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Bearings-Only Guidance of an Autonomous Vehicle Following a Moving Target with a Smaller Minimum Turning Radius

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Abstract— **This paper addresses the problem of following a moving target by an autonomous unmanned vehicle. The target may have higher maneuverability and a smaller minimum turning radius than the pursuing vehicle. The goal is to keep the autonomous vehicle as close as possible to the target all the time. We present a simple and constructive bearings-only guidance law and give its mathematically rigorous analysis.**

Index Terms— **Bearings-only guidance, navigation, UAV, mobile robots, target tracking**

I. Introduction

Problems of automatic guidance, navigation, control and coordination of autonomous unmanned vehicles for performing various tasks in remote or hazardous environments have attracted a lot of attention in recent years; see e.g. [12], [13], [2], [14], [3], [11], [9], [10], [4], [7] and references therein.

This paper addresses the problem of following a moving target by an autonomous unmanned vehicle. The kinematics of the autonomous vehicle under consideration is described by a standard model that is applicable to wheeled robots, unmanned aerial vehicles (UAVs), missiles and underwater vehicles. The proposed vehicle model satisfies standard design constraints on speed and maneuverability. On the other hand, maneuverability of the target that is followed by this autonomous vehicle is not constrained. In particular, the target kinematics may be described by a similar kinematics model but with a smaller minimum turning radius. Such a problem arises, for example, when moving ground vehicles are followed and observed by UAVs in numerous military and security applications [3], [9], [10]. Providing automatic following an assigned target on the ground without any human intervention is recognized as a key element in obtaining full autonomy for unmanned aerial vehicles [1]. Another possible example is a wheeled robot following a smaller and less powerful vehicle. It is obvious, that in such problems tracking with a zero steady-state error is impossible, and our goal is to keep the autonomous vehicle as close as possible in some sense to the target.

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In this paper, we introduce the concept of a following guidance law with a certain upper time period. Furthermore, we present a simple and easily implementable following guidance law and give its mathematically rigorous analysis. This guidance law belongs to the class of so-called bearings-only guidance algorithms. The effectiveness of the proposed guidance law is also confirmed by illustrative examples and simulations. Some guidance algorithms for similar problems were proposed in [3], [10], [9], however, no mathematically rigorous results were given.

The reminder of the paper is organized as follows. Section II presents basic definitions and states the problem under consideration. Section III introduces the proposed guidance law and gives its mathematically rigorous analysis. Illustrative examples and computer simulations are presented in Section IV. Finally, Section V gives brief conclusions and outlines directions for future research.

II. Problem Statement

We consider a target moving in a plane. Let $(x_T(t), y_T(t))$ be the Cartesian coordinates of the target in this plane. The target motion may be governed by any law, and the only constraint on the target motion is that the trajectory $(x_T(t), y_T(t))$ is smooth, and the target velocity is bounded, i.e.:

$$\sqrt{\dot{x}_T(t)^2 + \dot{y}_T(t)^2} \leq V_T \quad \forall t \geq 0 \qquad (2.1)$$

for some given constant $V_T > 0$. Let $\mathcal{T}[V_T]$ denote the class of all maneuvering target trajectories satisfying (2.1).

The target is pursued by an autonomous vehicle which is moving in the same plane with a constant speed V_V. Let $(x_V(t), y_V(t))$ be the Cartesian coordinates of the vehicle. Also, let $\theta(t)$ be the orientation of this vehicle with respect to the $x-$axis, that is $\theta(t)$ is measured from the $x-$axis in the counterclockwise direction, it takes values in the interval $[0, 2\pi)$. Then, the kinematic equations of the vehicle motion are given by

$$\begin{aligned} \dot{x}_V(t) &= V_V \cos(\theta(t)) \quad ; \\ \dot{y}_V(t) &= V_V \sin(\theta(t)) \quad ; \\ \dot{\theta}(t) &= \omega(t) \quad . \end{aligned} \qquad (2.2)$$

Here $\omega(t)$ is the control input. The equations (2.2) can describe the kinematics of tactical missiles, UAVs

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or wheeled mobile robots; see e.g. [15], [2], [5], [6], [4]. The standard design specifications impose the following input constraint:

$$-\omega_{max} \leq \omega(t) \leq \omega_{max} \qquad \forall t \geq 0 \tag{2.3}$$

where $\omega_{max} > 0$ is a given constant. In this case, the minimum turning radius of the vehicle is

$$R_{min} = \frac{V_V}{\omega_{max}}. \tag{2.4}$$

We assume that the information on the target that is available to the controller of the autonomous vehicle is the target coordinates $(x_T(\cdot), y_T(\cdot))$. Also, the vehicle's own coordinates and heading are measured. Hence, we wish to design for the autonomous a guidance law of the form:

$$\begin{aligned}\omega(t) \quad = \quad & \mathcal{F}(x_T(\tau)\mid_0^t, y_T(\tau)\mid_0^t, \\ & x_V(\tau)\mid_0^t, y_V(\tau)\mid_0^t, \theta(\tau)\mid_0^t)\end{aligned} \tag{2.5}$$

Our control goal is to keep the autonomous vehicle as close as possible to the target. We will use the following assumption:

$$V_V > V_T. \tag{2.6}$$

It is obvious that if (2.6) does not hold then for any feedback guidance law (2.5) there exists a target motion such that

$$|x_T(t) - x_V(t)| + |y_T(t) - y_V(t)| \to \infty$$

as $t \to \infty$.

There is a vast literature on the problem of tracking for two wheeled mobile robots, see e.g. [2], [8]. Typically, in those papers both the robot-target and the robot-follower are described by the equations of the form (2.2), and it is assumed that the speed of the follower is greater than the speed of the target, and the minimum turning radius of the follower is smaller than the minimum turning radius of the follower. It is known that under these assumptions, the robot-follower can asymptotically track the robot-target, that is

$$|x_T(t) - x_F(t)| \to 0, \qquad |y_T(t) - y_F(t)| \to 0,$$
$$|\theta_T(t) - \theta_F(t)| \to 0$$

as $t \to \infty$. Those results are not applicable to the problem that is under consideration in this paper. First, we consider the very general class of targets (2.1) which includes targets that are more maneuvering than the vehicle. In particular, a very important subclass consists of targets that are described by the equations of the form (2.2) but might have a minimum turning radius r_{min} such that $r_{min} < R_{min}$. Such a situation arises, for example, in an important practical problem of surveillance of ground targets by UAVs because the minimum turning radius of a typical UAV is much larger than the minimum turning radius of a ground vehicle. It is obvious, that in such situations, the condition

$$\|x_T(t) - x_F(t)\| \to 0, \qquad \|y_T(t) - y_F(t)\| \to 0$$

is not achievable with any guidance law (2.5). Moreover, a standard UAV design requirement requires that the speed of UAVs cannot be below some minimum level V_V, known as stall speed, at any time. A typical tactical UAV may have the stall speed around 80 km/h, hence, when the ground target moves with a speed below 80 km/h, the asymptotic tracking is obviously impossible. Also, it is quite common for ground targets to be steady for some periods of time. To state our control goal, we first introduce a number of definitions.

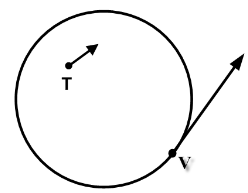

Fig. 1. Vehicle encircles the target along a minimum radius circle

Definition 2.1: Let $t_* \geq 0$ be some time. The time t_* is said to be a *time of encircling* for a guidance law of the form (2.5) and a target trajectory from the class $\mathcal{T}[V_T]$ defined by (2.1) if at this time the vehicle is moving along a minimum radius circle and the target lies either inside this circle (see Fig. 1) or on this circle.

Definition 2.2: Let $P > 0$ be a constant. A guidance law of the form (2.5) is said to be *following with the upper time period P* if for any target trajectory from the class $\mathcal{T}[V_T]$ defined by (2.1) and any target and vehicle initial conditions $(x_T(0), y_T(0), x_V(0), y_V(0), \theta(0))$ there exists a sequence of times of encircling $\{t_i\}_{i=1}^{\infty}$ where $t_{i+1} > t_i \geq 0$ such that

$$t_{i+1} - t_i \leq P \quad \forall i = 1, 2, 3, \ldots. \tag{2.7}$$

Problem Statement: Our problem is to find a constructive and easily implementable in real time following guidance law for the autonomous vehicle (2.2) and the class of target trajectories $\mathcal{T}[V_T]$ defined by (2.1) .

III. The Main Result

In this section, we propose and study a following guidance law. Let v_1 and v_2 be non-zero two-dimensional vectors, and let α be the angle between the vectors v_1 and v_2 measured from v_1 in the counterclockwise direction, $0 \leq \alpha < 2\pi$. Now introduce the following function:

$$f(v_1, v_2) = \begin{cases} 0 & \alpha = 0 \\ 1 & 0 < \alpha \leq \pi \\ -1 & \pi < \alpha < 2\pi \end{cases} \tag{3.8}$$

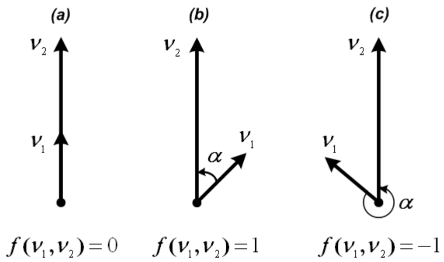

Fig. 2.

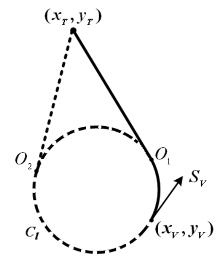

Fig. 3.

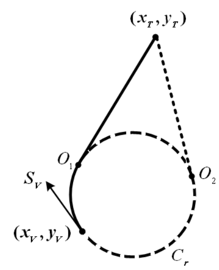

Fig. 4.

see Fig. 2. Also, introduce two-dimensional vectors $d(t)$ and $s_V(t)$ by

$$d(t) := \begin{pmatrix} x_T(t) - x_V(t) \\ y_T(t) - y_V(t) \end{pmatrix},$$
$$s_V(t) := \begin{pmatrix} V_V \cos(\theta(t)) \\ V_V \sin(\theta(t)) \end{pmatrix}. \quad (3.9)$$

We will consider the following guidance law of the form (2.5):

$$\omega(t) = \begin{cases} \omega_{max} f(s_V(t), d(t)) & d(t) \neq 0 \\ 0 & d(t) = 0 \end{cases} \quad (3.10)$$

Here $d(\cdot)$ and $s_V(\cdot)$ are defined by (3.9). It is obvious that the guidance law (3.8), (3.9), (3.10) satisfies the assumption (2.3).

Remark 3.1: The guidance law (3.8), (3.9), (3.10) is defined by the angle $\alpha(t)$ between the vehicle velocity vector $s_V(t)$ and the vector from the vehicle to the target, and the guidance law does not depend on the distance to the target. Therefore, the guidance law (3.8), (3.10) belongs to the class of so-called bearings-only guidance laws.

Now we are in a position to present the main result of this paper.

Theorem 3.1: Consider the autonomous vehicle described by the equations (2.2) and the class of target trajectories satisfying (2.1). Suppose that the assumption (2.6) holds. Then, the guidance law defined by (3.8), (3.9), (3.10) is a following guidance law with the upper period

$$P = \frac{2\pi R_{min}}{V_V - V_T}$$

where R_{min} is the minimum turning radius of the autonomous vehicle defined by (2.4).

Proof of Theorem 3.1 The proof of the theorem is based on properties of the Lyapunov function $W(x_T, y_T, x_V, y_V, \theta)$ defined as follows. Let d, s_V and $f(s_V, d)$ be defined by (3.8) and (3.9). If $d = 0$ then $W(x_T, y_T, x_V, y_V, \theta) := 0$. If $d \neq 0$ then the function $f(d, s_V)$ is defined. If $f(s_V, d) = 0$ then $W(x_T, y_T, x_V, y_V, \theta) := \|d\|$ where $\|\cdot\|$ is the standard Euclidean norm. In other words, in this case $W(x_T, y_T, x_V, y_V, \theta)$ is the distance between (x_T, y_T) and (x_V, y_V). If $f(s_V, d) = 1$ then consider the minimum radius circle C_l crossing the point (x_V, y_V) with the tangent vector s_V and located "to the left" from the straight line L defined by (x_T, y_T) and the vector s_V (see Fig. 3). Since $f(s_V, d) = 1$, C_l and (x_T, y_T) belong to the same half-plane defined by L. If (x_T, y_T) lies either inside C_l or on C_l, then $W(x_T, y_T, x_V, y_V, \theta) := 0$. Otherwise, if (x_T, y_T) lies outside C_l, consider two straight lines from (x_T, y_T) that are tangent to C_l. Let O_1 and O_2 be the intersection points between these straight lines and the circle C_l whereas O_1 will be the closest of them to (x_V, y_V) if we go along C_l in the counterclockwise direction (see Fig. 3). Furthermore, let $|(x_V, y_V), O_1|$ be the length of the corresponding arc of the circle C_l, and $\|O_1, (x_T, y_T)\|$ be the standard Euclidean distance between O_1 and (x_T, y_T). Then $W(x_T, y_T, x_V, y_V, \theta) := |(x_V, y_V), O_1| + \|O_1, (x_T, y_T)\|$. Analogously, if $f(s_V, d) = -1$ then $W(x_T, y_T, x_V, y_V, \theta)$ is defined by the same procedure with the circle C_l replaced by the circle the minimum radius circle C_r crossing the point (x_V, y_V) with the tangent vector s_V and located "to the right" from the straight line L defined by (x_T, y_T) and the vector s_V, and with the counterclockwise direction replaced by the clockwise direction (see Fig. 4). Let $t_2 > t_1$ be some times, and consider the guidance law (3.8), (3.9), (3.10) and a target satisfying the constraint (2.1). Assume that for any $t \in [t_1, t_2]$ the inequality $W(x_T(t), y_T(t), x_V(t), y_V(t), \theta(t)) > 0$ holds. Then, we prove that

$$W(x_T(t_2), y_T(t_2), x_V(t_2), y_V(t_2), \theta(t_2)) \leq$$

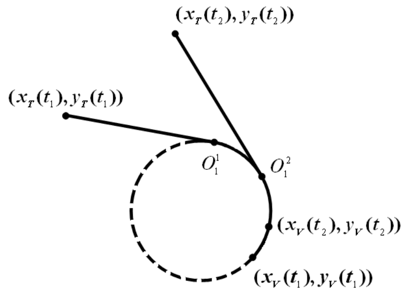

Fig. 5.

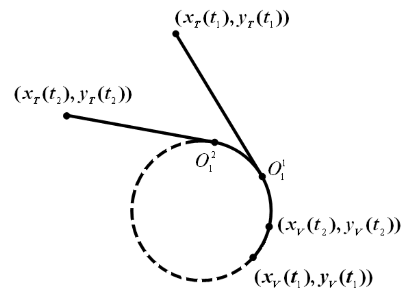

Fig. 6.

$$W(x_T(t_1), y_T(t_1), x_V(t_1), y_V(t_1), \theta(t_1)) - (t_2 - t_1)(V_V - V_T). \quad (3.11)$$

Indeed, it is obviously enough to prove (3.11) for $t_1,\ t_2$ such that $(t_2 - t_1)$ is small enough. By our definition of the Lyapunov function $W(x_T, y_T, x_V, y_V, \theta)$,

$$W(x_T(t_1), y_T(t_1), x_V(t_1), y_V(t_1), \theta(t_1)) = |(x_V(t_1), y_V(t_2)), O_1^1| + \|O_1^1, (x_T(t_1), y_T(t_1))\|,$$
$$W(x_T(t_2), y_T(t_2), x_V(t_2), y_V(t_2), \theta(t_2)) = |(x_V(t_2), y_V(t_2)), O_1^2| + \|O_1^2, (x_T(t_2), y_T(t_2))\|, \quad (3.12)$$

and if $(t_2 - t_1)$ is small enough, the point $(x_V(t_2), y_V(t_2))$ lies on the same minimum radius circle with $(x_V(t_1), y_V(t_1))$ and close enough to it. Two following two cases are possible.

Case 1. The point O_1^2 belongs to the arc $((x_V(t_2), y_V(t_2)), O_1^1)$, see Fig. 5. Here O_1^1 and O_1^2 are the point O_1 in the definition of $W(x_T(t_1), y_T(t_1), x_V(t_1), y_V(t_1), \theta(t_1))$ and $W(x_T(t_2), y_T(t_2), x_V(t_2), y_V(t_2), \theta(t_2))$ correspondingly. In this case,

$$\|O_1^2, (x_T(t_2), y_T(t_2))\| \leq |O_1^2, O_1^1| + \|O_1^1, (x_T(t_1), y_T(t_1))\| + \|(x_T(t_1), y_T(t_1)), (x_T(t_2), y_T(t_2))\|. \quad (3.13)$$

Furthermore, it follows from (2.1) that

$$\|(x_T(t_1), y_T(t_1)), (x_T(t_2), y_T(t_2))\| \leq (t_2 - t_1)V_T. \quad (3.14)$$

This and (3.13) imply that

$$\|O_1^2, (x_T(t_2), y_T(t_2))\| \leq |O_1^2, O_1^1| + \|O_1^1, (x_T(t_1), y_T(t_1))\| + (t_2 - t_1)V_T. \quad (3.15)$$

Moreover,

$$|(x_V(t_1), y_V(t_1)), (x_V(t_2), y_V(t_2))| = (t_2 - t_1)V_V. \quad (3.16)$$

Now (3.11) follows from (3.12), (3.15) and (3.16).

Case 2. The point O_1^2 is outside the arc $((x_V(t_2), y_V(t_2)), O_1^1)$, see Fig. 6. In this case, the set with the boundary consisting of the lines $(O_1^1, (x_T(t_1), y_T(t_1)))$, $((x_T(t_1), y_T(t_1)), O_1^2)$ and the arc (O_1^2, O_1^1) is convex. Furthermore, this convex set lies inside the triangle $(O_1^1, (x_T(t_2), y_T(t_2)), (x_T(t_1), y_T(t_1)))$ which is another convex set. This implies that

$$\|O_1^2, (x_T(t_2), y_T(t_2))\| + |O_1^1, O_1^2| \leq \|O_1^1, (x_T(t_1), y_T(t_1))\| + \|(x_T(t_1), y_T(t_1)), (x_T(t_2), y_T(t_2))\|. \quad (3.17)$$

Furthermore, (3.17) and (3.14) imply that

$$\|O_1^2, (x_V(t_2), y_V(t_2))\| + |O_1^1, O_1^2| \leq \|O_1^1, (x_T(t_1), y_T(t_1))\| + (t_2 - t_1)V_T. \quad (3.18)$$

Now (3.11) follows from (3.15), (3.18) and (3.16).

We have proved the inequality (3.11). Now (3.11) obviously implies that for any $t_1 > 0$ there exists a time $t_2 > t_1$ such that

$$W(x_T(t_2), y_T(t_2), x_V(t_2), y_V(t_2), \theta(t_2))) = 0.$$

Now let t_1 be a time such that

$$W(x_T(t_1), y_T(t_1), x_V(t_1), y_V(t_1), \theta(t_1)) = 0$$

and

$$W(x_T(t_1^+), y_T(t_1^+), x_V(t_1^+), y_V(t_1^+), \theta(t_1^+)) > 0$$

where

$$\nu(t^+) := \lim_{\epsilon > 0, \epsilon \to 0} \nu(t + \epsilon)$$

for any function $\nu(t)$. It is obvious by the definition of the Lyapunov function W that in this case

$$W(x_T(t_1^+), y_T(t_1^+), x_V(t_1^+), y_V(t_1^+), \theta(t_1^+)) \leq 2\pi R_{min}. \quad (3.19)$$

Now it follows from (3.11) and (3.19) that $W(x_T(t_2), y_T(t_2), x_V(t_2), y_V(t_2), \theta(t_2))) = 0$ for some $t_2 > t_1$ such that

$$(t_2 - t_1) \leq \frac{2\pi R_{min}}{V_V - V_T}.$$

The statement of the theorem obviously follows from this. This completes the proof of Theorem 3.1.

IV. ILLUSTRATIVE EXAMPLES

In this section, we present several examples of computer simulations for the proposed guidance law (3.8), (3.9) and (3.10). Simulation parameters of the vehicle and the target are shown in Table I. In the first experiment, we consider a target moving along a straight line with a constant speed $V_T = 15\ m/s$. Fig. (7) shows the simulation result for the guidance law (3.8), (3.9) and (3.10). The vehicle approaches the target and starts circling with its minimum radius circle, $R_{min} = \frac{V_V}{\omega_{max}} = 80m$. The second and third experiments are performed to ascertain the ability of the proposed

TABLE I
SIMULATION PARAMETERS 1

Parameter	Value	Comments
$x(0)$	$(0,0,0)$	Vehicle's initial posture
$x_T(0)$	$(-400m,0)$	target's initial position
V_v	$40\ m/s$	Vehicle's linear velocity
ω_{max}	$0.5\ rad/s$	Vehicle's maximum angular velocity

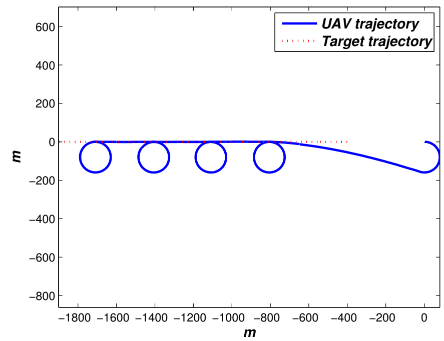

Fig. 7. Experiment 1

guidance strategy to work at different states of the target maneuvering. The vehicle is assigned to follow a ground target with a smaller turning radius than that of the vehicle. In the second scenario, the ground target moves along a curve shown in Fig. (7). Fig. (8) demonstrates the trajectory of the vehicle following a target which moves along a course with the turning radius $R_T = \frac{V_T}{\omega_{T max}} = 15m$. Similar to the previous cases, the vehicle approaches the target and follows it in a circular trajectory with the vehicle's minimum turning radius.

V. BRIEF CONCLUSION AND FUTURE RESEARCH

The concept of a following guidance law with a certain upper period has been introduced for the problem of following a maneuvering target by an autonomous vehicle. We have proposed a simple and easily implementable following guidance law and derived its upper period. An interesting and challenging problem for future research is to design a following guidance strategy with the smallest possible upper period. It is expected that the importance of this and similar problems will be increasing with the growing use of autonomous unmanned vehicles in surveillance applications.

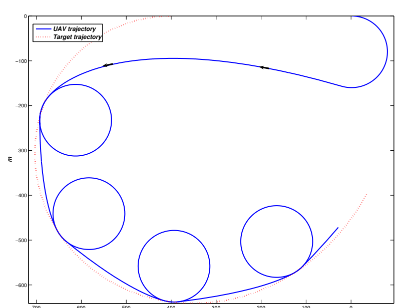

Fig. 8. Experiment 2

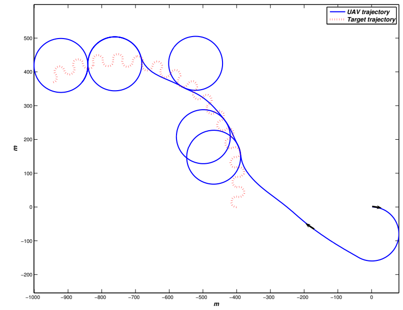

Fig. 9. Experiment 3

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