Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

Two-Degree-of-Freedom PI/PID Tuning Approach for *smooth* Control on Cascade Control Systems

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Abstract— **A design approach for Two-Degree-of-Freedom (2-DOF) PID controllers within a cascade control configuration that guarantees smooth control is presented in this paper. The rationale of operation associated to both, the inner and outer controllers, determines the need of good performance for disturbance attenuation (regulation) as well as set-point following (tracking). Therefore the use of 2-DOF controllers is introduced. However the use of 2-DOF controllers introduces additional parameters that need to be tuned appropriately. Specially for the case of PI/PID controllers there are not known clear auto-tuning guidelines for such situation. The approach undertaken in this paper provides the complete set of tuning parameters for the inner (2-DOF PI) controller and the outer (2-DOF PID) controller. The design equations are formulated in such a way that a non-oscillatory response is specified for both the inner and outer loop. A side advantage of providing the complete set of parameters is that it avoids the need for the usual identification experiment for the tuning of the outer controller.**

Index Terms— **PID Control, Cascade control, Two-Degree-of-Freedom**

I. Introduction

Cascade control is one of the most popular *multi-loop* control structures that can be found in the process industries, implemented in order to improve the disturbance rejection properties of the controlled system [6], [7]. The application of a cascade control structure is based on the introduction and use of an additional sensor that allows for a separation of the fast and slow dynamics of the process resulting in a nested loop configuration as it is shown in Fig. 1. The controller of the inner loop is called the secondary or slave controller whereas the controller of the outer loop as the primary or master controller, being the output of the primary loop the controlled variable of interest. The rationale behind this configuration is that the fast dynamics of the inner loop will provide faster disturbance attenuation and minimize the possible effect of the disturbances, before they affect the primary output.

As this set up includes two controllers, its tuning is therefore a more complicated design procedure than the one for a standard single-loop control system. The usual approach involves the tuning of the secondary controller while setting the primary controller in manual mode. On a second step, the primary controller is tuned by considering the secondary controller acting on the inner loop. Some existing studies provide approaches that help in the design of a cascade control system. In [2] a relay-feedback based autotuning method has been used. The procedure still needs of a sequential application of the usual relay based autotuning approach. Other results provide tuning rules for the primary and secondary controllers [5], [9] or suggest alternative control structures based on a modification of the conventional cascade configuration [4]. However there are no clear guidelines on how to automate the process and what should be the rationale behind both tunings.

Recently, in [10], an automated procedure is proposed. The main point of that approach is the approximation of the inner-loop dynamics on the basis of a First-Order-Plus-Dead-Time (FOPDT) dynamics, that allows the application of well known tuning rules. It is however needed, in order for this approximation to have validity, that the closed-loop system resulting from the application of the inner controller does not present oscillations. This is not guaranteed with the application of typical PID tuning rules. In addition, the model for the outer loop design is obtained via least squares approximation in the frequency domain. Therefore the method can not be considered completely automatic. In contrast the procedure presented here just needs to know the open-loop models. There will be no need for the obtention of a model for the outer loop design.

The purpose of this paper is to provide a completely automated design procedure within the framework of the usual PI/PID controllers. More precisely, the benefits of using the set-point weighting capabilities within the Two-Degree-of-Freedom PID controller will be highlighted. In order to guarantee the industrial applicability of the provided method, the design approach is formulated to fit within the usual industrial settings by using the ISA-PID [1]. The design approach will provide not only a rationale for selecting the inner and outer loop controllers but also a complete set of auto-tuning settings that will only need for open-loop information. Therefore, no additional experiment will be necessary. The adopted design approach is based on the specification of a non-oscillatory response for the inner loop,

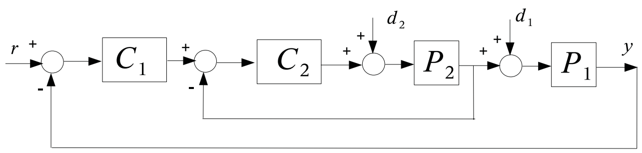

Fig. 1. Cascade Control Configuration

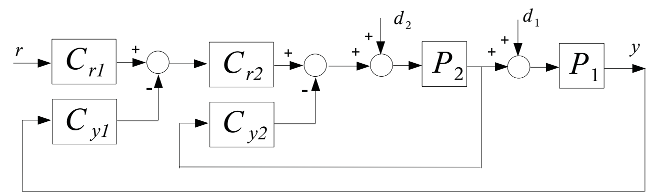

Fig. 2. Cascade Control Loop with 2-DOF Controllers

to obtain a smooth as possible behavior on the inner loop. As it will be seen, under mild conditions this can be achieved by using a PI controller. The same rationale is applied for the outer loop. However, this time a PID controller will be needed and the limitations imposed by dealing with an higher order system will be stated. In addition, the way the auto-tuning is formulated will allow for an automated computation and retuning of the outer loop controller if the inner loop controller settings are changed. This is a major feature not found on existing literature on cascade control.

The rest of the paper is organized as follows. Section II established the framework and notation to be used. Also the rationale and how design specifications should be posed for a cascade control configuration is discussed. Section III provides the derivation of the general tuning rules used for the inner and outer controllers. Section IV provides the cascade control system tuning strategy and autotuning formulae. Section V presents a simulation example that shows the performance of the method and the paper closes with summarizing the conclusions and pointing possible extensions and directions for future research.

II. Cascade Control

A typical configuration for cascade control is shown in Fig. 1, where an inner loop is originated from the introduction of an additional sensor in order to separate, as much as possible, the process fast and slow dynamics. As a result, the control system configuration has an inner controller $C_2(s)$ with inner loop process $P_2(s)$ and an outer loop controller $C_1(s)$ with outer loop process $P_1(s)$. Disturbance can enter at two possible distinct points: d_1 and d_2.

The rationale behind this configuration is to be able to compensate for the best, the possible disturbance d_2, before it is reflected to the outer loop output. In order to accomplish that purpose it is essential that the inner loop exhibits a faster dynamics that allows for such early compensation. This motivates the design of the inner loop controller to act as a regulator (in order to reject d_2) but with as fast as possible dynamics. However, tracking capabilities are also of interest for this inner loop. When a disturbance d_1 appears, at the slow part of the plant, the outer loop controller will react to it. This will introduce a variable set-point to be followed by the inner controller motivating the use of a Two-Degree-of-Freedom controller for the inner loop. On the other hand, the outer loop will be needed to compensate for disturbances not seen by the inner controller as well as to accommodate possible changes in the set-point input. It is therefore clear that in both cases (and especially for the inner loop) servo as well as regulatory performance is desired. In addition, if the controller structure is not allowed to be complicated by adding supplementary filters or models running in parallel with the plant models, the use of Two-Degree-of-Freedom PI/PID (2-DOF PI/PID) controllers is suggested. The use of such versions of the PI/PID controllers for cascade control is a novel feature of this paper. This will also make the results of the paper closer to the industrial application.

Based on the previous observations, the cascade control structure that we proppose is depicted in Fig. 2, where the outer loop controller will be a Two-Degree-of-Freedom PID controller (PID_2) and a Two-Degree-of-Freedom PI controller (PI_2) will be used as inner loop controller, both with the general structure given by

$$u_i(s) = C_{ri}(s)r_i(s) - C_{yi}(s)y_i(s) \quad (1)$$

and the following transfer functions

$$C_{r1}(s) = K_{c1}\left(\beta_1 + \frac{1}{T_{i1}s}\right) \quad (2)$$

$$C_{y1}(s) = K_{c1}\left(1 + \frac{1}{T_{i1}s} + \frac{T_{d1}s}{T_{d1}/Ns+1}\right) \quad (3)$$

for the outer loop controller, and

$$C_{r2}(s) = K_{c2}\left(\beta_2 + \frac{1}{T_{i2}s}\right) \quad (4)$$

$$C_{y2}(s) = K_{c2}\left(1 + \frac{1}{T_{i2}s}\right) \quad (5)$$

for the inner loop controller. Where the derivative filter constant N is to be taken $N = 10$ as it is usual practice in industrial controllers [3].

III. Analytical Tuning of 2-DOF PI and PID Controllers

The inner PI and outer PID controllers are to be designed on the basis of an Analytical Tuning (AT_2) method. The provided AT_2 approach attempts for a practical design of a Two-Degree-of-Freedom controller. It is presented here for a PI and a PID controllers. The formulation is based on the specification of a fast as possible disturbance attenuation target relation while assuring a First-Order-Plus-Dead-Time (FOPDT) resulting behavior for the reference to output closed-loop relation.

Consider the control system with a Two-Degree-of-Freedom controller of Fig. 3 whose output to a change in any of its inputs is given by

$$y(s) = \frac{C_r(s)P(s)}{1+C_y(s)P(s)}r(s) + \frac{P(s)}{1+C_y(s)P(s)}d(s) \quad (6)$$

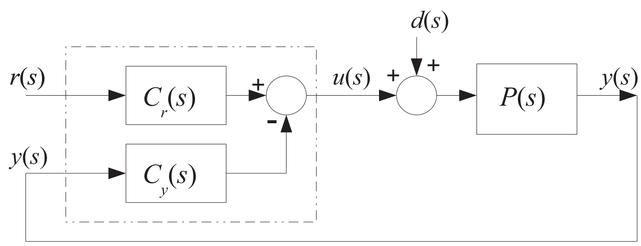

Fig. 3. Control System with a Two-Degree-of-Freedom Controller

The controller output is given by

$$u(s) = C_r(s)r(s) - C_y(s)y(s) \tag{7}$$

where

$$C_r(s) = K_c\left[\beta + \frac{1}{T_i s}\right] \tag{8}$$

is the *set-point controller* transfer function and

$$C_y(s) = K_c\left[1 + \frac{1}{T_i s} + T_d s\right] \tag{9}$$

the *feedback controller* transfer function.

The main objective of the AT_2 tuning is to obtain non-oscillatory control responses to set-point and load-disturbance changes. To obtain the tuning rules, an analytical procedure similar to the servo control synthesis in [8] was used for the regulatory control. The closed-loop transfer function from the set-point to the controlled variable is given by

$$\frac{y(s)}{r(s)} \doteq M_{yr}(s) = \frac{C_r(s)P(s)}{1 + C_y(s)P(s)} \tag{10}$$

and the closed-loop transfer function from the load-disturbance to the controlled variable is given by

$$\frac{y(s)}{d(s)} \doteq M_{yd}(s) = \frac{P(s)}{1 + C_y(s)P(s)} \tag{11}$$

which are related by

$$M_{yr}(s) = C_r(s)M_{yd}(s) \tag{12}$$

From (11) the required feedback controller can be synthesized for different controlled processes and target regulatory transfer function $M_{yd}^d(s)$, with the expression

$$C_y(s) = \frac{P(s) - M_{yd}^d(s)}{P(s)M_{yd}^d(s)} = \frac{1}{M_{yd}^d(s)} - \frac{1}{P(s)} \tag{13}$$

As can be seen from (13) a PI will be obtained for a first order process and a PID for a second order process.

Once, as a first step, the feedback controller $C_y(s)$, is obtained from (13), on a second step, the set-point controller $C_r(s)$ (8) free parameter (β) can be used in order to modify the servo control closed-loop transfer function (12).

The two subsections below apply this procedure in order to obtain a PI tuning for a FOPDT process and a PID tuning for a SOPDT process. These tunings will be the ones used for tuning, within in the cascade control structure, the inner (PI) controller and outer (PID) controller respectively.

A. PI_2 controller from a FOPDT process

Consider first a FOPDT controlled process given by

$$P(s) = \frac{K_p e^{-Ls}}{Ts + 1}, \; \tau_o = \frac{L}{T} \leq 1.0 \tag{14}$$

and a target regulatory control closed-loop transfer function

$$M_{yd}^d(s) = \frac{Kse^{-Ls}}{(\tau_c Ts + 1)^2} \tag{15}$$

where the design parameter τ_c is the relation between the closed-loop control system time constant and the controlled process time constant. By introducing $M_{yd}^d(s)$ and the FOPDT process (14) in (13), the required parameters for the feedback PI controller were obtained. The resulting tuning equations are

$$\kappa_c \doteq K_c K_p = \frac{2\tau_c - \tau_c^2 + \tau_o}{\tau_c^2(1 + \tau_o) + (2\tau_c - \tau_c^2 + \tau_o)\tau_o} \tag{16}$$

$$\tau_i \doteq \frac{T_i}{T} = \frac{2\tau_c - \tau_c^2 + \tau_o}{1 + \tau_o} \tag{17}$$

In this case the global output is computed as

$$y(s) = \frac{(\beta T_i s + 1)e^{-Ls}}{(\tau_c Ts + 1)^2} r(s) + \frac{Kse^{-Ls}}{(\tau_c Ts + 1)^2} d(s) \tag{18}$$

with

$$K = K_p\left[\tau_c^2 T + \frac{(2\tau_c - \tau_c^2\tau_o)T\tau_o}{1 + \tau_o}\right] \tag{19}$$

which will reduce to

$$y(s) = \frac{e^{-Ls}}{\tau_c Ts + 1} r(s) + \frac{Kse^{-Ls}}{(\tau_c Ts + 1)^2} d(s) \tag{20}$$

if the set-point weighting factor can be selected as $\beta = \tau_c T/T_i$.

This will provide tunable speed non-oscillatory responses to both, the set-point and the load-disturbance.

As indicated above, the target servo-control closed-loop transfer function

$$M_{yr}^d(s) = \frac{e^{-Ls}}{\tau_c Ts + 1} \tag{21}$$

may only be obtained if $\beta = \tau_c T/T_i$. As in commercial controllers the set-point weighting factor adjustment is restricted to have values lower or equal to 1, its selection criteria was stated as

$$\beta = \min\left\{\frac{\tau_c T}{T_i}, 1\right\} \tag{22}$$

Furthermore, in the development of the controller synthesis procedure was necessary to approximate the dead-time with a Maclaurin first order equation ($e^{-Ls} \approx 1 - Ls$). Due to the use of this approximation, the obtained response may deviate from the target system output. Therefore, to reduce this deviation it will be needed to restrict the selection range for the design parameter τ_c.

B. PID_2 controller from a SOPDT process

By using a similar procedure as the one presented for the PI controller, we will start right now with a Second-Order-Plus-Dead-Time (SOPDT) model of the form

$$P(s) = \frac{K_p e^{-L'' s}}{(T'' s+1)(aT'' s+1)},\ \tau_o = \frac{L''}{T''} \tag{23}$$
$$0.1 \le \tau_o \le 1.0,\ 0.15 \le a \le 1.0$$

In this situation, by using a Two-Degree-of-Freedom PID controller, the control system target response takes the form

$$\begin{aligned} y(s) = &\frac{(\beta T_i s+1)\, e^{-Ls}}{(\tau_c T'' s+1)^2(T_{cx}s+1)} r(s) \\ &+ \frac{K s e^{-Ls}}{(\tau_c T'' s+1)^2(T_{cx}s+1)} d(s) \end{aligned} \tag{24}$$

where T_{cx} is the time constant of the third pole of the closed-loop transfer function. This time constant is selected as $T_{cx} = 0.1\tau_c T''$ to reduce its influence on the control system dynamic behavior.

Regarding equation (24) we have

$$K = \frac{K_p T''[(21\tau_c + 10\tau_o)\tau_o^2 + \tau_c^2(\tau_c + 12\tau_o)]}{10[(1+a)\tau_o + a + \tau_o^2]} \tag{25}$$

and, as before, τ_c is the design parameter that expresses the relation between the closed-loop control system time constant and the controlled process time constant.

The following tuning equations were developed for a Two-Degree-of-Freedom PID controller

$$\kappa_c = \frac{10\tau_i}{21\tau_c + 10\tau_o - 10\tau_i} \tag{26}$$

$$\tau_i = \frac{(21\tau_c + 10\tau_o)[(1+a)\tau_o + a] - \tau_c^2(\tau_c + 12\tau_o)}{10(1+a)\tau_o + 10a + 10\tau_o^2} \tag{27}$$

$$\tau_d = \frac{12\tau_c^2 + 10\tau_i\tau_o - (1+a)(21\tau_c + 10\tau_o - 10\tau_i)}{10\tau_i} \tag{28}$$

$$\beta = \min\left\{\frac{\tau_c T''}{T_i}, 1\right\} \tag{29}$$

IV. Tuning of Cascade PI/PID Controllers

We will proceed with the application of the AT_2 tuning method presented in Section III for tuning the Two-Degree-of-Freedom master and slave controllers of the cascade control system.

A. Inner Loop Controller Tuning

As the main contribution of the cascade control is to reduce the influence of the d_2 disturbance over the controlled variable y, the inner loop (slave) controller needs to be tuned for fast load-disturbance rejection and fast response reaction to the set-point received from the outer loop (master) controller.

The controlled process transfer functions are supposed of First-Order-Plus-Dead-Time (FOPDT) as

$$P_1(s) = \frac{K_1 e^{-L_1 s}}{T_1 s + 1} \tag{30}$$

and

$$P_2(s) = \frac{K_2 e^{-L_2 s}}{T_2 s + 1} \tag{31}$$

with $T_1 + L_1 > T_2 + L_2$.

The AT_2 tuning equations for the slave PI_2 controller from the P_2 model are

$$\kappa_{c2} = K_{c2}K_2 = \frac{2\tau_{c2} - \tau_{c2}^2 + \tau_{o2}}{\tau_{c2}^2(1+\tau_{o2}) + (2\tau_{c2} - \tau_{c2}^2 + \tau_{o2})\tau_{o2}} \tag{32}$$

$$\tau_{i2} = \frac{T_{i2}}{T_2} = \frac{2\tau_{c2} - \tau_{c2}^2 + \tau_{o2}}{1 + \tau_{o2}} \tag{33}$$

$$\beta_2 = \min\left\{\frac{\tau_{c2} T_2}{T_{i2}}, 1\right\} \tag{34}$$

where $\tau_{o2} = L_2/T_2$ is the model normalized dead-time and $\tau_{c2} = T_{c2}/T_2$ the design parameter (the control closed-loop relative speed) and T_{c2} is the inner loop control system time constant.

To allow the use of the AT_2 tuning equations for the master PID_2 controller an over-damped Second-Order-Plus-Dead-Time (SOPDT) model is needed, then it is necessary to guarantee that the closed-loop transfer function of the cascade inner loop is of FOPDT. For this, the set-point weighting-factor (34) must be selected as

$$\beta_2 = \frac{\tau_{c2} T_2}{T_{i2}} \le 1 \tag{35}$$

Besides, as it is desirable to have a fast inner loop, τ_{c2} must be as small as possible.

Using (33) in (35) it is found that the lower limit for the design parameter is

$$\tau_{c2} = 1 - \tau_{o2} \tag{36}$$

Then the allowed range for the inner loop design parameter is

$$1 - \tau_{o2} \le \tau_{c2} \le 1 \tag{37}$$

Equation (36) also states that the method may be applied only to time constant dominated processes ($\tau_{o2} < 1$).

Using (36) into equations (32) to (34) the slave controller tuning equations to obtain the fastest response are

$$\kappa_{c2} = 1 + \tau_{o2} - \tau_{o2}^2 \tag{38}$$

$$\tau_{i2} = \frac{1 + \tau_{o2} - \tau_{o2}^2}{1 + \tau_{o2}} \tag{39}$$

$$\beta_2 = \frac{1 - \tau_{o2}^2}{1 + \tau_{o2} - \tau_{o2}^2} \tag{40}$$

These will guarantee that the closed-loop transfer function of the inner control-loop is a FOPDT given by

$$M_{yr2}(s) = \frac{e^{-L_2 s}}{(1 - \tau_{o2})T_2 s + 1} \tag{41}$$

B. Outer Loop Controller Tuning

By application of the previous PI_2 controller and by guaranteeing that the inner closed-loop has a FOPDT form, the resulting process seen by the master controller takes the form of the following SOPDT process given by

$$P(s) = \left[\frac{e^{-L_2 s}}{(1-\tau_{o2})T_2 s + 1}\right]\left[\frac{K_1 e^{-L_1 s}}{T_1 s + 1}\right] \quad (42)$$

that can be rearranged as

$$P(s) = \frac{K e^{-Ls}}{(Ts+1)(aTs+1)} \quad (43)$$

with $K = K_1$, $L = L_1 + L_2$, $T = T_1$, $a = (1-\tau_{o2})T_2/T_1$ and $\tau_{o1} = L/T$.

From (43), the master controller can be tuned using the above presented AT_2 method. The resulting equations for the normalized parameters take the form:

$$\kappa_{c1} = \frac{10\tau_{i1}}{21\tau_{c1} + 10\tau_{o1} - 10\tau_{i1}} \quad (44)$$

$$\tau_{i1} = \frac{(21\tau_{c1} + 10\tau_{o1})[(1+a)\tau_{o1} + a] - \tau_{c1}^2(\tau_{c1} + 12\tau_{o1})}{10(1+a)\tau_{o1} + 10a + 10\tau_{o1}^2} \quad (45)$$

$$\tau_d = \frac{12\tau_{c1}^2 + 10\tau_{i1}\tau_{o1} - (1+a)(21\tau_{c1} + 10\tau_{o1} - 10\tau_{i1})}{10\tau_{i1}} \quad (46)$$

$$\beta = \min\left\{\frac{\tau_{c1} T}{T_{i1}}, 1\right\} \quad (47)$$

from where the controller parameters can be found as

$$\begin{aligned} K_{c1} &= \kappa_{c1}/K \\ T_{i1} &= \tau_{i1} T \\ T_{d1} &= \tau_{d1} T \end{aligned} \quad (48)$$

If necessary, the performance-robustness *tradeoff* for the cascade control system may be resolved estimating a lower limit for the control system design parameter considering by example its Maximum Sensitivity M_s. Obviously, lower values for τ_c will provide less robust systems. However, a more detailed analysis is needed and the incorporation of M_s itself as a design parameter is now under study.

C. Taking Into Consideration the Dead-Time Approximation

As indicated above in the analytical deduction of the AT_2 tuning equations for processes with dead-time was necessary to approximate it by a Mclaurin first order series. This would make the actual system response to deviate from the desired one if very fast responses are requested (τ_c small).

As can be seen from (36), a very fast response is specified for process with normalized dead-time in the upper range, that may deviate the inner-loop behavior from the one of the FOPDT supposed.

It was found that use of (36) must be restricted to $\tau_{o2} \leq 0.4$ and that for process with normalized dead times over this limit, the design parameter for the inner-loop must be increased. Simulations of the control system allows to state the following design criteria for the slave controller

$$\tau_{c2} = \begin{cases} 1 - \tau_{o2} & \text{if } \tau_{o2} \leq 0.4 \\ 0.2 + \tau_{o2} & \text{if } 0.4 \leq \tau_{o2} \leq 1.0 \end{cases} \quad (49)$$

Therefore, just knowing the controlled process information given by its model (30) and (31) and the design criteria for the overall cascade control system τ_{c1}, both controllers may be tuned using (49) and (32) to (34) for the slave controller, and (44) to (48) for the master controller. No other test or information is needed, allowing and automatic tuning of the cascade control system.

V. EXAMPLE

Consider the controlled system given by the following transfer functions

$$P_1(s) = \frac{e^{-1.5s}}{5s+1},\ P_2(s) = \frac{e^{-0.3s}}{s+1}\ (\tau_{o2} = 0.3) \quad (50)$$

The overall controlled process model is then

$$P(s) = \frac{e^{-1.8s}}{(5s+1)(s+1)} \quad (51)$$

From (43) the transfer function of the controlled process seen by the master controller shall be

$$P'(s) = \frac{e^{-1.8s}}{(5s+1)(0.7s+1)} \quad (52)$$

The performance of the proposed cascade control system tuning will be compared with the one obtained from a standard single-loop control system also tuned with the AT_2 method, and with the cascade control system tuned using the equations presented by Lee *et. al.* in [5].

In order to include some robustness considerations, the M_s value of the designed system was evaluated in order to guide the selection of the design parameter. As a robust control system is desired ($M_s \approx 1.4$), a design parameter $\tau_{c1} = 0.95$ was used for the master controller in the cascade system configuration, and a $\tau_c = 1.10$ for the single-loop design.

Using the design method outlined in IV-C following parameters were obtained for the PI_2 slave controller: $K_{c2} = 1.210$, $T_{i2} = 0.931$ and $\beta_2 = 0.752$, and for the PID_2 master controller: $K_{c1} = 1.051$, $T_{i1} = 6.034$, $T_{d1} = 0.863$ and $\beta_1 = 0.787$.

For the single-loop PID_2 based control system, the controller parameters are: $K_c = 1.030$, $T_i = 6.773$, $T_d = 1.333$ and $\beta = 0.812$.

Using the equations and recommended closed-loop time constant (λ_1 and λ_2) selection in [5], the cascade PID controllers parameters are: $K_{c2} = 2.440$, $T_{i2} = 1.101$ and $T_d = 0.091$ for the slave controller, and $K_{c1} = 2.130$, $T_{i1} = 5.750$ and $T_{d1} = 0.670$ for the master controller.

Fig. 4 shows the controlled variable output of the three systems to a unit step change in set-point applied at $t = 5$ followed by a unit step change in the disturbance d_2 applied

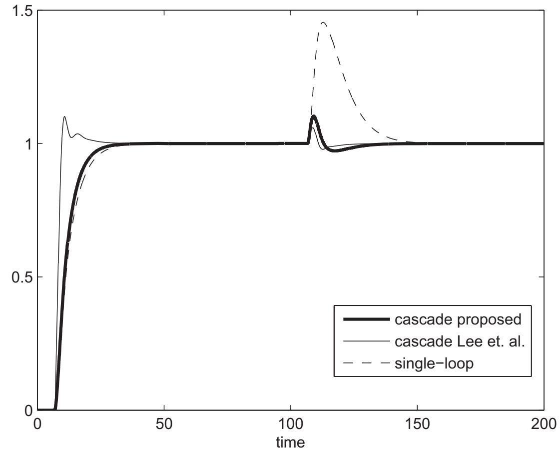

Fig. 4. Cascade and Single-Loop Controlled Variables

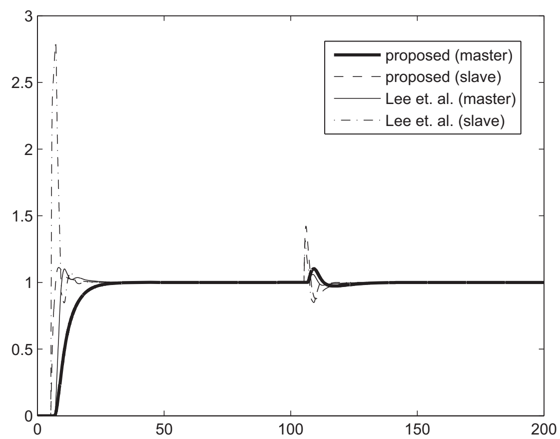

Fig. 5. Cascade loops controlled variables

at $t = 105$, Fig. 5 shows the inner and outer loop outputs (controlled variables).

As can be seen in these figures, the cascade inner loop effectively reduces the disturbance effect over the controller variable (Fig. 4); in the cascade control system the d_2 disturbance mainly affect only the inner loop controlled variable.

Fig. 4 may suggest a better performance of the cascade system tuned with [5] equations but it is evident from Fig. 5 that the variation of the slave controller output and of the slave controlled variable on this system are unrealistically high.

It was also found that its inner loop behavior is not of first order as supposed by [5] in the tuning method deduction and as it is evident in Fig. 4 nor the outer loop. To be able to have a first order behavior on the inner and outer loops it will be necessary to increase the closed-loop time constants, resulting in more slow responses to both the set-point and the load-disturbance.

VI. CONCLUSIONS

Complete autotuning settings for 2-DOF PI/PID controllers within a cascade control configuration are provided. The operation of the inner and outer controllers are analyzed and the need for good performance on both tracking and regulation modes determines the use of the corresponding 2-DOF version for the PI and PID controllers.

One of the major drawbacks of tuning a cascade control configuration (say the need for an additional experiment to determine the plant model of the inner loop and tune the outer controller) is overcome here by appropriate use of the second degree of freedom and providing autotuning formulae for both the inner and outer loop controller parameters.

Future research is conducted towards incorporate the control system performance-robustness tradeoff into the design parameter selection criteria and the use of higher order models as an extension of the proposed methodology. It is the authors' opinion that the approach will provide a considerable step towards fully automated cascade controller design.

VII. ACKNOWLEDGMENTS

This work has received financial support from the Spanish CICYT program under grant DPI2007-63356.

Also, the financial support from the University of Costa Rica and from the MICIT and CONICIT of the Government of the Republic of Costa Rica is greatly appreciated.

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