Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

Adaptive QFT Control using Hybrid Global Optimization and Constraint Propagation Techniques

P. S. V. Nataraj and Nandkishor Kubal

Abstract— **We propose a procedure for the online design of *adaptive* quantitative feedback theory (QFT) control system. The proposed procedure uses hybrid global optimization and interval constraint propagation techniques to automatically design online an adaptive QFT controller and prefilter as and when required. While the hybrid global optimization combines interval global optimization and nonlinear local optimization methods, the interval constraint propagation techniques accelerate the optimization search by very effectively discarding infeasible controller parameter regions. The proposed adaptive QFT control is experimentally demonstrated on a coupled tanks system in the laboratory. Experimental results show the superiority of the proposed adaptive QFT control over standard QFT control, in terms of both reduced error and reduced control effort.**

I. INTRODUCTION

In the standard QFT method of Horowtiz [3], the controller and prefilter are designed for some initially given maximum possible extent or full plant uncertainty. In situations where the plant uncertainty is reduced, the cost of feedback may be decreased by changing the parameters of the controller and prefilter using the adaptation algorithm in [8]. The work in [8] essentially introduces a new approach to QFT, by making the QFT control *adaptive* to the plant uncertainty. However, the variation of plant uncertainty is restricted to lie in a *subset* of either the current or initially defined uncertainty set.

This paper pursues the adaptive QFT control approach, and proposes certain enhancements over the one in [8]:

1) To start with, a standard QFT design of robust controller and prefilter is performed offline, for the performance specifications and initially given plant uncertainty intervals. This design is then implemented online to control the plant.
2) At each sampling instant, an online plant parameter estimator is used to continuously update the plant parameter values.
3) Whenever the updated parameter values are found to lie outside the current plant uncertainty intervals, then
   a) The plant uncertainty (intervals) are updated by assigning neighborhoods around the latest parameter values (The approach given in [2] can be used for this step).
   b) A complete redesign of the QFT controller and prefilter is done online for the newly constructed plant parameter intervals and given design specifications. The redesigned controller and prefilter are synthesized so as to be *optimum*, using the automatic synthesis algorithms proposed in [7] and [5].

Thus, in contrast to the work in [8], in the proposed adaptive QFT control method the variation of plant uncertainty is not restricted to lie in a subset of either the current or initially defined uncertainty set, but can be outside it. Further, whenever the latter situation arises, the QFT controller and prefilter are fully redesigned automatically online.

The paper is organized as follows: The proposed method for adaptive QFT control is presented in Section II, and experimentally demonstrated on a coupled tank setup described in Section III. This demonstration is believed to be the first one in the literature wherein interval optimization and constraint solver tools have been employed *online*, at least in the context of robust control applications. The conclusions of the work are drawn in Section IV.

II. A METHOD FOR ADAPTIVE QFT CONTROL

The proposed method for adaptive QFT control consists of three main tools (see Figure 1): an online parameter estimation technique, an algorithm for automated synthesis of QFT controller (see [7]) and an algorithm for automated synthesis of QFT prefilter (see [5]). The method is comprised of the following steps:

1) Initially, standard optimal QFT controller and prefilter of a desired structure are designed (offline) for the given stability and performance specifications and initially defined plant parameter interval $\mathbf{p_u}$. The algorithms in [7] and [5] can be used for the design. The designed controller and prefilter are implemented online on the plant.
2) At each sampling instant, the values of the plant parameters are updated using, for instance, the recursive least squares method [6]. If the identified parameter values fall outside $\mathbf{p_u}$, then Step 3 is executed, else Step 9 is executed.
3) New intervals of plant parameters are constructed around the identified parameter values, (see [2] for construction of such intervals), and $\mathbf{p_u}$ is reset to this vector of intervals.
4) At each design frequency, the QFT plant templates are computed as in [3].
5) At each design frequency, the QFT stability and performance bounds are computed using the quadratic inequality based approach [1].

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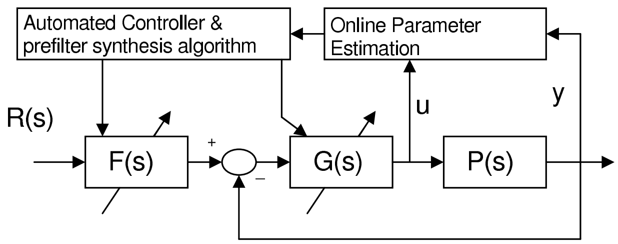

Fig. 1. Block diagram of the adaptive QFT control system with adaptive feedback and prefilter control. The controller $G(s)$ and prefilter $F(s)$ are designed using QFT principles.

6) For the desired structure of controller (as in Step 1), the optimum controller $G(s)$ is designed using the algorithm for automatic synthesis of QFT controller proposed in [7].
7) For the desired structure of prefilter (as in Step 1), the optimum prefilter $F(s)$ is synthesized using the method given in [5].
8) The newly synthesized $G(s)$ and $F(s)$ are implemented online to control the plant.
9) Go to Step 2.

III. CASE STUDY

The proposed method for adaptive QFT control is experimentally tested and compared with the standard QFT control on a coupled tank system in the laboratory. The implementation is done on a desktop PC in Microsoft FORTRAN 95 with interval arithmetic support using INTLIB [4].

A. Plant description

The coupled tank system, whose schematic is given in Fig. 2, consists of two hold-up tanks that are coupled by an orifice. Water is pumped into the first tank by a variable speed pump. The orifice allows this water to flow into the second tank and then out to a reservoir. The aim is to control the water level in the second tank by changing the flow rate of water to the first tank. The flow rate is manipulated by varying the voltage $(0-10\ \mathrm{V})$ to the variable speed pump. The water level in the tank is measured using a depth sensor whose output is voltage $(0-10\ \mathrm{V})$; this voltage is proportional to the water level.

Thus, in short, the plant input is the voltage $(0-10\ \mathrm{V})$ to the variable speed pump, and the plant output is the water level in the second tank in terms of the voltage signal $(0-10\ \mathrm{V})$.

An Advantech 5000 series data acquisition system is used, which is comprised of a 8-channel analog input module and a 4-channel analog output module. Communication between the data acquisition system and the digital computer is via the serial port of the computer.

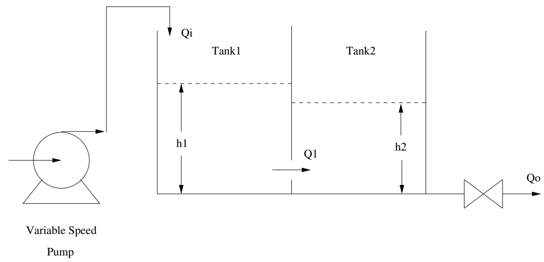

Fig. 2. Schematic of the coupled tank system.

B. Design specifications

For the design, the following closed-loop specifications are considered.

1) Stability margin specification: gain margin $\geq 4.5\,\mathrm{dB}$, phase margin $\geq 45°$, or

$$\left|\frac{L(j\omega)}{1+L(j\omega)}\right| \leq 3\,dB$$

2) Tracking performance specification: rise time $\leq$ 300 sec, maximum overshoot $\leq 10\%$, or

$$|T_L(j\omega)| \leq \left|\frac{F(j\omega)L(j\omega)}{1+L(j\omega)}\right| \leq |T_U(j\omega)|$$

where

$$T_U(s) = \frac{16.67s+1}{2140s^2+56.44s+1}$$

and

$$T_L(s) = \frac{1}{4.495\times 10^4 s^3+4740s^2+139.2s+1}$$

C. Synthesis of standard QFT control system

To identify a model of the coupled tank system, the plant is excited by a pseudo-random binary signal (PRBS) of ± 0.5 volts around three different operating points. The output-error identification technique [6] is then used to identify a model. Table II gives the details of the operating points and the three models obtained. Thus, the uncertain

TABLE I
MODELS IDENTIFIED FOR THE COUPLED TANK SYSTEM

Plant Operating Conditions		Identified model	MSE
Input: Flow rate (cm^3/min)	Output: water level in the tank 2 (cm)	$\frac{kab}{(s+a)(s+b)}$	
1500	7.0	$k = 1.4862$ $a = 0.0051$ $b = 0.1090$	7.4×10^{-4}
1900	9.6	$k = 0.5319$ $a = 0.0082$ $b = 0.1499$	5.9×10^{-4}
2300	14.2	$k = 0.9128$ $a = 0.0054$ $b = 0.0787$	2.6×10^{-3}

model of the coupled tank system over all the three operating points is

$$P(s) = \frac{kab}{(s+a)(s+b)} \quad (1)$$

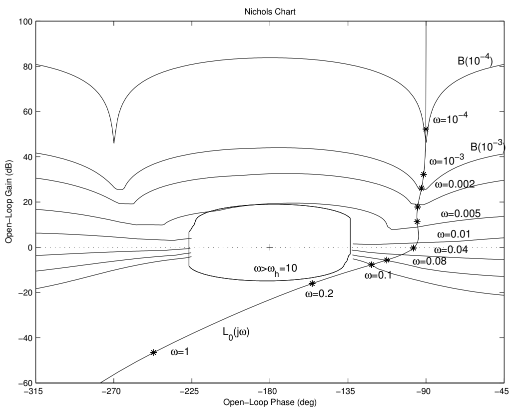

Fig. 3. Plot of the nominal loop transmission function $L_0(s)$ for the coupled tank system (1) with the standard QFT controller (4).

with,

$$k \in [0.532, 1.4862]$$

$$a \in [0.0051, 0.0082], b \in [0.0787, 0.1499]$$

Now, the QFT bounds corresponding to the stability and tracking specifications in Section III-B are generated. To design the standard QFT controller that satisfies the obtained QFT bounds, a second order controller structure

$$G(s) = \frac{k_c(s+z)}{s(s+p)} \tag{2}$$

with a very large search domain, is considered. When the algorithm in [7] is applied for automatic synthesis of a QFT controller, the algorithm terminates with the message "No feasible solution exists in the given search domain". Therefore, a third order controller structure of the form

$$G(s) = \frac{k_c(s+z_1)(s+z_2)}{s(s+p_1)(s+p_2)} \tag{3}$$

is next attempted. For this structure, the algorithm for automatic synthesis of controller is applied. The algorithm generates the optimum controller as

$$G_{fix}(s) = \frac{12.3619(s+0.0513)(s+0.00819)}{s(s+0.8328)(s+0.1091)} \tag{4}$$

while the prefilter synthesis algorithm in [5] generates the optimum prefilter (of two pole structure) as

$$F_{fix}(s) = \frac{1}{(s/0.0277328+1)(s/0.0277328+1)} \tag{5}$$

Figs. 3 and 4 show the resulting nominal loop transmission function and the closed loop frequency responses for several sample plants from the plant family (1). The standard QFT controller (4) and prefilter (5) are then experimentally implemented online on the coupled tank system. Figs. 5 and 6 gives the control effort and the closed loop responses obtained experimentally. It is seen that the obtained experimental responses satisfy the prescribed closed-loop time domain specifications.

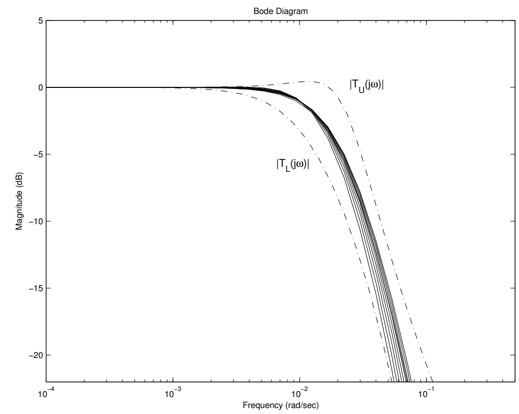

Fig. 4. Closed loop frequency responses (solid lines) obtained for some sample plants from the coupled tank uncertain model (1) with the standard QFT controller (4) and prefilter (5).

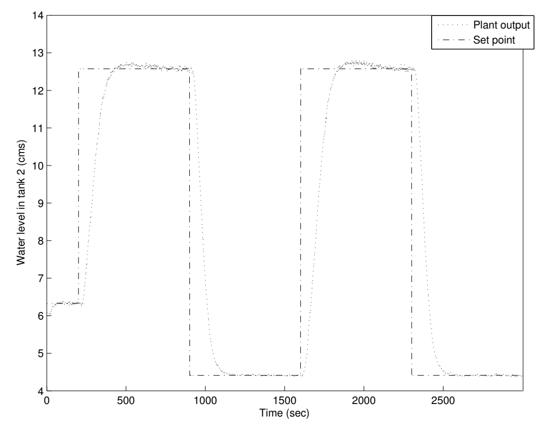

Fig. 5. Plot of the experimental closed-loop output response (water level in tank 2) for the coupled tank system with the standard QFT controller (4) and prefilter (5).

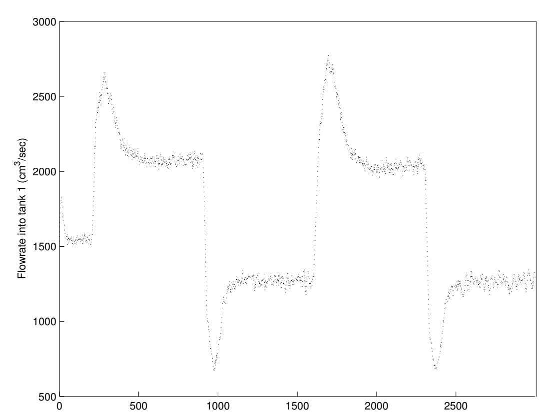

Fig. 6. Plot of the control input (flowrate to tank 1) required for the coupled tank system with the standard QFT controller (4) and prefilter (5).

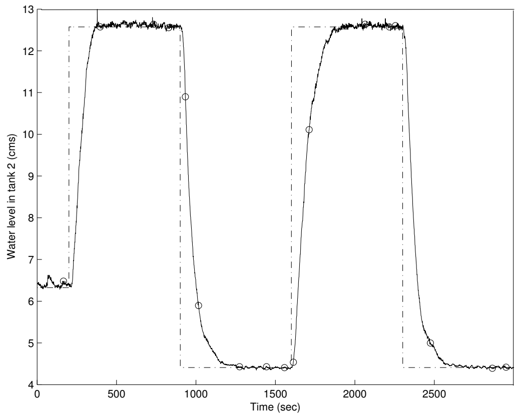

Fig. 7. Plot of the experimental closed-loop output response (water level in tank 2) for the coupled tank system with adaptive QFT control. Points marked with '∘' show the instances at which the complete redesign of controller and prefilter is done online.

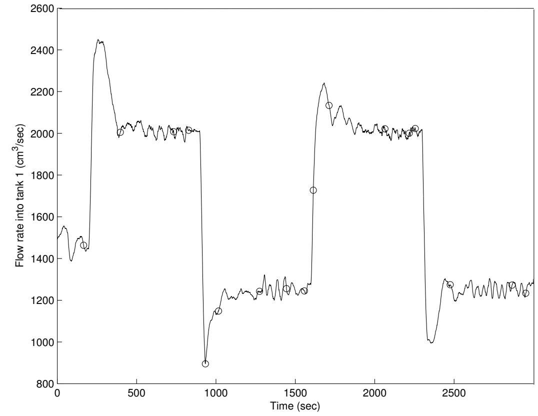

Fig. 8. Plot of the control input (flow rate to tank 1) required for the coupled tank system with adaptive QFT control. Points marked with '∘' show the instances at which the complete redesign of QFT controller and prefilter is done online.

D. Synthesis of adaptive QFT control system

The proposed adaptive QFT control is implemented on the coupled tank system, using the steps given in Section II. The implementation is started with a second order controller structure (2) with $k_c = 2.4690, z = 0.0045, p = 0.3557$. As mentioned earlier, whenever the identified plant parameter values fall outside the current parameter intervals, a complete redesign of controller and prefilter is done. Such instances are reported in Table 2, and shown as circles in Figs. 7 and 8 which plot the experimental closed loop response and the control effort required. It is seen that the obtained experimental responses satisfy the prescribed closed-loop time domain specifications.

TABLE II

PLANT PARAMETER VALUES WHEN ANY NEW IDENTIFIED MODEL PARAMETER FALL OUTSIDE THE CURRENT PARAMETER VALUES BY 30%.

Sampling Instant	Identified Plant Parameter Values
166	$k = 0.4231$, $a = 0.0103$, $b = 1.0579$
397	$k = 0.4687$, $a = 0.006$, $b = 0.8745$
733	$k = 0.4497$, $a = 0.0083$, $b = 0.5472$
828	$k = 0.4474$, $a = 0.01191$, $b = 0.6042$
933	$k = 0.4547$, $a = 0.01271$, $b = 0.9586$
1016	$k = 0.4503$ $a = 0.0164$ $b = 1.3773$
1274	$k = 0.4542$, $a = 0.0079$, $b = 1.2133$
1444	$k = 0.4455$, $a = 0.0103$, $b = 0.9479$
1557	$k = 0.4405$, $a = 0.0111$, $b = 0.6330$
1613	$k = 0.4369$, $a = 0.0149$, $b = 0.7837$
1712	$k = 0.4382$, $a = 0.0170$, $b = 2.2213$
2064	$k = 0.4499$, $a = 0.0075$, $b = 0.3564$
2217	$k = 0.4457$, $a = 0.0093$, $b = 0.2395$
2255	$k = 0.4509$, $a = 0.0109$, $b = 0.3179$
2475	$k = 0.4590$, $a = 0.0107$, $b = 2.2030$
2865	$k = 0.4525$, $a = 0.0102$, $b = 0.7968$
2951	$k = 0.4469$, $a = 0.0134$, $b = 0.7780$

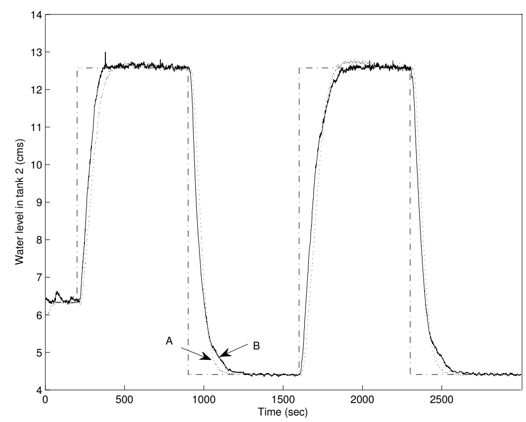

Fig. 9. Comparison of (experimental) output responses (water level in tank 2) for the coupled tank system. (A) with standard QFT control, and (B) with adaptive QFT control.

E. Comparison of standard and adaptive QFT control systems

Figs. 9 and 10 show the experimental closed loop responses and the control effort obtained with standard and adaptive QFT control. While both the control methods lead to satisfaction of the prescribed specifications, the control effort required by adaptive QFT is much less than that of standard QFT control. This is further evident from Tables III and IV, which compare the performances of standard and adaptive QFT in terms of 1-norm, 2-norm and ∞-norm of the error signals and control input. The tables clearly show the benefits of the proposed adaptive QFT control over standard QFT control.

IV. CONCLUSIONS

A method is proposed for *adaptive* QFT control. The proposed method consists of three main tools: online parameter estimation technique, automated synthesis of controller, and automated synthesis of prefilter. The proposed method for

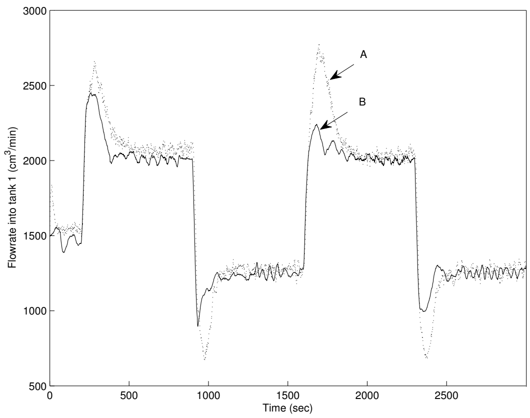

Fig. 10. Comparison of the control input (flowrate to tank 1) required for the coupled tank system. (A) with standard QFT control, and (B) with adaptive QFT control.

TABLE III

COMPARISON OF THE PROPOSED ADAPTIVE QFT CONTROL AND THE STANDARD QFT CONTROL BASED ON THE ERROR SIGNAL.

Performance Criterion	Standard QFT control	Adaptive QFT control
$\|e(t)\|$	700.7	638.2
$\|e(t)\|_2$	26.08	23.08
$\|e(t)\|_\infty$	1.514	1.508

adaptive QFT control is experimentally tested and compared with the standard QFT control, on a coupled tank system in the laboratory. The experimental results show the superiority of the adaptive QFT control method in that it yields less error while using lesser control input (as measured by the 1, 2, and ∞ norms) compared to the standard QFT control method.

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TABLE IV

COMPARISON OF THE PROPOSED ADAPTIVE QFT CONTROL AND THE STANDARD QFT CONTROL BASED ON THE CONTROL EFFORT REQUIRED.

Performance Criterion	Standard QFT control	Adaptive QFT control
$\|u(t)\|$	4533.5	3878.2
$\|u(t)\|_2$	85.17	70.07
$\|u(t)\|_\infty$	3.21	2.37