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A "Globally" Convergent Controller for Transient Stability of Multi–Machine Power Systems using Structure–Preserving Models

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Abstract— **The approach adopted in this paper for the problem of transient stabilization of multimachine power systems sees the entire network as the (structure-preserving) interconnection of the network components, described by well known models. These *structure-preserving models* preserve the identity of the network components and allow for a more realistic treatment of the loads. Our main contribution is the *explicit computation* of a control law that, under a detectability assumption, ensures that *all trajectories* converge to the desired equilibrium point, provided that they start and remain in the region where the model makes physical sense.**

I. INTRODUCTION

Classical research on transient stabilization of power systems has relied on the use of aggregated *reduced network* models that represent the system as an n–port described by a set of ordinary differential equations. Several excitation controllers that establish Lyapunov stability of the desired equilibrium of these models have been reported. The nonlinear controller design techniques that have been considered include feedback linearization [15], damping injection [8], [16], [17], as well as, the more general, interconnection and damping assignment passivity–based control, see [12], [11] and [10].

Aggregated models erase the identity of the network components and impose an unrealistic treatment of the loads. In this paper, we abandon the aggregated n–port view of the network and consider the more natural and widely popular structure–preserving models (SPM), first proposed in [2]. Since these models consist of differential algebraic equations (DAE) they require the development of some suitably tailored tools for controller synthesis and stability analysis. Another original feature of the present work is that we do not aim at Lyapunov stability, but establish instead a *"global" convergence* result.[1]

In [5] SPM with nonlinear loads have been studied using singular perturbation approach in which the algebraic equations are considered as a limit of the fast dynamics. This approach is used in order to circumvent the singular properties in DAE system. See [18] for more details. Furthermore, in [4] SPM were used to identify—in terms of feasibility of a LMI—a class of power systems with nonlinear (so-called ZIP) loads and leaky lines for which a *linear time–invariant* controller renders the overall *linearized* system dissipative with a (locally) positive definite storage function, thus ensuring stability of the desired equilibrium for the nonlinear system. Unfortunately, a full–fledged *nonlinear* analysis of the problem was not possible due to the difficulty in handling the complicated interdependence of the variables appearing in the algebraic constraints of the DAEs. The Lyapunov function in that paper is obtained by adding a quadratic term in the rotor angle to the classical energy function of [14]. This quadratic term is needed to compensate for a linear term (in rotor angle) appearing in the energy function of [14] and render the new storage function positive definite. To obtain our "global" convergence result we observe that removing the linear term from the energy function of [14] and increasing the quadratic term in bus voltages yields a function whose time derivative can be *arbitrarily assigned* with a "globally" defined static state feedback. Furthermore, although this new function is not positive definite, it is *bounded from below* and has some suitable radial unboundedness properties—features that are essential to establish boundedness of trajectories. We then select a control law that renders "globally" attractive the level set of this function that contains the desired equilibrium point. If, furthermore, the function defines a *detectable output*, then all trajectories will asymptotically converge to the equilibrium. The only critical assumption required to establish this result is that the loads are constant impedances—a condition that is implicitly assumed in all controllers derived for aggregated models.

The structure of the paper is as follows. Section II presents the mathematical model of the various elements comprising the power system. Then, we formulate the control problem in Section III and give a key preliminary lemma. Section IV contains our main "global" convergence result that relies on the aforementioned detectability assumption. Section V includes the application of the proposed technique to a classical example. We wrap up the paper with some concluding remarks in Section VI. Proofs of some of the Lemmas are presented in the appendices.

Notation All vectors in the paper are *column* vectors, even the gradient of a scalar function: $\nabla_x = \frac{\partial}{\partial x}$. For any function $f : \mathbb{R}^n \to \mathbb{R}$, we define $\nabla_{z_j} f(z) := \frac{\partial f}{\partial z_j}(z)$, and for vector functions $g : \mathbb{R}^n \to \mathbb{R}^n$, we define the Jacobian $\nabla g(z) := [\nabla g_1(z), \ldots, \nabla g_n(z)]^\top \in \mathbb{R}^{n\times n}$. To simplify notation we

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[1]The precise meaning of the qualifier "global" will be given in the sequel. It essentially boils down to restricting the analysis to the trajectories that remain in the region where the model makes physical sense.

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introduce the sets

$$\mathbb{M}^n := \mathbb{S}^n \times \mathbb{R}^n \times \mathbb{R}^n_{>0} \times \mathbb{S}^n \times \mathbb{R}^n_{>0},\ n \in \bar{n} := \{1,..,n\},$$

where $\mathbb{S}$ is the unitary circle and $\mathbb{R}^n_{>0} := \{x \in \mathbb{R}^n \mid x_j > 0\}$.

II. Structure–Preserving Modelling

In this section we recall the well–known *structure–preserving* model reported in [14]. To simplify the presentation of our results we assume a simplified network topology where attached to each bus there is a machine *and* a load.[2] Each bus, and their corresponding machine and load, have an associated identifier $j \in \bar{n} := \{1,..,n\}$. Buses are interconnected through transmission lines that are identified by the double subindex $jk \in \Omega \subset \bar{n} \times \bar{n}$, indicating that the line jk connects the bus $j \in \bar{n}$ with the bus $k \in \bar{n}$; the set avoids obvious repetitions, e.g., if $jk \in \Omega$ then $kj \notin \Omega$. All elements share as port variables the angle θ_j and the magnitude V_j of the bus voltage phasor $y_j = \mathrm{col}(\theta_j, V_j) \in \mathbb{S} \times \mathbb{R}_{>0}$. Associated to each bus are the active and reactive powers entering the machine, the load or the transmission lines, that are denoted

$$\begin{bmatrix} P_j^M \\ Q_j^M \end{bmatrix}, \begin{bmatrix} P_j^L \\ Q_j^L \end{bmatrix}, \begin{bmatrix} P_{jk} \\ Q_{jk} \end{bmatrix} \in \mathbb{R}^2, \tag{1}$$

respectively. Following standard convention, we take active and reactive powers as positive when entering their corresponding component.

A. Synchronous machines model

Each synchronous machine is described by a set of third order DAE's, [14]:

$$\begin{aligned} \dot{\delta}_j &= \omega_j \\ M_j \dot{\omega}_j &= P_{m_j} - D_j \omega_j + P_j^M \\ \tau_j \dot{E}_j &= -\frac{x_{d_j}}{x'_{d_j}} E_j + \frac{x_{d_j} - x'_{d_j}}{x'_{d_j}} V_j \cos(\delta_j - \theta_j) + E_{F_j}, \end{aligned} \tag{2}$$

$$\begin{aligned} P_j^M &= -\frac{E_j V_j}{x'_{d_j}} \sin(\delta_j - \theta_j) - Y_{2j} V_j^2 \sin(2(\delta_j - \theta_j)) \\ Q_j^M &= (Y_{V_j} - Y_{2j} \cos(2(\delta_j - \theta_j)))V_j^2 - \frac{E_j V_j}{x'_{d_j}} \cos(\delta_j - \theta_j), \end{aligned} \tag{3}$$

where, to simplify notation, we defined the constants

$$Y_{2j} := \frac{x'_{d_j} - x_{q_j}}{2 x_{q_j} x'_{d_j}},\ Y_{V_j} := \frac{x'_{d_j} + x_{q_j}}{2 x_{q_j} x'_{d_j}}.$$

The *state variables* $x_j := \mathrm{col}(\delta_j,\ \omega_j,\ E_j) \in \mathbb{S} \times \mathbb{R} \times \mathbb{R}_{>0}$ denote the rotor angle, the rotor speed and the quadrature axis internal e.m.f., respectively. The control variable is the field voltage E_{F_j}, which is split in two terms $E_{F_j}^\star + v_j$. The first is constant and fixes the equilibrium value, while the second one is the *control action*. The parameters are denoted as in [14], and are fairly standard.

[2] As will become clear below the derivations are also applicable for other network topologies—at the expense of a more cluttered notation.

B. Loads model

Loads are described by the standard ZIP model, see [7],

$$\begin{aligned} P_j^L &= P_{Z_j} V_j^2 + P_{I_j} V_j + P_{0_j} \\ Q_j^L &= Q_{Z_j} V_j^2 + Q_{I_j} V_j + Q_{0_j}, \end{aligned} \tag{4}$$

which explicitly represent the contribution of each type of load (constant impedance, current or power). As will become clear below, to state our main result we must consider a simplified model for the loads. Namely, we assume only constant impedance loads:

$$\begin{aligned} P_j^L &= P_{Z_j} V_j^2 \\ Q_j^L &= Q_{Z_j} V_j^2 \end{aligned} \tag{5}$$

This simplification, which is necessary to obtain the lumped parameter model used in most transient stability controller design studies, allows us to transform the algebraic constraints into a set of *linear* equations for which we can give conditions for solvability.

C. Transmission lines model

The transmission lines are modeled with the standard lumped Π circuit, see [1],

$$\begin{aligned} P_{jk} &= G_{jk} V_j^2 + B_{jk} V_j V_k \sin(\theta_j - \theta_k) - \\ &\quad G_{jk} V_j V_k \cos(\theta_j - \theta_k) \\ Q_{jk} &= (B_{jk} - B_{jk}^c) V_j^2 - B_{jk} V_j V_k \cos(\theta_j - \theta_k) - \\ &\quad G_{jk} V_j V_k \sin(\theta_j - \theta_k) \end{aligned} \tag{6}$$

where $jk \in \Omega$, while G_{jk}, B_{jk} and B_{jk}^c denote the lines conductance, series and shunt susceptance, respectively.

Remark 1: In contrast with reduced network models G_{jk} here is the effective line conductance and not the *transfer conductance* that lumps the effects of the line conductance *and* the load impedances. While G_{jk} may, sometimes, be neglected it is impermissible to neglect the transfer conductances [10]. We are interested in this paper in the more realistic case of leaky lines with *capacitive* effects.

D. Bus equations

From Kirchhoff's laws, at each bus we have

$$\begin{aligned} 0 &= \textstyle\sum_{k \in \Omega_j} P_{jk} + P_j^M + P_j^L \\ 0 &= \textstyle\sum_{k \in \Omega_j} Q_{jk} + Q_j^M + Q_j^L \end{aligned} \tag{7}$$

where $\Omega_j := \{k \in \bar{n} \mid \exists\ jk \in \Omega\}$, the set of buses that are linked to the bus j through some transmission line.

Remark 2: We bring to the readers attention the fact that V_j, being a magnitude of a phasor, is non–negative. Similarly, due to physical considerations, $E_j > 0$. These fundamental physical constraints of the model are assumed for our derivations.

III. Control Problem and a Key Lemma

To obtain the overall model we group all the algebraic constraints and write the system equations in the compact form

$$\begin{cases} \dot{x} &= f(x,y) + L_v v \\ 0 &= g(x,y), \end{cases} \tag{8}$$

where $(x := \mathrm{col}(x_j), y := \mathrm{col}(y_j)) \in \mathbb{M}^n$, $v := \mathrm{col}(v_j) \in \mathbb{R}^n$, the matrix $L_v := \mathrm{diag}\{\mathrm{col}(0, 0, \frac{1}{\tau_j})\} \in \mathbb{R}^{3n\times n}$, and the functions $f : \mathbb{M}^n \to \mathbb{R}^{3n}$, and $g : \mathbb{M}^n \to \mathbb{R}^{2n}$ are defined by (2), and the replacement of (3), (5) and (6) into (7), respectively.

A. Problem formulation

Assumption A1. There exists an isolated asymptotically stable *open loop equilibrium* $(x^\star, y^\star)$ of the system (8).

Asymptotic Convergence Problem. Consider the system (8) satisfying Assumptions A1 and A2. Find a control law $v = \hat{v}(x, y)$ such that:

$$(x(t), y(t)) \in \mathbb{M}^n,\ \forall t \geq 0 \ \Rightarrow\ \lim_{t\to\infty} (x(t), y(t)) = (x^\star, y^\star).$$

Consequently, $(x^\star, y^\star)$ is an attractive equilibrium of the closed–loop provided trajectories start, and remain, in $\mathbb{M}^n$—the set where the model is physically valid.

Remark 3: Assumption A1 is standard in transient stability studies where v is included to enlarge the domain of attraction of the operating point. On the other hand, the requirement of convergence to the equilibrium point is very stringent for engineering applications, see [6]. Due to space limitations we do not dwell here on a "more practical" stability requirement instead, aiming at the average control reader, stick to this classical property.

B. Proposed solution strategy

The solution to the problem stated in section III-A proceeds along the following steps:

1) Give an explicit solution of the power balance equations $g(x, y) = 0$.
2) Representation of the system dynamics as a *perturbed* port–Hamiltonian system using a Hamiltonian function with desired characteristics.
3) Construction of a control signal that, assigning the derivative of the Hamiltonian function, ensures that trajectories will converge to the level set of the Hamiltonian that contains the equilibrium point. Trajectories will then converge to the equilibrium if the Hamiltonian function defines a detectable output.
4) Prove that the resulting controller is well defined and convergence is guaranteed—provided the trajectories remain in $\mathbb{M}^n$.

The second and the third steps can be carried out for the model with the general ZIP loads (4). Invoking the existence of an isolated local minimum of Assumption A1, using some continuity arguments and assuming detectability we can, therefore, conclude that the proposed controller renders the equilibrium locally attractive. This kind of local results are easily obtained using linearization, and known in the power systems community as small–signal stability. In this paper we are interested in the nonlinear transient stability phenomenon,i.e., the large–signal stability problem, therefore, the last step is indispensable. To complete it, the first step is essential—unfortunately, this imposes the restrictive requirement of constant impedance loads (5).

IV. Main Result

This section contains our main "global" convergence result, which is derived proceeding along the steps delineated in Subsection III-B.

A. Solution of $g(x, y) = 0$

In this subsection we present an explicit solution to the algebraic constraints $g(x, y) = 0$, a result which is of interest on its own. To simplify the presentation we define, for $j \in \bar{n}$, the complex variables

$$\mathbf{V}_j := V_j e^{i\theta_j} \in \mathbb{C}, \qquad \mathbf{V} := \mathrm{col}(\mathbf{V}_j)_{j\in\bar{n}} \in \mathbb{C}^n, \tag{9}$$

and

$$E := \mathrm{col}(E_j)_{j\in\bar{n}} \in \mathbb{R}^n, \quad \delta := \mathrm{diag}\{\delta_j\}_{j\in\bar{n}} \in \mathbb{R}^{n\times n}.$$

Lemma 1: Consider the algebraic equations $g(x, y) = 0$ of the power systems model (8) defined by (3), (5), (6) and (7). If

$$\frac{1}{x'_{d_j}} + Q_{Z_j} > \sum_{k\in\Omega_j} B^c_{jk}, \quad j \in \bar{n}, \tag{10}$$

$g(x, y) = 0$ has a "globally" defined solution. That is, there exists a function $\hat{y} : \mathbb{S}^n \times \mathbb{R}^n_{>0} \to \mathbb{S}^n \times \mathbb{R}^n_{>0}$ such that $g(x, \hat{y}(x)) = 0$. Furthermore, this function can be written in the form

$$\mathbf{V} = W(\delta)E, \tag{11}$$

where $W : \mathbb{R}^{n\times n} \to \mathbb{C}^{n\times n}$ is *bounded and invertible*, with elements are rational functions of $\cos(\delta_j)$ and $\sin(\delta_j)$ [3]. The proof is given in Appendix I.

Remark 4: Note that condition (10) is always verified and realistic, since we are considering the low voltage terminals of the generators, which is usually connected through a single step-up transformer to the network. Also, it is clear that the construction of $\hat{y}$ directly follows from (9) and (11), and is omitted for brevity.

Remark 5: Since the solution of the algebraic equations $g(x, y) = 0$ is globally defined, and it will be used in the design of the controller, we overcome the classical assumption made about the invertibility of $\nabla_y g(x, y)$, *i.e.* the continuity of the trajectories restricted to $g(x, y) = 0$, see [18], [5]. Moreover, using equation (11) all the external perturbations will be taken implicitly into consideration in the model since the matrix $W(\delta)$ represents in some way the network.

B. Perturbed port–Hamiltonian representation

The j-th synchronous machine model dynamics (2) can be written as a perturbed port–Hamiltonian system

$$\dot{x}_j = (J_j - R_j)\nabla_{x_j} H_j(x_j, y_j) + L_{v_j} v_j + \xi_j \tag{12}$$

with the Hamiltonian functions $H_j : \mathbb{M}^1 \to \mathbb{R}$,

$$\begin{aligned} H_j &:= \frac{1}{2}M_j\omega_j^2 + \frac{1}{2}Y_{E_j}E_j^2 + \frac{1}{2}[\Delta_j + Y_{V_j}]V_j^2 - Y_{F_j}E_{F_j}^{\star}E_j \\ &\quad - \frac{Y_{2j}}{2}\cos 2(\theta_j - \delta_j)V_j^2 - \frac{E_j V_j}{x'_{d_j}}\cos(\theta_j - \delta_j) \end{aligned} \tag{13}$$

and we defined the matrices

$$J_j := \begin{bmatrix} 0 & \frac{1}{M_j} & 0 \\ -\frac{1}{M_j} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -J_j^\top,\ R_j := \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{D_j}{M_j^2} & 0 \\ 0 & 0 & \frac{1}{\tau_j Y_{F_j}} \end{bmatrix},$$

$L_{v_j} := [0, 0, \frac{1}{\tau_j}]^\top$, $\xi_j := [0, \frac{P_{mj}}{M_j}, 0]^\top$, and the constants

$$Y_{E_j} := \frac{x_{d_j}}{x'_{d_j}(x_{d_j} - x'_{d_j})}, \qquad Y_{F_j} := \frac{1}{x_{d_j} - x'_{d_j}},$$

where $R_j \geq 0$ and $\Delta_j \geq 0$ is a key *design parameter*.

One important property of the Hamiltonian H_j is that it is *quadratic* in $Z_j := \mathrm{col}(\omega_j, E_j, V_j)$ and, furthermore, *bounded from below*. (Consequently, if H_j is non–increasing, we can conclude that all signals are bounded—because Z_j will be bounded and θ_j and δ_j live in compact sets.) To prove this fact, let us write the function in the form

$$H_j = \frac{1}{2}Z_j^\top T_j(\theta_j - \delta_j)Z_j + Z_j^\top b_j \tag{14}$$

where we have defined

$$T_j := \begin{bmatrix} M_j & 0 & 0 \\ 0 & Y_{E_j} & \frac{-\cos(\theta_j - \delta_j)}{x'_{d_j}} \\ 0 & \frac{-\cos(\theta_j - \delta_j)}{x'_{d_j}} & \Delta_j + Y_{V_j} - Y_{2j}\cos 2(\theta_j - \delta_j) \end{bmatrix}$$

and $b_j := \mathrm{col}(0, -Y_{F_j}E_{F_j}^{\star}, 0)$. Using the fact that (for all $\Delta_j \geq 0$)

$$\Delta_j + Y_{V_j} > Y_{2j},$$

it is possible to show that, *uniformly* in $\theta_j - \delta_j$, there exists $\epsilon_j > 0$ such that $T_j \geq \epsilon_j I$. Consequently, after some basic bounding, we can prove that

$$H_j \geq -\frac{(Y_{F_j}E_{F_j}^{\star})^2}{2\epsilon_j}. \tag{15}$$

Remark 6: The functions H_j defined in (13) should be contrasted with the energy functions used in [13], see also [4]. On one hand, the latter includes an additional term $-P_{mj}\delta_j$.[3] On the other hand, we have included a term $\Delta_j V_j^2$ that, as will come clear below, is essential for the construction of the control law.

Remark 7: To handle the linear term $-P_{mj}\delta_j$ in a Lyapunov–like analysis we must take care of some delicate theoretical issues that have, unfortunately, been overlooked in the literature and we discuss in detail here—see also discussion in [10] and [16]. Since this function is not defined in $\mathbb{S}$, but in $\mathbb{R}$, if we look at the system as evolving in $\mathbb{M}^n$ it will be a *discontinuous* function and (standard) Lyapunov arguments will not hold true. To avoid this difficulty, we should consider that δ_j evolves in $\mathbb{R}$, instead of $\mathbb{S}$. In this case, the function H_j is not lower bounded anymore, stymying the establishment of the property of trajectory boundedness needed for LaSalle–based arguments.[4]

Remark 8: Due to the presence of the term ξ_j in (12) it is clear that the set of open–loop equilibria and the set of minima of H_j *are disjoint*. Therefore, the new Hamiltonian cannot qualify as a Lyapunov function candidate (for the desired equilibrium).

C. "Global" assignment of $\dot{H}(x, \hat{y}(x))$

Besides being lower bounded and quadratic (in Z_j) we prove in the paper another fundamental property of the function H_j, namely, that the derivative of the function

$$H(x, y) := \sum_{j \in \bar{n}} H_j(x_j, y_j) \tag{16}$$

restricted to the set $g(x, y) = 0$, can be *arbitrarily assigned* with a suitable selection of the control v. Towards this end, compute

$$\dot{H} = -\nabla_x^\top H R \nabla_x H + \tilde{\xi}(x, y) + \nabla_E^\top H \tau^{-1} v + \nabla_y^\top H \dot{y} \tag{17}$$

where $R := \mathrm{diag}\{R_j\}_{j \in \bar{n}} \in \mathbb{R}^{3n \times 3n}$, $\tilde{\xi}(x, y) := \sum_{j \in \bar{n}} \omega_j P_{mj} \in \mathbb{R}$ and $\tau := \mathrm{diag}\{\tau_j\}_{j \in \bar{n}} \in \mathbb{R}^{n \times n}$. Lemma 1 shows that the set $g(x, y) = 0$ is equivalent to $\mathbf{V} = W(\delta)E$. Therefore, to evaluate $\dot{y}$ it is convenient to express the Hamiltonian function (13), (16) in terms of the complex variables $\mathbf{V}$ defined in (9). Hence, noticing that

$$V_j^2 \cos 2(\theta_j - \delta_j) = \mathrm{Re}\{e^{-2i\delta_j}\,\mathbf{V}_j^2\},$$

we define

$$\begin{aligned} H_{\mathbb{C}}(x, \mathbf{V}) &:= \frac{1}{2}\left(w^\top M w + E^\top Y_E E + \mathbf{V}^{\mathcal{H}}(Y_V + \Delta)\,\mathbf{V}\right) \\ &\quad -\frac{1}{2}\mathrm{Re}\{\mathbf{V}^\top Y_2 e^{-2i\delta}\,\mathbf{V}\} - \mathrm{Re}\{\mathbf{E}^{\mathcal{H}} X\,\mathbf{V}\} - E^\top Y_F E_F \end{aligned}$$

where $E_F := \mathrm{col}(E_{F_j}^{\star})_{j \in \bar{n}}$, Y_V, Y_2, X are defined in Appendix I and

$$\begin{aligned} M &:= \mathrm{diag}\{M_j\}_{j \in \bar{n}},\ Y_E := \mathrm{diag}\{Y_{E_j}\}_{j \in \bar{n}}, \\ Y_F &:= \mathrm{diag}\{Y_{F_j}\}_{j \in \bar{n}},\ \Delta := \mathrm{diag}\{\Delta_j\}_{j \in \bar{n}} \end{aligned}$$

are defined in $\mathbb{R}^{n \times n}$.

[3]Removing this term induces the appearance of the constant term ξ_j in (12), but we show in the sequel that its effect can be compensated with a suitable selection of the control law.

[4]This unfortunate mistake is made in many papers. For instance, in Proposition 1 of [5], where the interesting idea of damping injection for structure preserving models is proposed, boundedness of trajectories is never established—nor assumed.

The following lemma, whose proof is given in Appendix I, is instrumental to compute the required derivative.

Lemma 2: Consider the quadratic function $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$,

$$f(\mu_1, \mu_2) = c_1\mu_1^2 + 2c_3\mu_1\mu_2 + c_2\mu_2^2,$$

with $c_i \in \mathbb{R}$ and $\mu_1, \mu_2 : \mathbb{R} \to \mathbb{R}$. Define $z = \mu_1 + i\mu_2 \in \mathbb{C}$ and the function $f_{\mathbb{C}} : \mathbb{C} \to \mathbb{R}$ such that

$$f_{\mathbb{C}}(z) = f(\mu_1, \mu_2).$$

Then,

$$\dot{f} = \mathrm{Re}\left\{\left(\frac{\partial f_{\mathbb{C}}}{\partial z}\right)^* \dot{z}\right\},$$

where $\frac{\partial f_{\mathbb{C}}}{\partial z} := \frac{\partial f_{\mathbb{C}}}{\partial \mu_1} + i\frac{\partial f_{\mathbb{C}}}{\partial \mu_2}$.

From the lemma it is clear that, to compute the time derivative of H, we require the term $\mathrm{Re}\left\{(\nabla_{\mathbf{V}} H_{\mathbb{C}})^{\mathcal{H}}\dot{\mathbf{V}}\right\}$. It is easy to see that

$$\nabla_{\mathbf{V}} H_{\mathbb{C}} = (Y_V + \Delta)\,\mathbf{V} - Y_2 e^{2i\delta}\,\mathbf{V}^* - X\,\mathbf{E}$$

where $(\cdot)^*$ denotes complex conjugation. We recall now the identity (23) established in Appendix I that, for ease of reference, we recall here

$$A_0\,\mathbf{V}^* - X\,\mathbf{E}^* - Y_2 e^{-2i\delta}\,\mathbf{V} = 0.$$

Substituting the complex conjugate of the latter in $\nabla_{\mathbf{V}} H_{\mathbb{C}}$ above we get

$$\nabla_{\mathbf{V}} H_{\mathbb{C}} = (Y_V + \Delta - A_0^*)\,\mathbf{V},$$

and by definition of A_0, given in (24), we get $\nabla_{\mathbf{V}} H_{\mathbb{C}} = D\,\mathbf{V}$, where

$$D := \Delta - Q_Z - B^d + B + i(-P_Z - G^d + G) \in \mathbb{C}^{n\times n}.$$

We recall that the matrices Δ, Q_Z, P_Z, B^d and G^d are diagonal. All these matrices are defined in Appendix I.

Let us now compute $\dot{\mathbf{V}}$. In Lemma 1 it is shown that $\mathbf{V} = WE$, where $W : \mathbb{R}^{n\times n} \to \mathbb{C}^{n\times n}$ is *bounded and invertible*. Therefore,

$$\dot{\mathbf{V}} = \dot{W}E + W\dot{E}$$

The function $\dot{W}$ depends on δ and ω, but is independent of v, while $\dot{E}$ will bring along terms on v. We now come back to $\dot{H}$, that takes the form

$$\dot{H} = -\nabla_x^\top HR\nabla_x H + \tilde{\xi} + \nabla_E^\top H\tau^{-1}v + \mathrm{Re}\left\{(\nabla_{\mathbf{V}} H_{\mathbb{C}})^{\mathcal{H}}\dot{\mathbf{V}}\right\}.$$

That, replacing the computations above, can be compactly written as

$$\dot{H} = -\nabla_x^\top HR\nabla_x H + \Xi(\delta, w, E, \mathbf{V}) + \mathcal{L}^\top(\delta, w, E, \mathbf{V})\tau^{-1}v \tag{18}$$

where we defined the (real valued) functions

$$\begin{aligned} \Xi &:= \tilde{\xi} + \mathrm{Re}\left\{(\nabla_{\mathbf{V}} H_{\mathbb{C}})^{\mathcal{H}}\left[\dot{W}E \;\; -W(\tau Y_F)^{-1}\nabla_E H\right]\right\} \\ \mathcal{L}^\top &:= \left\{\nabla_E^\top H + \mathrm{Re}\left\{(\nabla_{\mathbf{V}} H_{\mathbb{C}})^{\mathcal{H}} W\right\}\right\}. \end{aligned} \tag{19}$$

Let us take a brief respite to analyze (18). It is clear that, wherever the vector $\mathcal{L}(\delta, w, E, \mathbf{V})$ is bounded away from zero, we can easily select a control law v that assigns an arbitrary function to $\dot{H}$.

Proposition 1: Consider the power systems model (8) with Assumption A1 and the Hamiltonian function (13). There exists $\Delta_j^{min} > 0$ such that, for all $\Delta_j \geq \Delta_j^{min}$ we have

$$\mathcal{L}^\top(x, \hat{y}(x))E > 0 \qquad \text{for all} \quad x \in \mathbb{S}^n \times \mathbb{R}^n \times \mathbb{R}^n_{>0},$$

where $\mathcal{L}^\top$ is given in (19). Therefore, for any function $\alpha : \mathbb{M}^n \to \mathbb{R}$, the "globally" defined control law

$$v = \frac{1}{\mathcal{L}^\top E}[\alpha(x, y) + \nabla_x^\top HR\nabla_x H - \Xi]\tau E \tag{20}$$

ensures $\dot{H} = \alpha$.

Proof: By definition,

$$\mathcal{L}^\top E = \mathrm{Re}\left\{(\nabla_{\mathbf{V}} H_{\mathbb{C}})^{\mathcal{H}} W(\delta)E\right\} + \nabla_E^\top HE. \tag{21}$$

Let us consider the first term. Since $\mathbf{V} = W(\delta)E$ and $\nabla_{\mathbf{V}} H_{\mathbb{C}} = D\,\mathbf{V}$ we have

$$\mathrm{Re}\left\{(\nabla_{\mathbf{V}} H_{\mathbb{C}})^{\mathcal{H}} WE\right\} = \mathrm{Re}\left\{\mathbf{V}^{\mathcal{H}}\,D\,\mathbf{V}\right\} = \mathbf{V}^{\mathcal{H}}\,\frac{D + D^*}{2}\,\mathbf{V}$$

The matrix D is symmetric (not Hermitian self conjugate). Therefore,

$$\frac{D + D^*}{2} = \Delta - Q_Z - B^d + B,$$

where Q_Z, B^d and B are constant matrices defined in Appendix I. The quadratic form above can then be made *arbitrarily large* by choosing a large $\Delta > 0$.

Using again $\mathbf{V} = WE$ and invertibility of W we see that the second term in (21) is also a quadratic function of $\mathbf{V}$, that can be written in the form

$$\nabla_E^\top HE = \mathbf{V}^{\mathcal{H}}\,S(\delta)\,\mathbf{V} + \mathrm{Re}(\mathbf{V}^{\mathcal{H}}\,s(\delta)E_F^\star),$$

for some suitable matrices $S, s : \mathbb{R}^{n\times n} \to \mathbb{C}^{n\times n}$. From boundedness of W^{-1} we have that S and s are also *bounded* and we can conclude that, throughout $\mathbb{M}^n$, the first term in (21) can be made strictly greater than the second. Therefore, the denominator in (20) is always larger than zero, completing the claim.

D. A "globally" convergent controller

In this subsection we propose to select the function α such that trajectories converge to $(x^\star, y^\star)$ under the following.

Assumption A2. The function $H(x, y) - H^\star$ defines a *detectable output* for the closed–loop system.

Proposition 2: Consider the power systems model (8) with Assumption A1 in closed–loop with the control (20) with

$$\alpha(x, y) = -\lambda[H(x, y) - H^\star], \tag{22}$$

where $H^\star := H(x^\star, y^\star)$, $\lambda > 0$, $\Delta_j \geq \Delta_j^{min}$, and Δ_j^{min} is as in Proposition 1.

(i) Assume $(x(t), y(t)) \in \mathbb{M}^n$, $\forall t \geq 0$. Then, trajectories are *bounded*.

(ii) Furthermore, if Assumption A2 holds $\lim_{t\to\infty}(x(t), y(t)) = (x^\star, y^\star)$.

Proof: First, note that

$$\frac{d}{dt}[H(x,y) - H^\star] = -\lambda[H(x,y) - H^\star].$$

Hence H is bounded, ensuring boundedness of trajectories. Furthermore, we have that $H(x(t), y(t)) \to H^\star$. The proof is completed invoking LaSalle's Invariance Principle and the definition of detectability.

Remark 9: The controller of Proposition 1 drives the trajectories towards the level set $\{(x,y) \in \mathbb{M}^n \mid H(x,y) = H^\star\}$. The analysis of the dynamics restricted to this set is quite involved. However, we prove in [3] that the assumption is verified for the classical single machine infinite bus system.

Remark 10: A practically interesting property of the control law (20) is that it is "almost" *decentralized*. Indeed, it is of the form $v_i = \beta(x,y)E_i$, where the scalar function $\beta : \mathbb{M}^n \to \mathbb{R}$ is the only information that needs to be transferred among the generators.

Remark 11: We recall that the minima of H are not equilibria of the system—hence, it is not a Lyapunov function candidate and the property $\dot{H} \le 0$ is not sufficient to guarantee some stability/convergence properties.

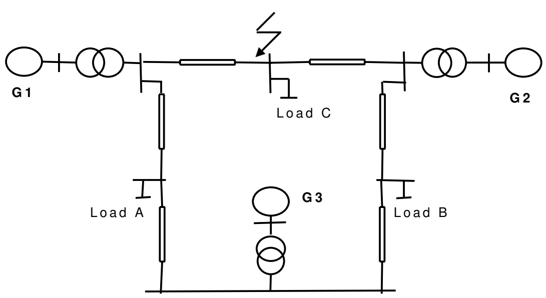

Fig. 1. Three-machines, nine-buses system.

V. A Benchmark Simulation Example

We consider here the classical 3-machines, 9-buses system considered in [1] and depicted in Fig. 1. We assume that the active and reactive components of the loads have constant impedance characteristics. Computations were done with the software package PSAT [9]. We analyze the response of (8) to a short circuit which consists of a zero-impedance three phase fault as shown in Fig. 1. The fault is introduced at $t = 0.5$ s and removed after a certain time (called the clearing time and denoted t_{cl}), after which the system is back to its pre-disturbance topology.

To tune the controller there is a compromise between the choices of Δ that, as indicated in Proposition 1, should be big enough to ensure that the denominator of the controller will stay away from zero, and λ that determines the speed of convergence to the desired level set. Indeed, Δ appears in H as ΔV^2 where V represents, in some way, the perturbation. Then, the bigger Δ is, the bigger H will be in the transient phase, and we have to decrease λ to eliminate impulsive responses in the controller during the perturbation.

Fig. 2 depicts the transient response of the nonlinear systems with $t_{cl} = 0.1$ s, $\Delta_j = 10$, $j = 1, 2, 3$, and $\lambda = 3$. As it can be seen, the controller is able to significantly improve the system's stability and provide damping.

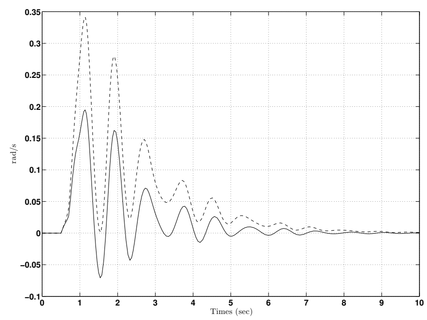

Fig. 2. *Speed w_1. Open loop (dashed), closed loop (continuous).*

VI. Conclusions and Future Work

We have presented in this paper an excitation controller to improve the transient stability properties of multi–machine power systems described by SPM with leaky lines including capacitive effects. Our main contribution is the explicit computation of a control law that ensures *"global" asymptotic convergence* to the desired equilibrium point of all trajectories starting and remaining in the physical domain of the system—provided a detectability assumption is satisfied. Note that this assumption was numerically verified for the standard SMIB system in [3]. To the best of our knowledge, no equivalent result is available in the literature at this level of generality. The usefulness of the technique for synthesis was illustrated with its application to a classical example.

Similarly to most developments reported by the control theory community on the transient stability problem, it is clear that the complexity of the proposed controller—as well as its high sensitivity to the system parameters and the assumption of full state measurement—severely stymies the practical application of this result. This kind of work pertains, however, to the realm of fundamental research where basic issues like existence of solutions are addressed. The present paper proves that, under a detectability assumption, a solution to the "global" convergence problem for the more natural SPM can indeed be *explicitly constructed*. The equilibrium stabilization formulation of the problem was done for mathematical convenience. However, physically convergence of the rotor angles to a fixed point is not required, it suffices to keep their difference bounded [1].

VII. Acknowledgments

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APPENDIX I
PROOF OF LEMMAS

Proof of Lemma 1. In order to motivate the reader and due to space limitations, we show in this section the first steps of the proof of Lemma 1, where the complete version can be found in [3]. Using equation (3), (5), (6), and after some simple calculations, we obtain

$$\begin{aligned} P_j^M + iQ_j^M &= iY_{V_j}|\mathbf{V}_j|^2 - \frac{i\,\mathbf{V}_j}{x'_{dj}}\mathbf{E}_j^* - iY_{2j}(\mathbf{V}_j)^2 e^{-2i\delta_j}, \\ P_j^L + iQ_j^L &= (P_{Z_j} + iQ_{Z_j})|\mathbf{V}_j|^2, \\ P_{jk} + iQ_{jk} &= [-G_{jk} - iB_{jk}]\,\mathbf{V}_j\,\mathbf{V}_k^* + \\ &\quad [G_{jk} + i(B_{jk} - B_{jk}^c)]|\mathbf{V}_j|^2. \end{aligned}$$

Replacing the expressions above in (7) we see that the j-th bus equation takes the form

$$i\,\mathbf{V}_j\left[\left(Q_{Z_j} - iP_{Z_j} + B_j^d - iG_j^d\right)\mathbf{V}_j^* - \frac{\mathbf{E}_j^*}{x'_{dj}} + Y_{V_j}\,\mathbf{V}_j^* \right.$$
$$\left. - Y_{2j}e^{-2i\delta_j}\,\mathbf{V}_j + i\sum_{k\in\Omega_j}(iB_{jk} + G_{jk})\,\mathbf{V}_k^*\right] = 0,$$

where we introduced the scalars

$$B_j^d := \sum_{k\in\Omega_j}(B_{jk} - B_{jk}^c), \quad G_j^d := \sum_{k\in\Omega_j} G_{jk}.$$

Since the voltage $V_j \in \mathbb{R}_{>0}$, the term in brackets should be zero leading to the following linear equation

$$\left(Y_{V_j} + Q_{Z_j} - iP_{Z_j} + B_j^d - iG_j^d\right)\mathbf{V}_j^* - Y_{2j}e^{-2i\delta_j}\,\mathbf{V}_j$$
$$-\frac{\mathbf{E}_j^*}{x'_{dj}} + \sum_{k\in\Omega_j}(-B_{jk} + iG_{jk})\,\mathbf{V}_k^* = 0.$$

Grouping all bus equations ($j \in \bar{n}$) leads to a square linear system for the complex vector $\mathbf{V}$ that can be written in the form

$$A_0\,\mathbf{V}^* - X\,\mathbf{E}^* - Y_2 e^{-2i\delta}\,\mathbf{V} = 0, \tag{23}$$

with

$$A_0 := Y_V + Q_Z - iP_Z + B^d - iG^d - B + iG \in \mathbb{C}^{n\times n}, \tag{24}$$

where we have defined the $n \times n$ real matrices

$$\begin{aligned} &Y_V := \operatorname{diag}\{Y_{V_j}\},\ Y_2 := \operatorname{diag}\{Y_{2j}\},\ B := \{B_{jk}\}, \\ &P_Z := \operatorname{diag}\{P_{Z_j}\},\ Q_Z := \operatorname{diag}\{Q_{Z_j}\},\ G := \{G_{jk}\}, \\ &G^d := \operatorname{diag}\{G_j^d\},\ X := \operatorname{diag}\{\frac{1}{x'_{dj}}\},\ B^d := \operatorname{diag}\{B_j^d\}, \end{aligned}$$

where $j, k \in \bar{n}$, $B_{jk} = G_{jk} = 0$ if $k \notin \Omega_j$.
Furthermore, to compute equation (11), one should solve the algebraic equation (23). See [3] for further details.
Proof of Lemma 2. By definition $f_{\mathbb{C}}$ can be written in the following form:

$$f_{\mathbb{C}}(z) = \frac{c_1 + c_2}{2}|z|^2 + \frac{c_1 - c_2}{2}\operatorname{Re}\{z^2\} - \operatorname{Re}\{ic_3 z^2\}$$

Where $\operatorname{Re}\{ic_3 z^2\} = -2c_3\mu_1\mu_2$. Then

$$\begin{aligned} \frac{\partial f_{\mathbb{C}}}{\partial z} &= (c_1 + c_2)z + (c_1 - c_2)z^* + 2ic_3 z^* \\ &= 2\left[(c_1\mu_1 + c_3\mu_2) + i(c_2\mu_2 + c_3\mu_1)\right] \end{aligned}$$

Hence,

$$\begin{aligned} \dot{f}(\mu_1, \mu_2) &= 2(c_1\mu_1 + c_3\mu_2)\dot{\mu_1} + 2(c_2\mu_2 + c_3\mu_1)\dot{\mu_2} \\ &= 2\operatorname{Re}\left\{\left[(c_1\mu_1 + c_3\mu_2) + i(c_2\mu_2 + c_3\mu_1)\right]^*\dot{z}\right\}. \end{aligned}$$