Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

Generation of Optimal Linear Parametric Models for LFT-based Robust Stability Analysis and Control Design

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Abstract— **We present a general approach to generate a linear parametric state-space model, which approximates a nonlinear system with high accuracy. It is optimally suited for LFT-based robust stability analysis and control design. At the beginning a Jacobian-based linearization is applied to generate a set of linearized state-space systems describing the local behavior of the nonlinear plant about the corresponding equilibrium points. These models are then approximated using multivariable polynomial fitting techniques in combination with global optimization. The objective is to find a linear parametric model, which allows the transformation into a Linear Fractional Representation (LFR) of least possible order. A gap metric constraint is included during the optimization in order to guarantee a specified accuracy of the transfer function of the linear parametric model. The effectiveness of the proposed method is demonstrated by a robust stability analysis for a nonlinear generic missile model.**

I. INTRODUCTION

Various dynamic systems can be described by nonlinear differential equations

$$\begin{aligned} \dot{x} &= f(x,u,p) \\ y &= g(x,u,p) \end{aligned} \tag{1}$$

with state vector $x(t)$ confined to some operating region $X \subset \mathbb{R}^n$, input vector u and output vector y. These systems may depend on a parameter vector p which is either not exactly known (i.e., uncertain) or is time-varying and belongs to an admissible parameter value set Π, i.e., $p \in \Pi$. The analysis or control design for such systems ensuring the stability and performance requirements for all allowable parameter variations and over the whole range of operating conditions is a highly complex task and can be addressed only by employing advanced techniques like μ-analysis/synthesis [11] or Linear Parameter Varying (LPV) control.

Therefore (1) is usually approximated by a linear parametric model of the form

$$\begin{aligned} \dot{x} &= A(\delta)x + B(\delta)u \\ y &= C(\delta)x + D(\delta)u, \end{aligned} \tag{2}$$

where the matrices $A(\delta)$, $B(\delta)$, $C(\delta)$, $D(\delta)$ depend rationally on δ. The artificial parameter vector δ may also depend on state variables, which allows to cover state dependent nonlinearities in the representation given by (2), i.e. $\delta \in \Pi \times X$. Finally, (2) is transformed into a Linear Fractional Representation (LFR) [11], which is required to apply modern robust control techniques like μ-analysis/synthesis. These techniques are usually computationally demanding and therefore LFRs of high accuracy and low complexity (order) are required. Once (2) is available, very powerful and efficiently implemented techniques [4], [5] exist for the transformation of (2) into an nearly least order LFR. However, the minimal achievable order of the resulting LFR mainly depends on the complexity (order of rational or polynomial approximations) and structure of (2). Thus the main emphasis must be put on an optimal generation of the linear parametric model (2) such that it is of high accuracy and optimally fits for low order LFR generation.

For this purpose we present a general procedure, starting with the generation of a set of linear, time-invariant (LTI) state-space systems obtained by linearization of (1) at certain equilibrium points (trim points). Least-squares multivariable polynomial fitting is used to approximate the single elements of the state-space matrices and to find a single parametric state-space system (2) covering the whole set of LTI equilibrium models. Within the fitting process, sophisticated methods are employed to reduce the complexity of the polynomials by eliminating monomials with low influence on the accuracy of the polynomials. Furthermore, the element-wise polynomial fitting is part of a global optimization loop, where a genetic algorithm tunes the polynomial order of each state-space element with the objective to minimize the overall order of the resulting LFR model. To guarantee the accuracy of the transfer function of the parametric model, gap-metrics between the set of equilibrium models and the parametric model are calculated and included as additional cost functions in the optimization loop.

It is important to emphasize that we directly choose to minimize the overall achievable LFR order for the optimization of the linear parametric model instead of reducing the polynomial order/complexity of its single elements. This may allow to increase the accuracy, while keeping the same order for the resulting LFR. To see this, consider an arbitrary matrix with two parameters

$$A(\delta) = \begin{bmatrix} \delta_1^2 & a_{12}(\delta_1,\delta_2) \\ 0 & \delta_2^2 \end{bmatrix} \tag{3}$$

where the entry a_{12} may only have small variations within a given parameter value set. In order to reduce the complexity of $A(\delta)$ one may therefore decide to choose a_{12} to be constant. However, in terms of LFRs, one may also choose a_{12} as a second order polynomial (e.g., $a_{12} = \delta_1 + \delta_1^2 + \delta_2^2$) without increasing the resulting LFR order for $A(\delta)$.

Note, that in several cases one may directly derive (2)

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from (1) via symbolic calculations. However, especially in aeronautical applications the nonlinear models usually include highly nonlinear functions (neural networks, tables) or may only be given for a discrete set of conditions (linear aeroelastic models) such that the generation and approximation of a set of LTI state-space models is the only way to apply modern robust control techniques.

In sections II and III we will describe the overall procedure for the generation of "optimal" linear parametric models and in section IV the efficiency of the procedure is demonstrated within a robust stability analysis for a highly nonlinear missile model.

II. BASIC PROCEDURE FOR THE GENERATION OF LINEAR PARAMETRIC MODELS

The starting point for the generation of linear parametric models is a nonlinear parametric model as given in (1). For this model, a grid of m linear, time-invariant state-space systems for a pre-specified set of equilibrium constraints and parameter values is generated. Note, that it must be guaranteed that all elements of the grid have coherent state, input and output vectors. In the following the grid-point state-space matrices with transfer matrix $G_k = D_k + C_k(sI - A_k)^{-}1B_k$ are represented in concatenated form as

$$S_k = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix},\ k = 1 \dots m. \tag{4}$$

The goal is now to calculate a matrix $S(\delta)$, which approximates all matrices S_k as a function of the parameter vector δ. The basic fitting procedure is divided into three steps:

- A Check if an element $s_{i,j}$ of S_k has either only an insignificant influence on the solution or is nearly constant within all linear equilibrium point models.
- B If an element $s_{i,j}$ is neither constant nor insignificant, find a polynomial approximation of the element $s_{i,j}$ as a function of the parameter vector δ. This step includes a so-called rank deficient trimming of the polynomial based on QR decomposition with column pivoting.
- C Finally, perform a full rank trimming of the polynomials using measurements for the significance of individual monomials.

A. Element-wise significance check

For the first step, for each element $s_{i,j}$ of the set of matrices S_k a so-called influence coefficient $IC_{i,j}$ is determined. An element has a low influence coefficient if it does not significantly contribute to the solution (in terms of the ν-gap metric [10]) or if it is sufficiently accurate to approximate an element with its mean values over all equilibrium points. Therefore for each $s_{i,j}$ a set of concatenated state-space matrices $S_{k_{i,j}},\ k = 1, \dots, m$ with transfer matrix $G_{k_{i,j}}$ is generated, where all entries are equal to the entries of the set S_k except the entry $s_{i,j}$, which is chosen as the mean value of the m grid point values $s_{i,j}$. Finally, the influence coefficient $IC_{i,j}$ of $s_{i,j}$ is defined as

$$IC_{i,j} = \max_k(\delta_\nu(G_k, G_{k_{i,j}})),\ .k = 1, \dots, m. \tag{5}$$

Note, that in (5) the δ_ν denotes the ν-gap metric, which is introduced by [10] as a measurement of the distance between two systems in terms of their closed-loop behavior. A ν-gap metric of the value one states that two systems are far apart, whereas zero means that they are identical.

If the influence coefficient $IC_{i,j}$ of a certain element $s_{i,j}$ is below a pre-specified threshold, then the constant mean value is used for this entry and no polynomial fitting is performed.

B. Multivariable Polynomial Fitting

The algorithm for finding polynomial approximations of the single matrix elements is based on a least squares fitting by employing singular value decomposition. In addition, sophisticated methods for trimming of redundant and insignificant terms of the approximation as described in [6] are implemented to ensure the simplest possible solution.

In the following the vector x_i contains the numerical values of the parameter δ_i, y the values of the element over all equilibrium points and b the polynomial coefficients. In a first step, a matrix X will be built, which considers all possible terms for a multivariable polynomial of a given order. In (6) X for a two parametric, second order polynomial is given.

$$X = \begin{bmatrix} 1 & x_{1,1} & x_{2,1} & x_{1,1}^2 & x_{1,1}x_{2,1} & x_{2,1}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{1,m} & x_{2,m} & x_{1,m}^2 & x_{1,m}x_{2,m} & x_{2,m}^2 \end{bmatrix} \tag{6}$$

With the above defined matrix X the following least squares minimization problem is solved.

$$\min_b \frac{1}{2}\|y - Xb\|_2 \tag{7}$$

$$\frac{1}{2}\frac{\partial\|y - Xb\|_2}{\partial b} = X^T Xb - X^T y = 0 \tag{8}$$

Since the assumption of X having full rank does not hold especially for higher order polynomial approximations, (8) cannot be directly solved for b by inverting $X^T X$. Instead, the optimal coefficients b^* of (7) are found by means of singular value decomposition (*SVD*) of X, which is defined in the following way. For an arbitrary matrix $X \in \mathbb{R}^{m\times n}$, there exist unitary, orthogonal matrices $U \in \mathbb{R}^{m\times m}$ and $V \in \mathbb{R}^{n\times n}$, such that (9) is valid [11].

$$X = U\Sigma V^T \tag{9}$$

The matrix Σ is a diagonal matrix containing the singular values of X in descending order. The rank r of the matrix X corresponds to the non-zero singular values, so that a diagonal matrix $\Sigma_r \in \mathbb{R}^{r\times r}$ can be defined which only contains the non-zero singular values. Further, the first r columns of U and V can be written in $U_r \in \mathbb{R}^{m\times r}$ and $V_r \in \mathbb{R}^{n\times r}$ respectively, since the neglected columns do not contribute to the result of the *SVD* [8].

With the so defined matrices Σ_r, U_r and V_r, (7) can finally be solved for the optimal coefficients b^*.

$$b^* = V_r\Sigma_r^{-1}\boldsymbol{U}_r^T y \tag{10}$$

In case of a nearly rank deficient X, some of its singular values are almost zero, which drastically reduces the quality of the solution of (10). A way to deal with nearly rank deficient matrices in case of a least squares problem is to use a truncated *SVD* solution [3]. Instead of the real rank a numerical rank is used by setting all singular values below a specified low value to zero.

The algorithm developed in the present work iteratively increases the polynomial order of the approximation and computes the new polynomial coefficients by solving (10) until either the root mean square error *RMSE* defined in (11) drops below a specified maximum or the improvement in the *RMSE* becomes insignificant.

$$RMSE = \frac{\|e\|_2}{\|y\|_2} \qquad \text{with} \qquad e = y - Xb^* \tag{11}$$

In addition to the already described polynomial fitting based on a singular value decomposition, sophisticated trim algorithms are integrated, in order to use as few monomials for the approximation as possible. Since the generation of the data matrix X includes all possible factors, some of them might be redundant and can be omitted. For this purpose a so-called rank deficient or nearly rank deficient trimming is performed.

In case of nearly rank deficient trimming, a subset of columns of X, which is numerically most linear independent shall be found and used for solving the least square problem [3], [6]. It is therefore possible to eliminate the redundant columns of X which reduces the number of monomials needed for the approximation. The rank deficient trimming is based on *QR* decomposition with column pivoting. First, the matrix of the right singular vectors V is partitioned, so that $V_{11} \in \mathbb{R}^{r\times r}$ and $V_{21} \in \mathbb{R}^{n-r\times r}$ [6].

$$V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \tag{12}$$

$$\bar{V} = \begin{bmatrix} V_{11}^T & V_{21}^T \end{bmatrix} \operatorname{diag}(b_1^*, \ldots, b_n^*) \tag{13}$$

Then, the *QR* decomposition with column pivoting of $\bar{V}$ can be computed.

$$\bar{V} = Q \begin{bmatrix} R \\ 0 \end{bmatrix} P^T \tag{14}$$

With the permutation matrix P a new data matrix Z is calculated according to (15) and the first r columns of Z are used to refit the polynomial approximation [6].

$$Z = XP \tag{15}$$

C. Full Rank Trimming

The full rank trimming is conducted as a final step of the fitting algorithm after a feasible solution has been found. Some terms of this solution may not have a significant influence on the quality of the solution, so that they can be eliminated. A good measurement of the significance of certain monomials is the magnitude of the whole term, i.e. the product of the coefficient with its corresponding column of X. For this reason a utility factor u for a term j is defined according to (16) [6].

$$u_j = \frac{\|b_j^* X_j\|_2}{\|y\|_2} \tag{16}$$

During the full rank trimming, the column corresponding to the lowest utility factor is ommitted from X and a new set of polynomial coefficients is calculated with the reduced X. This procedure is repeated until the root mean square error of the trimmed result has significantly increased with respect to the original approximation. The threshold is specified as the relative increase of the original $RMSE$ in percentage.

The presented approach via singular value decomposition and trimming introduced by [6] and implemented in this work has been compared to classical linear regression methods as featured in the Matlab Statistics Toolbox. Both algorithms yield similar results but the SVD-based one has performed faster and numerically more robust in case of rank deficient and nearly rank deficient problems in several test cases.

III. OPTIMIZATION OF THE LINEAR PARAMETRIC MODEL

The structured singular value computation and similar LFT based stability analysis are computational time intensive. Thus for an efficient analysis the availability of low order LFRs is vital. Such an optimal, low order model which still possesses a sufficient quality can be obtained by solving an optimization problem. Instead of minimizing the LFR order directly, it is approximated by a lower bound as described in [4], which reduces the computational effort drastically.

For a given linear parametric model $S(\delta)$ with $\delta \in \mathbb{R}^l$ the lower bound can be calculated as follows: Substitute all but one parameter δ_i with random values and compute a minimal order, one parametric LFR with order m_i. Note, that for single parametric systems one can always calculate a minimal order LFR. Repeat this procedure for all parameters. Finally, the lower bound m is given by $m = \sum_{i=1}^{l} m_i$.

The above defined lower bound can then be minimized over the following optimization parameters:

- Maximum allowed $RMSE$ for polynomial approximation
- Minimum required improvement in $RMSE$ to increase order of polynomial approximation
- Maximum allowed increase in $RMSE$ during the full rank trimming
- Tolerance for significance check of a single element

Those four parameters are written in the vector ρ for simplification. In short terms, ρ defines the general structure of the approximation, e.g. which elements shall be approximated by polynomials, which order shall the element wise fitted polynomials possess and which terms can be omitted.

For given numerical values of the optimization parameters, it is first decided which elements are significant and therefore are considered for the following polynomial approximation.

For those elements with a sufficiently high influence coefficient multivariable polynomials are fitted by means of SVD and trim algorithms described in the previous section.

A. Optimization with ν-Gap Metric Constraint

In addition to the minimization of the LFR order via its lower bound, an easily verifiable criteria for the fitness of the linear parametric model is required, in the form of a measurement of the distance between the linear parametric model $S(\delta)$ and the grid-point LTI models. For this, the ν-gap metric δ_ν already introduced at the element-wise significance check is used, which has the advantage that it can also be used for unstable systems.

By introducing the maximum ν-gap metric between the linear parametric model and the set of LTI models as an additional minimization objective $\delta_{\nu,max}$, the optimization problem can be written in the following way:

$$\min_t \left(m\left(\rho\right) + w_{gap}\delta_{\nu,max}\left(\rho\right)\right). \tag{17}$$

The weighting factor w_{gap} is used to find a balance between the complexity and quality of the approximation.

Due to the fact that neither $m(\rho)$ nor $\delta_\nu(\rho)$ are continuously differentiable in ρ, a gradient based optimization algorithm is unsuitable and instead a global search algorithm is used. In the present work the differential evolution algorithm proposed by [9] and implemented in [7] is applied.

B. Optimization of the Polynomial Coefficients

Following the optimization of (17), a further reduction of the gap between the linear parametric and the LTI models is achieved by additional minimization of $\delta_{\nu,max}$ over the polynomial coefficients. So far, aside from the influence coefficient computation, each element has been treated individually in the algorithm. In the end, however, the system's behavior reflects the quality of the solution, not the best individual approximation. Therefore, the following optimization problem is proposed, which can be solved by means of gradient based algorithms.

$$\min_{\text{Coefficients}} \delta_{\nu,max} \tag{18}$$

In our procedure, the polynomial coefficients are constraint within a ten percent band of the original solution, in order to preserve the structure of the system.

IV. EXAMPLE: ROBUST STABILITY ANALYSIS FOR A MISSILE MODEL

A. Structure of the Model

The missile model features a fully nonlinear description of the plant, linear representations of the actuators and sensors and a control system, which is scheduled by the actual flight condition. The control system consists of a proportional output feedback (eigenstructure assignment) as well as an integral feedback of the control error. The command signals are the accelerations in the z- and y-direction and the bank angle in the aerodynamic frame. Since the sensors measure the angular rates in the body-fixed frame, a coordinate transformation to the aerodynamic frame is also required. The actuating signals of the plant are the deflection angles of rudder, aileron and elevator. The basic structure of the missile model is shown in Fig. 1. In the coordinate transformation block the computation of the feedback error is also incorporated.

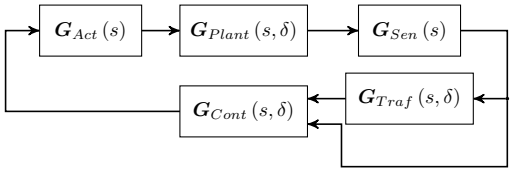


Fig. 1. Basic Structure of the Missile Model

Due to the high degree of non-linearity, it is only possible to analyze a limited region of the flight envelope. In the present work, for a constant Mach number and altitude the envelope is spanned by the incidence angle ϑ and polar angle φ. This region is further divided into three subregions (see Fig. 2), in order to allow the generation of simple parametric models.

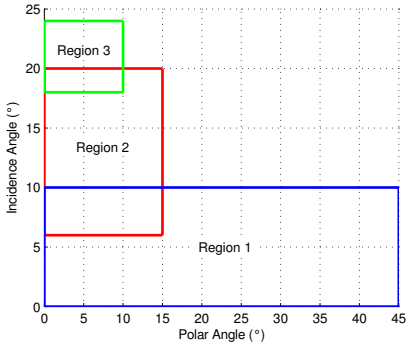


Fig. 2. Regions of the Flight Envelope

For the parameter dependent parts of the missile model linear parametric models have to be generated, which depend rationally on φ and ϑ. Those parts are the plant, the control system and the coordinate transformation. The coordinate transformation, thereby, only contains simple trigonometric functions, which can be expressed by Taylor series expansion and truncation after a sufficient high order, whereas no analytical knowledge of the plant and control system is available (highly nonlinear functions given in tables). For those two parts, the generation of the linear parametric model is conducted according to the optimal approach described in the previous sections.

In addition to the parametric dependence on φ and ϑ, multiplicative complex uncertainties $\delta_{Act,i} \in \mathbb{C}$, with $\|\delta_{Act,i}\| \leq 1$ are added to the output of each of the three actuators individually as described by (19) and illustrated in Fig. 3. $W(s)$ is thereby a weighting function and describes dynamic, high frequency uncertainties.

$$y_{Act,i} = \left(1 + \delta_{Act,i} W\left(s\right)\right) G_{Act,i}\left(s\right) u_{Act,i} \tag{19}$$

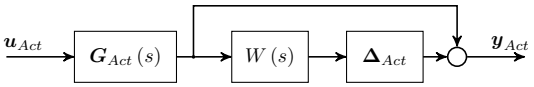


Fig. 3. Multiplicative Complex Output Uncertainty

B. System Validation with Nonlinear Simulation

The ν-gap metric, which has been used so far as a measurement of the fitness of the approximated linear parametric model, is limited to a comparison between the linear parametric and LTI models at the grid points of the linearization. Additionally, only the single submodels (e.g. plant, controller) were verified and not the whole missile model. For the purpose of a final validation of the optimized linear parametric models, time simulations are compared with the nonlinear simulation model making use of the tools provided by [2].

One result of such a validation simulation is exemplarily shown in Fig. 4. As shown in the figure, the dynamic behavior of the linear parametric model matches the one of the nonlinear model quite acurately.

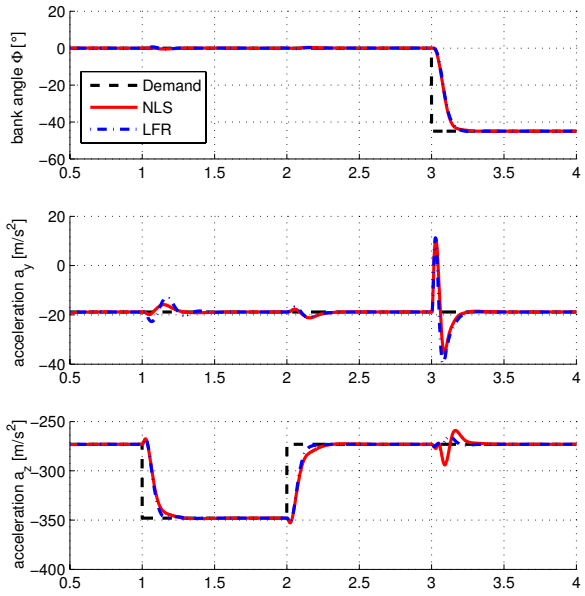


Fig. 4. Comparison between Nonlinear and Linear Parametric Model covering Region 3

C. Linear Fractional Transformation of the Linear Parametric Missile Model

Parametric models $S(\delta)$ are generated independently for the plant, controller and coordinate transformation systems and are transformed into an upper LFR $\mathcal{F}_u$ by "pulling out" the parameters δ from the systems and inserting new virtual in- and outputs w and v as illustrated in Fig. 5 and described in (20).

$$\mathcal{F}_u(M,\Delta) = M_{22} + M_{21}\Delta\left(I - M_{11}\Delta\right)^{-1} M_{12} \qquad (20)$$

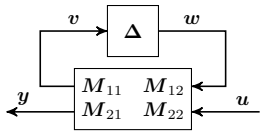


Fig. 5. Upper Linear Fractional Transformation

For the generation of the low order LFRs, the LFR toolbox of [5] is used. The generation is carried out in three steps [4]:

- Symbolic preprocessing of $S(\delta)$
- Object orientated LFR generation
- Numerical order reduction

Finally, the single LFRs are joined together by LFR operations, which are described for example in [4], [11]. Fig. 6 shows the built-up of the complete LFR consisting of the different LFR submodels augmented by an LTI sensor model. In order to show the efficiency of the proposed

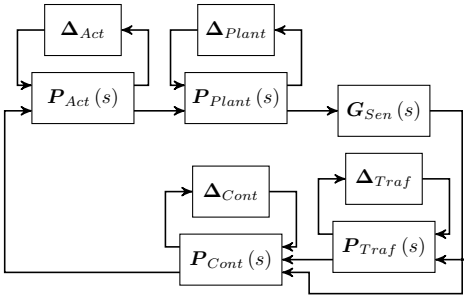


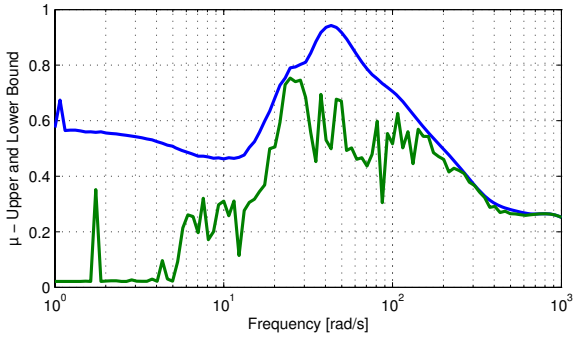
Fig. 6. Missile Model with Structured Uncertainties

optimal generation strategy for the linear parametric models, the results for the model covering region 3 of Fig. 2 are given in the following. Using the optimization process it was possible to reduce the lower bound for the LFR order of the plant and the controller from 25 and 11 to 15 and 7 respectively, while keeping the same maximum ν-gap value. By employing the advanced symbolic preprocessing methods from [4], the actual LFR model could be generated with exactly this minimal achievable order. In addition, with the help of a gradient based optimization of $(\delta_{\nu,max})$ the maximum ν-gap value could be reduced by further 15%. The matrix Δ of this LFR model is given as
$\Delta = \mathrm{diag}\left(\delta_{Act,1}, \delta_{Act,2}, \delta_{Act,3}, \varphi I_{8\times 8}, \vartheta I_{20\times 20}\right)$.

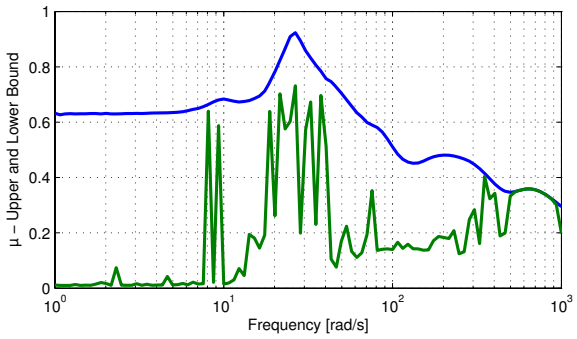
D. Robust Stability Analysis

The three LFR models covering the whole flight envelope of the closed-loop generic missile model are the basis of the following stability analysis. The structured singular value μ [11] is calculated, which provides a necessary and sufficient condition for robust stability. Unfortunately, an exact value for μ can not be calculated. However, numerical methods for calculating lower and upper bounds exist, which are implemented in the Matlab "Robust Control Toolbox" [1]. This toolbox is utilized in the present work to conduct the robust stability analysis of the generic missile.

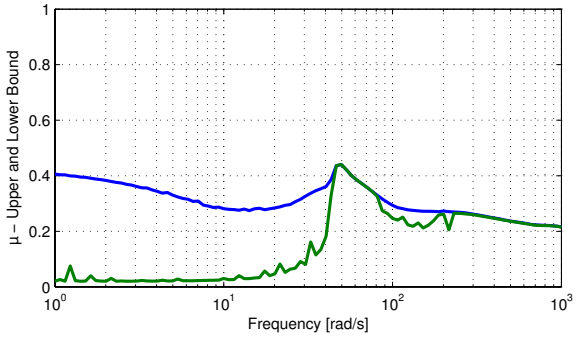
With the LFRs covering the regions depicted in Fig.2 and employing the μ-analysis, only three analyses are required to prove stability over the whole flight envelope for a constant Mach number and altitude. The results of these stability analyses are shown in Fig 7. It can be seen that the controlled missile model is stable ($\mu < 1$) over the considered region.



(a) High Incidence Angle Region 3



(b) Medium Incidence Angle Region 2



(c) Low Incidence Angle Region 1

Fig. 7. μ-Analysis of the Regions Specified in Fig.2

V. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

A very general algorithm for generating linear parametric models has been developed, which can be applied to arbitrary nonlinear systems, as long as the system behavior can be accurately described/approximated with rational parametric state-space systems. The bases for efficient LFT based methods like μ-analysis are lean and accurate models in LFR form. Hence, an optimization problem is derived to find an accurate parametric approximation of the nonlinear system, which provides an optimal structure in terms of least order LFR generation. Additionally, state-of-the-art algorithms proposed in [4] are used for the transformation of the linear parametric model into an LFR. In the present work, this algorithm has been successfully applied to a generic, highly nonlinear missile model, so that robust stability has been proven for a large region of the flight envelope by means of the structured singular value μ.

B. Future Works

In the future the incorporation of known physical relations in the linear parametric model generation process might further increase the quality of the approximation and even reduce the complexity. Furthermore, additional parametric uncertainties, such as Mach number or mass, shall be introduced into the system, to allow making further statements about the robustness of the system.

VI. ACKNOWLEDGMENTS

The authors would like to thank the German Aerospace Center and the LFK-Lenkflugkoerpersysteme GmbH, especially Gunther Michalka, for their support and the provision of the data for the missile model.

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