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Application of Wiener-Hammerstein System Identification in Electrically Stimulated Paralyzed Skeletal Muscle Modeling

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Abstract— **Electrical muscle stimulation has demonstrated potential for restoring functional movement and for preventing muscle atrophy after spinal cord injury (SCI). Control systems used to optimize delivery of electrical stimulation protocols depend upon algorithms generated using computational models of paralyzed muscle force output. The existing skeletal muscle models are either not accurate or too complicated to implement for real-time control. In this paper, we propose a Wiener-Hammerstein system, Linear-Saturation-Linear (LSL) model, to model the skeletal muscle dynamics under electrical stimulus conditions. Experimental data from the soleus muscles of an individual with SCI was used to quantify the performance of the model. We demonstrate that the proposed Wiener-Hammerstein system is comparable to, in terms of model fitting, and outperforms, in terms of prediction, the Hill Huxley model, the most advanced and accurate model previously reported. On the other hand, the proposed LSL model is much simpler in terms of the structure and involves a much smaller number of unknown coefficients. This has substantial advantages in identification algorithm analysis and implementation including computational complexity, convergence and also in real time model implementation for control purposes.**

I. INTRODUCTION

After spinal cord injury (SCI) the loss of volitional muscle activity triggers a range of deleterious adaptations. Muscle cross-sectional area declines by as much as 45% in the first six weeks after injury, with further additional atrophy occurring for at least six months [5], Muscle atrophy impairs weight distribution over bony prominences, predisposing individuals with SCI to pressure ulcers, a potentially life-threatening secondary complication [16]. The diminution of muscular loading through the skeleton precipitates severe osteoporosis in paralyzed limbs. The lifetime fracture risk for individuals with SCI is twice the risk experienced by the non-SCI population [21]. Rehabilitation interventions to prevent post-SCI muscle atrophy and its sequelae are an urgent need. Electrical muscle stimulation after SCI is an effective method to induce muscle hypertrophy [15], [11], fiber type and metabolic enzyme adaptations [8], [1], and improvements in torque output and fatigue resistance [18], [20]. New evidence suggests that an appropriate longitudinal dose of muscular load can be an effective anti-osteoporosis countermeasure [18], [17], [14]. Electrical muscle stimulation also has potential utility for restoration of function in tasks such as standing, reaching, and ambulating. The myriad applications for electrical stimulation after SCI have created a demand for control systems that adjust stimulus parameters in real-time to accommodate muscle output changes (potentiation, fatigue) or inter-individual force production differences. To facilitate the refinement of control system algorithms, mathematical models of muscle torque output are continuously being developed. To most successfully adapt stimulus parameters to real-time muscle output changes, an accurate and easy-to-implement model is essential.

Over the last decades, researchers have developed a number of muscle models aimed at predicting muscle force output [9], [10], [3]. The Hill Huxley model [10] is the most advanced and accurate model put forward to date [4]. Compared to other models, the Hill Huxley model represents muscle dynamics well. However, its complexity undermines its usefulness for real time implementation for control. Identification of a Hill Huxley model is non-trivial because it is time-varying, high dimensional and nonlinear. Local minimum versus global minimum is always a difficult issue for identification, and the user must tune identification algorithm parameters patiently (including the initial estimate) in order to have a good result.

Our goal has been to develop a model that is comparable to or outperforms the Hill Huxley model, but at a reduced complexity. We propose to use a Wiener-Hammerstein system that resembles the Hill Huxley structure but has the added advantage of greater simplicity. This approach was previously suggested by Hunt and colleagues [13] but was deemed inadequate for muscle modeling. By examining the experimental data sets and the Hammerstein system, we noted two problems. First, a linear block prior to the nonlinear block was missing and secondly, the static nonlinearity seemed suboptimal. The proposed Wiener-Hammerstein model overcomes these two deficiencies, enjoys a high degree of accuracy, and is comparable to or outperforms the Hill Huxley model. Most importantly, the proposed model is much simpler not only in the structure but also in the number of unknown coefficients. The purpose of this report is to describe a Wiener-Hammerstein system for modeling paralyzed skeletal muscle dynamics under electrical stimulation. By using actual soleus force data from a subject with SCI, we demonstrate that this model's advantages over previous models are theoretically justified and numerically verified. Equally important is a demonstration of the usefulness of block oriented nonlinear systems together with their identification algorithms. Identification of block oriented nonlinear systems including Wiener-Hammerstein systems has been extensively investigated recently in the control and

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identification literature and has become one of the most active research areas in identification [23], [22], [2], [24], [6]. However, almost all the works reported in the control and identification literature have been of a theoretical nature, e.g., identifiability, identification algorithms, convergence analysis and different types of static nonlinearities in the system. In contrast, we demonstrate an application of the block oriented nonlinear system in electrically stimulated paralyzed skeletal muscle modeling.

II. MODELING

A. Hill Huxley model

Among the several muscle models developed in recent decades [9], [10], [3], the Hill Huxley model [10] is the most advanced and accurate [4], [12]. The Hill Huxley nonlinear model describes the stimulated muscle behavior in the continuous time domain by means of two differential equations (1) and (2)

$$\frac{dC_N}{dt} = \frac{1}{\tau_c}\sum_{i=1}^{n} R_i \exp(-\frac{t-t_i}{\tau_c}) - \frac{C_N}{\tau_c}, \tag{1}$$

where $R_i = 1 + (R_0 - 1)\exp(-\frac{t_i - t_{i-1}}{\tau_c})$

$$\frac{dy}{dt} = A\frac{C_N}{K_m + C_N} - \frac{y}{\tau_1 + \tau_2 \frac{C_N}{K_m + C_N}}. \tag{2}$$

In (1) and (2), t_i is the time of *i*th stimulation input and C_N is the (internal) state variable, while $y(t)$ is the force output. Note no actual input amplitude is directly used but only the input time sequence t_i is used. The effect of the input amplitude is automatically adjusted by the parameters R_i and τ_c. The model incorporates six parameters A, R_0, and K_m as gains, and τ_1, τ_2 , τ_c as the time constants as well as a sequence of coefficients t_i's that describe the exact time and the interval of electrical pulse inputs.

B. Wiener-Hammerstein Model

The proposed Wiener-Hammerstein system is shown in Fig. 1(a). It is composed of two dynamic linear systems and a nonlinear block $w = f(v)$ denoted by $\frac{Av}{1+Bv}$, where B and A are unknown parameters which vary for each individual patient. Note that w will be "saturated" at A/B for positive v, therefore, we call it LSL (Linear-Saturation-Linear) model. The system is in the discrete time domain. The input $u(k) = u(kT)$ is the electrical stimulus (in volts) at time kT where $T = 1$ ms is the sampling interval and the output $y(k) = y(kT)$ is the muscle force at time kT. The internal signals are unavailable. The linear blocks prior and after the nonlinearity are first order dynamic systems. In theory, these linear systems can be replaced by high order linear systems to provide more flexibility. However, test results demonstrate that first order linear systems are sufficient. The LSL model resembles the structure of the Hill Huxley model but at a much reduced complexity.

Note that the parameters A and B in the nonlinear block are to be adjusted individually and are a necessary part of the system. The overall system in Fig. 1(a) is however unnecessarily complicated and in fact is ill-defined from an identification point of view. One of the most important properties in identification is the identifiability, i.e., whether there exists a different system that can generate the same input-output data. In the setting of Fig. 1(a) , the question is whether there exists two sets of parameters, a_0, a_1, b_0, b_1, A and B that could produce the same input-output data. To this end, it is easily verified that the system in Fig. 1(a) is not identifiable. Observe that the nonlinear block in Fig. 1(a) can be decomposed into three blocks, two constant gains and a known nonlinearity, as shown in Fig. 1(b). Further, the gains can be absorbed by the linear systems, which results in the following system in Fig. 1(c), where $a_2 = a_0 B$ and $b_2 = b_0\frac{A}{B}$. This normalization process not only makes the system identifiable but also greatly simplifies the identification problem, reducing the number of unknown parameters from six to four. Also, no additional sequence of t_i's is needed, which is not the case for the Hill Huxley model. It is important to comment that the system in Fig. 1(c) is identical to the system in Fig. 1(a) from input to output point of view, though the complexity is greatly reduced.

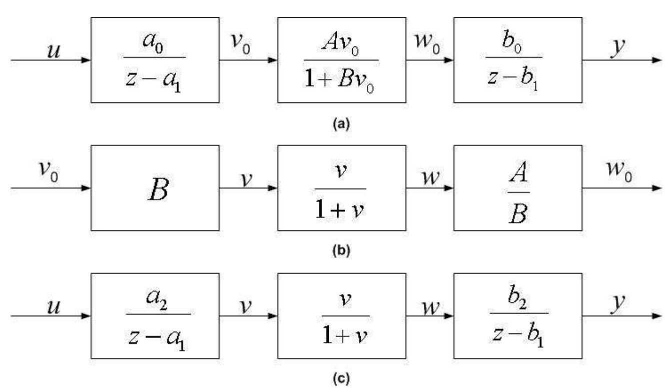

Fig. 1. (a): Wiener-Hammerstein muscle (LSL) model. (b): The middle nonlinear block of (a) can be decomposed into three parts. (c): The simplified Wiener-Hammerstein muscle (LSL) model.

III. IDENTIFICATION OF THE WIENER-HAMMERSTEIN MODEL

The proposed LSL Model appears in Fig. 1(c). Let $\theta = [a_1, a_2, b_1, b_2]$ denote the unknown system parameters. The purpose of identification is to determine their estimates $\hat{\theta} = [\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2]$ based on the available input-output measurements $u(k)$ and $y(k)$. Let $\hat{y}(k)$ be the predicted output calculated using the estimates

$$\hat{y}(k) = \frac{\hat{b}_2}{z - \hat{b}_1} f(\frac{\hat{a}_2}{z - \hat{a}_1} u(k)), \tag{3}$$

where z is the z-transform, i.e., $z^{-1}y(k) = y(k-1)$, and f is the known nonlinearity in the middle block of Fig. 3,

$$f(x) = \frac{x}{1+x}. \tag{4}$$

The identification problem is to find the best parameter set θ^* which minimizes the sum of squared errors between the actual output $y(k)$ and the predicted output $\hat{y}(k)$ of the proposed model

$$\theta^* = \arg\min_{\hat{\theta}}\{\sum_k (y(k) - \hat{y}(k,\hat{\theta}))^2\}. \quad (5)$$

Obviously, (5) is a nonlinear optimization problem because of the involvement of the nonlinear function. As for any nonlinear optimization problem, we face the issue of a local minimum versus the global minimum. We resolve this issue by the following approach.

If a nonlinear minimization algorithm starts at an initial estimate θ_0 that is very close to the optimal θ^*, then any reasonable nonlinear minimization algorithm should converge to the optimal θ^*. The question is how to find a good initial estimate. Suppose now the values of $\hat{a}_1$ and $\hat{a}_2$ are given, the internal signal $\hat{w}(k) = f(\frac{\hat{a}_2}{z-\hat{a}_1}u(k))$,$k = 1, 2, \cdots, N$, can be calculated. Based on this internal signal and the model $\hat{y}(k) = \frac{\hat{b}_2}{z-\hat{b}_1}\hat{w}(k)$, the values of $\hat{b}_1$ and $\hat{b}_2$ can be determined accordingly by the least squares method and the actual output $y(k)$,

$$[\hat{b}_1, \hat{b}_2]^T = (R^T R)^{-1} R^T Y, \quad (6)$$

where

$$Y = \begin{pmatrix} y(2) \\ y(3) \\ \vdots \\ y(N) \end{pmatrix} \text{ and } R = \begin{pmatrix} y(1) & \hat{w}(1) \\ y(2) & \hat{w}(2) \\ \vdots & \vdots \\ y(N-1) & \hat{w}(N-1) \end{pmatrix},$$

provided that the matrix R is full rank which is indeed the case for actual patient data sets. Therefore, $[\hat{b}_1, \hat{b}_2]$ is no longer independent but a deterministic function of $\hat{a}_1$ and $\hat{a}_2$

$$[\hat{b}_1, \hat{b}_2]^T = h(\hat{a}_1, \hat{a}_2), \quad (7)$$

and the minimization problem (5) of four parameters becomes the minimization problem of two parameters

$$\min J(a_1, a_2, h(a_1, a_2)). \quad (8)$$

We make three observations in regard to this approach: First, the minimization problem (5) is reduced to the minimization problem (8) that is still nonlinear. However, it is two-dimensional and the cost function J versus $\hat{a}_1$ and $\hat{a}_2$ can now be easily plotted and visualized. Second, physiological constraints require that $a_1, a_2 \geq 0$ and $|a_1| < 1$ because the force is always non-negative and bounded. Third, as illustrated by Fig. 2, electrical stimulation is not an arbitrary input. Although the input frequency can be altered, the pattern of pulses is unvarying. Under such an input and the corresponding actual muscle force output, the cost function of (8) versus a_1, a_2 is shown in Fig. 3 for input frequencies at 10 Hz and 20 Hz respectively. It can be seen clearly from Fig. 3 that the cost function $J(a_1, a_2, h(a_1, a_2))$ is not necessarily convex but has one and only one local (global) minimum. This makes optimization (8) very efficient and always convergent.

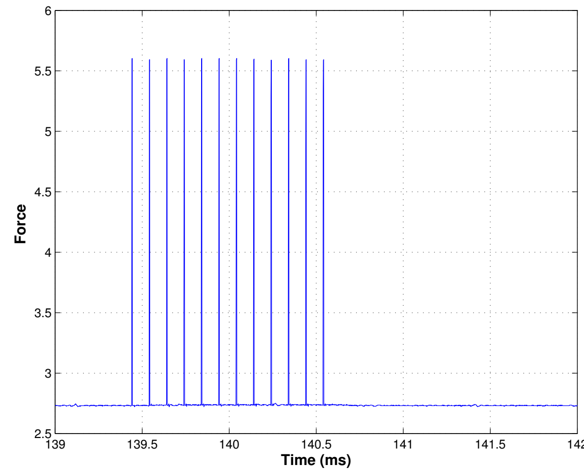

Fig. 2. Electrical stimulation input at frequency 10 Hz.

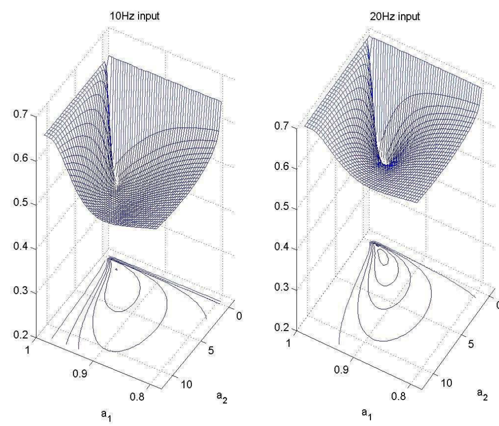

Fig. 3. The cost function $J(a_1, a_2, h(a_1, a_2))$ versus a_1 and a_2 for inputs at frequencies 10Hz and 20Hz respectively

Let the solution of (8) be $\hat{a}_{10}$, $\hat{a}_{20}$, $\hat{b}_{10}$ and $\hat{b}_{20}$. It seems as if the minimization problem (5) has been solved by (8). By a closer look, however, we notice that the solution $\hat{b}_{10}$ and $\hat{b}_{20}$ of (8) are the solution of the equation error type and the original cost function (5) is of output error type $y(k) = \hat{b}_1 y(k-1) + \hat{b}_2 \hat{w}(k-1)$. Thus $\hat{a}_{10}$, $\hat{a}_{20}$, $\hat{b}_{10}$ and $\hat{b}_{20}$ of (8) is not exactly a solution of (5) but it does provide a very good initial estimate for the four-dimensional minimization problem (5). The next step is to find the optimal parameter θ^* set using (5) and the obtained initial estimates $[\hat{a}_{10}, \hat{a}_{20}, \hat{b}_{10}, \hat{b}_{20}]$ from (8). To this end, many nonlinear optimization packages can be used. In our work, the program "fminsearch", which is Nelder-Meade simplex approach based and embedded in MATLAB (Ver 7.3.0.267 (R2006b)), is used to solve the nonlinear optimization problems (5) and (8). The main reason to apply such an algorithm is that the algorithm is very simple and is embedded in MATLAB, a widely available numerical package. A large number of numerical simulations verify that the global minimum is always achieved by this approach independent of the initial values in the two-dimensional optimization (8) which provides a good initial estimate of the four-dimensional problem (5).

IV. Experimental Results

Data collection and performance comparisons are made against the Hill Huxley nonlinear model, the most accurate model available in the literature. We identified both the LSL model and the Hill Huxley model using actual soleus force data from the individual with SCI. The LSL model is identified based on the algorithm presented above. They all converge to the global minimum quickly. For identification of the Hill Huxley nonlinear model, much care has to be taken to avoid a local minimum. To this end, the estimates are first manually tuned until the model output has a relatively good fit to the actual force output. Then the resultant parameters are used as initial values for "fminsearch", which refines the fitness further. The output of "fminsearch" is further perturbed to generate a cluster of initial estimates which are fed into "fminsearch" again. The best solution is considered as the Hill Huxley model. For comparison, standard goodness-of-fit (gof) and the normalized mean absolute error are used. Goodness-of-fit is calculated as

$$\text{gof} = 1 - \sqrt{\frac{\sum_{k=1}^{N}(y(k) - \hat{y}(k))^2}{\sum_{k=1}^{N}(y(k) - \bar{y})^2}}, \tag{9}$$

where N is the total number of data length, and $\hat{y}(k)$ and $\bar{y}$ are the predicted force output by the model and the mean of the actual force output, respectively. The normalized mean absolute error is the mean of absolute difference between the predicted and actual force outputs, divided by the maximum actual force.

$$\text{nmae} = \frac{\frac{1}{N}\sum_{k=1}^{N}|y(k) - \hat{y}(k))|}{max\{y(k)\}}. \tag{10}$$

A. Collection of SCI patient data

One male subject with chronic SCI ($>$ 2 years) provided written informed consent, as approved by the University of Iowa Human Subjects Institutional Review Board. A detailed description of the stimulation and force transducing system has been previously reported [18], [19], [20] (Fig. 4). In brief, the subject sat in a wheelchair with the knee and ankle positioned at ninety degrees. The foot rested upon a rigid metal plate, and the ankle was secured with a soft cuff and turnbuckle connectors. Padded straps over the knee and forefoot ensured isometric conditions. The tibial nerve was supramaximally stimulated in the popliteal fossa using a nerve probe and a custom computer-controlled constant-current stimulator. Stimulation was controlled by digital pulses from a data-acquisition board (Metrabyte DAS 16F, Keithley Instruments Inc., Cleveland, OH) housed in a microcomputer under custom software control. Ten 15 Hz stimulus trains were given to maximally potentiate the soleus muscle. Next, 10-pulse stimulus trains (2 ms pulse duration) were delivered at either 10 Hz or 20 Hz. The ensuing soleus isometric plantar flexion torques were measured via a load cell (Genisco AWU-250) positioned under the first metatarsal head. Torque was amplified 500 times (FPM 67, Therapeutics Unlimited) and input to a 12-bit resolution analog-to-digital converter at a sampling rate of 1000 samples per second. The digitized signals were analyzed with Datapac 2K2 software (RUN Technologies, Mission Viejo, CA).

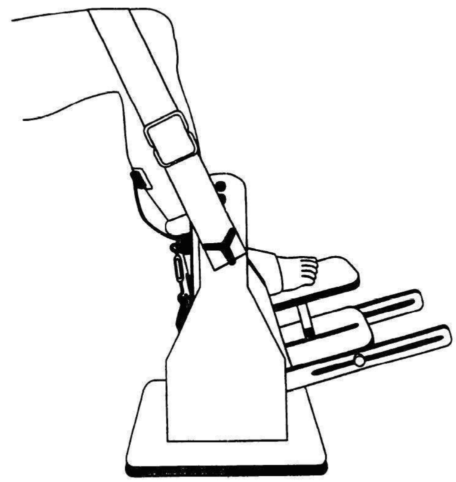

Fig. 4. Schematic representation of the limb fixation and force measurement system.

B. Fitting performance

Fig. 5 shows the outputs of the LSL model and the Hill Huxley model under 10 Hz and 20 Hz electrical stimulations. The corresponding goodness-of-fit and the mean absolute error are shown in Table I. Both the LSL model and the Hill Huxley model provide excellent results with the goodness-of-fit above 0.9000. The Hill Huxley model has a better performance than the LSL model for 10 Hz data (0.9361 vs. 0.9175); while the LSL model performs a little better than the Hill Huxley model for 20 Hz data (0.9100 vs. 0.9052).

TABLE I

Goodness-of-fit (gof) and normalized mean absolute error (nmae) of the proposed LSL model and the Hill Huxley model, respectively

Input Frequency	LSL model		Hill Huxley model	
	gof	nmae	gof	nmae
10 Hz	0.9175	2.21%	0.9361	1.67%
20 Hz	0.9100	2.03%	0.9052	2.37%

C. Prediction performance

Since the purpose of the model is to predict the force for different input stimuli, we compare the prediction performance of the two models under four cases: (a) using

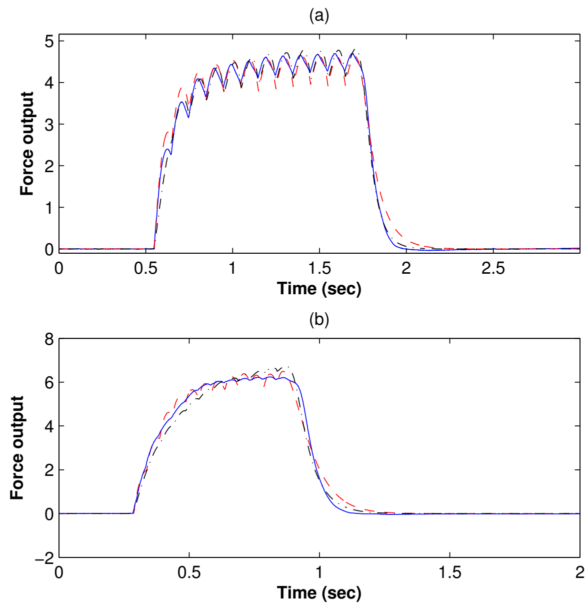

Fig. 5. The force outputs of the Wiener-Hammerstein (LSL) model (red), the Hill Huxley model (black), and the actual force output (blue) under (a) 10 Hz and (b) 20 Hz electrical stimulation.

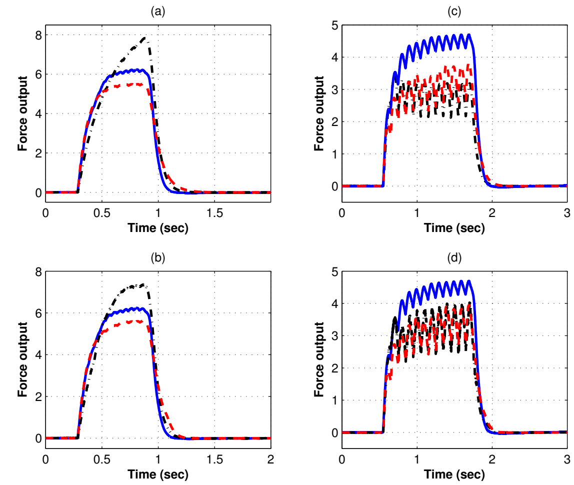

Fig. 6. Comparison of the prediction results of the LSL model (red dashed) and the Hill Huxley model output (black dash dot). The blue solid curves denote the actual force output. (a) using the models identified from 10 Hz data sets to predict the force output at 20 Hz input, (b) using the models identified from 10 Hz data sets to predict the force output at 20 Hz input assuming the force output is available after 150 steps (0.15 sec), (c) using the models identified from 20 Hz data sets to predict the force output at 10 Hz input, and (d) using the models identified from 20 Hz data sets to predict the force output at 10 Hz input assuming the force output is available after 150 steps (0.15 sec).

the models identified from 10 Hz data sets to predict the force output at 20 Hz input, (b) using the models identified from 10 Hz data sets to predict the force output at 20 Hz input assuming the actual force output is available after 150 step delay (0.15 sec, note that the sampling interval is 0.001 sec), (c) using the models identified from 20 Hz data sets to predict the force output at 10 Hz input, and (d) using the models identified from 20 Hz data sets to predict the force output at 10 Hz input assuming the actual force output is available after 150 step delay (0.15 sec, again note that the sampling interval is 0.001 sec)). The results are shown in Fig. 6 and Table II. It can be seen that both models identified at a frequency predict the force output excited by a different frequency input reasonably well. Also, the proposed LSL model outperforms the Hill Huxley model in all the cases, for example, by 10% in the case (a) and by 11% in the case (c), respectively, in terms of goodness-of-fit.

TABLE II

PREDICTION ABILITY OF THE PROPOSED LSL MODEL AND THE HILL HUXLEY MODEL, RESPECTIVELY.

Freq. A	Freq. B	Delay steps L	LSL model		Hill Huxley model	
			gof	nmae	gof	nmae
10 Hz	20 Hz	infinity	0.8507	3.93%	0.7563	5.90%
10 Hz	20 Hz	150	0.8677	2.45%	0.8158	3.31%
20 Hz	10 Hz	infinity	0.6328	10.17%	0.5289	12.46%
20 Hz	10 Hz	150	0.6580	3.93%	0.6457	9.30%

"Freq. A " is the frequency of which the data set used to identify the model, and "Freq. B" is the frequency at which the force we are going to predict. "Delay steps L" means that the actual L-step-delayed-output is available and used in prediction. "infinity" means no actual output is used in prediction.

V. DISCUSSION AND CONCLUSIONS

The utility of control systems for electrical stimulation of paralyzed muscle depends in large measure on the fit and predictive capabilities of the underlying muscle model. Although the Hill Huxley model is the most advanced and accurate model in the literature, it is not easy to identify and is difficult to incorporate into control algorithms due to its complicated structure and the number of parameters involved [12]. A simpler model with comparable predictive and fit capabilities would have greater usefulness in real-world control applications. Compared to the Hill Huxley model, the proposed LSL model possesses the following attractive properties.

- **Simplicity**: The proposed system consists of two time invariant first order linear systems and a known static nonlinearity and has only four parameters to identify. On the other hand, the Hill Huxley model is described by two time varying nonlinear differential equations with six unknown parameters and a sequence of t_i's that describe the exact time and the interval of electrical pulses.
- **Competitive performance**: Both the LSL model and the Hill Huxley model provide good performance. In terms of model fitting, the Hill Huxley model is about 1% better and in terms of prediction, the LSL model outperforms the Hill Huxley model by about 10%, a significant improvement because of its simplicity. In general, it is safe to say that the proposed LSL model is at least comparable to the Hill Huxley model in terms

of performance but at a much reduced complexity.

- **Easy to Identify**: The proposed LSL model is very easy to identify by the identification algorithms developed in this paper. It does not rely on the initial estimates due to the global convergence property of Approach 1 and/or Approach 2, while identification performance of the Hill Huxley model relies heavily on the initial estimates. Therefore, some time-consuming and very fine adjustments have to be made in order to avoid the local minimum problem.
- **Easy to implement**: The ultimate goal of the skeletal muscle model is to be implemented in the control algorithm. The proposed LSL model possesses a much simpler structure and a smaller number of unknown parameters and requires much less computational expense. This makes the proposed system much easier to incorporate into a control scheme.

In summary, the proposed LSL model performs well compared to the most advanced and accurate Hill Huxley model. In addition, it has a much simpler structure and a smaller number of unknowns. It is a very competitive alternative in modeling skeletal muscle dynamics and has great potential to be incorporated into control systems, which we are working on currently. We emphasize again that the convergence analysis carried out in this report is targeted at the particular application and is based on the observations of actual electrical stimulation inputs and muscle force outputs. The identification algorithms developed accordingly are very efficient for paralyzed skeletal muscle modeling based on the proposed Wiener-Hammerstein system.

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