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Identification of LPV Systems Using Successive Approximations

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***Abstract*— In this paper a successive approximation approach for MIMO linear parameter varying (LPV) systems with affine parameter dependence is proposed. This new approach is based on an algorithm previously introduced by the authors, which elaborates on a convergent sequence of linear deterministic-stochastic state-space approximations. In the previous algorithm the bilinear term between the time varying parameter vector and the state vector is allowed to behave as a white noise process when the scheduling parameter is a white noise sequence. However, this is a strong limitation in practice since, most often than not, the scheduling parameter is imposed by the process itself and it is typically a non white noise signal. In this paper, the bilinear term is analysed for non white noise scheduling sequences. It is concluded that its behaviour depends on the input sequence itself and it ranges from acting as an independent colored noise source, mostly removed by the identification algorithm, down to a highly input correlated signal that may be incorrectly assumed as being part of the system subspace. Based on the premise that the algorithm performance can be improved by the noise energy reduction, the bilinear term is expressed as a function of past inputs, scheduling parameters, outputs, and states, and the linear terms are included in a new extended input.**

I. Introduction

The increasing importance of LPV systems in control system design motivated, by the end of the last decade, the emergence of a new identification problem. With vasts applications ([3], [9]), LPV system identification research is still an area of research in its infancy. There are several approaches to this problem such as methods based on subspace techniques ([12], [13], [14], [2], [15]), basis functions ([10], [11]), stochastic framework based methods ([5], [6]), Linear Matrix Inequalities based optimization ([8]), parameter estimation based gradient searches ([4]) and simple Least Means Square approaches ([1]). Recently the authors proposed an iterative algorithm based on a convergent sequence of linear deterministic-stochastic state-space approximations ([5], [6]). The authors proved that it works with general inputs and zero mean white noise scheduling sequences. Here the proposed algorithm is adapted for general scheduling sequences.

The paper is organized as follows. In section 2 preliminary results are given. In section 3 the original algorithm is briefly described. The new algorithm is formulated in sections 4 and 5. In section 6 we present some numerical simulations and in section 7 we draw some conclusions.

II. A preliminary result

In this section we state a result that is the basis for the algorithm in [5], [6].

Lemma 1: If $p(t)$ is a zero mean white noise sequence and $x(t)$ is a zero mean quasi-stationary signal independent of $p(t)$, then $z(t) = p(t) \otimes x(t)$, where the operator $\otimes$ stands for the Kronecker product, is a second order zero mean white noise sequence

Proof: $z(t)$ is a zero mean second order white noise process if $\mathbb{E}\{z(t)\} = 0$ and $\mathbb{E}\{z(t)z(t-\tau)^T\} = 0$.
The zero mean condition follows from both the independence and the zero mean of $p(t)$ and $x(t)$. Since, from the Kronecker product properties

$$\begin{aligned}\mathbb{E}\left\{z(t)z(t-\tau)^T\right\} &= \\ &\mathbb{E}\left\{[p(t)\otimes x(t)]\left[p^T(t-\tau)\otimes x^T(t-\tau)\right]\right\} = \\ &\mathbb{E}\left\{\left[p(t)p^T(t-\tau)\right]\otimes\left[x(t)\otimes x^T(t-\tau)\right]\right\},\end{aligned}$$

the whiteness of $z(t)$ arises from the independence of $p(t)$ and $x(t)$, and from both the zero mean and the whiteness of $p(t)$.
□

III. LPV Systems With Zero Mean White Noise Scheduling Sequences

In this paper we consider LPV systems with affine parameter dependence described by

$$\begin{aligned} x(t+1) &= A_0x(t) + A_p[p(t)\otimes x(t)] \\ &\quad +B_0u(t) + B_p[p(t)\otimes u(t)] + q(t) \\ y(t) &= C_0x(t) + C_p[p(t)\otimes x(t)] \\ &\quad +D_0u(t) + D_p[p(t)\otimes u(t)] + r(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $p(t) \in \mathbb{R}^s$, $y(t) \in \mathbb{R}^\ell$, $A_0 \in \mathbb{R}^{n\times n}$, $B_0 \in \mathbb{R}^{n\times m}$, $C_0 \in \mathbb{R}^{\ell\times n}$, $D_0 \in \mathbb{R}^{\ell\times m}$ and

$$\begin{aligned} A_p &= \begin{bmatrix} A_1 & A_2 & \cdots & A_s \end{bmatrix} \in \mathbb{R}^{n\times sn} \\ B_p &= \begin{bmatrix} B_1 & B_2 & \cdots & B_s \end{bmatrix} \in \mathbb{R}^{n\times sm} \\ C_p &= \begin{bmatrix} C_1 & C_2 & \cdots & C_s \end{bmatrix} \in \mathbb{R}^{\ell\times sn} \\ D_p &= \begin{bmatrix} D_1 & D_2 & \cdots & D_s \end{bmatrix} \in \mathbb{R}^{\ell\times sm}, \end{aligned} \tag{2}$$

with $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C_i \in \mathbb{R}^{\ell\times n}$ and $D_i \in \mathbb{R}^{\ell\times m}$, $i = 1, \ldots, s$. $u(t)$ and $p(t)$ are independent zero mean quasi-stationary signals and $q(t)$ and $r(t)$ are zero mean white noise sequences independent of both $u(t)$ and $p(t)$. If we define

$$\begin{aligned} u_{g_0}(t) &= \begin{bmatrix} u(t) \\ p(t)\otimes u(t) \end{bmatrix} \\ B_{g_0} &= \begin{bmatrix} B_0 & B_p \end{bmatrix} \\ D_{g_0} &= \begin{bmatrix} D_0 & D_p \end{bmatrix} \end{aligned}$$

we may write the system of equations as

$$\begin{aligned} x(t+1) &= A_0x(t) + B_{g_0}u_{g_0}(t) + \\ &\quad +A_p\left[p(t)\otimes x(t)\right] + q(t) \qquad (3)\\ y(t) &= C_0x(t) + D_{g_0}u_{g_0}(t) + \\ &\quad +C_p\left[p(t)\otimes x(t)\right] + r(t), \qquad (4) \end{aligned}$$

In [5] an LPV identification algorithm is proposed that estimates an innovation model of the form

$$\begin{aligned} x(t+1) &= A_0x(t) + B_{g_0}u_{g_0}(t) + \\ &\quad A_p\left[p(t)\otimes x(t)\right] + Ke(t) \\ y(t) &= C_0x(t) + D_{g_0}u_{g_0}(t) + \\ &\quad C_p\left[p(t)\otimes x(t)\right] + e(t) \end{aligned}$$

from a record of input-output data. It is an iterative process where, in the first iteration, the bilinear signals $A_p\left(p(t)\otimes x(t)\right)$ and $C_p\left(p(t)\otimes x(t)\right)$ are seen as process and measurement noises, respectively. As a result, the process is modeled as a linear time invariant (LTI) system and is identified by a deterministic-stochastic subspace identification algorithm. The state sequence is estimated by a stationary Kalman filter which is then used to estimate the bilinear signal $p(t)\otimes x(t)$. In the next iteration the LPV system is described by the LTI model

$$x(t+1) = A_0x(t) + B_eu_e^{(1)}(t) + q^{(1)}(t) \qquad (5)$$

$$y(t) = C_0x(t) + C_eu(t)e^{(1)}(t) + r^{(1)}(t) \qquad (6)$$

where

$$u_e^{(1)} = \begin{bmatrix} u_{g_0}(t) \\ p(t)\otimes \hat{x}^{(0)}(t) \end{bmatrix} \qquad (7)$$

$$q^{(1)}(t) = A_p\left[p(t)\otimes \tilde{x}^{(0)}(t)\right] + q(t) \qquad (8)$$

$$r^{(1)}(t) = C_p\left[p(t)\otimes \tilde{x}^{(0)}(t)\right] + r(t) \qquad (9)$$

and

$$B_e = \begin{bmatrix} B_{g_0} & A_p \end{bmatrix} \qquad (10)$$

$$D_e = \begin{bmatrix} B_{g_0} & C_p \end{bmatrix} \qquad (11)$$

Here $\hat{x}^{(0)}(t)$ and $\tilde{x}^{(0)}(t)$ denote the state and state error estimate, while $q(t)$ and $r(t)$ stand for the process and measurement noises, respectively. This model is also estimated by the previously referred deterministic-stochastic subspace identification algorithm, and the process is repeated until convergence. If $p(t)$ is a zero mean white noise sequence independent of $u(t)$ then, from lemma 1, the signals $p(t)\otimes x(t)$ and $p(t)\otimes \tilde{x}^{(i)}(t)$, $i = 0, 1, \ldots$ are second order zero mean white noise sequences. Under this condition the algorithm converges if, for all iterations, the estimated models fulfil the stationary condition [5]

$$\rho\left(A_0^{(i)}\otimes A_0^{(i)} + (R_p)_{jk}\sum_{j=1}^{s}\sum_{k=1}^{s}A_k^{(i)}\otimes A_j^{(i)}\right) < 1 \qquad (12)$$

where $\rho(M)$ is the matrix spectral radius of M and $(R_p)_{jk}$ is the jk^{th} entry of the matrix

$$R_p = \lim_{N\to\infty}\frac{1}{N}\sum_{t=0}^{N-1}u(t)u^T(t)$$

IV. LPV Systems With General Scheduling Sequences

Unfortunately we do not have the freedom to choose the scheduling sequence $p(t)$. Most of the times it is imposed by the process itself and is typically a non white noise signal. When this happens the term $p(t)\otimes x(t)$ is not white noise anymore. As a result, the initial linear model is disturbed by colored noise correlated with both $u_{g_0}(t)$ and $x(t)$. Therefore, the algorithm identifies a biased model during the first iteration. In the worst case scenario, some noise components can surpass some state components in the SVD (Singular Value Decomposition) step of the subspace identification algorithm, preventing convergence. To see when this happens, let us decompose the sate vector in the following way

$$x(t) = x_1(t) + x_2(t) + x_3(t)$$

where

$$\begin{aligned} x_1(t+1) &= A_0x_1(t) + B_0u(t) + A_p\left[p(t)\otimes x_1(t)\right] \\ x_2(t+1) &= A_0x_2(t) + B_p\left[p(t)\otimes u(t)\right] + \\ &\quad A_p\left[p(t)\otimes x_2(t)\right] \\ x_3(t+1) &= A_0x_3(t) + A_p\left[p(t)\otimes x_3(t)\right] + q(t). \end{aligned}$$

Since $u(t)$ is independent of both $p(t)$ and $x_3(t)$, then

$$\begin{aligned} \mathbb{E}\left\{u(t)\left[p(t)\otimes x_3^T(t)\right]\right\} &= 0_{m\times sn} \\ \mathbb{E}\left\{\left[p(t)\otimes u(t)\right]\left[p^T(t)\otimes x_3^T(t)\right]\right\} &= 0_{sm\times sn} \end{aligned}$$

and from the Kronecker product properties

$$\begin{aligned} &\mathbb{E}\left\{u(t)\left[p^T(t)\otimes x^T(t)\right]\right\} = \qquad (13)\\ &\mathbb{E}\left\{\left[u(t)p^T(t)\right]\otimes x_1^T(t)\right\} + \mathbb{E}\left\{\left[u(t)p^T(t)\right]\otimes x_2^T(t)\right\} \end{aligned}$$

$$\begin{aligned} \mathbb{E}\left\{\left[p(t)\otimes u(t)\right]\left[p^T(t)\otimes x^T(t)\right]\right\} &= \qquad (14)\\ &\mathbb{E}\left\{\left[p(t)p^T(t)\right]\left[u(t)x_1^T(t)\right]\right\} + \\ &\mathbb{E}\left\{\left[p(t)p^T(t)\right]\left[u(t)x_2^T(t)\right]\right\} \end{aligned}$$

Now, we can state the following lemma.

Lemma 2: If $u(t)$ is a zero mean white noise sequence and if $x(t)$ is a stationary signal, then the bilinear signal $p(t)\otimes x(t)$ is uncorrelated with $u_{g_o}(t) = \begin{bmatrix} u^T(t) & p^T(t)\otimes u^T(t) \end{bmatrix}^T$.

Proof: If $u(t)$ is a zero mean white noise sequence independent of $p(t)$, then it is also independent of both $x_1(t)$ and $x_2(t)$. Consequently,

$$\begin{aligned}\mathbb{E}\left\{\left[u(t)p^T(t)\right]\otimes x_1^T(t)\right\} &= 0_{m\times sn}\\ \mathbb{E}\left\{\left[p(t)p^T(t)\right]\left[u(t)x_1^T(t)\right]\right\} &= 0_{sm\times sn}\\ \mathbb{E}\left\{\left[u(t)p^T(t)\right]\otimes x_2^T(t)\right\} &= 0_{m\times sn}\\ \mathbb{E}\left\{\left[p(t)p^T(t)\right]\left[u(t)x_2^T(t)\right]\right\} &= 0_{sm\times sn},\end{aligned}$$

and from (13) and (14) it follows that

$$\mathbb{E}\left\{u_{g_0}(t)\left[p^T(t)\otimes x^T(t)\right]\right\} = 0_{(s+1)m\times sn}$$

and this completes the proof.
□.

From this lemma, we can see that for a zero mean white noise input sequence, the signal $p(t)\otimes x(t)$ behaves like an independent colored noise source. The projection onto the input and past output space performed by the subspace identification algorithm will remove most of this noise, retaining only its components that are correlated with $x(t)$. Hence, it is not likely that this noise surpasses the state components, but it will introduce a bias into the A_0 matrix estimate. If $u(t)$ is a non white noise sequence, a non zero correlation appears between $p(t)\otimes x(t)$ and $u_{g_0}(t)$. This correlation will increase the amount of noise and cannot be removed by the subspace identification algorithm. If this is too high, the algorithm may not be able to distinguish the state vector from the noise and it will not succeed in estimating a useful model. From this we can see that zero mean white noise sequences are the optimal input signals for this algorithm, while periodic sequences are the worst. This is not a severe limitation in practice because most of the times we have the freedom to choose $u(t)$. However, there are still frequent occasions where this freedom does not exist. In the sequel we propose a change to the first iteration of the algorithm that reduces the colored noise thus improving the overall algorithm performance.

Let us write the bilinear term as

$$\begin{aligned}p(t)\otimes x(t) &= p(t)\otimes\left[A_0x(t-1)+B_{g_0}u_{g_0}(t-1)+\right.\\ &\qquad \left.A_p\left[p(t-1)\otimes x(t-1)\right]+q(t-1)\right].\end{aligned}$$

If we now define

$$u_{g_1}(t) = \begin{bmatrix} u_{g_0}(t)\\ p(t)\otimes u_{g_0}(t-1)\end{bmatrix} \quad (15)$$

$$p_1(t) = \begin{bmatrix} p(t)\\ p(t)\otimes p(t-1)\end{bmatrix} \quad (16)$$

$$q_1(t) = \begin{bmatrix} I_n & A_p\end{bmatrix}\begin{bmatrix} q(t)\\ p(t)\otimes q(t-1)\end{bmatrix} \quad (17)$$

$$r_1(t) = \begin{bmatrix} I_\ell & D_p\end{bmatrix}\begin{bmatrix} r(t)\\ p(t)\otimes q(t-1)\end{bmatrix} \quad (18)$$

and

$$\begin{aligned}A_{p_1} &= A_p\begin{bmatrix}(I_s\otimes A_0) & (I_s\otimes A_p)\end{bmatrix} &(19)\\ B_{g_1} &= \begin{bmatrix}B_{g_0} & A_p\left(I_s\otimes B_{g_0}\right)\end{bmatrix} &(20)\\ C_{p_1} &= C_p\begin{bmatrix}(I_s\otimes A_0) & (I_s\otimes A_p)\end{bmatrix} &(21)\\ D_{g_1} &= \begin{bmatrix}D_{g_0} & C_p\left(I_s\otimes B_{g_0}\right)\end{bmatrix}, &(22)\end{aligned}$$

we can rewrite the system equations as

$$\begin{aligned}x(t+1) &= A_0x(t)+B_{g_1}u_{g_1}(t)+ &(23)\\ &\quad A_{p_1}\left[p_1(t)\otimes x(t-1)\right]+q_1(t)\\ y(t) &= C_0x(t)+D_{g_1}u_{g_1}(t)+ &(24)\\ &\quad D_{p_1}\left[p_1(t)\otimes x(t-1)\right]+r_1(t)\end{aligned}$$

Although these equations seem similar to (3)-(4) there is a fundamental difference consisting on the fact that $q_1(t)$ and $r_1(t)$ are dependent of $x(t)$ (via $p(t)$ and $q(t-1)$). The correlation between $x(t)$ and $q_1(t)$ is given by

$$\begin{aligned}\mathbb{E}\left\{x(t)q_1^T(t)\right\} &= \mathbb{E}\left\{Ax(t-1)q_1^T(t)\right\}+\\ &\quad \mathbb{E}\left\{B_{g_0}u_{g_0}(t-1)q_1^T(t)\right\}+\\ &\quad \mathbb{E}\left\{A_p\left[p(t-1)\otimes x(t-1)\right]q_1^T(t)\right\}+\\ &\quad \mathbb{E}\left\{q(t-1)q_1^T(t)\right\}\end{aligned}$$

The first three terms are zero because $q(t)$ and $q(t-1)$ are independent of $x(t-1)$, $u(t-1)$ and $p(t-1)$. On the other hand

$$\begin{aligned}&\mathbb{E}\left\{q(t-1)q_1^T(t)\right\} =\\ &\mathbb{E}\left\{\begin{bmatrix} q(t)q^T(t-1)\\ \left[p(t)\otimes q(t-1)\right]q(t-1)^T\end{bmatrix}^T\right\}\begin{bmatrix} I_n\\ A_p^T\end{bmatrix} = 0_{n\times n}\end{aligned}$$

because $q(t-1)$ is independent of both $q(t)$ and $p(t)$ and these are zero mean signals. As a result $x(t)$ is uncorrelated with $q_1(t)$. If we identify the LPV system by the model (23)-(24) (considering $A_{p_1}\left[p_1(t)\otimes x(t-1)\right]$ and $C_{p_1}\left[p_1(t)\otimes x(t-1)\right]$ as noise), the estimated model will only exhibit a bias due to the colored *noise* terms proportional to $p_1(t)\otimes x(t-1)$. However we will get a better model than the one identified from (3)-(4) because the colored *noise* is smaller. If we keep developing this noise term until a lag d we will arrive at the following model

$$\begin{aligned}x(t+1) &= A_0x(t)+B_{g_d}u_{g_d}(t)+ &(25)\\ &\quad A_{p_d}\left[p_d(t)\otimes x(t-d)\right]+q_d(t)\\ y(t) &= C_0x(t)+D_{g_d}u_{g_d}(t)+ &(26)\\ &\quad C_{p_d}\left[p_d(t)\otimes x(t-d)\right]+r_d(t)\end{aligned}$$

where, for $k=1,\ \ldots,\ d$,

$$u_{g_k}(t) = \begin{bmatrix} u_{g_{k-1}}(t)\\ p_{k-1}(t)\otimes u_{g_0}(t-k)\end{bmatrix} \quad (27)$$

$$p_k(t) = \begin{bmatrix} p_{k-1}(t)\\ p_{k-1}(t)\otimes p(t-k)\end{bmatrix} \quad (28)$$

$$q_k(t) = \begin{bmatrix} I_n & A_{p_{k-1}}\end{bmatrix}\begin{bmatrix} q_{k-1}(t)\\ p_{k-1}(t)\otimes q(t-k)\end{bmatrix} \quad (29)$$

$$r_k(t) = \begin{bmatrix} I_\ell & D_{p_{k-1}}\end{bmatrix}\begin{bmatrix} r_{k-1}(t)\\ p_{k-1}(t)\otimes q(t-k)\end{bmatrix} \quad (30)$$

and

$$\begin{aligned}A_{p_k} &= A_{p_{k-1}}\begin{bmatrix}\left(I_{s_{k-1}}\otimes A_0\right) & \left(I_{s_{k-1}}\otimes A_p\right)\end{bmatrix} &(31)\\ B_{g_k} &= \begin{bmatrix}B_{g_{k-1}} & A_{p_{k-1}}\left(I_{s_{k-1}}\otimes B_{g_0}\right)\end{bmatrix} &(32)\\ C_{p_k} &= C_{p_{k-1}}\begin{bmatrix}\left(I_{s_{k-1}}\otimes A_0\right) & \left(I_{s_{k-1}}\otimes A_p\right)\end{bmatrix} &(33)\\ D_{p_k} &= \begin{bmatrix}D_{g_{k-1}} & C_{p_{k-1}}\left(I_{s_{k-1}}\otimes B_{g_0}\right)\end{bmatrix} &(34)\end{aligned}$$

with s_k being the $p_k(t)$ dimension. If both A_0 and A_p have a spectral radius less than unity, then the noise term $p_d \otimes x(t-d)$ vanishes for a sufficiently high lag. Although the similarities with a linear model, we can never obtain unbiased estimates of it because $q_d(t)$ and $r_d(t)$ are non white noise sequences anymore. Hopefully, the terms that make them colored noise sequences are attenuated by the powers of A_p. As a result we can expect a small bias. The development of the *noise* term is an attractive approach but it is limited in practice by the curse of dimensionality problem. In fact, the matrices dimensions grow exponentially with the lag d. Typically we can only go up to a lag of 1 or 2. Consequently, the estimated model will be affected by the amount of the colored *noise* term and it must be refined by the iterative process of the previous section. In the first iteration the system is identified by the model

$$x(t+1) = A_0x(t) + B_{g_d}u_{g_d}(t) + q_d^{(0)}(t) \quad (35)$$

$$y(t) = C_0x(t) + D_{g_d}u_{g_d}(t) + r_d^{(0)}(t) \quad (36)$$

where

$$q_d^{(0)}(t) = A_{p_d}\left[p_d(t) \otimes x(t-d)\right] + q_d(t) \quad (37)$$

$$r_d^{(0)}(t) = D_{p_d}\left[p_d(t) \otimes x(t-d)\right] + r_d(t) \quad (38)$$

V. Successive Approximations

The identification algorithm estimates the parameters of a state estimator of (35)-(36) given by

$$\hat{x}^{(0)}(t+1) = A_0\hat{x}^{(0)}(t) + B_{g_d}u_{g_d}(t) + K^{(0)}\left[y(t) - C_0\hat{x}^{(0)}(t) - D_{g_d}u_{g_d}(t)\right]. \quad (39)$$

If $p(t)$ is a zero mean white noise sequence, the noise terms $q_d(t)$ and $r_d(t)$ are second-order zero mean white noise sequences, and this is a Kalman filter of $x(t)$ in the sense that $K^{(0)}$ minimizes the state error covariances, assuming that $q_d(t)$ and $r_d(t)$ are unknown signals. For general scheduling sequences, this optimally is lost due to the non-whiteness of both $q_d(t)$ and $r_d(t)$. However, if the pairs $(A_0,\ B_0)$ and $(C_0,\ A_0)$ are stabilizable and detectable, respectively, the state estimator can produce reasonable estimates of $x(t)$, that can in turn be used to approximate the bilinear signal $p(t) \otimes x(t)$. In the sequel, for the sake of convenience, we will refer to this and other state estimators as Kalman filters. Let us now write the system equations (3)-(4) as

$$\begin{aligned} x(t+1) &= A_0x(t) + B_{g_0}u_{g_0}(t) + q(t) \\ &\quad A_p\left[p(t) \otimes \hat{x}^{(0)}(t)\right] + A_p\left[p(t) \otimes \tilde{x}^{(0)}(t)\right] \\ y(t) &= C_0x(t) + D_{g_0}u_{g_0}(t) + r(t) \\ &\quad C_p\left[p(t) \otimes \hat{x}^{(0)}(t)\right] + C_p\left[p(t) \otimes \tilde{x}^{(0)}(t)\right] \end{aligned}$$

where $\tilde{x}^{(0)}(t) = x(t) - \tilde{x}^{(0)}(t)$ is the state estimate error also given by

$$\tilde{x}^{(0)}(t+1) = A_0\tilde{x}^{(0)}(t) + q_d^{(0)}(t) - K^{(0)}\left[y(t) - C_0\hat{x}^{(0)}(t) - D_{g_d}u_{g_d}(t)\right]$$

Since we know $\hat{x}^{(0)}(t)$, we can consider $p(t) \otimes \hat{x}^{(0)}(t)$ as an additional input and the unknown terms $A_p\left[p(t) \otimes \tilde{x}^{(0)}(t)\right]$ and $C_p\left[p(t) \otimes \tilde{x}^{(0)}(t)\right]$ as process and measurement noises, respectively. This will lead us to the LTI model (5)-(11) which will be identified in the next iteration. At iteration i, the algorithm identifies the LTI system

$$x(t+1) = A_0x(t) + B_eu_e^{(i)}(t) + q^{(i)}(t) \quad (40)$$

$$y(t) = C_0x(t) + C_eu(t)e^{(i)}(t) + r^{(i)}(t) \quad (41)$$

where

$$u_e^{(i)} = \begin{bmatrix} u_{g_0}(t) \\ p(t) \otimes \hat{x}^{(i-1)}(t) \end{bmatrix} \quad (42)$$

$$q^{(i)}(t) = A_p\left[p(t) \otimes \tilde{x}^{(i-1)}(t)\right] + q(t) \quad (43)$$

$$r^{(i)}(t) = C_p\left[p(t) \otimes \tilde{x}^{(i-1)}(t)\right] + r(t) \quad (44)$$

and $\hat{x}^{(i-1)}(t)$ and $\tilde{x}^{(i-1)}(t)$ are the Kalman filter state and error estimates of the previous iteration. The algorithm can be summarized in the following steps:

Algorithm 1: **Successive Approximations.**

- Inputs
  - Input-output data record: $u(t)$, $p(t)$ and $y(t)$, $t = 1, \ldots, N$.
  - u, p and y dimensions: m, s and ℓ.
  - System order: n.
  - Subspace algorithm prediction horizon: j
  - number of past data lags: d
- **Step 1**: Initialization
  - Build $u_{g_d}(t)$ using (27) for $k = 1, \ldots, d$.
  - Identify the state-space LTI model (35)-(36) with a deterministic-stochastic subspace identification algorithm.
  - set $i = 1$
- **Step 2:** Successive Approximations
  Repeat (a)-(d) until convergence
  - (a) Estimate the Kalman filter estimates $\hat{x}^{(i-1)}(t)$ for $t = 1, ..., N$ using the Kalman filter identified in the previous iteration
  - (b) Compute
    $$u_e^{(i)}(t) = \begin{bmatrix} u_{g_0}(t) \\ p(t) \otimes \hat{x}^{(i-1)}(t) \end{bmatrix}$$
    for $t = 1, \ldots, n$.
  - (c) Identify the state-space LTI model (40)-(41) with a deterministic-stochastic subspace identification algorithm.
  - (d) set $i = i + 1$
- **Step 3:** Parameter extraction
  - $B_0 = B_e(:, 1 : m)$, $B_p = B_e(:, m+1 : m+ms)$, $D_0 = D_e(:, 1 : m)$, $D_p = D_e(:, m+1 : m+ms)$ $A_p = B_e(:, m+ms+1 : m+ms+ns)$, $C_p = B_e(:, m+ms+1 : m+ms+ns)$, ($A_0$ and C_0 are directly given by the subspace identification algorithm).

□

The convergence of this algorithm is closely related to the stationarity of the state sequence. If

$$\left\|\mathbb{E}\left\{x(t)x^T(t)\right\}\right\|_2 < C, \tag{45}$$

where C is a positive finite constant, does not hold, the algorithm does not converge. This is a stability condition for LPV system that depends on the autocorrelation of the scheduling sequence $p(t)$. When $p(t)$ is a zero mean white noise sequence, this condition is equivalent to (12) (see [6], [7] for a detailed analysis). It turns out to be more restrictive when $p(t)$ is a general sequence because the state covariance becomes dependent on the high order cumulants of $p(t)$. Since the LPV system stability is ensured when the spectral radius of $A_0 + p_1(t)A_1 + \cdots + p_s(t)A_s$ is within the unit circle for every t. Thus

$$\max\left|\lambda\left[A_0 + p_1(t)A_1 + \cdots + p_s(t)A_s\right]\right| < 1 \tag{46}$$

is a sufficient condition for convergence. However, this is too strong a condition because the stationarity of $x(t)$ allows the LPV system to become temporally unstable.

The algorithm can enter a state of limitless convergence, alternating between stable and unstable models, forming thus a precision barrier. To handle this situation the algorithm limits the number of iterations. When this number is exceeded it picks the model with the smallest prediction error variance.

VI. Numerical Example

In order to test the algorithm three sets of 100 Monte-Carlo simulations were performed using the following LPV system

$$\begin{aligned}
A_0 &= \begin{bmatrix} 0 & 1 \\ -0.1 & 0.7 \end{bmatrix}, A_p = \begin{bmatrix} 0 & 0 \\ 0.4276 & -0.51 \end{bmatrix}, \\
B_0 &= \begin{bmatrix} 1 \\ 0.50 \end{bmatrix}, \qquad B_p = \begin{bmatrix} 1.073 \\ 1.075 \end{bmatrix}, \\
C_0 &= \begin{bmatrix} 0.443 & 0.06 \end{bmatrix}, C_p = \begin{bmatrix} 0 & 0 \end{bmatrix}, \\
D &= 0.5, \qquad D_p = 0.
\end{aligned}$$

We performed these simulations in an attempt to approximate the algorithm worst scenario. Hence, we used the sinusoidal scheduling sequence $p(t) = 0.5\sin\left(\frac{2\pi}{100}t\right)$ and the input signal $u(t) = p(t) + w(t)$ where $w(t)$ is a white noise binary sequence with an amplitude of 0.5 (see Figure 1). This way,

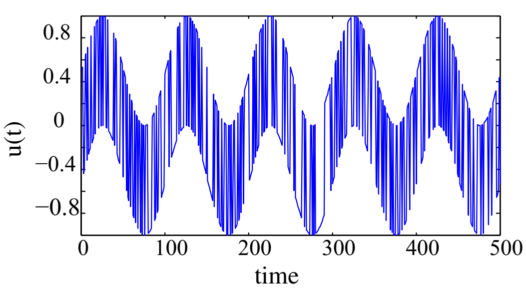

Fig. 1. Input signal.

we ensured a significant correlation between $u(t)$ and $p(t)$.

We used a record data length of 500 in each simulation. The first set was without noise and the others were with output white noise with SNR values of 30 dB and 20 dB. The corresponding output noise standard deviations were 0.021 and 0.066, respectively. In the noiseless simulations the algorithm always converged to the true model with a number of iterations between 7 and 14 (see the iterations histogram in Figure 2). This was not the case for experiments with

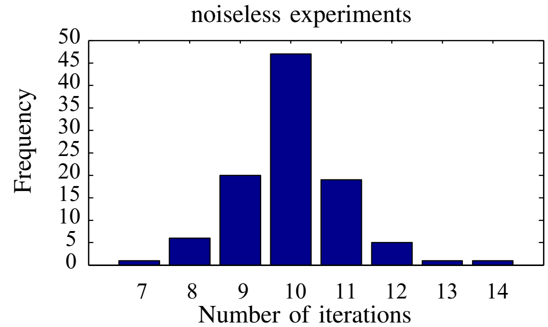

Fig. 2. Histogram of the number of iteration in the noiseless simulations experiments.

noise. From the histograms depicted in Figures 3 and 4 for the experiments with SNRs of 30 dB and 20 dB, respectively,

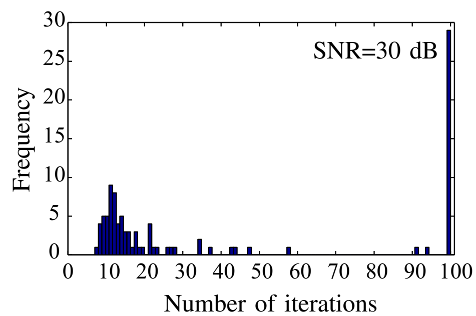

Fig. 3. Histogram of the number of iteration in the SNR=30 dB experiments.

we can see that the algorithm reached the upper limit number of 100 iterations in about 27 % of the 30 dB and 60 % of the 20 dB SNR experiments. Figures 5 and 6 show the percentage prediction error histograms. We can see that in more than 90 % of the experiments the error remained close to its theoretical value which is about 3 % and 10 % for the experiments with SNR= 30 dB and SNR= 20 dB, in that order. This is an evidence that the algorithm at least weekly converged in almost cases. This may be confirmed by the A_0 and A_p eigenvalues scatter plots, where it is evident that, in most cases, the algorithm produced acceptable estimates, in particular for the A_0 matrix. We also tried several lag values but there was no significant differences. Thus, in order to improve the convergence we have to improve the accuracy of the estimated models in the different iterations by removing the correlation between the noise and the state vector. This will be a topic of future research work.

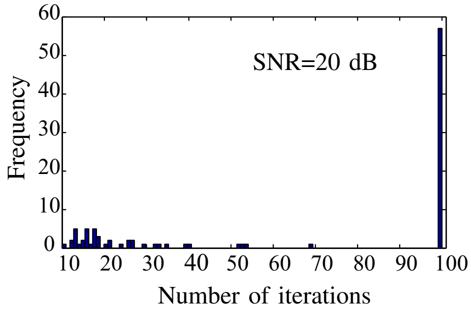

Fig. 4. Histogram of the number of iteration in the SNR=20 dB experiments.

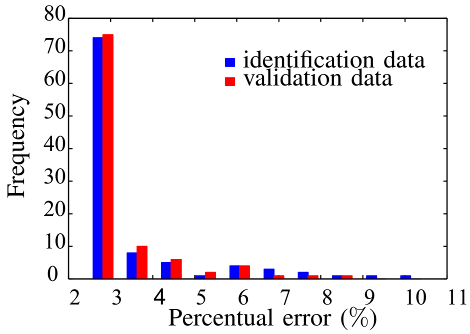

Fig. 5. percentage prediction errors histogram in the SNR=30 dB experiments.

VII. Conclusions

In this paper we analyzed the performance of an LPV system identification algorithm for general scheduling sequences. This algorithm was designed for zero mean white noise sequences. It is concluded that its behaviour depends on the input sequence itself and it ranges from acting as an independent colored noise source, mostly removed by the identification algorithm, down to a highly input correlated signal that may be incorrectly assumed as being part of the system subspace. Based on the premise that the algorithm performance can be improved by the noise energy reduction, the bilinear term was expressed as a function of past inputs, scheduling parameters, and states, and the linear terms were included in a new extended input. The algorithm was tested with Monte Carlo simulations that mimicked the worst case scenario. The results show that these changes did not bring any significant improvement to the algorithm's performance. But they also show that it is a viable alternative for LPV systems identification. In future research work we will try to remove the correlation between the noise and the state vector in order to improve the algorithm's convergence properties.

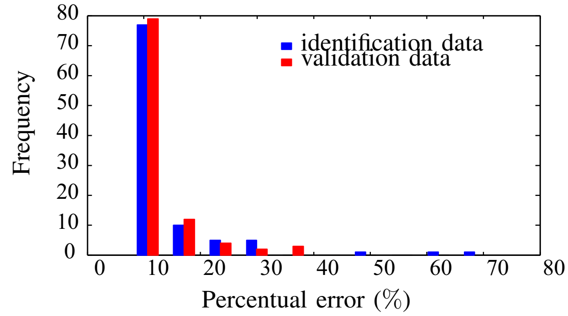

Fig. 6. percentage prediction errors histogram in the SNR=20 dB experiments.

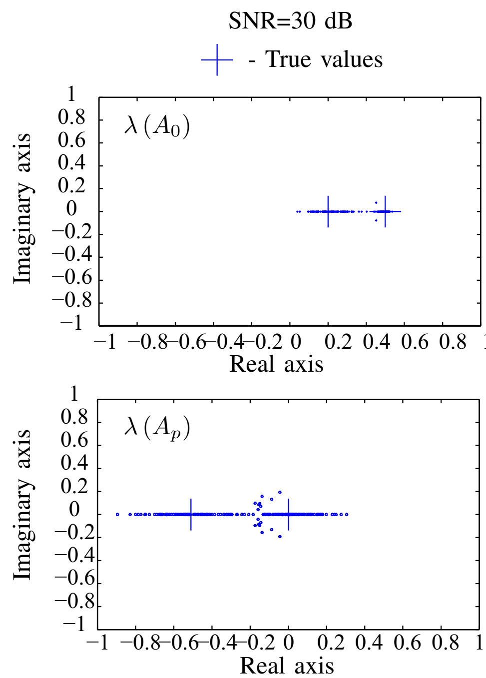

Fig. 7. Eigenvalues of A_0 and A_p in the SNR=30 dB experiments.

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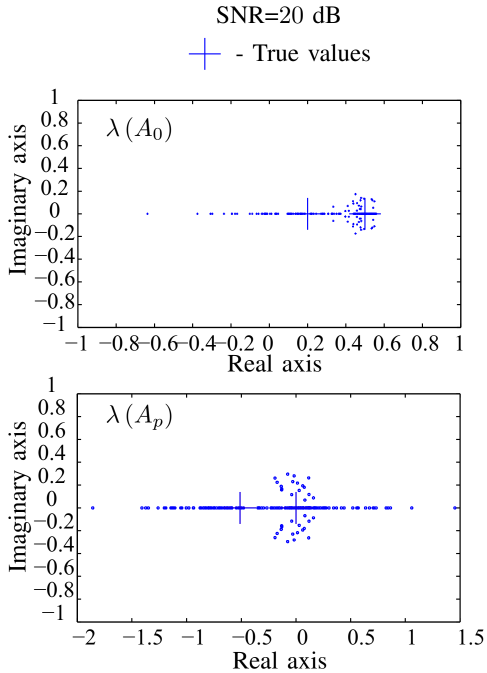

Fig. 8. Eigenvalues of A_0 and A_p in the SNR=20 dB experiments.

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