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A disturbance rejection anti-windup framework and its application to a substructured system

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Abstract— **In this paper, we consider the disturbance rejection problem of stable systems with input saturation based on the anti-windup (AW) framework developed by Weston and Postlethwaite (W&P) [17]. The performance is improved by explicitly incorporating a transfer function representing the effect of the disturbance on the nonlinear loop during the AW compensator synthesis. The suggested AW-design approach improves disturbance rejection performance over the design framework usually suggested for the coprime-factorization based W&P-approach. For this, an extra degree of freedom is exploited for the coprime factorization which usually results in an implicitly computed multivariable algebraic loop for the AW-implementation. Suitable suggestions are made to overcome the algrabraic loop through explicit computations.**

The approach is applied to the control of a dynamic substructured system (DSS) subject to a measurable excitation/disturbance signal and actuator limits. The benefit of this approach is demonstrated for a quasi-motorcycle DSS simulation.

I. Introduction

To improve the stability and performance of control systems subject to control input saturation, anti-windup (AW) control has been extensively studied, see, e.g., [8], [14], [17], [1], [7], [15] etc. The main feature of this strategy is that a two-step design procedure is involved in the controller synthesis: a nominal (linear) controller is first designed ignoring the input saturation; then a linear compensator is synthesized to cope with the "windup" problem. Among the existing AW approaches, the linear conditioning scheme proposed by Weston and Postlethwaite (W&P) [17] is comparatively easy to implement and its design objective is to recover the system's linear behavior quickly when input saturation occurs. Successful applications of the W&P scheme to aerospace and hard-disk drive problems have been reported (see, e.g., [5], [3]).

The current studies of the W&P scheme mainly focus on stability and recovery of linear control performance, mainly for tracking problems. However, disturbances are always present and can deteriorate the performance significantly. Hence, although it is not necessary to consider the disturbances when studying the system stability alone, the system performance with the presence of disturbances is nontrivial and not much attention has been paid to this topic with respect to the W&P-scheme. This paper aims to develop an AW framework for the disturbance rejection problem based on the W&P scheme. The performance is improved by explicitly incorporating a transfer function representing the effect of the disturbance on the nonlinear loop in the AW synthesis. Furthermore, a generic method is proposed to cope with the algebraic loop introduced into the framework.

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As an important application, we illustrate this AW-design framework for the problem of DSS, which are currently attracting worldwide interest in the field of real-time experimental tests [10]. The DSS approach allows, in a realistic way, the testing of an engineering component, or subsystem, of a complete system. For this reason, an interaction interface, i.e. actuators, introduces forces and torques on the part to be tested. These forces are usually the result of the interaction of the physical substructure with the remaining numerical substructure. In general, the overall dynamic system is subject to disturbances which are measurable as they are introduced numerically. The control objective of DSS is to synchronize the interaction interface between the numerical and physical substructures subject to the disturbance/testing signal. The existing DSS control strategies include Linear Substructuring Control (LSC) and Minimal Control Synthesis (MCS) [11], [12]. However, the nonlinearity from the actuator limits has not been considered in previous analytical work. Actuators usually used in DSS can have significant limits; it is therefore important to design AW compensators for DSS control. Furthermore, since the testing signal for the DSS in the controller design can be assumed to be a measured disturbance, the AW compensator for disturbance rejection developed in this paper is more suitable for DSS than the original W&P AW compensator. We use a simulation case study on a quasi-motorcycle suspension system, based at the University of Bristol, to show the advantage of the AW approach developed in this paper.

II. A generic framework for the disturbance rejection anti-windup compensator

A. W&P scheme

The AW scheme proposed by W&P is shown in Fig. 1, where the matrix E is usually the identity (e.g.[15]). w denotes an external (disturbance) signal. The transfer functions for the plant and controller are

$$\mathcal{P}(s) = \begin{bmatrix} \mathcal{P}_w(s) & \mathcal{P}_u(s) \end{bmatrix} \quad \mathcal{K}(s) = \begin{bmatrix} \mathcal{K}_w(s) & \mathcal{K}_y(s) \end{bmatrix} \quad (1)$$

Suppose the right coprime factorization of $\mathcal{P}_u(s)$ is $\mathcal{P}_u(s) = N(s)M^{-1}(s)$, so that

$$\begin{bmatrix} M(s) - I \\ N(s) \end{bmatrix} \sim \left[\begin{array}{c|c} A_p + B_{p,u}F & B_{p,u} \\ \hline F & 0 \\ C_p + D_{p,u}F & D_{p,u} \end{array} \right] \quad (2)$$

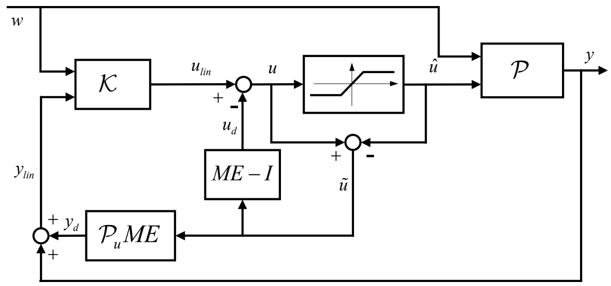

Fig. 1. Anti-windup with disturbance

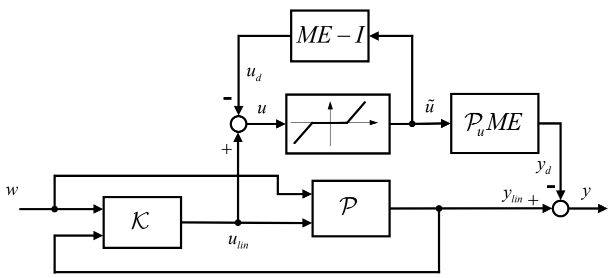

Fig. 2. Equivalent representation of Fig. 1

then the system conditioning is achieved by tuning F. From simple block diagram manipulation, Fig. 1 can be equivalently represented by Fig. 2 for the study of system stability and performance (with the assumption $M(\infty) = I$ and $E = I$ in Fig. 2).

This scheme has been extensively investigated, where the system conditioning is interpreted by minimizing the L_2 gain from $u_{\rm lin}$ to y_d, while the signals satisfy the saturation nonlinearity (or deadzone) condition and the value of an associated Lyapunov function is decreasing. This synthesis approach can be expressed as the following lemma:

Lemma 1 (W&P approach [17], [16], [15]): Given the AW framework as in Figs 1 and 2, with the right coprime factorization of $\mathcal{P}_u(s)$ as (2) and the assumption $M(\infty) = I$, then the L_2 gain from $u_{\rm lin}$ to y_d is less than γ_u if the following LMI is satisfied:

$$\begin{bmatrix} M_{11} & B_{p,u}U - L^T & 0 & QC_p^T + L^T D_p^T \\ \star & -2U & I_{n_u} & UD_p^T \\ \star & \star & -\gamma_u I_{n_u} & 0 \\ \star & \star & \star & -\gamma_u I_{n_y} \end{bmatrix} < 0 \tag{3}$$

with $M_{11} = A_pQ + QA_p^T + B_{p,u}L + L^T B_{p,u}^T$, $Q = Q^T > 0$, $L = FQ$, diagonal matrix $U > 0$ and scalar $\gamma_u > 0$. •

B. Design of AW-compensators for disturbance rejection

In this paper, we aim to reduce the influence of the external disturbance signal w. Hence the control objective is modified by minimizing the L_2 gain from the external signal w to y_d directly. To do this, an extra transfer function $\mathcal{P}_d$ from w to $u_{\rm lin}$ is included. The explicit inclusion of this transfer function is nontrivial, because it would complicate the LMI convexification procedure, although it would lead to better performance than the original method.

The transfer function from w to $u_{\rm lin}$ in Fig. 2 is derived as $\mathcal{P}_d(s) = (I - \mathcal{K}_y\mathcal{P}_u)^{-1}(\mathcal{K}_w + \mathcal{K}_y\mathcal{P}_w) \sim (A_d, B_d, C_d, D_d)$,

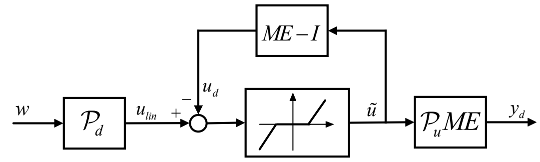

Fig. 3. W&P anti-windup framework for disturbance rejection

hence Fig. 2 can be simplified to Fig. 3 for the AW-compensator design.

The right coprime factorization of $\mathcal{P}_u(s)$ is $\mathcal{P}_u(s) = N(s)M^{-1}(s) = (NE)(ME)^{-1}$ with E invertible, so that

$$\begin{bmatrix} M(s)E - I \\ N(s)E \end{bmatrix} \sim \left[\begin{array}{c|c} A_p + B_{p,u}F & B_{p,u}E \\ \hline F & E - I \\ C_p + D_{p,u}F & D_{p,u}E \end{array}\right] \tag{4}$$

In Fig. 3, we suppose the states of $\mathcal{P}_d$ and $\mathcal{P}_u$ are $x_d \in \mathbb{R}^{n_d}$ and $x_p \in \mathbb{R}^{n_p}$; the dimensions of the signals and matrices are $u_{\rm lin}, u_d, \tilde{u} \in \mathbb{R}^{n_u}$, $w \in \mathbb{R}^{n_w}$, $y_d \in \mathbb{R}^{n_y}$, $E \in \mathbb{R}^{n_u \times n_u}$ and $F \in \mathbb{R}^{n_u \times n_p}$. Note that an extra variable E is introduced in the coprime factorization, which is key in the convexification procedure using the Projection Lemma in Theorem 1, as follows.

Given the AW framework as shown in Fig. 3, we have the following theorem for AW compensator synthesis:

Theorem 1: The L_2 gain from w to y_d is less than γ_d if there exists a symmetric positive definite matrix

$$P := \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \in \mathbb{R}^{n_p + n_d} \tag{5}$$

such that the following two LMIs are satisfied

$$\begin{bmatrix} PA_o + A_o^T P + PW_A + W_A^T P & W_C + PW_B \\ W_C^T + W_B^T P & W_D \end{bmatrix} < 0 \tag{6}$$

with

$$A_o = \begin{bmatrix} A_p & 0 \\ 0 & A_d \end{bmatrix} \qquad B_o = \begin{bmatrix} 0_{n_p \times n_u} & 0_{n_p \times n_w} \\ 0_{n_d \times n_u} & B_d \end{bmatrix}$$

$$C_{do} = \begin{bmatrix} 0_{n_u \times n_p} & C_d \\ 0_{n_w \times n_p} & 0_{n_w \times n_d} \end{bmatrix} \qquad D_{do} = \begin{bmatrix} 0_{n_u} & D_d \\ 0_{n_w \times n_u} & 0_{n_w} \end{bmatrix}$$

$$C_{po} = \begin{bmatrix} C_p & 0_{n_y \times n_d} \end{bmatrix} \qquad W_A = \begin{bmatrix} 0_{n_p} & B_{p,u}C_d \\ 0_{n_d \times n_p} & 0_{n_d} \end{bmatrix}$$

$$W_B = \begin{bmatrix} B_{p,u}D_d & 0_{n_p \times n_y} \\ B_d & 0_{n_d \times n_y} \end{bmatrix} \qquad W_C = \begin{bmatrix} 0_{n_p \times n_w} & C_p^T \\ 0_{n_d \times n_w} & C_d^T D_p^T \end{bmatrix}$$

$$W_D = \begin{bmatrix} -\gamma_d I_{n_w} & D_d^T D_{p,u}^T \\ D_{p,u}D_d & -\gamma_d I_{n_y} \end{bmatrix}$$

and

$$\begin{bmatrix} A_d^T P_{22} + P_{22}A_d & P_{22}B_d \\ B_d^T P_{22} & -\gamma_d I_{n_w} \end{bmatrix} < 0 \tag{7}$$

with $\gamma_d > 0$. ♢

Proof : The proof can be conducted using the Projection Lemma by following a similar approach to those given in, e.g. [1], [4]. ■

Remark 1: It has been shown that the framework of W&P falls into the more generic one of Grimm *et al.* [1], but the W&P framework is much less complicated to implement [4]. Theorem 1 in this paper, as an extension of the W&P

framework, can also be subsumed into the framework of Grimm *et al.* [1]; the emphasis here is the parameterization of the AW compensator via a coprime factorization approach, which is simpler and more tractable than [1]. ◦

Remark 2: It is possible to further reduce the conservatism by using other conditions on the saturation/deadzone, e.g. [9], [6], although the design complexity might be increased. ◦

Remark 3: We summarize the AW compensator construction procedure as follows:
(1) Given matrix variable $P = P^T > 0$, solve $\gamma_d^* := \min \gamma_d > 0$ subject to LMIs (6) and (7) to yield P^* and γ_d^*.
(2) Substituting P^* and γ_d^* with some chosen diagonal positive definite U, solve the LMI:

$$\Psi + H^T \Lambda G + G^T \Lambda^T H < 0 \tag{8}$$

for Λ, with $\Lambda := \begin{bmatrix} F & E \end{bmatrix}$ and

$$\Psi = \begin{bmatrix} A_o^T P + P A_o & P B_o + C_{do}^T \tilde{W} & C_{po}^T \\ B_o^T P + \tilde{W} C_{do} & \tilde{W} D_{do} + D_{do}^T \tilde{W} - \gamma_d \tilde{I}_{n_w} & 0 \\ C_{po} & 0 & -\gamma_d I_{n_y} \end{bmatrix}$$

$$H = \left[\begin{array}{cc|cc|c} B_{p,u}^T & 0_{n_u \times n_d} & -I_{n_u} & 0_{n_u \times n_w} & D_p^T \end{array}\right] \mathrm{diag}(P, \tilde{W}, I)$$

$$G = \left[\begin{array}{cc|cc|c} I_{n_p} & 0_{n_p \times n_d} & 0_{n_p \times n_u} & 0_{n_p \times n_w} & 0_{n_p \times n_y} \\ 0_{n_u \times n_p} & 0_{n_u \times n_d} & I_{n_u} & 0_{n_u \times n_w} & 0_{n_u \times n_y} \end{array}\right]$$

$$\tilde{W} = \begin{bmatrix} W & 0 \\ 0 & I_{n_w} \end{bmatrix}$$

◦

If the Lyapunov function candidate is chosen as

$$V = x_p^T P_1 x_p + x_d^T P_2 x_d \tag{9}$$

with $P_1 = P_1^T > 0$ and $P_2 = P_2^T > 0$, then a simpler version of Theorem 1 is derived as follows without proof:

Corollary 1: The L_2 gain from w to y_d is less than γ_d if the following LMI is satisfied

$$\begin{bmatrix} M_{11} & 0 & B_{p,u} E U - L^T & 0 & L^T D_p^T + Q_1 C_p^T \\ * & M_{22} & Q_2 C_d^T & B_d & 0 \\ * & * & -EU - UE^T & D_d & U E^T D_p^T \\ * & * & * & -\gamma_d I & 0 \\ * & * & * & * & -\gamma_d I_{n_y} \end{bmatrix} < 0 \tag{10}$$

with $Q_1 > 0$, $Q_2 > 0$ and

$$M_{11} = A_p Q_1 + Q_1 A_p^T + B_{p,u} L + L^T B_{p,u}^T$$
$$M_{22} = A_d Q_2 + Q_2 A_d^T$$

Here $U = W^{-1}$, $\gamma_d > 0$ and $L = F Q_1$. ♠

Remark 4: Compared with the approach developed in Theorem 1, Corollary 1 provides a much simpler approach for implementation and the algebraic loop can be resolved directly by setting $E = I$. This is possible since the gain for the LMI-optimization process in Corollary 1 is independent of E. However, the approach from Theorem 1 is less conservative than the one from Corollary 1, due to the different candidate Lyapunov functions employed. In the following section, we discuss how to cope with the algebraic loops when $E \neq I$. ◦

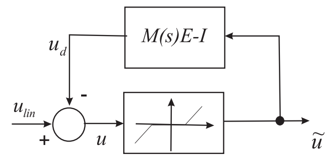

Fig. 4. Algebraic loop in anti-windup compensation

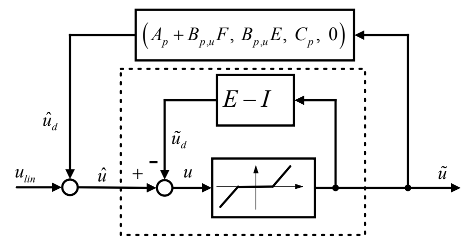

Fig. 5. Equivalent two-loop representation of the algebraic loop

III. RESOLVING ALGEBRAIC LOOPS

Although the scheme suggested in Theorem 1 in Section II-B can provide superior performance, one of the significant problems for implementation is the issue of algebraic loops, due to the matrix $E \neq I$. For AW-compensator implementation, it is necessary to explicitly compute the signals in the partial AW-structure of Fig. 4.

A proposal for resolving scalar algebraic loops has been given in [2]. It was shown that the scalar algebraic loop is easily solved explicitly, rather than through implicit numerical algorithms, assuming that a saturation nonlinearity limits the control signal. This idea can be extended to algebraic loops with multiple signals.

The algebraic loop of Fig. 4 can be decomposed as in Fig. 5 so that we obtain a purely static operator $\hat{u} \mapsto \tilde{u}$ containing the algebraic loop, while an outer loop contains the dynamics of the system which do not contribute to the algebraic loop problem. Hence, to resolve the algebraic loop problem, it is sufficient that the static operator $\hat{u} \mapsto \tilde{u}$ is analyzed. The following Lemmas will establish an approach to resolve the algebraic loop issue through explicit computations:

Lemma 2: Assume that the deadzone limits are given by $\bar{u} = [\bar{u}_1, \bar{u}_2, \cdots, \bar{u}_m]^T$, E is invertible and $\tilde{u}_i \neq 0$, $\forall\ i = 1, ..., m$, then

$$E^{-1}(\hat{u} - \mathrm{Sign}(u)\bar{u}) = \tilde{u} \tag{11}$$

with $\mathrm{Sign}(u) = \mathrm{diag}\left(\mathrm{sign}(u_1), \mathrm{sign}(u_2), \cdots, \mathrm{sign}(u_m)\right)$. •

Proof : From $\tilde{u}_i \neq 0$, $\tilde{u} + \mathrm{Sign}(u)\bar{u} = u$. Moreover, $u = \hat{u} - (E - I)\tilde{u}$. Hence, the assertion follows. ■

This implies, that for scalar algebraic loops, the loop can be explicitly solved [2]:

Corollary 2: Assuming the algebraic loop is scalar, i.e. $E = e$ and $\mathrm{sign}(\hat{u}) = \mathrm{sign}(u)$, then $\frac{\mathrm{dz}(\hat{u})}{e} = \tilde{u}$. ♠

Thus, the scalar algebraic loop has a very simple explicit solution.

For multi-variable algebraic loops, it would be desirable to exploit the relationship in (11). Hence, we wish to compute $\tilde{u}$ for a given $\hat{u}$. However, this is only possible with equation (11) if we have knowledge of $\mathrm{Sign}(u)$. The following Lemma establishes the necessary condition:

Lemma 3: Assume that the deadzone limits are given by $\bar{u} = [\bar{u}_1, \bar{u}_2, \cdots, \bar{u}_m]^T$, E is invertible and $\tilde{u}_i \neq 0$, $\forall\ i = 1, ..., m$, then $\mathrm{Sign}(E^{-1}\hat{u}) = \mathrm{Sign}(\tilde{u}) = \mathrm{Sign}(u)$ if for any diagonal matrix $\mathcal{D} = \mathrm{diag}\,(d_1, d_2, \cdots, d_m)$, satisfying $|d_i| = 1$, $d_i \in \mathbb{R}$ the following holds:

$$\mathrm{Sign}(E^{-1}\mathcal{D}\bar{u}) = \mathcal{D} \tag{12}$$

•

Proof : It has been established using (11) that $E^{-1}\hat{u} = \tilde{u} + E^{-1}\mathrm{Sign}(u)\bar{u}$. Moreover, $\mathrm{Sign}(u) = \mathrm{Sign}(\tilde{u})$. Hence, $E^{-1}\hat{u} = \tilde{u} + E^{-1}\mathrm{Sign}(\tilde{u})\bar{u}$, and $\mathrm{Sign}(E^{-1}\mathrm{Sign}(\tilde{u})\bar{u}) = \mathrm{Sign}(\tilde{u})$. Thus, the assertion follows. ■

Equation (12) is satisfied if and only if the matrix $(E^{-1}\mathrm{diag}(\bar{u}))$ is strictly diagonally dominant. Hence, the following fact can be used:

Corollary 3: Assume that the conditions of Lemma 2 hold and $(E^{-1}\mathrm{diag}(\bar{u}))$ is strictly diagonally dominant, then $E^{-1}(\hat{u} - \mathrm{Sign}(E^{-1}\hat{u})\bar{u}) = \tilde{u}$. ♠

Hence, Corollary 3 is necessary for resolving an algebraic loop and is a generalization of Corollary 2 of [2].

Note that the LMIs of (10) and (8) guarantee that the algebraic loop has a solution which is unique [16]. Hence, it is now possible to resolve an algebraic loop for a strictly diagonally dominant matrix $(E^{-1}\mathrm{diag}(\bar{u}))$. As an example, an algebraic loop with two constrained signals, $m = 2$, shall suffice:

$$E = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix}$$

The following steps are to be taken for given $\hat{u} = [\hat{u}_1\ \hat{u}_2]^T$:

1) Compute $\tilde{u} = E^{-1}(\hat{u} - \mathrm{Sign}(E^{-1}\hat{u})\bar{u})$, $\tilde{u} = [\tilde{u}_1\ \tilde{u}_2]^T$. If $\tilde{u}_1 \neq 0$, $\tilde{u}_2 \neq 0$ and $\mathrm{Sign}(E^{-1}\hat{u}) = \mathrm{Sign}(\tilde{u})$, then a solution of the algebraic loop is found.
   Otherwise, go to step 2):
2) Assume $\tilde{u}_1 = 0$. This is satisfied if $\mathrm{dz}_1(\hat{u}_1 - e_{12}\tilde{u}_2) = 0$ for $\tilde{u}_2 = \frac{\mathrm{dz}_2(\hat{u}_2)}{e_{22}}$.
   Otherwise, go to step 3):
3) Assume $\tilde{u}_2 = 0$. This is satisfied if $\mathrm{dz}_2(\hat{u}_2 - e_{21}\tilde{u}_1) = 0$ for $\tilde{u}_1 = \frac{\mathrm{dz}_1(\hat{u}_1)}{e_{11}}$.

This procedure will guarantee the solution of the algebraic loop and can be easily extended to $m > 2$.

IV. SUBSTRUCTURING - AN APPROACH FOR REAL-TIME TESTING OF ELECTRO-MECHANICAL COMPONENTS

The principal idea of substructuring is to test the complex critical subcomponents of a large engineering system (represented as a physical substructure) in real-time while the remainder of the engineering system is simultaneously represented as a numerical model (called the numerical substructure). Hence, a DSS consists of two components,

- a physical subcomponent which has to be tested practically with respect to mechanical reliability, stresses etc, together with actuators, which exert the necessary forces or torques on the physical test specimen, called the transfer system.
- a numerical simulation of the forces/torques and the dynamics of the remaining parts of the system.

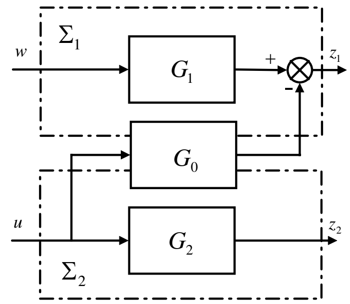

Fig. 6. The substructured system [12]

The substructuring approach can be more advantageous than traditional testing methods such as full-size testing of the entire system, scale-model testing, pseudo-dynamic testing and purely numerical testing [18]. An important issue of the substructuring method is the synchronization of the physical and numerical substructures, which significantly affects the testing accuracy of the entire system. This demands a high fidelity of control to reduce the error of the interface between the two substructures. However, dynamical interaction between the two substructures, together with the dynamics of the transfer system, will normally cause problems with synchronization. Successful control strategies that specifically cope with these problems include LSC and MCS [11], [12]. However, the actuator limits in the transfer system have not been explicitly part of the dynamic synthesis procedure so far. From this point of view, it is novel to use AW compensation to improve the interaction interface between the numerical and physical substructures, where the excitation signal (testing signal) is assumed a measured disturbance.

A general DSS can be expressed by [12] (see Fig. 6)

$$z_1 = G_1 w - G_0 u \tag{13}$$

$$z_2 = G_2 u \tag{14}$$

where G_1 and G_2 represent the dynamics of the numerical and physical substructures, and G_0 the interaction dynamics between the two substructures. We use the generalized set $\{\Sigma_1, \Sigma_2\}$ to represent the numerical and physical substructures $\{\Sigma_N, \Sigma_P\}$ respectively, or conversely $\{\Sigma_P, \Sigma_N\}$. The control objective is to use a synchronizing control signal u to make the output z_2 of Σ_2 track the output z_1 of Σ_1, subject to the external excitation signal w. The smaller the tracking error $e = z_1 - z_2$, the closer the DSS to the real system. For more details about the substructuring, see [12] and the references therein.

V. SUBSTRUCTURING FOR A MOTORCYCLE SUSPENSION

We consider a quasi-motorcycle suspension system currently under investigation at the University of Bristol in the UK. In this case study, we separate the system into the following parts: the quasi-motorcycle body with two suspension struts, and the front and rear wheels/tires modeled numerically, as shown in Fig. 7. We call this a *single mode* substructure. We can also model one wheel/tire numerically and the other physically, or two wheels/tires physically and

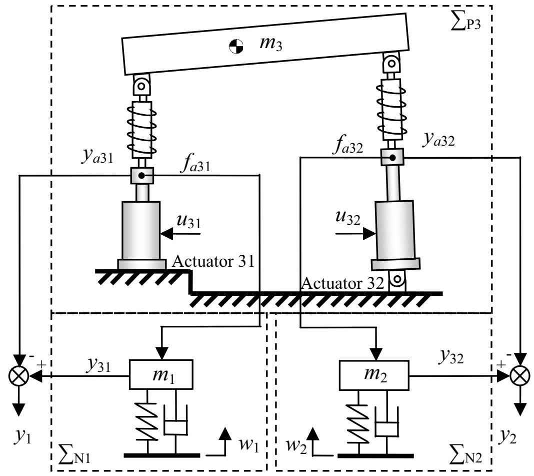

Fig. 7. Substructuring for a motor cycle suspension

the body with two suspension struts numerically, depending on the problems that we are interested in. The control objective is to synchronize the physical and numerical substructures by minimizing the displacement errors $\{y_1, y_2\}$ between the front/rear suspension struts $\{y_{a31}, y_{a32}\}$ and front/rear wheel hubs $\{y_{31}, y_{32}\}$, subject to external testing signals $\{d_1, d_2\}$ (which can be viewed as road disturbances). The model for this system can be established and represented in the standard DSS framework, so that G_1 only contains the numerical substructure parameters, G_2 the substructure parameters of the physical components, i.e. the quasi-motor cycle. The interaction transfer function G_0 contain elements of both the numerical and physical substructures. See [13] for the details of the model establishment, the parameter values, as well as the LSC and MSC control designs. In this paper, in contrast to [13] and other substructuring controller designs, we take into account the actuator limits on $\{u_{31}, u_{32}\}$ using the AW compensators. We first design an LQG controller for the system ignoring the actuator limits, then design AW compensators respectively based on Theorem 1, Corollary 1 and the original W&P approach (Lemma 1).

A. LQG controller

Suppose that the transfer functions $G_0(s)$, $G_1(s)$ and $G_2(s)$ are strictly proper and their state space matrices are $G_i(s) \sim (A_i, B_i, C_i, 0)$ with $i = 0, 1, 2$, then the state space realization for the whole system can be written as

$$\dot{x} = Ax + B_u u + B_w w \tag{15a}$$
$$y = Cx \tag{15b}$$

with $x = \begin{bmatrix} x_0^T & x_1^T & x_2^T \end{bmatrix}^T \in \mathbb{R}^{n_x}$, $y \in \mathbb{R}^{n_u}$ and

$$A = \begin{bmatrix} A_0 & 0 & 0 \\ 0 & A_1 & 0 \\ 0 & 0 & A_2 \end{bmatrix} \quad B_u = \begin{bmatrix} B_0 \\ 0 \\ B_2 \end{bmatrix} \quad B_w = \begin{bmatrix} 0 \\ B_1 \\ 0 \end{bmatrix}$$
$$C = \begin{bmatrix} -C_0 & C_1 & -C_2 \end{bmatrix}$$

The equations for a linear observer are

$$\dot{\hat{x}} = A\hat{x} + B_u u + B_w w + L(y - \hat{y}) \tag{16a}$$
$$\hat{y} = C\hat{x} \tag{16b}$$

Suppose the feedback gain K is computed from the algebraic Ricatti equation so that

$$u = -K\hat{x} \tag{17}$$

Substituting (17) and (16b) into (16a) leads to the LQG controller-observer equations:

$$\dot{\hat{x}} = (A - LC - B_u K)\hat{x} + B_w w + Ly \tag{18a}$$
$$u = -K\hat{x} \tag{18b}$$

Therefore,

$$A_c = A - LC - B_u K \qquad B_{c,w} = B_w \qquad B_{c,y} = L$$
$$C_c = -K \qquad D_{c,w} = 0 \qquad D_{c,y} = 0$$

B. Simulation results

The substructured quasi-motorcycle suspension system can be represented by the standard framework [13] as shown in Fig. 6 where

$$G_0 = \begin{bmatrix} G_{0(1,1)} & G_{0(1,2)} \\ G_{0(2,1)} & G_{0(2,2)} \end{bmatrix} \quad G_1 = \begin{bmatrix} G_{1(1,1)} & 0 \\ 0 & G_{1(2,2)} \end{bmatrix}$$
$$G_2 = \begin{bmatrix} \frac{8.3}{s+8.3} & 0 \\ 0 & \frac{8.3}{s+8.3} \end{bmatrix}$$

with

$$G_{0(1,1)} = G_{0(2,2)}$$
$$= \frac{413.4s^3 + 2953s^2}{s^5 + 52.57s^4 + 1358s^3 + 1.778e4s^2 + 1.26e5s + 3.873e5}$$
$$G_{0(1,2)} = G_{0(2,1)}$$
$$= \frac{206.3s^3 + 1474s^2}{s^5 + 52.57s^4 + 1358s^3 + 1.778e4s^2 + 1.26e5s + 3.873e5}$$
$$G_{1(1,1)} = G_{1(2,2)} = \frac{30.27s + 466.7}{s^2 + 30.27s + 466.7}$$

The weights of the Kalman filter when designing the observer are chosen as $Q_n = 10^5 I_{n_y}$ and $R_n = I_{n_u}$; The weights for the algebraic Ricatti equation are $Q = 5 \times 10^3 \times C_p^T C_p$ and $R = I_{n_u}$. We use a pulse signal with amplitude 0.01m, period 2s and pulse width 0.2s as the testing signal. The limits for both actuators are $[-0.012, 0.012]$m.

Based on the LQR controller, we make a comparison of four cases: (a) without AW compensator; (b) with AW compensator – minimizing the L_2 gain from u_{lin} to y_d (Lemma 1); (c) with AW compensator – minimizing the L_2 gain from w to y_d (Corollary 1); and (d) with AW compensator – minimizing the L_2 gain from w to y_d (Theorem 1);

For case (b), the L_2 gain from u_{lin} to y_d is $\gamma_u = 1.3903$.

For case (c), if set $E = I_{n_u}$, then the L_2 gain from w to y_d is $\gamma_d = 4.2565$.

For case (d), the L_2 gain from w to y_d and the variable E are

$$\gamma_d = 1.2761 \qquad E = \begin{bmatrix} 4.5424 & -0.4778 \\ -0.4779 & 4.5426 \end{bmatrix}$$

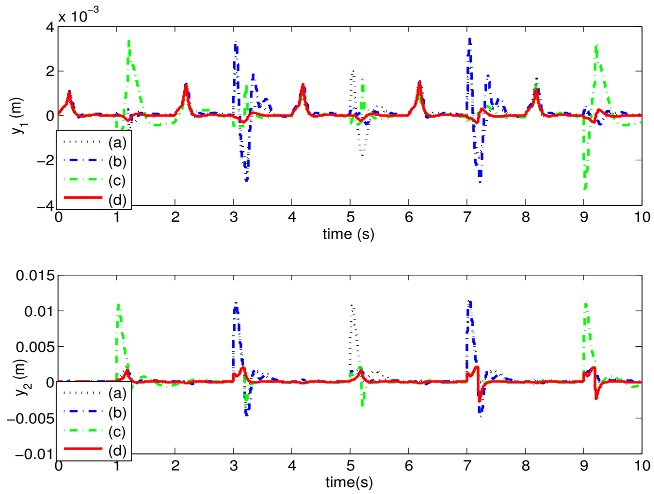

Fig. 8. Comparison of the DSS outputs of the four cases (a)∼(d).

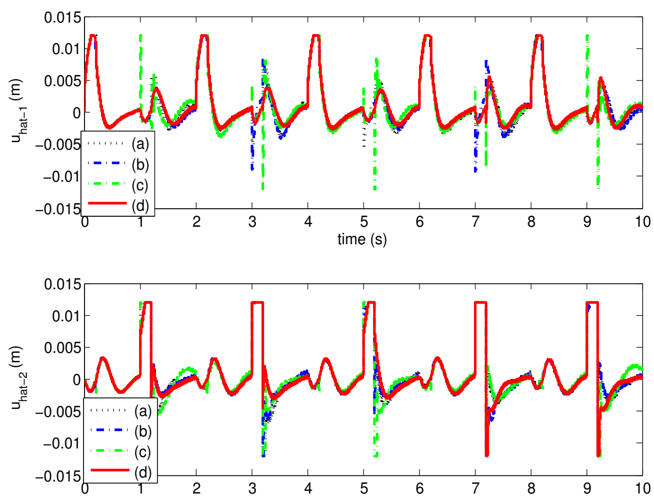

Fig. 9. Comparison of the DSS control signals of the four cases (a)∼(d).

Here $E^{-1}\,\mathrm{diag}(0.012, 0.012)$ is strictly diagonally dominant, hence the algebraic loop can be resolved using the approach in Section III.

From the results, we note that the L_2 gain γ_d is greatly reduced when using the approach based on Theorem 1, compared with the one based on Corollary 1. Fig. 8 shows the two outputs, i.e., the interaction interface errors of the DSS for 4 cases. We can see that the performance in case (d) is better than other three cases, while the performance of case (b) and (c) are not better than (a). This shows that the original W&P approach is not suitable for the disturbance rejection problem in this example and the approach based on Theorem 1 is much less conservative than the one based on Corollary 1. Fig. 9 shows the control inputs of the plant in the four cases, we can see that the control input magnitude of case (d) is less than the ones of other three cases.

VI. Conclusion

We have developed an approach to improve system performance for disturbance rejection problems based on the W&P scheme. The novel feature of this new approach is that a transfer function representing the effect of the disturbance on the nonlinear loop is considered in the compensator synthesis. This approach is applied to a DSS problem to cope with the actuator limits, which has not been taken into account in previous analytical work. The benefit of using this approach is shown in a simulation example.

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