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Global Trajectory Tracking Through Static Feedback for Robot Manipulators With Input Saturations

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Abstract— **In this work, two globally stabilizing bounded controllers for the tracking control of robot manipulators with saturating inputs are proposed. They may be seen as extensions of the so-called PD+ algorithm to the bounded input case. With respect to previous works, our approaches give a global solution to the problem through static feedback. Moreover, they are not defined using a specific sigmoidal function, but any one on a set of saturating functions. Furthermore, the bound of such functions is explicitly considered in their definition. Hence, the control gains are not tied to satisfy any *saturation-avoidance* inequality and may consequently take any positive value. The efficiency of the proposed schemes is corroborated through experimental results.**

I. INTRODUCTION

A fundamental scheme for the global trajectory tracking of robot manipulators is the well-known PD+ control law proposed in [6]. Such an algorithm considers a continuous calculation of a special form of the robot dynamics, where the current position vector is considered at every of its terms (gravity, inertial, and centrifugal and Coriolis calculated force vectors), the desired acceleration vector is involved in the computed inertial force vector, and both the current and desired velocity vectors are considered in the Coriolis and centrifugal calculated force vector. This gives rise to a strategic closed loop form wherefrom it is clear that the desired trajectory is a solution of the closed-loop system. But such terms do not guarantee, by themselves, the (global) stabilization towards the desired trajectory. This is achieved through the additional consideration of position-error (P) and velocity-error (D) linear correction terms. Nevertheless, because of the linearity of such P (*proportional* on the position error) and D (proportional on the *derivative* of the position error) terms and that of the computed Coriolis and centrifugal force vector on the current velocity vector, the PD+ controller turns out to be unbounded. Consequently, when such an algorithm is implemented in an actual application, the resulting control signals may try to force the actuators to go beyond their natural capabilities, undergoing the well-known phenomenon of *saturation*. Unfortunately, this may give rise to undesirable effects, as pointed out for instance in [1] and [4].

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In order to avoid the above mentioned problem, a bounded dynamical extension of the PD+ algorithm has been proposed in [5]. To begin with, the current velocity vector is replaced by the desired velocity trajectory in the computed Coriolis and centrifugal force vector. Hence, by considering twice continuously differentiable desired position trajectories whose 1st and 2nd time-derivative (*i.e.* velocity and acceleration) vectors are uniformly bounded, the computed (special form of the) system dynamics turns out to be bounded. Further, the P and D gains are applied to sigmoidal functions —specifically, the hyperbolic tangent— of the closed loop error variables, giving rise to bounded nonlinear P and D terms. Moreover, an auxiliary (internal) dynamical subsystem is considered for the asymptotic estimation of the system velocity error variables. Consequently, only position measurements are involved in the developed algorithm. In a frictionless setting, such a control scheme was proven to semi-globally stabilize the closed-loop system.

By considering viscous friction in the open-loop dynamics, a globally stabilizing version of the control law in [5] was achieved in [8]. The developed scheme keeps the structure of the controller in [5], but the viscous friction force vector is added to the computed robot dynamics, replacing the current velocity vector by the desired velocity trajectory. Under such considerations, global tracking is achieved for suitable trajectories.

Two alternative dynamical approaches were proposed in [1]. Both consider P and D correction terms where the hyperbolic tangent of the tracking error and filtered tracking error variables, respectively, are involved. The first one includes a bounded adaptive compensation of the robot dynamics involving position and velocity measurements. The second one, on the contrary, is free of velocity measurements, keeping a *Computed-Torque*-like structure [2, Ch. 10]. It considers the same form of the gravity, viscous friction, and Coriolis and centrifugal calculated force vectors used in [8], but a special form of inertial (complemented) force vector where the bounded nonlinear P and D terms are included. Semi-global tracking is achieved by both controllers.

Let us note that by the way the bounded nonlinear P and D terms are defined in the previous works, the P and D gains are tied to satisfy a *saturation-avoidance* inequality (since these define the bounds of the P and D terms). Consequently, such control gains cannot take *any* (positive) value, which restricts their *performance-adjustment* natural role.

When velocity measurements are unavailable or highly noisy, the algorithms in [5], [8], and the second one in [1] may be considered to give an appropriate solution to the tracking problem. On the contrary, the first algorithm in [1] may be suitably applied when the system parameters are uncertain. Nevertheless, none of the above-cited works solve the global tracking problem through a static controller involving all the system states (positions and velocities) and parameters. The design of such a scheme does not only represent an analytical challenge, but its implementation would give rise to faster closed-loop responses. Indeed, involving dynamic estimations of some states or parameters in the control system generally slows down the stabilization time and adds inertial effects that commonly give rise to oscillating transient responses. From this point of view, static controllers expressed in terms of the whole system data (states and parameters) remain an important choice when acceptable estimations of such information are available.

In this work, two globally stabilizing bounded static controllers for the trajectory tracking of robot manipulators with saturating inputs are proposed. They may be seen as extensions of the PD+ algorithm to the bounded input case. With respect to the above-mentioned previous works, they (both) give a global solution to the problem through static feedback. Moreover, they are not defined using a specific sigmoidal function, but any one on a set of *saturating* functions. Furthermore, the bound of such functions is explicitly considered in their definition. These are consequently applied to the whole linear P and D expressions, giving the P and D gains the liberty to adopt any positive value.

The work is organized as follows. Section II states the general n-degree-of-freedom (n-DOF) serial rigid robot manipulator open-loop dynamics and some of its main properties, as well as considerations and definitions that are involved throughout the study. In Section III, the proposed controllers are presented. Section IV states the main results, where the stability analyses are developed and the control objective is proved to be achieved (for both proposed controllers). Experimental results are presented in Section V. Finally, conclusions are given in Section VI.

II. Preliminaries

The following notation is used throughout the paper. $\mathbb{R}$ and $\mathbb{R}_+$ denote the set of real and nonnegative real numbers respectively, $\mathbb{R}^n$ and $\mathbb{R}^n_+$ represent the set of n-dimensional vectors whose elements are real and nonnegative real numbers respectively, and $\mathbb{R}^{n\times m}$ is the set of $n\times m$ matrices whose elements are real numbers. We denote 0_n the origin of $\mathbb{R}^n$. Let $x\in\mathbb{R}^n$ and $A\in\mathbb{R}^{n\times m}$. x_i represents the i^{th} element of x. $\|\cdot\|$ stands for the standard Euclidean vector norm and induced matrix norm, *i.e.* $\|x\| = \left[\sum_{i=1}^n x_i^2\right]^{1/2}$ and $\|A\| = \left[\lambda_{\max}(A^TA)\right]^{1/2}$, where $\lambda_{\max}(A^TA)$ represents the maximum eigenvalue of A^TA. Let $\mathcal{A}$ and $\mathcal{E}$ be subsets of some vector spaces $\mathbb{A}$ and $\mathbb{E}$ respectively. We denote $\mathcal{C}^m(\mathcal{A};\mathcal{E})$ the set of m-times continuously differentiable functions from $\mathcal{A}$ to $\mathcal{E}$. Consider a continuous-time function $h\in\mathcal{C}^2(\mathbb{R}_+;\mathcal{E})$. The time-derivative and second-time-derivative of h are respectively represented as $\dot h$ and $\ddot h$, *i.e.* $\dot h : t\mapsto \frac{d}{dt}h$ and $\ddot h : t\mapsto \frac{d^2}{dt^2}h$.

Let us consider the general n-DOF serial rigid robot manipulator dynamics with viscous friction [9, §6.2]:

$$D(q)\ddot q + C(q,\dot q)\dot q + F\dot q + g(q) = \tau \tag{1}$$

where $q, \dot q, \ddot q\in\mathbb{R}^n$ are, respectively, the position (generalized coordinates), velocity and acceleration vectors, $D(q)\in\mathbb{R}^{n\times n}$ is the inertia matrix, and $C(q,\dot q)\dot q$, $F\dot q$, $g(q)$, $\tau\in\mathbb{R}^n$ are, respectively, the vectors of Coriolis and centrifugal, viscous friction, gravity, and external input generalized forces, with F being a constant, positive definite, diagonal (viscous friction coefficient) matrix, *i.e.* $F = \mathrm{diag}[f_1,\dots,f_n]$, with $f_i>0$, $\forall i\in\{1,\dots,n\}$. The terms of such a dynamical model satisfy some well-known properties (see for instance [2, Ch. 4]). Some of them are recalled here.

Property 1: The inertia matrix $D(q)$ is a positive definite symmetric matrix satisfying $d_mI\le D(q)\le d_MI$, $\forall q\in\mathbb{R}^n$, where I denotes the $n\times n$ identity matrix, for some positive constants $d_m\le d_M$.

Property 2: The Coriolis matrix $C(q,\dot q)$ satisfies:

2.1. $x^T\left[\frac{1}{2}\dot D(q,\dot q) - C(q,\dot q)\right]x = 0$, $\forall x,q,\dot q\in\mathbb{R}^n$;
2.2. $\dot D(q,\dot q) = C(q,\dot q) + C^T(q,\dot q)$, $\forall q,\dot q\in\mathbb{R}^n$;
2.3. $C(w,x+y)z = C(w,x)z + C(w,y)z$, $\forall w,x,y,z\in\mathbb{R}^n$;
2.4. $C(x,y)z = C(x,z)y$, $\forall x,y,z\in\mathbb{R}^n$;
2.5. $\|C(x,y)z\|\le k_c\|y\|\|z\|$, $\forall x,y,z\in\mathbb{R}^n$, for some constant $k_c\ge 0$.

Property 3: The gravity vector satisfies $\|g(q)\|\le\gamma$, $\forall q\in\mathbb{R}^n$, for some positive constant γ, or equivalently, every element of the gravity vector, $g_i(q), i=1,\dots,n$, satisfies $|g_i(q)|\le\gamma_i$, $\forall q\in\mathbb{R}^n$, for some positive constants $\gamma_i, i=1,\dots,n$.

Property 4: The viscous friction coefficient matrix satisfies $f_m\|x\|^2\le x^TFx\le f_M\|x\|^2$, $\forall x\in\mathbb{R}^n$, where $0<f_m\triangleq\min_i\{f_i\}\le\max_i\{f_i\}\triangleq f_M$.

Let us suppose that the absolute value of each input τ_i is constrained to be smaller than a given saturation bound $T_i>0$, *i.e.* $|\tau_i|\le T_i$, $i=1,\dots,n$. In other words, if u_i represents the control signal (controller output) relative to the i^{th} DOF, then

$$\tau_i = T_i\mathrm{sat}\left(\frac{u_i}{T_i}\right) \tag{2}$$

$i=1,\dots,n$, where $\mathrm{sat}(\cdot)$ is the standard saturation function, *i.e.* $\mathrm{sat}(\varsigma) = \mathrm{sign}(\varsigma)\min\{|\varsigma|,1\}$.

The control scheme proposed in this work involves a special type of functions fitting the following definition.

Definition 1: Given a positive constant M, a function $\sigma:\mathbb{R}\to\mathbb{R}$ is said to be a **generalized saturation** with bound M, if it is locally Lipschitz, nondecreasing, and satisfies:

1) $\varsigma\sigma(\varsigma)>0$, $\forall\varsigma\ne 0$;
2) $|\sigma(\varsigma)|\le M$, $\forall\varsigma\in\mathbb{R}$.

A strictly increasing continuously differentiable function fulfilling Definition 1 has the following properties.

Lemma 1: Let $\sigma:\mathbb{R}\to\mathbb{R}:\varsigma\mapsto\sigma(\varsigma)$ be a strictly increasing continuously differentiable generalized saturation function with bound M, k and k_0 be positive constants, and

$\sigma'(\varsigma)$ denote the derivative of σ with respect to its argument, *i.e.* $\sigma'(\varsigma) = \frac{d\sigma}{d\varsigma}(\varsigma)$. Then

1) $y[\sigma(x+y) - \sigma(x)] > 0$, $\forall y \neq 0$, $\forall x \in \mathbb{R}$;
2) $\sigma(y)[\sigma(x+y) - \sigma(x)] > 0$, $\forall y \neq 0$, $\forall x \in \mathbb{R}$;
3) $\lim_{|\varsigma|\to\infty} \sigma'(\varsigma) = 0$;
4) $\sigma'(\varsigma)$ is positive and bounded, *i.e.* there exists a constant $\sigma'_M \in (0, \infty)$ such that $0 < \sigma'(\varsigma) \leq \sigma'_M$, $\forall \varsigma \in \mathbb{R}$;
5) $\frac{\sigma^2(k\varsigma)}{2k\sigma'_M} \leq \int_0^\varsigma \sigma(kr)dr \leq \frac{k\sigma'_M \varsigma^2}{2}$, $\forall \varsigma \in \mathbb{R}$;
6) $\int_0^\varsigma \sigma(kr)dr > 0$, $\forall \varsigma \neq 0$;
7) $\int_0^\varsigma \sigma(kr)dr \to \infty$ as $|\varsigma| \to \infty$;
8) $|\sigma(kx + k_0 y) - \sigma(k_0 y)| \leq \sigma'_M k|x|$, $\forall x, y \in \mathbb{R}$.
9) $|\sigma(kx)| \leq \sigma'_M k|x|$, $\forall x \in \mathbb{R}$.

Proof:

1) Let $x, y, z \in \mathbb{R}$. Since σ is strictly increasing, we have that $\sigma(z) > \sigma(x) \iff z > x$ and $\sigma(z) < \sigma(x) \iff z < x$. Let $z = y + x$. Then $\sigma(y+x) - \sigma(x) > 0 \iff y > 0$ and $\sigma(y+x) - \sigma(x) < 0 \iff y < 0$, $\forall x \in \mathbb{R}$, wherefrom it follows that $y[\sigma(y+x) - \sigma(x)] > 0$, $\forall y \neq 0$, $\forall x \in \mathbb{R}$.
2) Let $x, y \in \mathbb{R}$. From the proof of item 1 of the Lemma and item 1 of Definition 1, we have that $\sigma(y+x) - \sigma(x) > 0 \iff y > 0 \iff \sigma(y) > 0$ and $\sigma(y+x) - \sigma(x) < 0 \iff y < 0 \iff \sigma(y) < 0$, $\forall x \in \mathbb{R}$, wherefrom it follows that $\sigma(y)[\sigma(y+x) - \sigma(x)] > 0$, $\forall y \neq 0$, $\forall x \in \mathbb{R}$.
3) Since σ is a continuous function that keeps the sign of its argument (according to item 1 of Definition 1), and is strictly increasing and bounded by M, there exists a positive constant $c \leq M$ such that $\lim_{|\varsigma|\to\infty} |\sigma(\varsigma)| = c$, or equivalently $\lim_{|\varsigma|\to\infty} \sigma(\varsigma) = c \cdot \mathrm{sign}(\varsigma)$. Hence, we have that
$$\begin{aligned}\lim_{|\varsigma|\to\infty} \sigma'(\varsigma) &= \lim_{|\varsigma|\to\infty} \lim_{h\to 0} \frac{\sigma(\varsigma+h) - \sigma(\varsigma)}{h} \\ &= \lim_{h\to 0} \lim_{|\varsigma|\to\infty} \frac{\sigma(\varsigma+h) - \sigma(\varsigma)}{h} \\ &= \lim_{h\to 0} \frac{c \cdot \mathrm{sign}(\varsigma) - c \cdot \mathrm{sign}(\varsigma)}{h} = 0\end{aligned}$$
4) Since σ is a continuously differentiable and strictly increasing function, we have that $\sigma'(\varsigma)$ exists and is continuous on $\mathbb{R}$, and $\sigma'(\varsigma) > 0$, $\forall \varsigma \in \mathbb{R}$. Furthermore, in view of its continuity, $\sigma'(\varsigma)$ is bounded on any compact subset of $\mathbb{R}$. Thus, its boundedness will be uniform if $\lim_{|\varsigma|\to\infty} \sigma'(\varsigma) < \infty$. Since $\lim_{|\varsigma|\to\infty} \sigma'(\varsigma) = 0$ (according to item 3 of the Lemma), we conclude that $\sigma'(\varsigma)$ is uniformly bounded, *i.e.* $\exists \sigma'_M > 0$ such that $\sigma'(\varsigma) \leq \sigma'_M$, $\forall \varsigma \in \mathbb{R}$.
5) From continuous differentiability —implying Lipschitz-continuity— of σ and item 4 of the Lemma, it follows that
$$\frac{\sigma'(k\varsigma)}{\sigma'_M} |\sigma(k\varsigma)| \leq |\sigma(k\varsigma)| \leq \sigma'_M |k\varsigma|$$
$\forall \varsigma \in \mathbb{R}$, wherefrom, considering that σ has the sign of its argument (according to item 1 of Definition 1), we have that
$$\int_0^\varsigma \frac{\sigma(kr)}{\sigma'_M} \sigma'(kr)dr \leq \int_0^\varsigma \sigma(kr)dr \leq \int_0^\varsigma \sigma'_M kr dr$$
wherefrom we get
$$\frac{\sigma^2(k\varsigma)}{2k\sigma'_M} \leq \int_0^\varsigma \sigma(kr)dr \leq \frac{k\sigma'_M \varsigma^2}{2}$$
$\forall \varsigma \in \mathbb{R}$.
6) Strict positivity of $\int_0^\varsigma \sigma(kr)dr$ on $\mathbb{R} \setminus \{0\}$ follows from item 5 of the Lemma, by noting (from item 1 of Definition 1) that $\sigma^2(k\varsigma) > 0$, $\forall \varsigma \neq 0$.
7) From the continuous differentiability and strictly increasing characters of σ, and its satisfaction of item 4 of the Lemma, we have that $\sigma'(k\sigma)$ is continuous, positive, and bounded on $[-a, a]$, for any $a > 0$, in such a way that
$$0 < \inf_{r\in[-a,a]} \sigma'(kr) \leq \sigma'(k\varsigma) \leq \sup_{r\in[-a,a]} \sigma'(kr) \leq \sigma'_M \tag{3}$$
$\forall \varsigma \in [-a, a]$. Let us consider a positive constant $k_a \leq \inf_{r\in[-a,a]} \sigma'(kr)$. Then, from (3), we have that
$$\left| k_a a \,\mathrm{sat}\left(\frac{\varsigma}{a}\right) \right| \leq |\sigma(k\varsigma)|$$
$\forall \varsigma \in \mathbb{R}$, wherefrom we get
$$S_a(\varsigma) = \int_0^\varsigma k_a a \,\mathrm{sat}\left(\frac{r}{a}\right) dr \leq \int_0^\varsigma \sigma(k\varsigma)dr$$
$\forall \varsigma \in \mathbb{R}$, with
$$S_a(\varsigma) \triangleq \begin{cases} \frac{k_a}{2}\varsigma^2 & \forall |\varsigma| \leq a \\ k_a a \left(|\varsigma| - \frac{a}{2}\right) & \forall |\varsigma| > a \end{cases}$$
Thus, from these expressions we observe, on the one hand, that $\lim_{|\varsigma|\to\infty} S_a(\varsigma) \leq \lim_{|\varsigma|\to\infty} \int_0^\varsigma \sigma(kr)dr$, and, on the other, that $S_a(\varsigma) \to \infty$ as $|\varsigma| \to \infty$, wherefrom we conclude that $\int_0^\varsigma \sigma(kr)dr \to \infty$ as $|\varsigma| \to \infty$.
8) Let $w, x, y, z \in \mathbb{R}$. From continuous differentiability of σ and item 4 of the Lemma, we have that σ satisfies the Lipschitz condition globally on $\mathbb{R}$ with σ'_M as Lipschitz constant (see for instance [3, Lemma 3.3]), *i.e.* $|\sigma(w) - \sigma(z)| \leq \sigma'_M |w - z|$, $\forall w, z \in \mathbb{R}$. By taking $w = kx + k_0 y$ and $z = k_0 y$, we get $|\sigma(kx + k_0 y) - \sigma(k_0 y)| \leq \sigma'_M k|x|$, $\forall x, y \in \mathbb{R}$.
9) From item 8 of the Lemma with $y = 0$, we have that $|\sigma(kx)| \leq \sigma'_M k|x|$, $\forall x \in \mathbb{R}$.

■

We state the **control objective** as the global stabilization of the robot configuration vector variable, q, towards a desired trajectory vector, $q_d(t)$, through bounded control signals avoiding input saturations (*i.e.* such that $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \ldots, n$, $\forall t \geq 0$; see (2)).

III. Proposed Controllers

The following assumption turns out to be crucial within the analytical setting considered in this work:

Assumption 1: $T_i > \gamma_i$, $\forall i \in \{1, \ldots, n\}$.

Further, in order to guarantee the achievement of the stated control objective, the proposed scheme is restricted to desired trajectory vectors meeting the following:

Assumption 2: The desired trajectory vector, $q_d(t)$, is a twice continuously differentiable function, *i.e.* $q_d \in \mathcal{C}^2(\mathbb{R}_+; \mathbb{R}^n)$, satisfying

$$\sup_{t\geq 0} \|\dot{q}_d(t)\| \leq B_{dv} \tag{4a}$$

and

$$\sup_{t\geq 0} \|\ddot{q}_d(t)\| \leq B_{da} \tag{4b}$$

for some (velocity and acceleration vector) bounds such that

$$(B_{dv}, B_{da}) \in \mathcal{B}_1 \cup \mathcal{B}_2 \tag{5}$$

where

$$\mathcal{B}_i \triangleq \{(\xi, \zeta) \in \mathbb{R}_+^2 \mid \xi < B_{vi} \, , \, \zeta < B_{ai}\} \tag{6}$$

$i = 1, 2,$

$$B_{v1} \triangleq \begin{cases} \min\left\{\frac{f_m}{k_c} \, , \, B_{11}\right\} & \text{if } k_c > 0 \\ B_{10} & \text{if } k_c = 0 \end{cases} \tag{7a}$$

$$B_{a1} \triangleq \frac{\Delta_m - k_c B_{dv}^2 - f_M B_{dv}}{d_M} \tag{7b}$$

$$B_{11} \triangleq -\frac{f_M}{2k_c} + \sqrt{\left(\frac{f_M}{2k_c}\right)^2 + \frac{\Delta_m}{k_c}} \tag{7c}$$

$$B_{10} \triangleq \frac{\Delta_m}{f_M} \tag{7d}$$

and

$$B_{a2} \triangleq \frac{\Delta_m}{d_M} \tag{8a}$$

$$B_{v2} \triangleq \begin{cases} \min\left\{\frac{f_m}{k_c} \, , \, B_{21}\right\} & \text{if } k_c > 0 \\ B_{20} & \text{if } k_c = 0 \end{cases} \tag{8b}$$

$$B_{21} \triangleq -\frac{f_M}{2k_c} + \sqrt{\left(\frac{f_M}{2k_c}\right)^2 + \frac{\Delta_m - d_M B_{da}}{k_c}} \tag{8c}$$

$$B_{20} \triangleq \frac{\Delta_m - d_M B_{da}}{f_M} \tag{8d}$$

with

$$\Delta_m \triangleq \min_i \{T_i - \gamma_i\} \tag{9}$$

Under Assumptions 1 and 2, we propose an SP-SD+ control scheme of the form

$$u = -s_2(K_2\bar{q}) - s_1(K_1\dot{\bar{q}}) + \tau_c(q, \dot{q}_d, \ddot{q}_d) \tag{10}$$

and an SPD+ control law of the form

$$u = -s_0(K_2\bar{q} + K_1\dot{\bar{q}}) + \tau_c(q, \dot{q}_d, \ddot{q}_d) \tag{11}$$

where

$$\tau_c(q, \dot{q}_d, \ddot{q}_d) = D(q)\ddot{q}_d + C(q, \dot{q}_d)\dot{q}_d + F\dot{q}_d + g(q)$$

$\bar{q} \triangleq q - q_d(t)$; K_1 and K_2 are positive definite diagonal matrices, *i.e.* $K_1 = \operatorname{diag}[k_{11}, \ldots, k_{1n}]$ and $K_2 = \operatorname{diag}[k_{21}, \ldots, k_{2n}]$ with $k_{1i} > 0$ and $k_{2i} > 0$ for all $i = 1, \ldots, n$; and $s_j : \mathbb{R}^n \to \mathbb{R}^n : x \mapsto s_j(x) = \big(\sigma_{j1}(x_1), \ldots, \sigma_{jn}(x_n)\big)^T$, $j = 0, 1, 2$, with $\sigma_{ji}(\cdot)$, $i = 1, \ldots, n$, being **strictly increasing continuously differentiable generalized saturation functions** with bounds M_{ji} satisfying

$$M_{1i} + M_{2i} < T_i - d_M B_{da} - k_c B_{dv}^2 - f_M B_{dv} - \gamma_i \tag{12}$$

(see Properties 1, 2.5, 3, and 4), $\forall i = 1, \ldots, n$, in the SP-SD+ case (controller (10)), and

$$M_{0i} < T_i - d_M B_{da} - k_c B_{dv}^2 - f_M B_{dv} - \gamma_i \tag{13}$$

$\forall i = 1, \ldots, n$, in the SPD+ case (controller (11)). Let us note that the satisfaction of Assumptions 1 and 2 guarantees the existence of (positive) bounds M_{1i} and M_{2i} fulfilling (12) and M_{0i} meeting (13).[1]

IV. Main Results

Proposition 1: Consider the system (1),(2) with the control law (10). Under Assumptions 1 and 2, and the satisfaction of inequalities (12), global asymptotic stabilization of the closed-loop system solutions $q(t)$ towards the desired trajectory vector $q_d(t)$ is guaranteed with $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \ldots, n$, $\forall t \geq 0$.

Proof: From (10), (12), and Properties 1, 2.5, 3 and 4, one sees that $|u_i(t)| \leq M_{1i} + M_{2i} + d_M B_{da} + k_c B_{dv}^2 + f_M B_{dv} + \gamma_i < T_i$, $i = 1, \ldots, n$, $\forall t \geq 0$. From this and (2) it follows that $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \ldots, n$, $\forall t \geq 0$. We now focus on the stability analysis. The closed-loop dynamics takes the form

$$D(q)\ddot{\bar{q}} + [C(q, \dot{q}) + C(q, \dot{q}_d)]\dot{\bar{q}} + F\dot{\bar{q}} + s_1(K_1\dot{\bar{q}}) + s_2(K_2\bar{q}) = 0_n \tag{14}$$

where Property 2.4 has been used (observe from the definition of $\bar{q}$, stated in Section III, that $q = \bar{q} + q_d(t)$ and $\dot{q} = \dot{\bar{q}} + \dot{q}_d(t)$). Let us define the scalar function

$$\begin{aligned} V_1(t, \bar{q}, \dot{\bar{q}}) = \frac{1}{2}\dot{\bar{q}}^T D(\bar{q} + q_d(t))\dot{\bar{q}} + \int_0^{\bar{q}} s_2^T(K_2 r)dr \\ + \varepsilon_1 s_2^T(K_2\bar{q}) D(\bar{q} + q_d(t))\dot{\bar{q}} \end{aligned} \tag{15}$$

[1] Observe that Assumption 1 guarantees positivity of B_{11} and B_{10} in (7a) (see (7c), (7d), and (9)) and of the right-hand-side expression in (8a) (see (9)). Furhter, the definitions of B_{v1} in (7a) and B_{a2} in (8a) respectively ensure positivity of the right-hand-side expression in (7b) and of B_{21} and B_{20} in (8b) (see (8c) and (8d)). Finally, the definitions of B_{a1} in (7b) and B_{v2} in (8b) guarantee positivity of the right-hand-side expression in (12) and (13).

where $\int_0^{\bar{q}} s_2^T(K_2 r)dr = \sum_{i=1}^n \int_0^{\bar{q}_i} \sigma_{2i}(k_{2i}r_i)dr_i$, and ε_1 is a positive constant satisfying[2]

$$\varepsilon_1 < \min\left\{ \frac{f_m - k_c B_{dv}}{k_c B_{2M} + d_M k_{2M}\sigma'_{2M} + \left(k_c B_{dv} + \frac{\sigma'_{1M}k_{1M}+f_M}{2}\right)^2}, \sqrt{\frac{d_m}{2d_M^2 k_{2M}\sigma'_{2M}}} \right\} \quad (16)$$

with $\sigma'_{jM} \triangleq \max_i\{\sigma'_{jiM}\}$ (see item 4 of Lemma 1) and $k_{jM} \triangleq \max_i\{k_{ji}\}$, $j = 1, 2$, and $B_{2M} \triangleq \left[\sum_{i=1}^n M_{2i}^2\right]^{1/2}$. Let us note, from Property 1 and items 5 and 9 of Lemma 1, that

$$\frac{1}{2}\int_0^{\bar{q}} s_2^T(K_2 r)dr + \frac{1}{2}\begin{pmatrix}\|s_2(K_2\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix}^T P_{11}\begin{pmatrix}\|s_2(K_2\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix} \leq V_1(t,\bar{q},\dot{\bar{q}}) \leq \frac{1}{2}\begin{pmatrix}\|\bar{q}\| \\ \|\dot{\bar{q}}\|\end{pmatrix}^T P_{12}\begin{pmatrix}\|\bar{q}\| \\ \|\dot{\bar{q}}\|\end{pmatrix}$$

where

$$P_{11} = \begin{pmatrix} \frac{1}{2k_{2M}\sigma'_{2M}} & -\varepsilon_1 d_M \\ -\varepsilon_1 d_M & d_m \end{pmatrix}$$

and

$$P_{12} = \begin{pmatrix} k_{2M}\sigma'_{2M} & \varepsilon_1 d_M k_{2M}\sigma'_{2M} \\ \varepsilon_1 d_M k_{2M}\sigma'_{2M} & d_M \end{pmatrix}$$

Further, since $\varepsilon_1 < \sqrt{\frac{d_m}{2d_M^2 k_{2M}\sigma'_{2M}}}$ (see (16)), one can verify (after several basic developments) that P_{11} and P_{12} are positive definite symmetric matrices. From this and items 6 and 7 of Lemma 1, one sees that $V_1(t,\bar{q},\dot{\bar{q}})$ is positive definite, radially unbounded, and decrescent. Its derivative along the system trajectories is given by

$$\begin{aligned}\dot{V}_1(t,\bar{q},\dot{\bar{q}}) = &-\dot{\bar{q}}^T C(\bar{q}+q_d(t),\dot{q}_d(t))\dot{\bar{q}} - \dot{\bar{q}}^T F\dot{\bar{q}} - \dot{\bar{q}}^T s_1(K_1\dot{\bar{q}}) \\ &- \varepsilon_1 s_2^T(K_2\bar{q})C(\bar{q}+q_d(t),\dot{q}_d(t))\dot{\bar{q}} \\ &- \varepsilon_1 s_2^T(K_2\bar{q})F\dot{\bar{q}} - \varepsilon_1 s_2^T(K_2\bar{q})s_1(K_1\dot{\bar{q}}) \\ &- \varepsilon_1 s_2^T(K_2\bar{q})s_2(K_2\bar{q}) \\ &+ \varepsilon_1\dot{\bar{q}}^T C(\bar{q}+q_d(t),\dot{\bar{q}})s_2(K_2\bar{q}) \\ &+ \varepsilon_1\dot{\bar{q}}^T C(\bar{q}+q_d(t),\dot{q}_d(t))s_2(K_2\bar{q}) \\ &+ \varepsilon_1\dot{\bar{q}}^T D(\bar{q}+q_d(t))s'_2(K_2\bar{q})K_2\dot{\bar{q}}\end{aligned}$$

with $s'_2(K_2\bar{q}) = \mathrm{diag}[\sigma'_{21}(k_{21}\bar{q}_1),\ldots,\sigma'_{2n}(k_{2n}\bar{q}_n)]$, where $D(\bar{q}+q_d(t))\ddot{\bar{q}}$ has been replaced by its equivalent expression from the closed-loop dynamics (14), and Properties 2.1–2.3 have been used. From Properties 1, 2.5 and 4, item 9 of Lemma 1, and the satisfactions of inequalities (4), one gets, after several basic developments, that

$$\dot{V}_1(t,\bar{q},\dot{\bar{q}}) \leq -\dot{\bar{q}}^T s_1(K_1\dot{\bar{q}}) - \begin{pmatrix}\|s_2(K_2\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix}^T Q_1\begin{pmatrix}\|s_2(K_2\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix}$$

with

$$Q_1 = \begin{pmatrix}\varepsilon_1 & Q_{112} \\ Q_{112} & Q_{122}\end{pmatrix}$$

where

$$Q_{112} = -\varepsilon_1\left(k_c B_{dv} + \frac{\sigma'_{1M}k_{1M}+f_M}{2}\right)$$

$$Q_{122} = f_m - k_c B_{dv} - \varepsilon_1(k_c B_{2M} + d_M k_{2M}\sigma'_{2M})$$

and the facts that $\|s_2(K_2\bar{q})\| \leq \left[\sum_{i=1}^n M_{2i}^2\right]^{1/2} \triangleq B_{2M}$ and $\|s'_2(K_2\bar{q})\| \leq \max_i\{\sigma'_{2iM}\} \triangleq \sigma'_{2M}$, $\forall\bar{q}\in\mathbb{R}^n$, have been considered. Further, since $\varepsilon_1 < \frac{f_m - k_c B_{dv}}{k_c B_{2M} + d_M k_{2M}\sigma'_{2M} + \left(k_c B_{dv} + \frac{\sigma'_{1M}k_{1M}+f_M}{2}\right)^2}$ (see (16)), one can verify (after several basic developments) that Q_1 is a positive definite symmetric matrix. From this and item 1 of Definition 1, one sees that $\dot{V}_1(t,\bar{q},\dot{\bar{q}})$ is negative definite. Thus, from Lyapunov's stability theory (applied to non-autonomous systems; see for instance [3, Th. 4.9]), the proposition follows. ■

Proposition 2: Consider the system (1),(2) with the control law (11). Under Assumptions 1 and 2, and the satisfaction of inequalities (13), global asymptotic stabilization of the closed-loop system solutions $q(t)$ towards the desired trajectory vector $q_d(t)$ is guaranteed with $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1,\ldots,n$, $\forall t \geq 0$.

Proof: From (11), (13) and Properties 1, 2.5, 3 and 4, one sees that $|u_i(t)| \leq M_{0i} + d_M B_{da} + k_c B_{dv}^2 + f_M B_{dv} + \gamma_i < T_i$, $i = 1,\ldots,n$, $\forall t \geq 0$. From this and (2) it follows that $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1,\ldots,n$, $\forall t \geq 0$. The stability analysis is now developed. The closed-loop dynamics takes the form

$$D(q)\ddot{\bar{q}} + [C(q,\dot{q}) + C(q,\dot{q}_d)]\dot{\bar{q}} + F\dot{\bar{q}} + s_0(K_1\dot{\bar{q}} + K_2\bar{q}) = 0_n \quad (17)$$

where Property 2.4 has been used (recall that $q = \bar{q} + q_d(t)$ and $\dot{q} = \dot{\bar{q}} + \dot{q}_d(t)$). Let us define the scalar function

$$V_2(t,\bar{q},\dot{\bar{q}}) = \frac{1}{2}\dot{\bar{q}}^T D(\bar{q}+q_d(t))\dot{\bar{q}} + \int_0^{\bar{q}} s_0^T(K_2 r)dr + \varepsilon_2 s_0^T(K_2\bar{q})D(\bar{q}+q_d(t))\dot{\bar{q}} \quad (18)$$

where $\int_0^{\bar{q}} s_0^T(K_2 r)dr = \sum_{i=1}^n \int_0^{\bar{q}_i}\sigma_{0i}(k_{2i}r_i)dr_i$ and ε_2 is a positive constant satisfying[3]

$$\varepsilon_2 < \min\left\{ \frac{f_m - k_c B_{dv}}{k_c B_{0M} + d_M k_{2M}\sigma'_{0M} + \left(k_c B_{dv} + \frac{\sigma'_{0M}k_{1M}+f_M}{2}\right)^2}, \sqrt{\frac{d_m}{2d_M^2 k_{2M}\sigma'_{0M}}} \right\} \quad (19)$$

[2]Observe that the satisfaction of Assumption 2 —wherefrom we have that $f_m > k_c B_{dv}$ (see (5), (6), and the definitions of B_{v1} in (7a) and B_{v2} in (8b)), no matter what the nonnegative constant value of k_c be— ensures positivity of the first term within the braces in (16), thus guaranteeing the existence of a positive ε_1 fulfilling (16).

[3]Observe that —by the same arguments furnished in Footnote 2— the satisfaction of Assumption 2 ensures positivity of the first term within the braces in (19), guaranteeing the existence of a positive ε_2 fulfilling (19).

where $\sigma'_{0M} \triangleq \max_i\{\sigma'_{0iM}\}$, $k_{jM} \triangleq \max_i\{k_{ji}\}$, $j = 1, 2$, and $B_{0M} \triangleq \left[\sum_{i=1}^n M_{0i}^2\right]^{1/2}$. Notice that $V_2(t, \bar{q}, \dot{\bar{q}})$ in (18) adopts the same form of $V_1(t, \bar{q}, \dot{\bar{q}})$ in (15) (by simply replacing s_2 in V_1 by s_0). Thus, following a procedure analog to the one developed for $V_1(t, \bar{q}, \dot{\bar{q}})$ in the proof of Proposition 1, we get

$$\frac{1}{2}\int_0^{\bar{q}} s_0^T(K_2 r)dr + \frac{1}{2}\begin{pmatrix}\|s_0(K_2\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix}^T P_{21}\begin{pmatrix}\|s_0(K_2\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix}$$

$$\leq V_2(t, \bar{q}, \dot{\bar{q}}) \leq \frac{1}{2}\begin{pmatrix}\|\bar{q}\| \\ \|\dot{\bar{q}}\|\end{pmatrix}^T P_{22}\begin{pmatrix}\|\bar{q}\| \\ \|\dot{\bar{q}}\|\end{pmatrix}$$

where

$$P_{21} = \begin{pmatrix}\frac{1}{2k_{2M}\sigma'_{0M}} & -\varepsilon_2 d_M \\ -\varepsilon_2 d_M & d_m\end{pmatrix}$$

and

$$P_{22} = \begin{pmatrix}k_{2M}\sigma'_{0M} & \varepsilon_2 d_M k_{2M}\sigma'_{0M} \\ \varepsilon_2 d_M k_{2M}\sigma'_{0M} & d_M\end{pmatrix}$$

Further, since $\varepsilon_2 < \sqrt{\frac{d_m}{2d_M^2 k_{2M}\sigma'_{0M}}}$ (see (19)), one can verify (after several basic developments) that P_{21} and P_{22} are positive definite symmetric matrices. From this and items 6 and 7 of Lemma 1, one sees that $V_2(t, \bar{q}, \dot{\bar{q}})$ is positive definite, radially unbounded, and decrescent. Its derivative along the system trajectories is given by

$$\begin{aligned}\dot{V}_2(t, \bar{q}, \dot{\bar{q}}) = & -\dot{\bar{q}}^T C(\bar{q} + q_d(t), \dot{q}_d(t))\dot{\bar{q}} - \dot{\bar{q}}^T F\dot{\bar{q}} \\ & - \dot{\bar{q}}^T\left[s_0(K_1\dot{\bar{q}} + K_2\bar{q}) - s_0(K_2\bar{q})\right] \\ & - \varepsilon_2 s_0^T(K_2\bar{q})C(\bar{q} + q_d(t), \dot{q}_d(t))\dot{\bar{q}} \\ & - \varepsilon_2 s_0^T(K_2\bar{q})F\dot{\bar{q}} - \varepsilon_2 s_0^T(K_2\bar{q})s_0(K_2\bar{q}) \\ & - \varepsilon_2 s_0^T(K_2\bar{q})\left[s_0(K_1\dot{\bar{q}} + K_2\bar{q}) - s_0(K_2\bar{q})\right] \\ & + \varepsilon_2\dot{\bar{q}}^T C(\bar{q} + q_d(t), \dot{\bar{q}})s_0(K_2\bar{q}) \\ & + \varepsilon_2\dot{\bar{q}}^T C(\bar{q} + q_d(t), \dot{q}_d(t))s_0(K_2\bar{q}) \\ & + \varepsilon_2\dot{\bar{q}}^T D(\bar{q} + q_d(t))s'_0(K_2\bar{q})K_2\dot{\bar{q}}\end{aligned}$$

with $s'_0(K_2\bar{q}) = \mathrm{diag}[\sigma'_{01}(k_{21}\bar{q}_1), \ldots, \sigma'_{0n}(k_{2n}\bar{q}_n)]$, where $D(\bar{q}+q_d(t))\ddot{q}$ has been replaced by its equivalent expression from the closed-loop dynamics (17), and Properties 2.1–2.3 have been used. From Properties 1, 2.5 and 4, item 8 of Lemma 1, and the satisfaction of inequilities (4), one gets, after several basic developments, that

$$\begin{aligned}\dot{V}_2(t, \bar{q}, \dot{\bar{q}}) \leq & -\dot{\bar{q}}^T\left[s_0(K_1\dot{\bar{q}} + K_2\bar{q}) - s_0(K_2\bar{q})\right] \\ & - \begin{pmatrix}\|s_0(K_2\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix}^T Q_2\begin{pmatrix}\|s_0(K_2\bar{q})\| \\ \|\dot{\bar{q}}\|\end{pmatrix}\end{aligned}$$

with

$$Q_2 = \begin{pmatrix}\varepsilon_2 & Q_{212} \\ Q_{212} & Q_{222}\end{pmatrix}$$

where

$$Q_{212} = -\varepsilon_2\left(k_c B_{dv} + \frac{\sigma'_{0M}k_{1M} + f_M}{2}\right)$$

$$Q_{222} = f_m - k_c B_{dv} - \varepsilon_2(k_c B_{0M} + d_M k_{2M}\sigma'_{0M})$$

and the facts that $\|s_0(K_2\bar{q})\| \leq \left[\sum_{i=1}^n M_{0i}^2\right]^{1/2} \triangleq B_{0M}$ and $\|s'_0(K_2\bar{q})\| \leq \max_i\{\sigma'_{0iM}\} \triangleq \sigma'_{0M}$, $\forall\bar{q} \in \mathbb{R}^n$, have been considered. Further, since $\varepsilon_2 < \frac{f_m - k_c B_{dv}}{k_c B_{0M} + d_M k_{2M}\sigma'_{0M} + \left(k_c B_{dv} + \frac{\sigma'_{0M}k_{1M} + f_M}{2}\right)^2}$ (see (19)), one can verify (after several basic developments) that Q_2 is a positive definite symmetric matrix. From this and item 1 of Lemma 1, one sees that $\dot{V}_2(t, \bar{q}, \dot{\bar{q}})$ is negative definite. Thus, from Lyapunov's stability theory (see for instance [3, Th. 4.9]), the proposition follows. ■

V. EXPERIMENTAL RESULTS

Both proposed algorithms, the SP-SD+ controller in (10) and the SPD+ scheme in (11), were implemented on a well-identified direct-drive robot arm. The experimental manipulator is a two-axis robot with the same mechanical structure of that in [7], but with different dynamic parameters. The entries of the corresponding terms in (1) are given by

$$D(q) = \begin{bmatrix}3.511 + 0.191\cos q_2 & 0.072 + 0.096\cos q_2 \\ 0.072 + 0.096\cos q_2 & 0.072\end{bmatrix}$$

$$C(q, \dot{q}) = \begin{bmatrix}-0.096\dot{q}_2\sin q_2 & -0.096(\dot{q}_1 + \dot{q}_2)\sin q_2 \\ 0.096\dot{q}_1\sin q_2 & 0\end{bmatrix}$$

$$g(q) = \begin{bmatrix}40.888\sin q_1 + 2.079\sin(q_1 + q_2) \\ 2.079\sin(q_1 + q_2)\end{bmatrix}$$

$$F = \begin{bmatrix}0.764 & 0 \\ 0 & 0.328\end{bmatrix}$$

Thus, Properties 1, 2.5, 3, and 4 are satisfied with $d_m = 0.0638$ kg·m^2, $d_M = 3.71$ kg·m^2, $k_c = 0.3816$ kg·m^2, $\gamma_1 = 42.97$ N·m, $\gamma_2 = 2.08$ N·m, $f_m = 0.3279$ kg·m^2/sec, and $f_M = 0.7639$ kg·m^2/sec. The maximum torques allowed are $T_1 = 150$ N·m and $T_2 = 15$ N·m for the first and second links respectively. Observe that Assumption 1 is fulfilled.

Additionally, simulations were run considering the PD+ controller of [6] in an unbounded input context, *i.e.* $\tau \equiv u = -K_2\bar{q} - K_1\dot{\bar{q}} + D(q)\ddot{q}_d + C(q, \dot{q})\dot{q}_d + g(q)$. The desired trajectory vector, for all the controllers, was defined as $q_d(t) = (q_{d1}(t), q_{d2}(t)) = (\pi + 0.5\sin t\,,\, 0.5\cos t)$ [rad]. For such a desired trajectory, inequalities (4) are satisfied with $B_{dv} = 0.5$ rad/sec and $B_{da} = 0.5$ rad/sec^2. The initial conditions at every (experimental and simulation) test were $q_i(0) = \dot{q}_i(0) = 0$, $i = 1, 2$. The generalized saturation functions were defined as

$$\sigma_{ji}(\varsigma) = \begin{cases}-L_{ji} + \frac{(M_{ji} - L_{ji})(\varsigma + L_{ji})}{\sqrt{(M_{ji} - L_{ji})^2 + (\varsigma + L_{ji})^2}} & \forall\varsigma < -L_{ji} \\ \varsigma & \forall|\varsigma| \leq L_{ji} \\ L_{ji} + \frac{(M_{ji} - L_{ji})(\varsigma - L_{ji})}{\sqrt{(M_{ji} - L_{ji})^2 + (\varsigma - L_{ji})^2}} & \forall\varsigma > L_{ji}\end{cases} \tag{20}$$

with $L_{ji} < M_{ji}$, $\forall(i, j) \in \{1, 2\} \times \{0, 1, 2\}$. The control gains were adjusted as follows: $k_{11} = 229.2$, $k_{12} = 12$, $k_{21} = 3437.7$, and $k_{22} = 171.9$ (with k_{1i} in N·m·sec and k_{2i} in N·m, $i = 1, 2$), for all the controllers. The bounds of the saturation functions were defined as: $M_{11} = 40$, $M_{21} =$

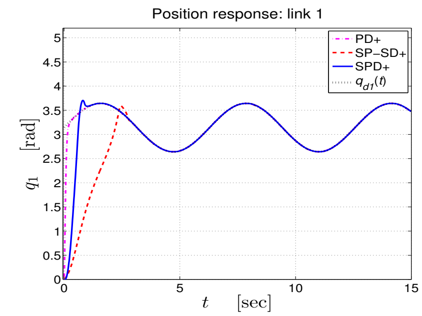

Fig. 1. Position response of link 1: $q_1(t)$

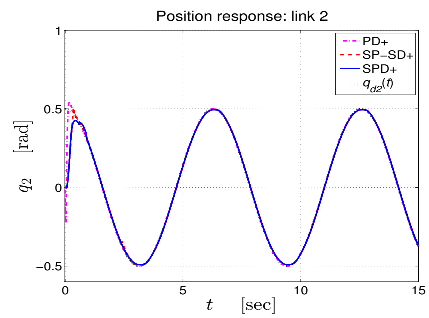

Fig. 2. Position response of link 2: $q_2(t)$

60, $M_{12} = 2.5$, and $M_{22} = 5$, for the SP-SD+ controller; $M_{01} = 100$ and $M_{02} = 7.5$, for the SPD+ scheme; and $L_{ji} = 0.9M_{ji}$, $\forall(i,j) \in \{1,2\}\times\{0,1,2\}$, for both algorithms (with M_{ji} and L_{ji} in N·m, $i = 1,2$, $j = 0,1,2$). One can easily verify that Assumption 2 as well as inequalities (12) and (13) are satisfied.

Figs. 1 and 2 respectively show the shoulder and elbow joint position responses, *i.e.* $q_1(t)$ and $q_2(t)$, for all the controllers. Observe that the stabilization objective is achieved in every case. It is worth to note the closed-loop performance differences resulting from the implementation of the proposed schemes: while the system trajectories due to the SPD+ algorithm are closer to those obtained in simulation with the PD+ unbounded controller, the responses due to the SP-SD+ scheme are slower.[4] Furthermore, Figs. 3 and 4 show the applied inputs τ_1 and τ_2, for all the tested schemes. Observe that the control signals generated through the SP-SD+ and SPD+ algorithms are clearly within the input bounds considered at every link. On the contrary, the control signal generated in simulation through the unbounded PD+ algorithm reach maximum absolute values (not shown in the graphs) that are larger than 10000 N·m for the first link and around 300 N·m for the second link. Hence, with this controller, the control inputs would undergo saturation in the actual bounded input context considered for the schemes proposed in this work.

[4]Recall that the same control gains are used at every tested scheme. This way, it is the different controller structures that are responsible for the resulting closed-loop performance differences.

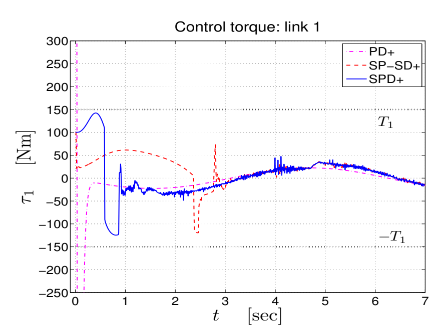

Fig. 3. Control torque applied to link 1: $\tau_1(t)$

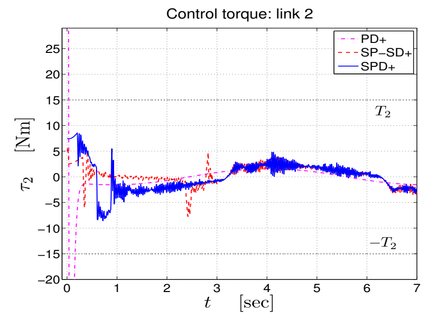

Fig. 4. Control torque applied to link 2: $\tau_2(t)$

VI. Conclusions

In this work, two globally stabilizing bounded controllers for the trajectory tracking of robot manipulators with saturating inputs were proposed. With respect to previously proposed algorithms, our approaches gave a global solution to the problem through static feedback. Moreover, they were not defined using a specific sigmoidal function, but any one on a set of saturating functions. This permitted the proposed controllers to adopt a suitable structure where the control gains were able to take any positive value, which may be considered beneficial for performance-adjustment purposes. The efficiency of the proposed schemes was corroborated through experimental results.

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