Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

Vehicle yaw control using a fast NMPC approach

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***Abstract*— A model predictive control approach to improve vehicle yaw rate dynamics by means of a rear active differential is introduced. In particular, the use of nonlinear predictive controllers is investigated to show their effectiveness in the vehicle stability control context. In order to allow the online implementation of the designed predictive control law, a fast technique based on Set Membership approximation methodologies using a Nearest Point approach is adopted. Enhancements on stability in demanding conditions such as μ-split braking and damping properties in impulsive maneuvers are shown through simulation results performed on an accurate nonlinear model of the vehicle. Improvements over a well assessed approach which employ an enhanced IMC structure to handle input constraints will be shown too.**

I. Introduction

Vehicle yaw dynamics may show unexpected dangerous behavior in presence of unusual external conditions such as lateral wind force, different left-right side friction coefficients and steering steps needed to avoid obstacles. Moreover, in standard turning maneuvers understeering phenomena may deteriorate handling performances in manual driving and cause uncomfortable feelings to the human driver. Vehicle active control systems aim to enhance driving comfort characteristics ensuring stability in critical situations. Several solutions to active chassis control have appeared in the last years. All the proposed strategies modify the vehicle dynamics by means of suitable yaw moments that can be generated in different ways (see e.g. [1], [2], [3], [4], [5], [6]). Common to all solutions is the fact that they are able to generate limited values of the yaw moment. The immediate consequence is that the input variable may saturate and this could deteriorate the control performances. Moreover, good damping properties and vehicle safety (i.e. stability) performance can be considered as well by imposing suitable constraints on the yaw rate $\dot{\psi}(t)$ and on the sideslip angle $\beta(t)$ values as described in [7]. Therefore, considering the presence of such constraints, the employment of Model Predictive Control (MPC) (see e.g. [8]) techniques appears to be an appropriate approach to solve the problem. However, the realization of MPC control laws require the online solution of an optimization problem. Indeed, the sampling times required for such kind of application may not allow to perform the related optimization problem online. Nevertheless, predictive control has been successfully employed in vehicle lateral control and vehicle stability control by means of suitable solutions aimed at improving the online computational times. In particular, in [9], predictive control techniques have been used in active steering control for an autonomous vehicle where online linearization of the vehicle model gave rise to an effective suboptimal solution which allowed the online implementation. Moreover, in [10] an interesting contribution to the problem of control allocation in yaw stabilization has been introduced by means of nonlinear multiparametric programming where an approximate solution obtained by means of a piecewise linear function is used for the online implementation of the controller. In this paper, the problem of efficient MPC implementation is solved using an approximated control function, with lower required computational time, derived using the Fast Model Predictive Control (FMPC) methodology introduced and described in [11] and [12]. In particular, the “Nearest Point” (NP) approach proposed in [12] is employed. Under this context, the approximating function which realizes the predictive controller is based on the offline computation of a finite number ν of exact MPC control solutions and guarantees stability as well as constraint satisfaction. A similar approach has been successfully applied also in the control of semi-active suspension systems (see [13]). In order to show in a realistic way the effectiveness of the proposed control approach, extensive simulation tests in demanding driving situations will be performed using a detailed nonlinear 14 degrees of freedom vehicle model. Finally, improvements over a well assessed approach which employ an enhanced IMC structure to handle input constraints will be shown too.

II. Vehicle modeling and control requirements

Vehicle dynamics can be described using the following single track model (see e.g. [14]):

$$\begin{aligned} & mv(t)\dot{\beta}(t) + mv(t)\dot{\psi}(t) = F_{yf}(t) + F_{yr}(t) \\ & J_z\ddot{\psi}(t) = aF_{yf}(t) - bF_{yr}(t) + M_z(t) \end{aligned} \tag{1}$$

In model (1) the inputs are the yaw moment M_z and the front steering angle δ. Moreover, m is the vehicle mass, J_z is the moment of inertia around the vertical axis, β is the sideslip angle, ψ is the yaw angle and v is the vehicle speed, a and b are the distances between the center of gravity and the front and rear axles respectively. F_{yf} and F_{yr} are the front and rear tyre lateral forces, which can be expressed as nonlinear functions of the other variables (see [6] and [15] for more details):

$$\begin{aligned} F_{yf} &= F_{yf}(\beta, \dot{\psi}, v, \delta) \\ F_{yr} &= F_{yr}(\beta, \dot{\psi}, v, \delta) \end{aligned} \tag{2}$$

Vehicle dynamics can be modified by means of suitable yaw moments generated by exploiting appropriate combinations of longitudinal and/or lateral tyre forces. In this paper, the

This research was supported in part by funds of Ministero dell'Università e della Ricerca under the Project “Advanced control and identification techniques for innovative applications”

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978-1-4244-3124-3/08/$25.00 ©2008 IEEE

required yaw moment is supposed to be generated by a Rear Active Differential (RAD) whose clutches are actuated by means of electric valves driven by the current $i(t)$ originated by the control algorithm (see [6] for a detailed description of such device). As a first approximation, the actuator behavior can be described by the model:

$$M_z(t) = K_A i(t - \vartheta) \tag{3}$$

where K_A and ϑ are the actuator gain and delay respectively. Equations (1), (2) and (3) can be rearranged in the state equation form:

$$\begin{bmatrix} \ddot{\psi}(t) \\ \dot{\beta}(t) \end{bmatrix} = f(\dot{\psi}(t), \beta(t), \delta(t), i(t - \vartheta)) \tag{4}$$

The input variable $i(t)$ is employed for control purposes, while $\delta(t)$ is not manipulable and describes the driver's maneuvering intention. The control requirements in terms of understeer characteristics improvements can be taken into account by a suitable choice of a reference signal $\dot{\psi}_{\text{ref}}(t)$ generated by means of a nonlinear static map

$$\dot{\psi}_{\text{ref}}(t) = \mathcal{M}(\delta(t), v(t)) \tag{5}$$

which uses the current values of the steering angle and of the vehicle speed as inputs. Details on the computation of the map $\mathcal{M}(\cdot)$ can be found in [6]. In order to take into account such reference following requirements, the control strategy can be designed by minimizing the amount of the error variable $e(t)$:

$$e(t) = \dot{\psi}_{\text{ref}}(t) - \dot{\psi}(t)$$

Moreover, good damping properties and vehicle safety (i.e. stability) performance can be considered as well by imposing suitable constraints on the yaw rate $\dot{\psi}(t)$ and on the sideslip angle $\beta(t)$ values as described in [7]. However, the amount of the yaw moment generated by the employed active device is subject to its physical limits. In particular, the considered device has an input current limitation of $\pm$ 1 A which correspond to the range of allowed yaw moment of $\pm$ 2500 Nm that can be mechanically generated (see [16] and [17]). Thus, saturation aspects of the control input (i.e. the actuator current $i(t)$) have to be carefully taken into account in the control design. Therefore, considering the presence of state and input constraints, the employment of (Nonlinear) Model Predictive Control (NMPC) techniques appears to be an appropriate approach to solve the problem. In the next Section, the details of the predictive approach to yaw control will be introduced.

III. NMPC STRATEGY FOR YAW CONTROL

In this Section it will be shown how Model Predictive Control strategies (see e.g. [8]) can be effectively employed in vehicle active control. The control move computation is performed at discrete time instants kT_s, $k \in \mathbb{N}$, defined by the sampling period T_s and on the basis of the following state equations obtained by discretization of (4), e.g. by means of forward difference approximation (for simplicity, the notation $k + j \triangleq (k + j)T_s$ will be used):

$$\begin{bmatrix} \dot{\psi}_{k+1} \\ \beta_{k+1} \end{bmatrix} = \tilde{f}(\dot{\psi}_k, \beta_k, \delta_k, i_{k-r}) \tag{6}$$

where r is the input delay of the current i which depends on the value of the actuator delay ϑ. Thus, at each sampling time k, the measured values of the state $\dot{\psi}_k, \beta_k$, supposed to be available, together with the requested value of the yaw rate reference $\dot{\psi}_{\text{ref},k}$, and of the input variables $\delta_k, i_{k-1}, \ldots, i_{k-r}$ are used to compute the control move through the optimization of the following performance index:

$$J(k) = \sum_{k=0}^{N_p-1} e^2_{k+j+1|k} + \rho i^2_{k+j|k} \tag{7}$$

where $N_p \in \mathbb{N}$ is the prediction horizon, $e_{k+j|k}$ is j^{th} step ahead prediction of the error variable obtained as

$$e_{k+j|k} \triangleq \dot{\psi}_{\text{ref},k} - \dot{\psi}_{k+j|k}$$

The value of $\dot{\psi}_{\text{ref},k}$ is computed using the current values of δ_k and v_k (see (5)). The predicted yaw rate $\dot{\psi}_{k+j|k}$ is obtained via the state equation (6), starting from the "initial condition":

$$\begin{bmatrix} \dot{\psi}_k \\ \beta_k \end{bmatrix}$$

and using the following values of the inputs i and δ:

$$\begin{bmatrix} \delta_{k|k} = \delta_{k+1|k} = \ldots = \delta_{k+N_p-1|k} \\ i_{k-r}, \ldots, i_{k-1}, i_{k|k}, \ldots, i_{k+N_c-1|k}, \ldots, i_{k+N_p-1|k} \end{bmatrix}$$

where $N_c \leq N_p$ is the control horizon and the assumption $i_{k+j|k} = i_{k+N_c-1|k}, N_c \leq j \leq N_p - 1$ is made. Thus, since during the prediction horizon the value of the steering angle δ is kept constant at the value $\delta_{k|k}$ measured at time k, the optimization of the index (7) is performed with respect to the variables $I = [i_{k|k}, \ldots, i_{k+N_c-1|k}]^T$. Therefore the performance index $J(k)$ depends on the vector $w_k \in \mathbb{R}^{4+r}$ of the measured variables:

$$w_k \triangleq \left[\dot{\psi}_k, \beta_k, \delta_k, v_k, i_{k-r}, \ldots, i_{k-1}\right]^T \tag{8}$$

Thus the predictive control law is computed using a receding horizon strategy:

1) At time instant k, get w_k.
2) Solve the optimization problem:

$$\min_I \quad J(k) \tag{9a}$$

subject to

$$I \in \mathbb{I} = \left\{i_{k+j|k} \,:\, |i_{k+j|k}| \leq \bar{i} > 0,\ j \in [0, N_c - 1]\right\} \tag{9b}$$

$$|\beta_{k+j|k}| \leq \bar{\beta} > 0,\ j \in [1, N_p - 1] \tag{9c}$$

3) Apply the first element of the solution sequence I as the actual control action $i_k = i_{k|k}$.
4) Repeat the whole procedure at the next sampling time $k + 1$.

Note that no constraints have been imposed on $\dot{\psi}$, as its limitation on the basis of criteria similar to the ones introduced in [7] have been implicitly taken into account in the computation of $\dot{\psi}_{\text{ref}}$ (see [6]). Besides, the constraint on β accounts for vehicle directional stability.

The obtained predictive controller results to be a nonlinear static function of the variable w_k defined in (8):

$$i_k = \kappa^0(w_k) \tag{10}$$

For a given w_k, the value of the function $\kappa^0(w_k)$ is computed by solving at each sampling time k the constrained optimization problem (9). However, such online optimization problem cannot be solved at the sampling period required for this application, which is of the order of 0.01 s. An approach to overcome this problem is to evaluate offline a certain number of values of $\kappa^0(w_k)$ to be used to find an approximation $\kappa^{\text{NP}}(w_k)$ of κ^0, suitable for online implementation. Such solution is discussed in the next Section.

IV. Fast NMPC implementation

The NMPC approach described in Section III is able to achieve optimal performance in the sense of cost function (7), while handling input and state constraints effectively. In the standard NMPC formulation, the solution $\kappa^0(w)$ is evaluated "implicitly", i.e. by solving the optimization problem (9) online. However, as just described, a limitation in the practical use of NMPC is given by the sampling period required by the considered yaw control application which may be too low for real-time optimization.

In this paper, the problem of efficient NMPC implementation is solved using an approximated control function, with lower required computational time, derived using the Fast Model Predictive Control (FMPC) methodology introduced and described in [11] and [12]. In particular, the "Nearest Point" (NP) approach proposed in [12] is employed: such technique makes it possible to systematically derive an approximation $\kappa^{\text{NP}} \approx \kappa^0$ with a desired guaranteed approximation accuracy, in terms of the pointwise error $\|\kappa^0(w) - \kappa^{\text{NP}}(w)\|_2$, and consequently with guaranteed stability and performance properties (see [12] for details).

A. Prior information

The a priori knowledge on the nominal control law κ^0 is now introduced. The approximating function κ^{NP} is computed over a compact subset $\mathcal{W} \subset \mathbb{R}^{4+r}$ of the domain of the exact function κ^0. Moreover, it is assumed that function κ^0 is continuous in $\mathcal{W}$. Results on the continuity of solutions of nonlinear optimization problems with respect to their parameters can be found in [18]. Note that stronger regularity assumptions (e.g. differentiability) cannot be made, since even in the simple case of linear dynamics, linear constraints and quadratic cost function, κ^0 is a piece-wise linear continuous function (see e.g. [19] and [20]). Inside $\mathcal{W}$, a finite number ν of points $\tilde{w}^\ell, \ell = 1, \ldots, \nu < \infty$ is suitably chosen, giving rise to the set: $\mathcal{W}_\nu = \{\tilde{w}^\ell \in \mathcal{W},\ \ \ell = 1, \ldots, \nu\}$. For each value of $\tilde{w} \in \mathcal{W}_\nu$, the corresponding value $\tilde{i} = \kappa^0(\tilde{w})$ is computed by solving offline the optimization problem (9), so that:

$$\tilde{i} = \kappa^0(\tilde{w}), \ \ \forall \tilde{w} \in \mathcal{W}_\nu \tag{11}$$

Such values of $\tilde{w}, \tilde{i}$ are stored to be used for the online computation of κ^{NP}. The set $\mathcal{W}_\nu$ is supposed to be chosen such that the following property holds:

$$\lim_{\nu \to \infty} d_H(\mathcal{W}, \mathcal{W}_\nu) = 0 \tag{12}$$

where $d_H(\mathcal{W}, \mathcal{W}_\nu)$ is the Hausdorff distance between $\mathcal{W}$ and $\mathcal{W}_\nu$, defined as (see e.g. [21]):

$$\begin{aligned} &d_H(\mathcal{W}, \mathcal{W}_\nu) = \\ &= \max\left(\sup_{w \in \mathcal{W}} \inf_{\tilde{w} \in \mathcal{W}_\nu} (\|w - \tilde{w}\|_2),\ \sup_{\tilde{w} \in \mathcal{W}_\nu} \inf_{w \in \mathcal{W}} (\|w - \tilde{w}\|_2)\right) \end{aligned} \tag{13}$$

Since both $\mathcal{W}$ and $\mathbb{I}$ are compact, the following Lipschitz continuity property holds:

$$\|\kappa^0(w^1) - \kappa^0(w^2)\|_2 \leq \gamma \|w^1 - w^2\|_2,\ \forall w^1,\ w^2 \in \mathcal{W} \tag{14}$$

All this prior information can be summarized by concluding that κ^0 belongs to the following Feasible Function Set (FFS):

$$\kappa^0 \in FFS = \{\kappa \in \mathcal{A}_\gamma : \kappa(\tilde{w}) = \tilde{i},\ \forall \tilde{w} \in \mathcal{W}_\nu\} \tag{15}$$

where $\mathcal{A}_\gamma$ is the set of all continuous functions $\kappa : \mathcal{W} \to \mathbb{I}$, such that (14) holds.

B. Nearest Point approximation

The approximating function κ^{NP} is computed as follows. For any $w \in \mathcal{W}$, denote with $\tilde{w}^{\text{NP}}$ a value such that:

$$\tilde{w}^{\text{NP}} \in \mathcal{W}_\nu : \|\tilde{w}^{\text{NP}} - w\|_2 = \min_{\tilde{w} \in \mathcal{W}_\nu} \|\tilde{w} - w\|_2 \tag{16}$$

Then, the NP approximation $\kappa^{\text{NP}}(x)$ is defined as:

$$\kappa^{\text{NP}}(w) = \kappa^0(\tilde{w}^{\text{NP}}) \tag{17}$$

As showed in [12], such approximation has the following properties:

i) the input constraints are always satisfied:

$$\kappa^{\text{NP}}(w) \in \mathbb{I},\ \forall w \in \mathcal{W} \tag{18}$$

ii) for a given ν, a bound $\zeta(\nu)$ on the pointwise approximation error can be computed:

$$\begin{aligned} &\|\kappa^0(w) - \kappa^{\text{NP}}(w)\|_2 \leq \zeta = \\ &= \gamma\, d_H(\mathcal{W}, \mathcal{W}_\nu),\ \forall w \in \mathcal{W} \end{aligned} \tag{19}$$

iii) $\zeta(\nu)$ is convergent to zero:

$$\lim_{\nu \to \infty} \zeta = 0 \tag{20}$$

Properties (18)–(20) suffice to guarantee that the stability and performance characteristics obtained with control law κ^{NP} are arbitrarily close to those of κ^0, in terms of Euclidean distance between their respective closed–loop trajectories (see [11] and [12] for further details on the stabilizing properties of FMPC control laws). As regards the computation of the Lipschitz constant γ, which is needed to compute the approximation error bound ζ^{NP}, an estimate $\widehat{\gamma}$ can be derived as follows:

$$\widehat{\gamma} = \inf\left(\tilde{\gamma} : \tilde{i}^h + \tilde{\gamma}\|\tilde{w}^h - \tilde{w}^k\|_2 \geq \tilde{i}^k,\ \forall k, h = 1, \ldots, \nu\right) \tag{21}$$

Such estimate guarantees that $FFS \neq \emptyset$ (since it follows as an extension of Theorem 1 in [22]). In [12] it is showed that:

$$\lim_{\nu \to \infty} \widehat{\gamma} = \gamma$$

C. Variable scaling

In the computation of the Hausdorff distance (13) and of NP control law (16), (17) , the Euclidean norm $\|\tilde{w}-w\|_2 = \sqrt{(\tilde{w}-w)^T(\tilde{w}-w)}$ is considered to measure the distance between $\tilde{w}$ and w. In [12], such choice gives good results on a numerical example. However, in practical applications it is usually needed to scale the variables w to adapt to the properties of data. This is obtained using a weighted Euclidean norm:

$$\|\tilde{w}-w\|_2^M = \sqrt{(\tilde{w}-w)^T M^T M(\tilde{w}-w)} \qquad (22)$$

where

$$M = \mathrm{diag}(m_i),\; i = 1,\ldots,4+r \qquad (23)$$

and $m_i \in (0,1) : \sum_{i=1}^{4+r} m_i = 1$ are suitable scalar weights. In [22] the issue of choosing the values of m_i is considered when the function to be approximated is differentiable. A similar approach is now presented in the case of Lipschitz continuous functions. For the sake of notation's simplicity, consider $\kappa^0(w): \mathbb{R}^{4+r} \to \mathbb{R}$.
Due to the continuity assumption, function κ^0 is Lipschitz continuous with respect to each component w_i of w, $i = 1,\ldots,n$. Thus, for each value of $w \in \mathcal{W}$ there exist Lipschitz constants $\gamma_i(w)$, $i = 1,\ldots,4+r$ such that:

$$|\kappa^0([v^1, w_{j\neq i}]) - \kappa^0([v^2, w_{j\neq i}])| \leq \gamma_i(w)|v^1 - v^2|,\; \forall v^1, v^2 \in \mathcal{V}_i$$

where $\mathcal{V}_i = \{v : [v, w_{j\neq i}] \in \mathcal{W}\}$. Consider now the constants:

$$\Gamma_i = \sup_{w\in\mathcal{W}} \gamma_i(w), i = 1,\ldots,4+r$$

Estimates of Γ_i can be computed e.g. by performing a preliminary differentiable approximation $\hat{\kappa} \approx \kappa^0$ (e.g. linear, neural networks...) and evaluating:

$$\Gamma_i \simeq \sup_{w\in\mathcal{W}} |\partial\hat{\kappa}(w)/\partial w_i|$$

Then, the values of m_i can be computed as:

$$m_i = \frac{\Gamma_i}{\sum_{i=1}^{4+r} \Gamma_i} \qquad (24)$$

equation (24) is derived applying normalization to the values given by Lemma 2 in [22].

D. Design procedure

The overall design procedure for the fast NMPC approach proposed in this paper can be resumed as follows:

1) Design the nominal NMPC control law according to (9).
2) Choose the set $\mathcal{W}$ where the FMPC control law is defined and collect the values $\tilde{w}^j$, $\tilde{i}^j$, $j = 1,\ldots,\nu$ (11) such that (12) holds, e.g. by performing simulations of suitably chosen maneuvers using the closed loop vehicle with the nominal NMPC controller.
3) Derive a preliminary smooth approximated control law $\hat{\kappa} \approx \kappa^0$ using some identification method and evaluate the weight matrix M (23) using (24).
4) Estimate the Lipschitz constant γ using (21), considering the scaled values $\tilde{v}^j = M\tilde{w}^j$, $j = 1,\ldots,\nu$.
5) Evaluate the guaranteed approximation error $\zeta(\nu)$ using (19), computing the Hausdorff distance $d_H(\mathcal{W}, \mathcal{W}_\nu)$ (13) with the weighted Euclidean norm $\|\cdot\|_2^M$ (22). Eventually tune the weight matrix M and/or increase the number ν of off-line computed values to reduce $\zeta(\nu)$.
6) Implement on-line the NP approximated control law using (16) and (17) with the weighted Euclidean norm $\|\cdot\|_2^M$ (22).

V. Simulation results

The nominal predictive controller κ^0 has been designed using model (1)–(2) with the following nominal parameter values:
$m = 1715\,\mathrm{kg}$ $\quad J_z = 2700\,\mathrm{kgm}^2$ $\quad a = 1.07\,\mathrm{m}$ $\quad b = 1.47\,\mathrm{m}$
$\vartheta_A = 20\,\mathrm{ms}$ $K_A = 2500\,\mathrm{Nm/A}$
To be used in the optimization algorithm, the vehicle model has been discretized using forward difference approximation, with sampling time $T_s = 0.01$ s. Therefore, since the nominal actuator delay value is $\vartheta = 20\mathrm{ms} = 2T_s$, at the generic time step k the past input values i_{k-1}, i_{k-2} (i.e. the number r of the current delay is 2) have to be used to compute the predicted vehicle behavior. The weight ρ in cost function (7) has been chosen as $\rho = 10^{-6}$, and the employed state and input constraints are $\overline{\beta} = 5^\circ$ and $\overline{i} = 1\,\mathrm{A}$. The chosen prediction and control horizons are $N_p = 100$ and $N_c = 5$ respectively. The nominal control move computation has been performed using a sequential constrained Gauss-Newton quadratic programming algorithm (see e.g. [23]), where the underlying quadratic programs have been solved using the MatLab® optimization function `quadprog`. Thus, the nominal control law at sampling time k results to be a static function of the variables $w_k = [\dot{\psi}_k\, \beta_k\, \delta_k\, v_k\, i_{k-1}\, i_{k-2}]^T \in \mathbb{R}^6$. Note that the reference yaw rate $\dot{\psi}_{\mathrm{ref},k}$ is not explicitly considered in the regressor vector w_k, since it is computed using a static function of δ_k and v_k (see [6]), which are already included in w_k. The values of $\tilde{w}$, $\tilde{i}$ in (11) have been computed performing simulations involving an extensive set of handwheel steps and sinusoids maneuvers. In this way, a number $\nu = 5.5\,10^5$ of values was collected in the set:

$$\mathcal{W} = \left\{ w : \begin{bmatrix} -0.5 \\ -0.1 \\ -0.1 \\ 22 \\ -1 \\ -1 \end{bmatrix} \preceq w \preceq \begin{bmatrix} 0.5 \\ 0.1 \\ 0.1 \\ 33 \\ 1 \\ 1 \end{bmatrix} \right\}$$

where the symbol $\preceq$ indicates element-wise inequalities. The weights m_i, $i = 1,\ldots,6$, $\sum_{i=1}^{6} m_i = 1$ (24) for the NP control approximation have been initially computed on the basis of a preliminary linear approximation of κ^0 (see [22]) and they have been tuned through simulations. The chosen values are $m_{1\div 6} = [0.107, 0.539, 0.352, 1.9\cdot 10^{-7}, 2.6\cdot 10^{-4}, 2.6\cdot 10^{-4}]$.
In order to test the performances obtained by the considered yaw control approach, simulations have been performed

using a detailed nonlinear 14 degrees of freedom Simulink model, which gives an accurate description of the vehicle dynamics as compared to actual measurements and includes nonlinear suspension, steer and tyre characteristics, obtained on the basis of measurements on the real vehicle. The following open loop (i.e. without driver's feedback) maneuvers have been chosen to test the control effectiveness:
- steer reversal test with handwheel angle of 50° performed at 100 km/h, with a steering wheel speed of 400°/s. This test aims to evaluate the controlled car transient and steady state performances: the employed handwheel course is showed in Fig. 1.
- $\mu-$split braking maneuver performed at 100 km/h with

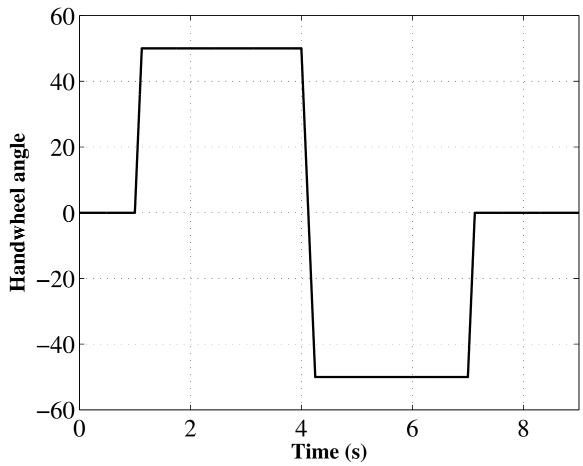

Fig. 1. Handwheel angle course for the 50° steer reversal test maneuver.

dry road on one side and icy road on the other, with braking pedal input corresponding to a deceleration value of 0.5 g on dry road. The objective of this test is to evaluate the system response in presence of strong disturbances. Note that the $\mu-$split maneuver implies a differential left-right change in the tyre–road friction coefficients, which was not taken into account in FMPC design, since the maneuvers considered in the off-line computation of the control moves were performed with a single track model.
- steering wheel frequency sweep performed at 90 km/h in the frequency range 0-7 Hz with steering wheel angle amplitude of 30°.

The performance obtained with the NP approximation technique have been compared to those of the uncontrolled vehicle, of the nominal MPC control law and of the enhanced IMC structure proposed in [6] for the same application, which proved to give quite good results.

The results of the 50° steer reversal test are reported in Fig. 2–3. In Fig. 2 it can be noted that the approximated MPC controller (solid line) and the nominal one (dashed-dotted) show a very similar behavior, with only a slight difference in the second part of the maneuver (see Fig. 2 at about $t = 6$ s). Moreover, the transient performances obtained with the proposed fast NMPC technique are better than those of the IMC controller (dashed line, see Fig. 2 at $t = 1$ s, $t = 4$ s and $t = 7$ s), which already showed very good performance with respect to the uncontrolled vehicle (Fig. 2, dotted line). The steady state yaw rate reference is reached and, according

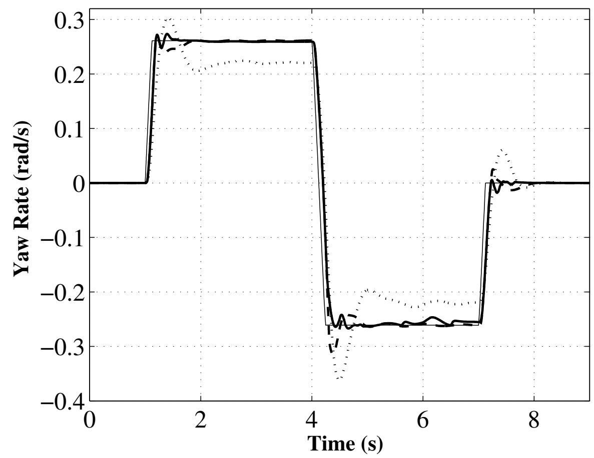

Fig. 2. 50° steer reversal test at 100 km/h. Comparison between the reference (thin solid line) vehicle yaw rate course and those obtained with the uncontrolled vehicle (dotted) and the IMC (dashed), nominal NMPC (dashed–dotted) and FMPC (solid) controlled vehicles.

to the reference map (see e.g. [6]), it is higher than the uncontrolled vehicle yaw rate, thus improving car maneuverability. The obtained sideslip angle $\beta(t)$ is kept inside the considered constraint, with a maximum absolute value of 2.8°, as well as the input variable u (Fig. 3, solid line). Note

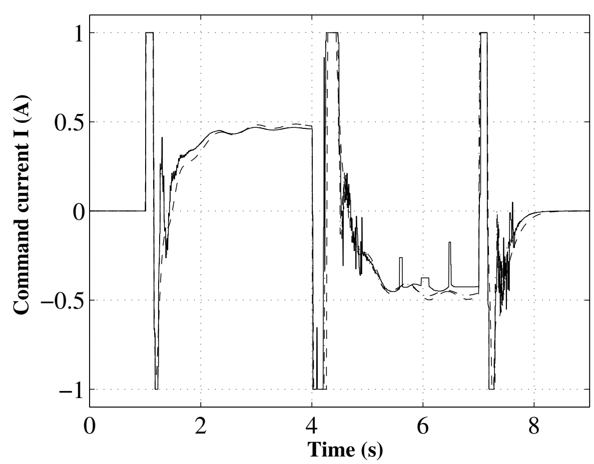

Fig. 3. 50° steer reversal test at 100 km/h. Comparison between the input variable u obtained with the IMC (dashed), nominal NMPC (dash-dotted) and FMPC (solid) controllers.

that some chattering of the input variable occurs with the FMPC control law: such phenomenon can be mitigated by increasing the number ν of off-line computed control moves (see [12]), at the expense of higher memory usage. Another possibility would be the use of a "local" set membership approximation, as described in [22], which can practically lead to good approximation accuracy with low values of ν. Indeed, the choice of the regressor values is a key-point in FMPC design, especially if the regressor dimension is relatively high, like in the considered application. A higher value of ν leads to better accuracy, but also to higher memory requirements and computational costs. With the employed NP approximation, the on-line computational time can be greatly reduced by suitably arranging the collected data and, in the case of uniform gridding of $\mathcal{W}$, the computational burden is independent on ν (see [12] for details). However, uniform gridding of $\mathcal{W}$ may lead to excessively high ν values and is not adopted in this application. The obtained mean computational time for the FMPC control law is 1 ms, using MatLab® 7 under MS Windows XP and an Intel® Core(tm)2 Duo T7700@2.4 GHz processor with 2 GB RAM. On the same machine, the mean computational time for the online optimization is 35 ms.

As regards the considered μ-split braking maneuver, Fig. 4 shows the vehicle trajectories obtained in the uncontrolled case (black), with the IMC controller (white) and with the FMPC controller (gray). It can be noted that the approximated predictive control law achieves the best performance, since the effects of the disturbance on the vehicle path is lower than the other cases, while the uncontrolled vehicle is not stable. Finally, the steering wheel frequency sweep

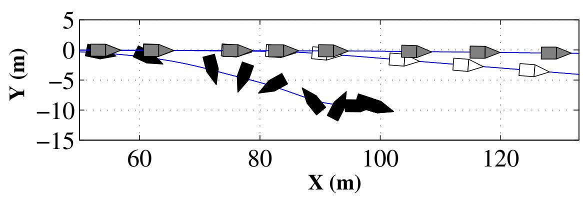

Fig. 4. $\mu - split$ braking maneuver at 100 km/h. Comparison between the trajectories obtained with the uncontrolled vehicle (black) and the IMC (white) and FMPC (gray) controlled ones.

maneuver aims to evaluate the improvement achieved by the FMPC controlled vehicle in terms of resonance peak reduction and bandwidth increase. In particular, the frequency course of the transfer ratio:$T_m(\omega) = (\overline{\dot{\psi}}(\omega))/(\overline{\dot{\psi}}(0))$ has been analyzed, where $\overline{\dot{\psi}}(\omega)$ is the steady state yaw rate amplitude obtained in presence of the sinusoidal 30° handwheel input at frequency ω, and $\overline{\dot{\psi}}(0)$ is the steady state yaw rate in presence of a constant handwheel input of 30°. The FMPC controlled vehicle has a lower resonance peak (1.3 dB) and slightly higher bandwidth (3.5 Hz) with respect to the case of IMC control (1.9–dB resonance peak and 3–Hz bandwidth) and of the uncontrolled vehicle (2.8–dB resonance peak and 2.2–Hz bandwidth). Finally, note that the enhanced IMC controller of [6] also employs a feedforward control contribution to enhance the system transient response, which is not needed in the case of NMPC.

VI. Conclusions

A Model Predictive Control approach to vehicle yaw control has been introduced. In the proposed approach the predictive controller has been realized by means of a Nearest Point approximation using a finite number of exact offline solutions. Simulation results performed on an accurate model of the considered vehicle demonstrate the effectiveness of the considered approach. In particular, it has been shown that a highly damped behaviour in reversal steer maneuvers has been obtained; stability is guaranteed in presence of demanding driving conditions like $\mu-$split braking and resonance peak has been significantly reduced in the frequency response. Finally, improvements over a well assessed approach which employ an enhanced IMC structure to handle input constraints have been shown too.

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