting of two segments.

The following establishes a decomposition property that holds for all three formulations.

Theorem IV.3 *The maximum lifetime routing problem under either the nominal, fat, or robust formulation for the network L of Fig. 1 can be decomposed into the corresponding problem for L_1 (or L_2).*

Proof: Consider either the nominal, fat, or robust formulation and let $T^*_{L_1}$, $T^*_{L_2}$, T^*_L be the optimal values for networks L_1, L_2, and, L respectively. Clearly, $T^*_{L_1} = T^*_{L_2} \leq T^*_L$ since by combining the optimal flow vectors for L_1 and L_2 we obtain a feasible flow vector for L.

Due to the symmetric structure of L there exists an optimal optimal flow vector which is symmetric about the center of L. Moreover, due to Thm. III.4 and Corollary III.7, such a flow vector is acyclic and has no outflows from sinks. Flows in the interface between the two segments L_1 and L_2 can fall into one out of two possible cases shown in Fig. 2 (top). In each case, we can reconstruct the optimal flows between nodes k and $k-1$ of L_1 and nodes $-k$ and $-k+1$ of L_2 as shown in 2 (bottom). This flow

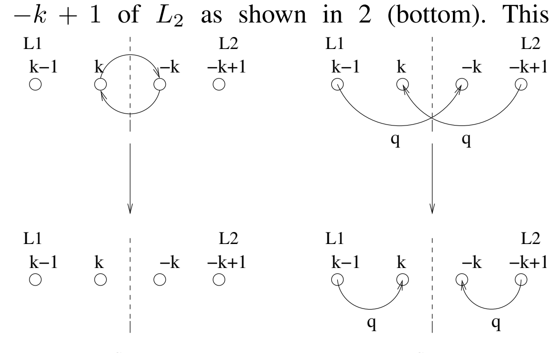

Fig. 2. Flow reconstruction for an optimal flow of L.

reconstruction process maintains feasibility and eliminates any communication between segments L_1 and L_2. Then $T_L^* = \min\{T_{L_1}, T_{L_2}\} \leq T_{L_1}^* = T_{L_2}^*$. Together with our earlier observation it follows $T_L^* = T_{L_1}^* = T_{L_2}^*$, which establishes the result. ■

The following is our main result for linear arrays. It establishes that the nominal formulation is not particularly useful since its predicted lifetime will be achieved with a diminishing probability as the size of the network increases.

Theorem IV.4 *Consider a linear network consisting of 2^n linear segments of the type shown in Fig. 1 and let E_i be i.i.d. and non-degenerate random variables (i.e., not equal to a constant). Then, as $n \to \infty$, $P_N \to 0$.*

Proof: By applying Theorem IV.3 n times, we can decompose the network into 2^n identical segments. Then we can establish the result by Proposition IV.2. ■

B. Square Arrays

We next consider a *square array* like the S_1 depicted in in Fig. 3, where the sink node is at the center. The vertical

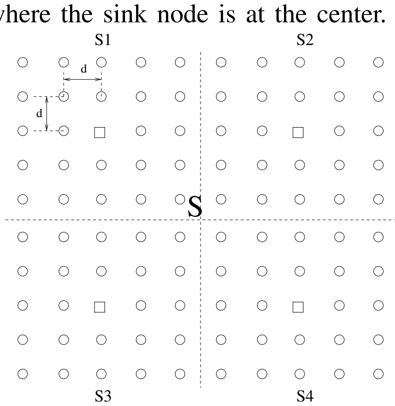

Fig. 3. A square array with four segments.

and horizontal distance between neighboring nodes is d and we assume that the radio range is less than $d\sqrt{5}$. As earlier, we grow a square network in both dimensions by stitching together arrays. Fig. 3 depicts a network S consisting of four segments S_1, S_2, S_3, and, S_4.

Theorem IV.5 *The maximum lifetime routing problem under either the nominal, fat, or robust formulation for the network S of Fig. 3 can be decomposed into the corresponding problem for S_1.*

Proof: Fix a particular formulation, fat, nominal, or robust. Let $T_{S_i}^*, T_S^*$ be the optimal values for network S_i, $i = 1, \ldots, 4$, and S, respectively. As in the proof of Thm. IV.3 $T_{S_i}^* \leq T_S^*$ for all i.

Due to the symmetry of S, Thm. III.4, and Corollary III.7, there exists an acyclic optimal flow vector for S with no flows out of sinks which is symmetric about the vertical line that separates (S_1, S_3) and (S_2, S_4). As in Thm. IV.3 we consider all possible cases and reconstruct the optimal flow as shown in Fig. 4, resulting in a flow with no communication between (S_1, S_3) and (S_2, S_4). A similar flow reconstruction process can result in a flow with no communication between (S_1, S_2) and (S_3, S_4). These flow reconstruction steps maintain flow conservation and do not violate the energy constraints, so the resulting flow is optimal. It follows that $T_S^* \leq T_{S_i}^*$ for all i which concludes the proof. ■

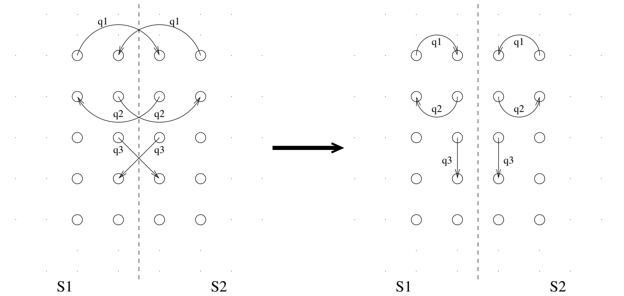

Fig. 4. Flow reconstruction for an optimal flow of S.

As in the linear array case we can now show that the nominal formulation does not provide a useful lifetime prediction. We omit the proof as it is similar to the proof of Thm. IV.4.

Theorem IV.6 *Consider a square network constructed by repeating n times the process of constructing S from S_1, ..., S_4. Let E_i be i.i.d. and non-degenerate random variables (i.e., not equal to a constant). Then, as $n \to \infty$, $P_N \to 0$.*

V. NUMERICAL EXAMPLES

In this section we present a set of numerical examples. For all these examples we adopt the following *communication energy consumption* model.

Let d^r be the transmission range of each node. Then $j \in \mathscr{S}_i$ if and only if $d_{ij} \leq d^r$, where d_{ij} is the distance between nodes i and j. The energy expenditure per data unit transmitted from node i to j satisfies

$$e_{ij}^t = e^\circ + \epsilon_{amp} d_{ij}^4, \qquad e_{ij}^r = e^R,$$

where $e^\circ = 50$ nJ/bit and $e^R = 150$ nJ/bit denote the energy consumed in the transceiver circuitry at the transmitter and the receiver respectively, and $\epsilon_{amp} = 100$ pJ/bit/m^4 is the energy consumed at the output transmitter antenna for transmitting a bit over one meter. The receiver circuitry in general consumes more energy than the transmitter circuitry within the same order of magnitude. The path loss exponent of four is chosen to account for the multipath reflection instead of using a free space model which uses two.

A. A 4-node WSNET

We start with a toy example to gain some intuition on the routing policies produced by each formulation. The WSNET consists of one origin node, two relay nodes, and one sink node. The origin node O has an information generation rate $Q_O = 500$ bits/sec. The radio range is 25 m. Thus, the origin node has to use relays R_1 and R_2 to reach the sink S (see Fig. 5). All E_i, e_{ij}^t, e_{ji}^r are uniformly distributed with $\bar{E}_i = 10$ J, $E_O \in [9, 11]$ J, $E_{R_1} \in [8.5, 11.5]$ J, $E_{R_2} \in [9.5, 10.5]$ J, $\Delta e_{ij}^t / \bar{e}_{ij}^t = 0.1$, and $\Delta e_{ji}^r / \bar{e}_{ji}^r = 0.1$. Note $\Gamma_O^e \in [0, 4]$, $\Gamma_{R_1}^e \in [0, 6]$, $\Gamma_{R_2}^e \in [0, 6]$, and $\Gamma_i^E \in [0, 1]$ for all i. Set $\Gamma_O^e = 1.6$, $\Gamma_{R_1}^e = 2.4$, $\Gamma_{R_2}^e = 2.4$, and $\Gamma_i^E = 0.5$ for all i. Fig. 5 depicts the optimal flows under each of the nominal, robust, and fat formulations and Table I lists the corresponding optimal values and the lifetime guarantee probability, where T_{real} denotes the lifetime achieved by implementing the corresponding policy.

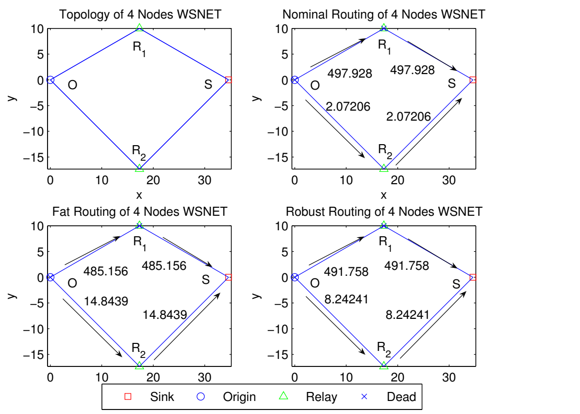

Fig. 5. A 4-node WSNET and optimal flows under the three formulations.

TABLE I

T^* AND $\mathbf{P}[T_{real} > T^*]$ FOR THE 4-NODE WSNET OF FIG. 5.

	Nominal	Fat	Robust
T^* $(\frac{T_R^*-T_F^*}{T_F^*} \times 100\%)$	1239.81	983.26	1056.01 (7.40%)
$\mathbf{P}[T_{real} > T^*]$	0.25	1.0	0.91

It can be seen that all three policies prefer the path (O, R_1, S) which is shorter, thus, less energy demanding. The lifetime prediction of the nominal policy has a low probability of being achieved. The robust policy provides a 7.4% higher lifetime quote with a high enough (0.91) probability of being achieved, so it is an interesting compromise between the nominal and the fat.

B. Linear and square arrays

In this example, every linear array has 10 origin nodes and one sink node. The distance between two neighboring nodes is 10 m. The radio range is 25 m, $Q_i = 500$ bits/sec, and $\bar{E}_i = 10$ J. We consider instances formed as the one in Fig. 1 with 1, 2, 4, and 8 segments, respectively. The results are in Table II where $\Gamma_i^e = (|J_i^t| + |J_i^r|) \cdot 30\%$ for all instances, and $\Gamma_i^E = 0.6, 0.7, 0.8, 0.9$ for each of the 4 instances, respectively. We observe that the nominal formulation does not provide a useful lifetime prediction while the robust provides a gain (in terms of quoted lifetime) over the fat (diminishing with the size of the network).

TABLE II

RESULTS FOR LINEAR ARRAYS WITH 1, 2, 4, AND 8 SEGMENTS.

LAs	T_N^*	P_N	T_F^*	P_F	T_R^*	P_R	$\frac{T_R^*-T_F^*}{T_F^*} \times 100\%$
1	3480.77	1.9×10^{-3}	2847.91	1.0	2976.25	0.87	4.51%
2	3480.77	8×10^{-6}	2847.91	1.0	2944.57	0.91	3.37%
4	3480.77	0.0	2847.91	1.0	2912.91	0.94	2.28%
8	3480.88	0.0	2847.99	1.0	2881.25	0.98	1.17%

Finally, we consider square array networks. Each square array has 48 origin nodes and one sink node where $d = 10$ m, the radio range is 21 m, $Q_i = 500$ bits/sec, and $\bar{E}_i = 10$ J. We consider instances formed as the one in Fig. 3 consisting of 1, 4, and 16 segments, respectively. The results are in Table III where $\Gamma_i^e = (|J_i^t|+|J_i^r|)\cdot 20\%$ for all instances, and $\Gamma_i^E = 0.75, 0.85, 0.9$ for each of the 3 instances, respectively.

TABLE III

RESULTS FOR SQUARE ARRAYS WITH 1, 4, AND 16 SEGMENTS.

SAs	T_N^*	P_N	T_F^*	P_F	T_R^*	P_R	$\frac{T_R^*-T_F^*}{T_F^*} \times 100\%$
1	1889.72	0.0	1546.14	1.0	1589.36	0.84	2.80%
4	1889.72	0.0	1546.14	1.0	1572.17	0.81	1.68%
16	1889.72	0.0	1545.82	1.0	1563.57	0.74	1.15%

VI. CONCLUSIONS

We considered a maximum lifetime routing problem in WSNETs which can be formulated as a linear programming problem. Our main observation is that in the presence of uncertainty in key network parameters a nominal – certainty equivalence – formulation that uses mean parameter values is not particularly useful. In particular, it provides a routing policy which almost never achieves the lifetime predicted by the formulation. To accommodate uncertainty, we develop a worst-case (fat) formulation that computes a policy yielding a lifetime that always exceeds the one predicted by the formulation. The fat formulation is "tight" in the sense that there exist random network parameter instances under which the fat policy will result in a lifetime equal to the predicted one. However, in many cases one may want to be more optimistic, that is, predict a lifetime with a reasonably high probability of being achieved by the corresponding policy. To that end, we develop a series of robust formulations with a tunable set of "robustness budgets" that allow the WSNET designer to trade-off the predicted lifetime with the probability it is achieved.

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