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Attitude Control of a Flexible Planar Space Robot

Tatsuo Narikiyo, Masanori Sakata and Michihiro Kawanishi

Abstract— **Rigid planar space robot is represented as Caplygin system and its base attitude can be stabilized by means of geometric phase. However, in cases when flexible arms are used for control of base attitude, inherent resonance modes are excited and instability is resulted. In this study we propose a stabilized controller is synthesized as adaptive tracking control system convined with sliding mode control to track to the arm trajectories derived from the geometric phase of the Caplygin system. Usefulness of the proposed controller is demonstrated by numerical simulations and Laboratory experiments.**

I. INTRODUCTION

In '90s, many research articles about the attitude control of the space robot have been published [1],[2],[3],[4]. However in these studies both theoretical and experimental consideration about stabilization of the planar space robot with flexible arms have not been sufficiently developed.

In this paper the attitude control problem of planar space robot with flexible dual arm is addressed. The proposed control scheme is based on the geometric phase approach [4] and the adaptive control scheme [6]. The adaptive control compensates the system uncertainties and difficulties arising from obtaining data for flexible dynamic co-ordinates. In this adaptive control, sliding mode control with Hybrid Sliding Surface (HSS) which is consisted of frequency shaped optimal sliding mode ($FSOSM$) and terminal sliding mode (TMS) is employed. The proposed control scheme does not directly guarantee the stability of the base satellite. However, since elastic vibration caused by the flexibility is sufficiently suppressed by the proposed control scheme, geometric phase works effectively. As a result of this control, base attitude can be regulated in a similar way to the case of rigid planar space robot.

The usefulness and validity of this control scheme can be demonstrated by hardware experiments. In our experiments free flying planar space robot with flexible dual arms floated by air on the horizontal plane has been employed.

II. MODELING OF A FLEXIBLE PLANAR SPACE ROBOT

Fig.1 shows a schematic model of a planar space robot consisting of two flexible arms connected with a base satellite via revolute joints. Coordinate system Σ_0 is global coordinate system and Σ_1 is moving coordinate system fixed on the center of gravity of the base satellite. Subscripts $\{1\}$ or $\{l\}$ symbolizes the left arm and $\{2\}$ or $\{r\}$ symbolizes the right arm. ϕ_i and τ_i denote the angle of rotation of the arm and the torque supplied by the motor. Flexible arm of length L has uniform mass density m_l and uniform flexural rigidity EI. Let J_i and J_h be the moment of motor inertia and the moment of base inertia, respectively. Let m_L be the mass of the tip of arm and (X_i, Y_i) be the position coordinate of the tip. θ denotes the base attitude. For derivation of equations of motion of the flexible planar space robot, the elastic displacement $y_l(x,t)$ and $y_r(x,t)$ at position x of the flexible arm is approximated by using assumed mode method as follows:

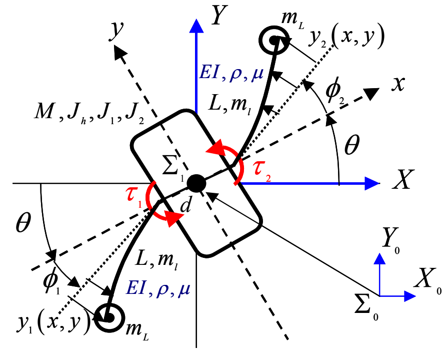

Fig. 1. Flexible planar space robot

$$y_l(x,t) = \varphi^T(x) q_{fl}(t), \quad y_r(x,t) = \varphi^T(x) q_{fr}(t) \qquad (1)$$

where $\varphi(x) = [\varphi_1(x) \quad \varphi_2(x) \quad \dots \quad \varphi_m(x)]^T$ and $q_{f*}(t) = [q_{f*1}(t) \quad q_{f*2}(t) \quad \dots \quad q_{f*m}(t)]^T$, $(* = l, r)$. x is any point along the undeformed link. $q_{f*j}(t)$ is the jth modal displacement for the flexible arm, $\varphi_j(x)$ is the jth assumed mode shape for the flexible arm,j=1,...,m. By using these parameters Lagrangian $\pounds$ can be obtained as follows[5]:

$$\begin{aligned}
\pounds \;=\; & \frac{J_h}{2}\dot{\theta}^2 + \frac{m_l}{L}\int_0^L d^2\dot{\theta}^2 dx + \frac{m_L}{2}\left(\dot{X_1}^2 + \dot{Y_1}^2\right) \\
& + \frac{m_L}{2}\left(\dot{X_2}^2 + \dot{Y_2}^2\right) + \frac{m_l}{2L}\int_0^L \left(x^2 + y_1^2\right) \\
& \times \left(\dot{\theta} + \dot{\phi_1}\right)^2 dx + \frac{m_l}{2L}\int_0^L \left(x^2 + y_2^2\right)\left(\dot{\theta} + \dot{\phi_2}\right)^2 dx \\
& + \frac{J_1}{2}\left(\dot{\theta} + \dot{\phi_1}\right)^2 + \frac{J_2}{2}\left(\dot{\theta} + \dot{\phi_2}\right)^2 + \frac{m_l}{2L}\int_0^L \dot{y_1}^2 dx \\
& + \frac{m_l}{2L}\int_0^L \dot{y_2}^2 dx + \frac{m_l}{L}\int_0^L x\varphi^T(x)\dot{q_{fl}} dx \left(\dot{\theta} + \dot{\phi_1}\right) \\
& + \frac{m_l}{L}\int_0^L x\varphi^T(x)\dot{q_{fr}} dx \left(\dot{\theta} + \dot{\phi_2}\right)
\end{aligned}$$

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T. Narikiyo, M. Sakata and M. Kawanishi are with the Department of Advanced Science and Technoligy, Toyota Technological Institute, 2-12-1 Hisakata, Nagoya 468-8511, Japan. T. Narikiyo is also visiting researcher at Bio-mimetic Control Research Center (RIKEN), Shimoshidami, Nagoya 463-0003, Japan. n-tatsuo@toyota-ti.ac.jp, kawa@toyota-ti.ac.jp

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$$+\frac{m_l}{L}\int_0^L xdx\dot{\theta}\left(\dot{\theta}+\dot{\phi}_1\right)d\cos\phi_1$$
$$+\frac{m_l}{L}\int_0^L xdx\dot{\theta}\left(\dot{\theta}+\dot{\phi}_2\right)d\cos\phi_2$$
$$-\frac{m_l}{L}\int_0^L \varphi^T(x)q_{fl}dx\dot{\theta}\left(\dot{\theta}+\dot{\phi}_1\right)d\sin\phi_1$$
$$-\frac{m_l}{L}\int_0^L \varphi^T(x)q_{fr}dx\dot{\theta}\left(\dot{\theta}+\dot{\phi}_2\right)d\sin\phi_2$$
$$-\frac{EI}{2}\int_0^L \left(\frac{d^2\varphi(x)}{dx^2}\right)^T\left(\frac{d^2\varphi(x)}{dx^2}\right)dxq_{fl}^Tq_{fl}$$
$$-\frac{EI}{2}\int_0^L \left(\frac{d^2\varphi(x)}{dx^2}\right)^T\left(\frac{d^2\varphi(x)}{dx^2}\right)dxq_{fr}^Tq_{fr} \quad (2)$$

For the simplicity we assume $d = 0$, distance between center of the base and the arm joint, and truncate the flexible mode on the 2nd mode, that is $m = 1$. From these assumptions we can obtain the dynamic equations of motion for the controller synthesis as follows:

$$M(q)\ddot{q} + h(q,\dot{q})\dot{q} + K(q)q = \tau \quad (3)$$

where $q = [\phi_1 \ \ \phi_2 \ \ q_{fl} \ \ q_{fr}]^T, \tau = [\tau_1 \ \ \tau_2 \ \ 0 \ \ 0]^T$,and $M(q) = \{M_{i,j}(q)\}, h(q,\dot{q}) = \{h_{i,j}(q,\dot{q})\}, K(q) = \{K_{i,j}(q)\}, i,j = 1,...,4$. In this equation q_{fl} and q_{fr} are used instead of q_{fl1} and q_{fr1}, respectively.

Base attitude θ does not appear in (3), though it is governed by the conservation law of angular momentum.

$$\dot{\theta} = \frac{1}{\lambda}\left(\tilde{\alpha}\dot{\phi}_1 + \tilde{\beta}\dot{\phi}_2 + \tilde{\gamma}\dot{q}_{fl} + \tilde{\delta}\dot{q}_{fr}\right) \quad (4)$$

Where

$$\begin{aligned}
\lambda &= J_h + J_1 + J_2 + 2m_L L^2 + \frac{m_l}{L}\int_0^L x^2dx \\
\tilde{\alpha} &= -m_L L^2 - \frac{m_l}{L}\int_0^L x^2dx - J_1 \\
\tilde{\beta} &= -m_L L^2 - \frac{m_l}{L}\int_0^L x^2dx - J_2 \\
\tilde{\gamma} &= -m_L L\varphi_1(x) - \frac{m_l}{L}\int_0^L x\varphi_1(x)dx \\
\tilde{\delta} &= -m_L L\varphi_1(x) - \frac{m_l}{L}\int_0^L x\varphi_1(x)dx
\end{aligned}$$

In equations (3) and (4), m_l/L can be replaced by mass density of the arms ρ.

By using these approximated model we consider the following problem.

Problem Suppose the dynamic physical parameters written in (3) is unknown. Synthesize the stabilizing controller of the flexible dual arm space robot to stabilize the base attitude and suppress the vibration caused by the arm flexibility. ■

III. CONTROLLER SYNTHESIS

A. Controller design for the rigid space robot

To solve this problem we turn back to the rigid space robot. For the rigid dual arm space robot conservation law of angular momentum can be written by the following Caplygin form.

$$\dot{\theta} = \alpha_{rigid}\dot{\phi_1} + \beta_{rigid}\dot{\phi_2} \quad (5)$$

Where α_{rigid} and β_{rigid} are similar form to the flexible case except the flexible terms. Applying the concept of the geometric phase to this Caplygin form, we can obtain the desired trajectories of the arm motions. The desired trajectories are synthesized as follows[4]:
Caplygin form (5) can be converted into

$$\theta = \oint_C \alpha_{rigid}d\phi_1 + \beta_{rigid}d\phi_2 \quad (6)$$

Suppose initial base attitude is θ_0. Then we can design closed contour C so that

$$-\theta_0 = \oint_C \alpha_{rigid}d\phi_1 + \beta_{rigid}d\phi_2 \quad (7)$$

From this closed contour we can obtain the desired trajectories of ϕ_1 and ϕ_2 as follows:

$$\begin{aligned}
\phi_{1d}(s(t)) &= -R\cos(ks(t)+\psi) + R\cos\psi \\
\phi_{2d}(s(t)) &= -R\sin(ks(t)+\psi) + R\sin\psi \quad (8)
\end{aligned}$$

Where R denotes a radius of the closed contour determined by relation (7), k denotes the direction of the closed contour and is assigned to 1 or -1. ψ is a constant arbitrarily assigned[4]. $s(t)$ is monotonically increasing function and is given by

$$s(t) = \Lambda t - \sin(\Lambda t). \quad (9)$$

$\Lambda > 0$ indicates the angular velocity of the arm motion along the closed contour and $T = 2\pi/\Lambda$ defines the one cycle of the contour trajectory $C_d = (\phi_{1d}(s(t)), \phi_{2d}(s(t)))$. Then

$$\frac{ds}{dt} = 0, \quad t = nT, \ (n = 0,1,2,...).$$

This property shows that the desired trajectory $C_d = (\phi_{1d}(s(t)), \phi_{2d}(s(t)))$ is at least once differentiable with respect to t. Indeed it can be easily shown that $C_d = (\phi_{1d}(s(t)), \phi_{2d}(s(t)))$ is twice continuously differentiable with respect to t.

B. Hybrid sliding surface (HSS)

Generally speaking, conventional sliding mode control is considered to be hardly applicable to the control of the flexible system because switching control input happens to excite the inherent resonance mode of the system. To overcome this problem $FSOSM$ has been developed [7]. Recently, Shahravi et al [6] and Xu et al [9] proposed the new sliding mode control method with hybrid sliding surface (HSS) to apply to the control of the single arm flexible space robot.

The HSS is written as

$$S_H = \alpha S_{FSOSM} + (1-\alpha)S_{TSM} \quad (10)$$

where subscripts $FSOSM$ and TSM indicate frequency shaped optimal sliding mode and terminal sliding mode respectively. Weighting coefficient α should be chosen properly to obtain required performance and satisfy stability conditions. In this subsection subscripts $\{* = l, r\}$ symbolize left and right arm.

The $FSOSM$ sliding surface is synthesized as

$$S_{FSOSM} = e_{2*} + c_e e_{1*} + c_1 z_{1*} + c_2 z_{2*} = 0, \quad (11)$$

where $e_{1l} = \phi_1 - \phi_{1d}, e_{2l} = \dot{\phi}_1 - \dot{\phi}_{1d}$ and $e_{1r} = \phi_2 - \phi_{2d}, e_{2r} = \dot{\phi}_2 - \dot{\phi}_{2d}$. z_{1*}, z_{2*} are filter states and are given by

$$\dot{z}_* = F z_* + E e_{2*} \quad (12)$$

where

$$z_* = \begin{bmatrix} z_{1*} \\ z_{2*} \end{bmatrix}, \ F = \begin{bmatrix} 0 & 1 \\ -\omega_c^2 & -2\omega_c \end{bmatrix}, \ E = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

$\omega_c > 0$ can be assigned arbitrarily. The main utilization of a filter would be to assure that unmodelled vibration modes in a given system are not excited in sliding mode[7]. c_e, c_1 and c_2 are design parameters synthesized by some optimal technique (see Appendix).

The TSM sliding surface is synthesized as

$$S_{TSM} = e_{2*} + c_p e_{1*}^p = 0 \quad (13)$$

where $c_p > 0$, $0 < p < 1$. Substituting (11) and (13) into (10), we can obtain the HSS as

$$S_{H*} = e_{2*} + \alpha c_e e_{1*} + (1-\alpha) c_p e_{1*}^p + \alpha c_1 z_{1*} + \alpha c_2 z_{2*} = 0 \quad (14)$$

C. Adaptive controller

The proposed controller consists of the reference controller and adaptive controller. Desired trajectories are obtained from the reference controller synthesized for the rigid space robot. To track to these desired trajectories we synthesize the adaptive controller as follows.

First we define the HSS($S_H = [S_{Hl} \ \ S_{Hr}]^T$) as

$$S_H = \begin{bmatrix} e_{2l} + \alpha c_e e_{1l} + (1-\alpha) c_p e_{1l}^p + \alpha c_1 z_{1l} + \alpha c_2 z_{2l} \\ e_{2r} + \alpha c_e e_{1r} + (1-\alpha) c_p e_{1r}^p + \alpha c_1 z_{1r} + \alpha c_2 z_{2r} \end{bmatrix}.$$

Substituting (8) and (9) into the first derivative of e_{2l} and e_{2r}, we obtain

$$\begin{aligned} \dot{e}_{2l} &= \ddot{\phi}_1 - k^2 R \Lambda^2 (1 - 2\cos\Lambda t + \cos^2 \Lambda t) \\ &\quad \times \cos(k\tau + \psi) - kR\Lambda^2 \sin(k\tau + \psi) \sin \Lambda t, \\ \dot{e}_{2r} &= \ddot{\phi}_2 - k^2 R \Lambda^2 (1 - 2\cos\Lambda t + \cos^2 \Lambda t) \\ &\quad \times \cos(k\tau + \psi) + kR\Lambda^2 \sin(k\tau + \psi) \sin \Lambda t. \end{aligned}$$

$\ddot{\phi}_1$ and $\ddot{\phi}_2$ are derived from (3) and substituted into above equations. Substituting $\dot{e}_{2l}$ and $\dot{e}_{2r}$ into the first derivative of S_H, we obtain

$$\begin{aligned} \dot{S}_H &= \begin{bmatrix} \dot{S}_{Hl} \\ \dot{S}_{Hr} \end{bmatrix} = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \left(\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \right. \\ &\quad \left. + \begin{bmatrix} W_{1l} & W_{2l} & \cdots & W_{12l} \\ W_{1r} & W_{2r} & \cdots & W_{12r} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_{12} \end{bmatrix} \right) \\ &= \lambda_{inv} \left(\tau + W^T \boldsymbol{Y} \right). \end{aligned} \quad (15)$$

Where

$$\lambda_{inv} = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix},$$

λ_{ij} is obtained from $M^{-1}(q) = \{\lambda_{ij}\}$. And

$$W = \begin{bmatrix} W_l^T \\ W_r^T \end{bmatrix}^T = \begin{bmatrix} W_{1l} & W_{2l} & \cdots & W_{12l} \\ W_{1r} & W_{2r} & \cdots & W_{12r} \end{bmatrix}^T$$

$$\boldsymbol{Y} = [Y_1 \ \ Y_2 \ \ \cdots \ \ Y_{12}]^T,$$

where $Y_1 = \dot{\phi}_1, Y_2 = \dot{\phi}_2, Y_3 = e_{1l}^{p-1} e_{2l}, Y_4 = e_{1r}^{p-1} e_{2r}, Y_5 = e_{2l}, Y_6 = e_{2r}, Y_7 = z_{1l}, Y_8 = z_{2l}, Y_9 = z_{1r}, Y_{10} = z_{2r}, Y_{11} = \ddot{\phi}_{1d}, Y_{12} = \ddot{\phi}_{2d}$. Since the filter states are governed by (12) and the desired trajectories are twice continuously differentiable, we can assume these parameters are bounded.

Finally from these parametorizations we synthesize the control laws and adaptive laws as follows:

$$\begin{cases} \tau_1 = -\left(a\hat{W}_l^T |\boldsymbol{Y}| + \epsilon \right) sat(S_{Hl}, \delta) \\ \tau_2 = -\left(b\hat{W}_r^T |\boldsymbol{Y}| + \epsilon \right) sat(S_{Hr}, \delta) \\ \dot{\hat{W}}_l = \beta_{l1} |\boldsymbol{Y}| |S_{Hl}| + \beta_{l2} |\boldsymbol{Y}| |S_{Hr}| \\ \dot{\hat{W}}_r = \beta_{r1} |\boldsymbol{Y}| |S_{Hr}| + \beta_{r2} |\boldsymbol{Y}| |S_{Hl}| \end{cases} \quad (16)$$

Where $a, b, \beta_{l1}, \beta_{l2}, \beta_{r1}, \beta_{r2}$ are synthesis parameters, $\epsilon > 0$, $\hat{W}$ is estimation of W^*, $W_i^* > |W_i|, i = 1, ..., 12$. W_i^* means upper bound of $|W_i|$ and is unknown constant. $|\boldsymbol{Y}|$ and $sat(S_{H*}, \delta)$ are

$$|\boldsymbol{Y}| = [|Y_1|, |Y_2|, \cdots, |Y_{12}|]^T,$$

$$sat(S_{H*}, \delta) = \begin{cases} sgn(S_{H*}) & |S_{H*}| > \delta \\ \frac{S_{H*}}{\delta} & |S_{H*}| \le \delta, \quad (* = l, r) \end{cases}$$

where δ is the size of dead zone.

Remark Since $p < 1$, Y_3 and Y_4 may happen to be infinite. To overcome this drawback we introduce small positive number ϵ_0. For sufficiently small ϵ_0 we replace Y_3, Y_4, S_{Hl} and S_{Hr} with followings.

$$\begin{aligned} Y_3 &= \epsilon_0^{p-1} e_{2l}, \quad (|e_{1l}| < \epsilon_0) \\ Y_4 &= \epsilon_0^{p-1} e_{2r}, \quad (|e_{1r}| < \epsilon_0) \end{aligned} \quad (17)$$

$$\begin{aligned} S_{Hl} &= e_{2l} + \alpha c_1 z_{1l} + \alpha c_2 z_{2l} + \alpha c_e e_{1l} \\ &\quad + c_p (1-\alpha) \epsilon_0^{p-1} e_{1l}, \quad (|e_{1l}| < \epsilon_0) \\ S_{Hr} &= e_{2r} + \alpha c_1 z_{1r} + \alpha c_2 z_{2r} + \alpha c_e e_{1r} \\ &\quad + c_p (1-\alpha) \epsilon_0^{p-1} e_{1r}, \quad (|e_{1r}| < \epsilon_0) \end{aligned} \quad (18)$$

Then the following Lemmas are holded.

Lemma 1: Suppose that the control laws (16) are applied to a flexible dual arm space robot whose dynamic equation model being described by (3) and (4). Consider the following Lyapunov function.

$$V = \frac{1}{2} S_H^T \lambda S_H + \frac{1}{2} \tilde{W}^T \tilde{W} \quad (19)$$

Where

$$\lambda = \begin{bmatrix} \frac{1}{\lambda_{11}} & 0 \\ 0 & \frac{1}{\lambda_{22}} \end{bmatrix}, \ \tilde{W} = \begin{bmatrix} \tilde{W}_l \\ \tilde{W}_r \end{bmatrix} = \begin{bmatrix} \hat{W}_l - W_l^* \\ \hat{W}_r - W_r^* \end{bmatrix}.$$

Then the first derivative of V is negative definite, $\dot{V} < 0$, and $S_H \to 0, \tilde{W} \to 0$ as $t \to \infty$.

Proof: Taking the time derivative of V and using equations (15) and (16) yield

$$\begin{aligned}
\dot{V} &= \frac{1}{\lambda_{11}} S_{Hl}\dot{S}_{Hl} + \frac{1}{\lambda_{22}} S_{Hr}\dot{S}_{Hr} + \tilde{W_l}^T\dot{\tilde{W_l}} + \tilde{W_r}^T\dot{\tilde{W_r}} \\
&= -a|S_{Hl}|\hat{W_l}^T|\boldsymbol{Y}| - |S_{Hl}|\epsilon + S_{Hl}W_l^T\boldsymbol{Y} \\
&\quad -\frac{\lambda_{12}}{\lambda_{11}} bS_{Hl}\hat{W_r}^T|\boldsymbol{Y}|sat(S_{Hr},\delta) \\
&\quad -\frac{\lambda_{12}}{\lambda_{11}} S_{Hl}\epsilon sat(S_{Hr},\delta) \\
&\quad +\frac{\lambda_{12}}{\lambda_{11}} S_{Hl}W_r^T\boldsymbol{Y} - \frac{\lambda_{21}}{\lambda_{22}} aS_{Hr}\hat{W_l}^T|\boldsymbol{Y}|sat(S_{Hl},\delta) \\
&\quad -\frac{\lambda_{21}}{\lambda_{22}} S_{Hr}\epsilon sat(S_{Hl},\delta) + \frac{\lambda_{21}}{\lambda_{22}} S_{Hr}W_l^T\boldsymbol{Y} \\
&\quad -b|S_{Hr}|\hat{W_r}^T|\boldsymbol{Y}| - \beta_{l2}W_l^{T*}|\boldsymbol{Y}||S_{Hr}| \\
&\quad -|S_{Hr}|\epsilon + S_{Hr}W_r^T\boldsymbol{Y} + \beta_{l1}\hat{W_l}^T|\boldsymbol{Y}||S_{Hl}| \\
&\quad -\beta_{l1}W_l^{*T}|\boldsymbol{Y}||S_{Hl}| + \beta_{l2}\hat{W_l}^T|\boldsymbol{Y}||S_{Hr}| \\
&\quad +\beta_{r1}\hat{W_r}^T|\boldsymbol{Y}||S_{Hr}| - \beta_{r1}W_r^{*T}|\boldsymbol{Y}||S_{Hr}| \\
&\quad +\beta_{r2}\hat{W_r}^T|\boldsymbol{Y}||S_{Hl}| - \beta_{r2}W_r^{*T}|\boldsymbol{Y}||S_{Hl}|. \qquad (20)
\end{aligned}$$

For the simplicity we set $a = 1, b = 1, \beta_{l1} = 1, \beta_{r1} = 1, \beta_{l2} = 1, \beta_{r2} = 1$. Then $\dot{V}$ is

$$\begin{aligned}
\dot{V} &< -\left(1 - \frac{|\lambda_{12}|}{\lambda_{11}}\right)|S_{Hl}|\epsilon + \frac{2|\lambda_{12}|}{\lambda_{11}}\hat{W_r}^T|\boldsymbol{Y}||S_{Hl}| \\
&\quad -\left(1 - \frac{|\lambda_{21}|}{\lambda_{22}}\right)|S_{Hr}|\epsilon + \frac{2|\lambda_{21}|}{\lambda_{22}}\hat{W_l}^T|\boldsymbol{Y}||S_{Hr}| \qquad (21)
\end{aligned}$$

From the definition of λ_{inv}, we can show $\lambda_{11} = \lambda_{22} > 0, \lambda_{11} > |\lambda_{12}|, \lambda_{22} > |\lambda_{21}|$. And we set $\epsilon > \max\left(\lambda_1^*\hat{W_r}^T|\boldsymbol{Y}|, \lambda_2^*\hat{W_l}^T|\boldsymbol{Y}|\right)$. Where

$$\lambda_1^* = \frac{2|\lambda_{12}|}{\lambda_{11} - |\lambda_{12}|}, \quad \lambda_2^* = \frac{2|\lambda_{21}|}{\lambda_{22} - |\lambda_{21}|}.$$

Substituting these parameters into (21), we can obtain $\dot{V} < 0$. Then, $S_H \to 0, \tilde{W} \to 0$ as $t \to \infty$. ■

Lemma 2: Suppose $S_H \to 0$ as $t \to \infty$. Then $\{e_{1l}, e_{1r}\} \to 0$ and $\{\dot{e}_{1l}, \dot{e}_{1r}\} \to 0$ as $t \to \infty$.

Proof: Since $S_H \to 0$, $S_{Hl} \to 0$ and $S_{Hr} \to 0$. This leads us

$$\begin{aligned}
e_{2l} &= -\alpha c_e e_{1l} - (1-\alpha)c_p e_{1l}^p - \alpha c_1 z_{1l} - \alpha c_2 z_{2l} \\
e_{2r} &= -\alpha c_e e_{1r} - (1-\alpha)c_p e_{1r}^p - \alpha c_1 z_{1r} - \alpha c_2 z_{2r}. \qquad (22)
\end{aligned}$$

Since following discussion is satisfied in both the l's case and r's case, we omit the subscript $\{*\}$. Substituting (22) into (12), we obtain the following state equation.

$$\begin{aligned}
\dot{x_e} &= A_c x_e - b_c u(y) \\
y &= c_c x_e \qquad (23)
\end{aligned}$$

Where

$$A_c = \begin{bmatrix} 0 & 1 & 0 \\ -\omega_c^2 - \alpha c_1 & -2\omega_c - \alpha c_2 & -\alpha c_e \\ -\alpha c_1 & -\alpha c_2 & -\alpha c_e \end{bmatrix},$$

$b_c = [0 \;\; 1 \;\; 1]^T, c_c = [0 \;\; 0 \;\; 1], x_e = [z_1 \;\; z_2 \;\; e_1]^T, u(e_1) = w(y) = c_p(1-\alpha)e_1^p$.

Stability of the system (23) is shown by the Popov's Stability Criterion[8]. If following three conditions are satisfied for an arbitrarily small ϵ_0, then the origin of the system (23) is globally asymptotically stable.

1)A_c is Hurwitz and $[A_c, b_c]$ is controllable.
2)The nonlinearity $u(y)$ belongs to the sector $[0, k_0]$.
3)There exists a strictly positive number η such that

$$Re\left[(1 + j\eta\omega)G(j\omega)\right] \geq \epsilon_0 - \frac{1}{k_0}, \quad \forall\omega \geq 0. \qquad (24)$$

Where $G(s) = c_c(sI - A_c)^{-1}b_c$.

The first condition can be checked by Routh-Hurwitz method. In order to check the stability of A_c we have following conditions.

$$\begin{aligned}
\bar{\omega}_c = 2\omega_c + c_2\alpha + c_e\alpha &> 0 \\
\bar{\omega}_c(\omega_c^2 + c_1\alpha + 2\omega_c c_e\alpha) - \omega_c^2 c_e\alpha &> 0 \\
\omega_c^2 c_e\alpha &> 0
\end{aligned}$$

Since $c_e > 0$ is guaranteed by the synthesis method of the $FSOSM$ sliding surface, if α is chosen as

$$0 < \alpha < min\left\{1, \frac{2\omega_c}{|c_2 + c_e|}\right\}, \qquad (25)$$

then A_c is Hurwitz. Furthermore transfer function $G(s) = c_c(sI - A_c)^{-1}b_c$ is given by

$$G(s) = \frac{s^2 + 2\omega_c s + \omega_c^2}{s^3 + \bar{\omega}_c s^2 + (\omega_c^2 + c_1\alpha + 2\omega_c c_e\alpha)s + \omega_c^2 c_e\alpha}.$$

There is no pole-zero cancellation except on the hyper plane $c_1 - (c_2 + c_e)\omega_c = 0$ in the parameter space. Therefore $[A_c, b_c]$ is controllable almost everywhere in the parameter space.

From the definition of $u(y)$ the 2nd condition can be easily checked. Since $0 < \alpha < 1$, $c_p > 0$ and $0 < p < 1$, then $w(y)y > 0$. Furthermore

$$\frac{\partial w}{\partial y} = (1-\alpha)pc_p e_1^{p-1} \in (0, +\infty).$$

Then $0 \leq w(y)y \leq ky^2$. This means the 2nd condition is satisfied.

The last condition can be checked as follows. Letting $\Theta = Re(G(j\omega))$ and $\Xi = \omega Im(G(j\omega))$, (24) can be rewritten as

$$\Xi \leq \frac{1}{\eta}\left(\Theta + \frac{1}{k_0}\right) - \frac{\epsilon_0}{\eta} < \frac{1}{\eta}\left(\Theta + \frac{1}{k_0}\right) \qquad (26)$$

Where

$$\Xi = \frac{-\omega^6 - (2\omega_c^2 + 2\omega_c c_2\alpha - c_1\alpha)\omega^4}{[\omega_c^2 c_e\alpha - (2\omega_c + c_2\alpha + c_e\alpha)\omega^2]^2} \quad \frac{-(\omega_c^4 + \omega_c^2 c_1\alpha)\omega^2}{+(\omega_c^2 + c_1\alpha + 2\omega_c c_e\alpha - \omega^2)^2\omega^2}.$$

If

$$\alpha < \frac{8\omega_c^2 c_1 - 8\omega_c^3 c_2}{4\omega_c^2 c_2^2 + c_1^2 - 4\omega_c c_1 c_2} = \alpha_{bound},$$

then $\Xi < 0, \forall\omega$. Therefore, if α is selected as $0 < \alpha < \min(1, \alpha_{bound})$, there exist η and k_0 such that (24) is satisfied. From Popov's criterion origin of (23) is globally asymptotically stable, $x_e \to 0$ as $t \to \infty$. This completes the proof. ∎

Lemma 1 and Lemma 2 are summerized to the following theorem.

Theorem 1: Suppose that the control laws (16) are applied to the flexible dual arm planar space robot described by (3) and (4). Then $\{e_{1l}, e_{1r}\} \to 0$ as $t \to \infty$ and $\{\dot{e_{1l}}, \dot{e_{1r}}\} \to 0$ as $t \to \infty$.

IV. SIMULATIONS

TABLE I

PHYSICAL PARAMETERS

Arm length	L‡ .0 [m]
Arm rigidity	EI0 [Nm]
Mass density of the arms	ρ 0 [kg m³]
Mass of the arms	m_l‡ .0 [kg]
Mass of the tip of the arms	m_L2 .0 [kg]
Base moment of inertia	J_H6 .6 × 0 ³ [kg m²]
Arm moment of inertia	J_1=$J_2$0 .5[kg m 2]
Distance between the center of the base and the joint	d0 .1 [m]
Dumping coefficient	μ 0 .0

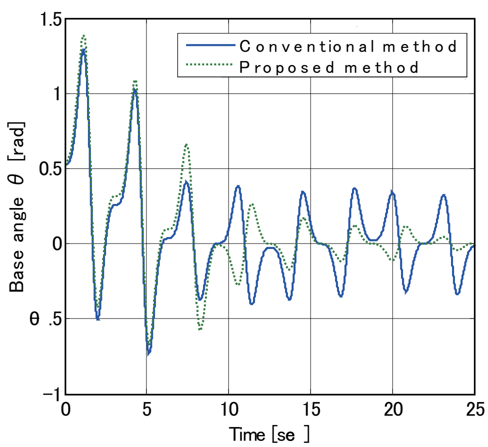

Fig. 2. Time histories of base angle

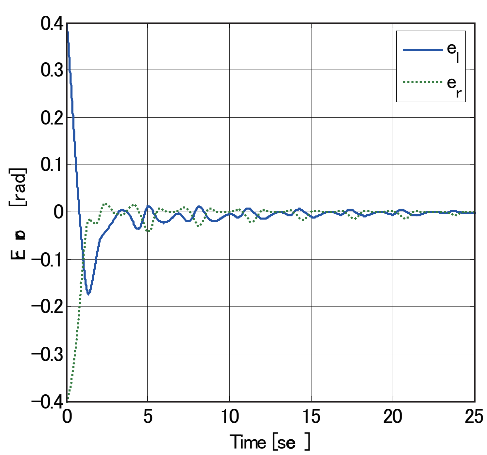

Fig. 3. Trajectory errors

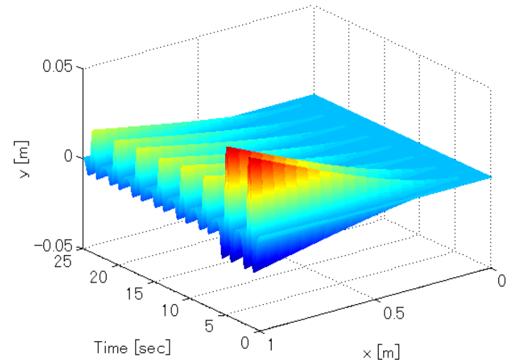

Fig. 4. Arm deflection(conventional)

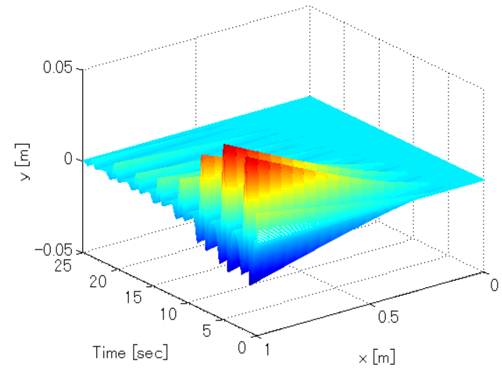

Fig. 5. Arm deflection(proposed)

In this section, results of the numerical simulations for the closed loop system are presented. Parameters used for simulations are shown in Table 1. Conservation law of angular momentum for the rigid planar space robot is written by (5), that is

$$\begin{aligned}\alpha_{rigid} &= -\frac{m_L L^2 + m_L dL \cos\phi_1}{J_{arm} + 2m_L dL(\cos\phi_1 + \cos\phi_2)} \\ &= -\frac{10000(10 + \cos\phi_1)}{202323 + 20000(\cos\phi_1 + \cos\phi_2)}, \\ \beta_{rigid} &= -\frac{m_L L^2 + m_L dL \cos\phi_2}{J_{arm} + 2m_L dL(\cos\phi_1 + \cos\phi_2)} \\ &= -\frac{10000(10 + \cos\phi_2)}{202323 + 20000(\cos\phi_1 + \cos\phi_2)},\end{aligned}$$

where $J_{arm} = J_H + 2m_L d^2 + 2m_L L^2$. In these calculations we assume that rigid arm link does not have mass and mass m_L is concentrated on the tip of the arm. For this system we can obtain the desired trajectories $\phi_{1d}(t), \phi_{2d}(t)$. Where

$$R = -1.818k|\theta|^{\frac{1}{3}}, \ k = sgn(\theta).$$

Control torques are calculated from the dynamic model whose arm is rigid without mass distribution and has point mass concentrated at the tip of arm.

On the other hand control parameters of the control laws (16) are given as $a = 1, b = 1, \beta_{l1} = 1, \beta_{r1} = 1, \epsilon = 30[Nm], \delta = 0.3[rad], \epsilon_0 = 0.000001[rad], \omega_c = 2[rad/sec], a' = 10, c_e = 0.01, c_1 = -3.7879, c_2 = -3.2456, c_p = 2, p = 3/5, \alpha = 0.6$. a' assigns corner frequency of the $FSOSM$.

Fig.2,3, 4 and 5 show the simulation results. In these simulations parameter $\Lambda = 2$ representing the angular velocity of the arm motion is set $2[rad/sec]$. Fig.2 shows the time histories of the base attitude θ. Solid line denotes the performance of the conventional controller which is proposed for the control of the rigid planar space robot[4]. Dashed line denotes the performance of the proposed controller. Base attitude controlled by the proposed controller converges to the origin. On the other hand base attitude controlled by the conventional controller cannot be converged and it is still remained vibration. Fig.3 shows the trajectory errors $\{e_l, e_r\}$. Both errors are asymptotically converged to the origin. Fig.4 and Fig.5 show the elastic deflections of the arm as a function of two independent variables $\{t, x\}$. In both figures the arm deflection is enlarged at the beginning of performance. However as shown in these figures, the elastic deflection caused by the proposed controller is vanished faster than that caused by the conventional controller.

V. EXPERIMENTS

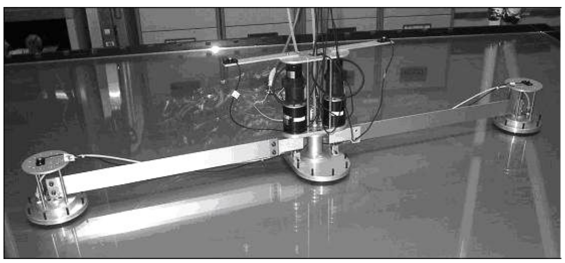

Fig. 6. Prototype of the planar space robot

In order to investigate real mechanical dynamics and also to demonstrate the validity and effectiveness of the proposed control laws in a practical system, the prototype of the planar space robot floated by air on the horizontal table with air is employed. Fig.6 shows a photograph of the prototype of the space robot with symmetric two arms connected to the base satellite with revolute joints. Two arms are made from aluminum beam and physical parameters of these arms are same, $450[mm]$ length, $2[mm]$ thickness and $40[mm]$ width. On the tips of two arms and center of the base air pads are equipped separately. Through these air pads, compressed air is exhausted on the horizontal table covered with smooth glass to float the prototype robot. Control parameters are same as those used for simulations except $\Lambda = 0.1$.

Fig.7 and 8 show the experimental results. Controller parameters are same as those in the simulations except $\Lambda = 0.1$. Fig.7 shows the time history of the trajectory tracking error e_r. From this result it can be said that the tracking error is sufficiently reduced. Fig.8 shows the deflection of the tip coordinate. These experimental results demonstrate the usefulness and validity of the proposed controller.

VI. CONCLUSION

A new method for attitude control of flexible dual arm planar space robot has been proposed. It is based on the geometric phase and adaptive sliding mode control with a synthesized hybrid sliding surface. The hybrid sliding surface makes it posible to minimize excitation of flexible modes caused by the conventional sliding mode control. The

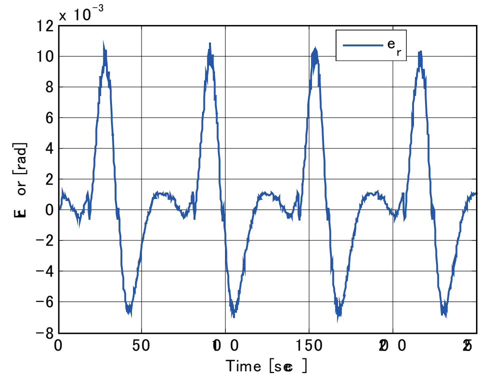

Fig. 7. Trajectory errors

proposed controller has been applied to the experimental system of the flexible planar robot floated on the flat plane by the air. By simulation and experimental results it has been demonstrated that the proposed control laws and adaptive laws overcome the problems caused by the arm flexibility and parameter uncertainty.

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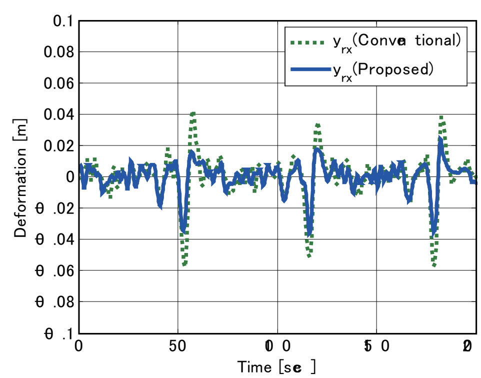

Fig. 8. Time histories of arm deflection