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Robust Limit Cycle Control in an Attitude Control System with Switching-Constrained Actuators

Alexandre Rodrigues Mesquita[1*], Karl Heinz Kienitz[2] and Erico Luis Rempel[3]

Abstract— **In this paper the robust behavior in some piecewise affine systems with minimally spaced transition times is studied. Such systems are found e.g. in satellites and satellite launchers. On-off thrusters are frequently used as actuators for attitude control and are typically subject to switching constraints. In these systems, persistent motions of different nature may occur, such as limit cycles, quasi-periodic-like and chaotic motions. Thus, in the presence of model uncertainties, the emergence of bifurcations can seriously affect performance. In this contribution, we use Tsypkin's method in order to investigate the robustness of the condition for the existence of limit cycles. Robustness frontiers in the space of control parameters are identified. These frontiers are verified via simulation and compared to those given by the describing function method, revealing the difficulties of this latter method to address the robustness analysis in this system. Moreover, we present a design method for robust controllers based on the Hamel locus. An evaluation of performance requirements such as fuel consumption, limit cycle amplitude and transient response is carried out in the identified regions of robust behavior.**

I. INTRODUCTION

Throughout the last decades, attitude control systems with switching actuators have been used in satellite and launching systems [1], [2], [3], [4], [5]. In the attitude stabilization phase, such systems typically have been operated in limit cycle conditions. As actuators, several types of on-off thrusters are employed, such as hydrazine, cold-gas and pulsed plasma thrusters [2]. These thrusters are typically affected by switching constraints, which have been a cause of concern about the degradation of the system's performance. As shown by Oliveira and Kienitz [4], non-conventional analysis/design problems arise when actuators are subject to switching-time restrictions. Certain conditions ensure that limit cycles exist. When these conditions do not hold, system motion may not be of limit cycle type.

During recent research on the issue of limit cycle control for a system with minimally spaced switching-times, we observed [6] that the optimal control parameter set, which guarantees minimum amplitude and minimum fuel consumption, lies on the frontier where the system bifurcates into nonperiodic persistent motions. Here arises the concern with the robustness of an optimal controller. In this paper, we are interested in the design of robust controllers that preserve a good performance for the uncertain system while guaranteeing operation in the limit cycle mode, i.e., controllers that mitigate the possibility of bifurcations. More specifically, we focus on the bifurcation that arises when the limit cycle frequency reaches a certain threshold.

Computing the limit cycle points of uncertain nonlinear systems has attracted the attention of researchers in the last decade. Most of them were simply concerned with the inhibition of limit cycles in order to prove stability. In the main papers available on this issue [7], [8], [9], [10], [11], [12], [13], [14], first harmonic approximation has been adopted. The deficiency of this approximated analysis for the studied system was shown in [6] and is further exemplified in section VI.

Tan and Atherton [11] present a method to compute magnitude and phase envelopes of uncertain transfer functions and apply describing function analysis to predict the existence of limit cycles. We adopt a similar approach in this paper, with the difference that we do not use the describing function approximation. Among the other possible approaches, the most consistent ones are proposed by Katebi and Zhang [14] and [13], which incorporate the dynamics neglected by the describing function approximation as an unstructured uncertainty into the problem description. Nevertheless, these approaches are not useful when dealing with some relay-type nonlinearities, for the uncertainty due to the approximation may be conservative to the point that a robust controller result may not exist.

A natural conclusion is that the intended robustness analysis should take advantage of the knowledge of higher-order harmonics. Hence, in this contribution we consider exact methods for limit cycle prediction. Using Tsypkin's method, we are able to address both parametric uncertainties and magnitude-phase envelopes of uncertain transfer functions. By analyzing each point in a grid of the space of control parameters, we can find a frontier that determines robust limit cycle behavior and, as a consequence, we are able to avoid nonperiodic regimes as we are looking for an amplitude minimization. On this frontier, we calculate an interval of amplitude variation and study the vanishing of transients. In addition, we propose a design procedure using a Hamel-type locus, which allows a decrease in the dimension of the space of control parameters.

II. PROBLEM DESCRIPTION

The problem description given here is akin to that in [4] and it is based on the attitude control design of the

[1] Center for Control, Dynamical Systems and Computation, University of California, Santa Barbara, CA 93106

[2] Departamento de Sistemas e Controle, Instituto Tecnológico de Aeronáutica, CTA, São José dos Campos, 12228-900, Brazil

[3] Departamento de Matemática, Instituto Tecnológico de Aeronáutica, CTA, São José dos Campos, 12228-900, Brazil

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Brazilian Satellite Launcher. We shall consider the single-axis problem only since coupling can be ignored under reasonable conditions as in [15, Sec. 6.3]. Yet, the analysis of the 3-axis problem could be carried out according to the same principle applied in this paper. Consider a simple rigid body (e.g. satellite or rocket in the upper atmosphere) whose attitude ϕ is to be controlled using sets of small thrusters, which are on-off actuators with switching-time restrictions. A simplified representation of the system is shown in Fig. 1, where the thrust F may assume final values $-F_{\max}$, 0 or $F_{\max}$.

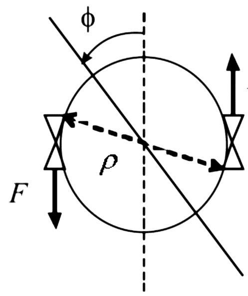

Fig. 1. Rigid body with a set of thrusters

A body inertia $J = 1500$ [kgm^2] is given. The small thruster actuators do have delays and switching-time restrictions:

- Maximum absolute torque: $\rho F_{\max} = 308$ [Nm].
- Switching-time restrictions:
  - minimum duration of pulses: $t_{\text{on}} = 100$ [ms].
  - minimum rest between successive pulses of the same sign: $t_{\text{s}} = 50$ [ms].
  - minimum rest between pulses of different sign: $t_{\text{off}} = 500$ [ms].
- Thrust build up dynamics (On):
  - 10% of maximum thrust: 10-30 [ms]
  - 90% of maximum thrust: 20-50 [ms]
- Thrust build up dynamics (Off):
  - 90% of maximum thrust: 9-16 [ms]
  - 10% of maximum thrust: 15-50 [ms]

The typical requirement for the controlled system is that initial conditions and attitude perturbations shall asymptotically die away into a well behaved limit cycle. For the purpose of achieving an appropriate performance, a tachometric feedback law (feedback of position and velocity) and a single-pole controller $C(s) = \frac{1}{s-p}$ are often added to the loop, resulting in the controlled system represented in Fig. 2.

The Actuators block of Fig. 2 is decomposed into a series structure with two sub blocks. The first one contains a relay with the above switching restrictions and with output in $\{k_r, -k_r, 0\}$, where $k_r = F_{\max}\rho/J$. The second one contains a linear dynamics which models thrust build up. In practice, actuator delays may vary during the operation of the system. Their value may depend on several parameters. Thus, the model is affected by uncertainty. All the gains

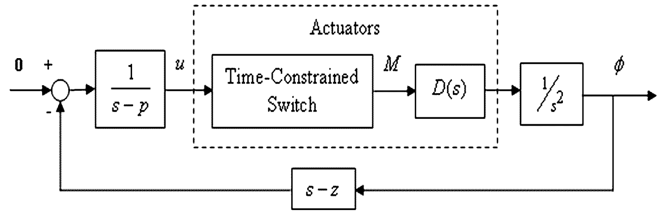

Fig. 2. Block diagram of the controlled system

in the system are rearranged to the output M of the Time-Constrained Switch block. Since the controller is linear, these gains affect only the amplitude of the response.

According to relay systems theory, for appropriate values of p and z, we should expect symmetric unimodal limit cycle behavior. Necessary conditions to the existence of this limit cycle can be provided either by approximate or exact methods (see [16]). The above switching-time restrictions impose another condition, which states the existence of a maximum value for the switching frequency f:

$$f \le f_{\max} = \frac{1}{2(t_{\text{on}} + t_{\text{off}})} \quad . \tag{1}$$

Since the controller is linear, the period for which the actuator is off at each half-cycle is always t_{off}. Thus, the fuel consumption will be minimum if the period for which the actuator is on is also minimum, that is, if the limit cycle frequency is maximum. Additionally, if we calculate $\phi(t)$ approximately by double-integrating the periodic train of pulses $M(t)$, one can intuitively see that the amplitude decreases monotonically with f as well. However, if the controller demands a switching frequency higher than $f_{\max}$, nonperiodic persistent motions arise [6] and the amplitude may vary significantly in the presence of uncertainties. In [17], we characterize a quasi-periodic-like motion that arises as a bifurcation from periodic motion.

Hence, the robust performance aimed in this paper consists of the occurrence of single-switching (unimodal) limit cycles that possess a set of possible frequencies with upper bound $f_{\max}$ and maximum lower bound.

III. AN EXACT METHOD TO PREDICT LIMIT CYCLES

In this section we obtain necessary conditions to the existence of limit cycles in the fashion of Tsypkin [18]. Suppose the existence of a single-switching (unimodal) periodic output $M(t)$ with period T as depicted in Fig. 3.

As noted in [4], this wave is equivalent to the sum of a square wave with amplitude $k_r/2$ and another square wave with the same amplitude but delayed by t_{off}. If we call $\omega_0 = 2\pi T^{-1}$, the following Fourier series decomposition can be verified:

$$M(t) = \sum_{k \text{ odd}} \frac{4k_r}{\pi k} \operatorname{Im}\left\{ \left(\frac{1 + e^{-jk\omega_0 t_{\text{off}}}}{2} \right) e^{jk\omega_0 t} \right\} \quad . \tag{2}$$

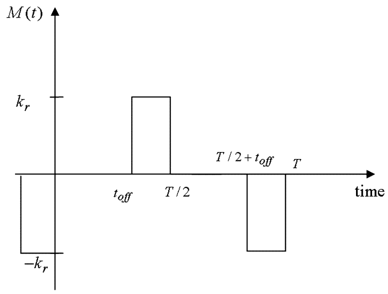

Fig. 3. Unimodal periodic actuators' output

Thus, in the case of single-switching periodic motion, the Time-Constrained Switch block can be replaced by a simple relay with output $M'(t)$ in $\{k_r, -k_r\}$ and followed by the transfer function $\left(1+e^{-st_{\text{off}}}\right)/2$.

Two necessary conditions to the existence of such output will be considered, see [18, Eq. 5.47]:

$$\begin{aligned} &u(T/2) = 0 \\ &\frac{du}{dt}(T/2) < 0 \ . \end{aligned} \tag{3}$$

These conditions are not sufficient because we do not take into account the possibility of intermediary crossings $u(t) = 0$ for $t < T/2$. The Tsypkin locus in the complex plane is defined in [18, Eq. 6.1] as

$$\Lambda(\omega) = \frac{1}{\omega}\frac{du}{dt}(T/2) + ju(T/2) \ , \tag{4}$$

where $\omega = 2\pi T^{-1}$. According to the conditions in (3), the existence of a limit cycle of angular frequency ω_0 requires that $\angle\Lambda(\omega_0) = \pi$. Defining the transfer function $L(s) = -U(s)/M'(s)$ and recurring to (2) and (4), we can verify the following expression for the Tsypkin locus

$$\Lambda(\omega) = \sum_{k \text{ odd}} \frac{4k_r}{\pi}\left[\text{Re}\{L(j\omega k)\} + j\frac{1}{k}\,\text{Im}\{L(j\omega k)\}\right] \ . \tag{5}$$

Thus, the Tsypkin locus is a useful tool for the determination of limit cycle properties such as frequency and amplitude. Though other exact methods exist, such as the state-space based method [19], Tsypkin's is a more convenient method when dealing with uncertain systems, since it is more convenient to express uncertainties in the frequency domain. A useful criterion to address the limit cycle stability graphically from the Tsypkin locus is provided by the following necessary condition from [18, Eq. 10.69]:

$$\text{Im}\left\{\frac{d\Lambda(\omega)}{d\omega}\right\} > 0 \ . \tag{6}$$

Another useful result from [18, Eq. 6.54] is the exact expression of the Tsypkin locus in the case the transfer function from the relay output to the relay input has the form:

$$L(s) = \frac{P(s)}{s^2 Q(s)}e^{-s\tau} \ , \tag{7}$$

where $P(s)$ and $Q(s)$ are polynomials with non-zero simple roots.

This expression is applied in the parametric robustness analysis and is also interesting since it provides qualitative knowledge on the Tsypkin's locus, such as the maximum number of possible limit cycles.

Notice that, there was not a minimal pulse duration, a limit cycle with frequency $\omega_0 = \pi/t_{\text{off}}$ would exist, since $\Lambda(\pi/t_{\text{off}}) = L(\pi/t_{\text{off}}) = 0$. At this frequency, however, the output $M(t)$ is always zero, that is, this limit cycle is an equilibrium point at the origin. If this limit cycle was proved to be stable, the system could be stabilized by the application of infinitesimal duration pulses. As these pulses are not allowed, an undesirable quasi-periodic-like motion would arise.

IV. MODEL UNCERTAINTIES

The main uncertainty that affects the dynamical behavior of the system is the thrust build up dynamics. Though build up dynamics are different when actuators switch on or off, one can verify that assuming symmetric build up dynamics will just make our robustness analysis more conservative. Therefore, we model these dynamics by the transfer function $D(s)$, whose step response must be contained in a time envelope given by the fastest and the slowest responses that we describe in section II.

In this paper we utilize both parametric and a nonparametric representations for the uncertainty in $D(s)$. A detailed derivation of these representations is provided in [20]. The parametric representation assumes the following structure for $D(s)$:

$$D(\alpha, \tau, s) = \frac{e^{-s\tau}}{\alpha s + 1} \ . \tag{8}$$

The parametric domain is given by $\tau \in [\tau_{\min}, \tau_{\max}]$ and $\alpha \in \left[\alpha_{\min}, \frac{\tau - 0.05}{\ln 0.1}\right]$, where $\tau_{\max} = 29.04$ [ms], $\alpha_{\max} = 18.66$ [ms] and $\tau_{\min} = 7.03$ [ms].

The unstructured representation is given by magnitude and phase envelopes defined by the transfer functions

$$\bar{D}(j\omega) = \begin{cases} \left|\frac{1}{\alpha_{\min} j\omega + 1}\right| \frac{D_F(j\omega)}{|D_F(j\omega)|} & \text{, if } \omega < 28 \\ \left|\frac{1}{\alpha_{\min} j\omega + 1}\right| & \text{, if } \omega \geq 28 \end{cases} \tag{9}$$

and

$$\bar{D}(j\omega) = \begin{cases} \left|\frac{1}{\alpha_{\max} j\omega + 1}\right| \frac{D_G(j\omega)}{|D_G(j\omega)|} & \text{, if } \omega < 30.5 \\ \left|\frac{1}{\alpha_{\max} j\omega + 1}\right| e^{-j\omega\tau_a} & \text{, if } \omega \geq 30.5 \end{cases} \ , \tag{10}$$

where $\tau_a = 65$ and $D_F(s)$ and $D_G(s)$ denote, respectively, the transfer functions related to the fastest and to the slowest curves in the time envelope described in section II.

V. LIMIT CYCLE ROBUSTNESS ANALYSIS

In this section we test the necessary conditions for the existence of limit cycles with respect to their robustness. Since these conditions are necessary only, we validate them via simulation. The robust controller to be designed is that for which the supremum of the set of predicted possible frequencies is not larger than $\omega_{\max} = 2\pi f_{\max}$, in such a way that (1) is satisfied. Therefore, a bifurcation frontier in the space of control parameters can be calculated by checking for values of z and p such that this supremum is $\omega_{\max}$. Let $\mathcal{B}$ be the family of possible Tsypkin loci for the uncertain system and assume that $\mathrm{Re}\{\Lambda_\xi(\omega_{\max})\} < 0$ for all $\Lambda_\xi \in \mathcal{B}$. Then, the periodic to quasi-periodic-like bifurcation frontier must lie on the curve

$$(z,p) : \min_{\Lambda_\xi \in \mathcal{B}} \{\mathrm{Im}\{\Lambda_\xi(\omega_{\max})\}\} = 0 \ . \qquad (11)$$

As shown in Fig. 4, a point of the bifurcation frontier occurs whenever the lower bound of the interval of possible $\mathrm{Im}\{\Lambda(\omega_{\max})\}$ crosses zero. However, the bifurcation frontier may not coincide with the above curve, given that we do not verify sufficient conditions for the existence of stable limit cycles.

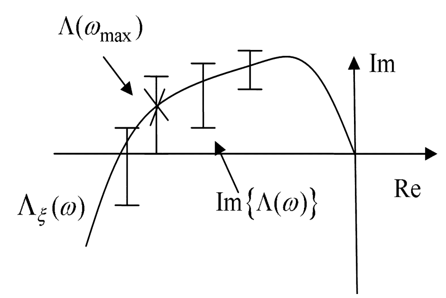

Fig. 4. Uncertain Tsypkin locus for p and z on the bifurcation frontier

In the case of parametric uncertainties, the bifurcation frontier is calculated by the evaluation of the exact Tsypkin locus expression in a grid of the parameter space. We have adopted $\Delta\alpha = 0.8$ [ms] and $\Delta\tau = 1.1$ [ms]. For each combination of z and p, if we find $\mathrm{Im}\{\Lambda_\xi(\omega_{\max})\} < 0$ for $\Lambda_{xi} \in \mathcal{B}$, we may interrupt the search and conclude that the limit cycle is not robust at this point. In fact, as $\Lambda_\xi(\pi/t_{\text{off}}) = 0$ and $\pi/t_{\text{off}} > \omega_{\max}$, the Tsypkin locus must cross the real axis positively for some $\omega > \omega_{\max}$, which violates the condition in (1).

In the case of unstructured uncertainty, we can establish a lower bound for $\min_{\Lambda_\xi \in \mathcal{B}} \{\mathrm{Im}\{\Lambda_\xi(\omega_{\max})\}\}$ by choosing $D(j\omega) = D_{\xi^*}(j\omega)$ inside the phase and magnitude envelopes in such a way that each harmonic contribution to $\mathrm{Im}\{\Lambda(\omega_{\max})\}$ in (5) is minimized.

A. ANALYSIS/DESIGN VIA HAMEL'S METHOD

In this section we present an alternative analysis method that can be useful in synthesis. It uses the Hamel locus, which allows for a more intuitive zero allocation. If we define $\epsilon = u(T/2)$ and $\dot{\epsilon} = \frac{du}{dt}(T/2)$, the Hamel locus is given by the curve in the phase plane:

$$H = (\epsilon, \dot{\epsilon}) = (\omega\,\mathrm{Re}\{\Lambda\}, \mathrm{Im}\{\Lambda\}) \ . \qquad (12)$$

The oscillation frequency is also determined by the crossing of the abscissa. On the other hand, we can interpret the placing of a block $(s - z)$ in the open loop as a change in the switching condition from $\epsilon = 0$ to $-z\epsilon + \dot{\epsilon}$. Therefore, the oscillation frequency can be found in the crossing of the line $\epsilon = \dot{\epsilon}/z$ by the Hamel locus of the system without the zero. This suggests that a robust controller synthesis can be done by the proper allocation of a line passing through the origin and tangent to the set of possible Hamel locus points at $\omega = \omega_{\max}$. If we consider this set to be rectangular and that $H(\omega_{\max})$ belongs to the second quadrant, we conclude

$$z = \frac{\min \epsilon(\omega_{\max})}{\min \dot{\epsilon}(\omega_{\max})} = \omega_{\max} \frac{\min\{\mathrm{Re}\{\Lambda_\xi(\omega_{\max})\}}{\min\{\mathrm{Im}\{\Lambda_\xi(\omega_{\max})\}} \ . \qquad (13)$$

The above procedure is illustrated in Fig. 5. Since one considers a rectangular set of possible $H(\omega_{\max})$, this procedure is expected to be somewhat more conservative than that we present using the Tsypkin's method.

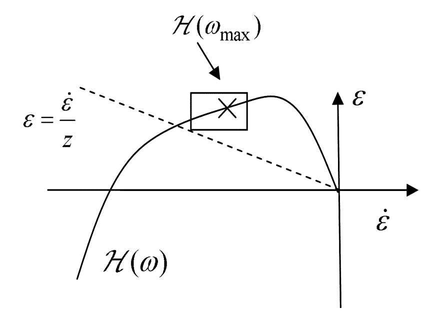

Fig. 5. Robust design by allocation of the switching line (dashed) on the Hamel locus (solid)

B. INSTABILITY FRONTIER

Besides the periodic to quasi-periodic-like bifurcation frontier, we can identify another important frontier given by the arising of instability. Indeed, the double integrator in $L(s)$ implies that $\lim_{\omega\to 0^+} \angle\Lambda(\omega) = \lim_{\omega\to 0^+} \angle L(\omega) = \pi$. If this limit cycle is stable, there will be trajectories with no switching at all, that is, there will be instability. Based on (6), the instability frontier is expressed as follows:

$$(z,p) : \max_{\Lambda_\xi \in \mathcal{B}} \left(\lim_{\omega\to 0^+} \mathrm{Im}\left\{ \frac{d\Lambda_\xi}{d\omega}(\omega) \right\} \right) = 0 \ . \qquad (14)$$

For each combination of z and p on the frontier, we must have $\lim_{\omega\to 0^+} \mathrm{Im}\left\{\frac{d\Lambda_\xi}{d\omega}(\omega)\right\} = 0$ in the worst case, which occurs if and only if $\lim_{\omega\to 0^+} \mathrm{Im}\left\{\frac{dL_\xi}{d\omega}(\omega)\right\}$. As $L_{\xi^*}(j\omega) \to \pi$ in the limit, the above condition is equivalent to

$$\lim_{\omega\to 0^+} \frac{d\angle L_{\xi^*}}{d\omega}(j\omega) = 0 \ . \qquad (15)$$

Replacing $L_{\xi^*}(j\omega)$, we obtain

$$\frac{1}{p} - \frac{1}{z} - \frac{t_{\text{off}}}{2} + \lim_{\omega\to 0^+} \frac{d}{d\omega}\angle D_{\xi^*}(j\omega) = 0 \ . \qquad (16)$$

According to the phase envelope, the limit in (16) must be in the interval [-53,-8.7] [ms]. In the case of parametric uncertainties, the limit is given by $-(\alpha+\tau)$, where $-(\alpha+\tau) \in [-38.1, -9.8]$ [ms], that is contained by the interval for unstructured uncertainty. As the derivative of the phase and of the imaginary part of $L_{\xi^*}(j\omega)$ have opposite signs in the limit $\omega \to 0^+$, we conclude that the instability frontier is determined by (16) with $D_{\xi^*}(j\omega)$ being such that $\lim_{\omega\to 0^+} \frac{d}{d\omega}\angle D(j\omega)$ is minimum.

VI. NUMERICAL RESULTS

The numerical assessment considers the space $z \times p = [-3, 0] \times [-60, -10]$ with grid resolution $\Delta z \times \Delta p = 0.2 \times 0.1$. The choice of the number N of harmonics in the truncation of the Tsypkin locus expression is empirical. In Fig. 6 we exhibit the bifurcation frontier given by Hamel locus approach for different N. From this we decided to adopt $N = 27$. The figure also indicates that a first order approximation would seriously affect a robust design.

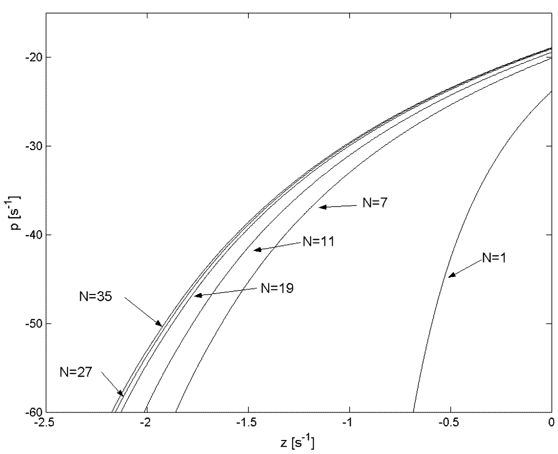

Fig. 6. Convergence of the bifurcation frontier as the truncation term N is increased

Fig. 7 exhibits the regions where the limit-cycle frequency is robustly bounded. The region for proper control is that where $0 < \omega_0 < \omega_{\max}$. Especially, the amplitude is minimized on the bifurcation frontier and increases indefinitely as we move towards the instability frontier. We should remark that for $z > 0$ limit cycles become unstable. In fact, when z changes sign the residue of the term $1/s^2$ in $L(s)$ also changes sign, which makes unstable the related closed-loop sampled-data system we use to assess limit cycle stability, as done in [18, Chapter 10].

In order to evaluate the conservativeness of the robustness frontier we use Fig. 8 to compare it with frontiers given by other approaches. The parametric analysis is carried on the domain of z and p with grid resolution $\Delta\alpha = 0.8$ [ms]

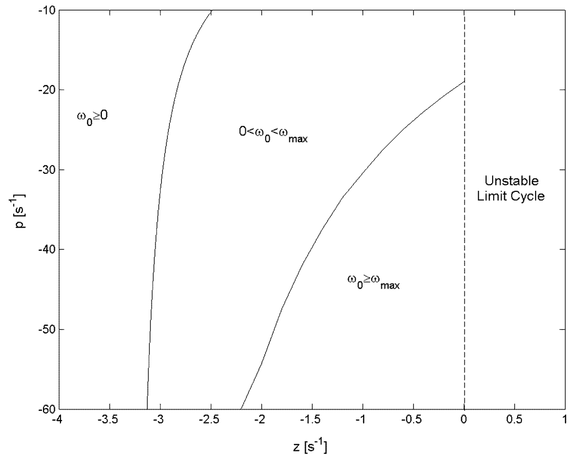

Fig. 7. Regions of robust limit cycle behavior (unstructured uncertainty)

and $\Delta\tau = 1.1$ [ms]. We compare the frontier obtained for the unstructured uncertainty to the one for the structured uncertainty. On the p axis the second curve may be at most 12 [s^{-1}] below the first one; in the z axis, at most 0.4 [s^{-1}] to the right. It is remarkable that the worst case $\Lambda_\xi(\omega_{\max})$ is always verified for the case of fastest thrust build up, that is, for $\alpha = \alpha_{\min}$ and $\tau = \tau_{\min}$. The frontier provided by the Hamel locus design was slightly more conservative than that given by unstructured uncertainties. On the p axis they differ at most by 1 [s^{-1}]; on the z axis, by 0.04 [s^{-1}]. At length, we trace the frontier obtained when we consider $D(j\omega) = D_F(j\omega)$. This frontier suggests that the envelope technique is an important cause of conservativeness, otherwise we would have the curve for $D_F(j\omega)$ closer to the frontier given by the unstructured uncertainty than to that given by parametric uncertainties.

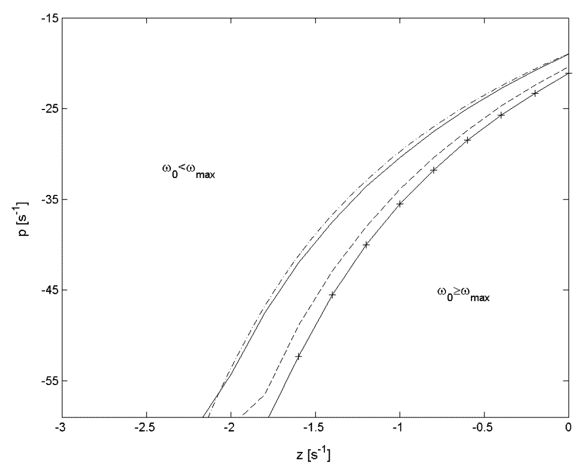

Fig. 8. Comparison of the bifurcation frontiers based on unstructured uncertainty (solid), on parametric uncertainties (cross), on the function $D_F(s)$ (dashed) and on Hamel locus approach (dot-dashed)

The large area of the region in Fig. 7 where $0 < \omega_0 < \omega_{\max}$, suggests that the desired robust controller will be subject to great intervals of frequency and amplitude variation. Indeed, one verifies that the largest intervals of amplitude variation occur nearby the instability frontier and the smallest intervals occur in the limit cycle stability boundary at $z = 0$. Choosing z in order to have a minimum amplitude variation has obviously a drawback, since in this case the limit cycle would be marginally stable. This drawback is the duration of transient responses, which increases unboundedly as z approaches 0. The maximum value of the roll angle amplitude settling time for 1% was obtained via simulation and plotted in Fig. 9. Thus, the designer can establish a trade off between length of the amplitude intervals and duration of transients.

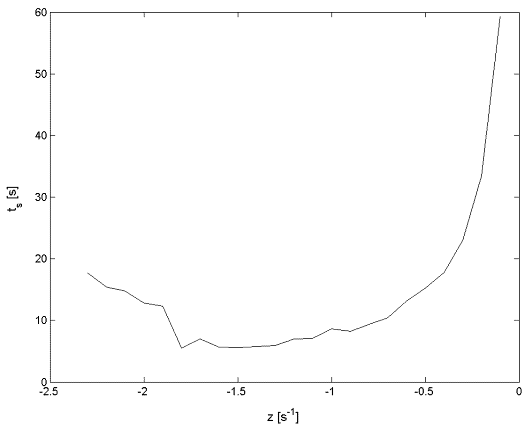

Fig. 9. Maximum roll angle settling time along the bifurcation frontier

As stated in the previous sections there is a series of hypotheses that must be verified in order that all points on the calculated bifurcation frontier be correct. Among the cited hypotheses are: the uniqueness of the stable limit cycle; the sufficiency of limit cycle stability condition; the absence of intermediary switches; the unimodal limit cycle is a global behavior. Though the designer need to validate only the chosen control parameter combination, it would be interesting to know whether the entire frontier is correct. In [20], we show via simulations that the calculated frontier is indeed correct.

VII. Conclusion

In this paper we presented a study of the robust limit cycle control in an attitude control system with relay-type actuators subject to minimally spaced transition times. As the emergence of bifurcations in this system can seriously affect performance, we developed analysis/synthesis techniques for robust prevention of bifurcations and efficient employment of actuators. The proposed techniques introduce a robust limit cycle control that relies in an exact limit cycle prediction. The discussed attitude control problem is an important instance for which an exact prediction would be noticeably advantageous in obtaining an improved performance. First-order linear controllers that robustly reduce both amplitude and fuel consumption can be obtained. However, the designer should establish a trade off between amplitude interval and transient duration. In addition, since most relay control systems are subject to similar time restrictions, the presented techniques may be useful to efficiently exploit actuators in other systems that alternate among unstable dynamics.

A disadvantage of exact methods in relation to the describing function methods is the low availability of control synthesis procedures. We believe that such a deficiency can be overcome through a joint analysis that uses describing function and exact methods.

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